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MHD FLOW IN INSULATING CIRCULAR DUCTS FOR FUSION BLANKETS*

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ABSTRACT

This paper treats steady laminar flows of liquid metals in electrically insulated circular ducts in the presence of strong, transverse nonuniform magnetic fields. The interaction parameter and Hartmann number are assumed to be large, whereas the magnetic Reynolds number is assumed to be small. This problem is of importance to MHD flows in fusion blankets especially when suitable insulating material is identified and developed for use in fusion reactor environments. Use of insulated duct could substantially reduce the MHD pressure drop and thus would simplify the desgin and enhance the performance of the liquid-metal-cooled blankets.

I. INTRODUCTION

In a deuterium-tritium (D-T) fusion reactor, the basic T (D,n) ^{4}He fusion reaction in the plasma releases its energy in a 14.1-MeV neutron, a 3.5 MeV alpha particle (helium) and electromagnetic radiation. The alpha particle is typically retained in the plasma and provides for self-heating. The energy from the neutron and electromagnetic radiation is recovered in a structure surrounding the plasma called "blanket" which, among many other purposes, also serves to breed tritium to replenish the supply of tritium fuel. Tritium is produced when the high energy neutron collides with a lithium atom. In previous blanket design studies such as the Blanket Comparison and Selection Study (BCSS)[1], lithium is a favorable choice for coolant and breeding material. It has ideal thermal conductivity and nuclear properties. The main disadvantage is its high electrical conductivity which leads to large magnetohydrodynamic (MHD) pressure gradient in the liquid metal when it is circulated in thin walled conducting ducts in the presence of strong magnetic fields. The resulting MHD pressure drop may cause excessive pumping power losses and large material stresses.

It is known that the use of electrically-insulated duct would reduce substantially the MHD pressure drop and, consequently, would simplify the design and enhance the performance of liquid-metal-cooled blankets. At the present, no suitable insulating materials with acceptable characteristics of radiation damage, compatibility with both liquid metal and structural materials have been developed. While such a development is a long term activity, capability for analyzing MHD flow in insulating ducts is needed now to quantify the impact of insulating duct on MHD-related issues.

This paper treats the steady flow of a liquid metal in insulating circular duct in nonuniform transverse magnetic field. The magnetic field in a fusion reactor is so strong that inertia effects can be neglected and viscous effects are confined to very thin boundary layers adjacent to the wall. The induced field produced by the circulating currents in the liquid metal is also negligible compared to the imposed magnetic field.

II. ANALYSIS

A. Governing Equations

Consider the flow of an incompressible liquid metal in a circular duct with electrically-insulated wall in the presence of an imposed nonuniform transverse magnetic field. The magnetic field, $\underline{B} = B_x(x,y)\,\hat{\underline{x}} + B_y(x,y)\,\hat{\underline{y}}$ where $\hat{\underline{x}}$ and $\hat{\underline{y}}$ are unit vectors, is assumed to be uniform in the plane of the duct cross section (Figure 1). It is also assumed that $\underline{B}$ is symmetric about the y = 0 plane and varies in the x direction over a characteristic axial length $L_B \gg L$, L being the radius of the duct. Then the function $\underline{B}$ is approximated by

$$\underline{B} = B_y(x)\,\hat{\underline{y}}$$

neglecting $O(L/L_B)$ terms.

The two important parameters in any general MHD problem are the interaction parameter, N, and Hartmann number, M, defined by

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$$N = \frac{\sigma B_o^2 L}{\rho U_o}$$

$$M = L B_o \left(\frac{\sigma}{\rho\nu}\right)^{1/2}$$

where ρ and ν are the fluid's density and kinematic viscosity, B_o is a characteristic magnetic flux density, and U_o is the average axial velocity of the fluid. The values for M, which is the square root of the ratio of the electromagnetic (EM) force to the viscous body force, and N, which is the ratio of EM force to the inertial body force, are typically of the order of 10^3 - 10^5 in a tokamak fusion reactor. Thus the EM force is the dominant force determining the flow and pressure distributions throughout the liquid-metal flow, except for thin boundary and possibly free shear layers.

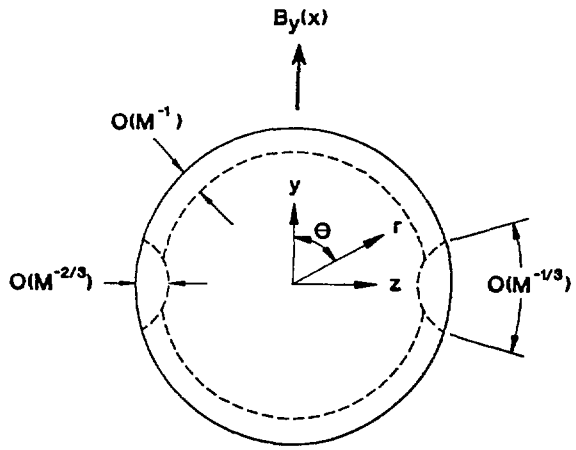

FIG. 1 A CROSS SECTIONAL VIEW

The inertialess, dimensionless equations governing the flow are:

$$\nabla p = j \times B + M^{-2} \nabla^2 v, \qquad (1a)$$

$$j = -\nabla\phi + v \times B \qquad (1,b)$$

$$\nabla \cdot v = 0, \; \nabla \cdot j = 0. \qquad (1c,d)$$

Here p, j, v, and ϕ are the pressure, electric current density, velocity, and electric potential, normalized by $\sigma U_o B_o^2 L$, $\sigma U_o B_o$, U_o, and $U_o B_o L$, respectively.

By symmetry, the solution is sought in one quadrant of the duct, namely for $0 \le y \le (1-z^2)^{1/2}$, $0 \le z \le 1$ (all lengths are nondimensionalized by the duct radius L).

B. Solution in the Core

In the core, the viscous term in Eq. (1a) can be neglected. Then the x, y, z core velocity components u_c, v_c, w_c, and electric current density components j_{xc}, j_{yc}, j_{zc}, which satisfy equation (1) and the symmetry conditions $j_{yc} = v_c = 0$ at $y = 0$ are:

$$u_c(x,y,z) = \beta \frac{\partial \phi_c}{\partial z} - \beta^2 \frac{\partial p}{\partial x} \qquad (2a), (Cf.1b)$$

$$w_c(x,y,z) = -\beta \frac{\partial \phi_c}{\partial x} - \beta^2 \frac{\partial p}{\partial z} \qquad (2b), (Cf.1b)$$

$$v_c(x,y,z) = -y\,\beta'(x) \frac{\partial \phi_w}{\partial z} + y \frac{\partial}{\partial x}\left[\beta^2 \frac{\partial p}{\partial x}\right]$$

$$+ y\left[\beta^2 + \frac{1}{2}\left(a^2 - \frac{y^2}{3}\right)\beta'^2\right] \frac{\partial^2 p}{\partial z^2} \qquad (2c), (Cf.1c)$$

$$j_{xc}(x,y,z) = \beta \frac{\partial p}{\partial z} \qquad (2d), (Cf.1a)$$

$$j_{zc}(x,y,z) = -\beta \frac{\partial p}{\partial x} \qquad (2e), (Cf.1a)$$

$$j_{yc}(x,y,z) = -y\beta' \frac{\partial p}{\partial z} \qquad (2f), (Cf.1b)$$

where p(x,z) is the pressure which is constant along magnetic field lines by virtue of Eq. (1a), $\beta(x) = B_y^{-1}(x)$ and $\beta' = d\beta/dx$. The electric potential in the core varies along the magnetic field lines according to

$$\phi_c(x,y,z) = \phi_w(x,z) - \frac{1}{2}(a^2 - y^2)\,\beta' \frac{\partial p}{\partial z}, \qquad (2g)$$

where $\phi_w(x,z)$ is the electric potential at the inside surface of the duct. Equation (2g) is obtained by integrating the y- component of equation (1b), and using equation (2f).

The three-dimensional problem with eight variables is reduced to solving for the functions p(x,z) and $\phi_w(x,z)$. The equations necessary for the determination of p(x,z) and $\phi_w(x,z)$ are derived in subsequent subsections.

C. Solution in the Hartmann Layer

For M >> 1, viscous effects are confined to the Hartmann layer which has $O(M^{-1})$ thickness and which separates the inviscid core region from the duct wall. In insulating ducts, the Hartmann layer is part of the electrical circuit through which currents circulate. Thus a complete analysis of the Hartmann layer together with that in the core is needed to guarantee that the flow and current leaving the core region is the same as that entering the Hartmann layer.

Introducing the (r,θ,x) cylindrical coordinate system (see Fig. 1), Eqs. (1) are equivalent to:

$$\frac{\partial p}{\partial r} = B \sin\theta\, j_x + M^{-2}\left[\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] \qquad (3a)$$

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = B \cos\theta\, j_x + M^{-2}\left[\nabla^2 v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2}\right] \qquad (3b)$$

$$\frac{\partial p}{\partial x} = -B \sin\theta\, j_r - B \cos\theta\, j_\theta + M^{-2} \nabla^2 v_x \qquad (3c)$$

$$j_r = -\frac{\partial\phi}{\partial r} + B\sin\theta v_x \quad (3d)$$

$$j_\theta = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} + B\cos\theta v_x \quad (3e)$$

$$j_x = -\frac{\partial\phi}{\partial x} - B\sin\theta v_r - B\cos\theta\, v_\theta \quad (3f)$$

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial v_x}{\partial x} = 0 \quad (3g)$$

$$\frac{\partial j_r}{\partial r} + \frac{j_r}{r} + \frac{1}{r}\frac{\partial j_\theta}{\partial\theta} + \frac{\partial j_x}{\partial x} = 0 \quad (3h)$$

Each of the variables in the Hartmann layer (subsequently denoted with subscript "h") is expanded asymptotically in power series of M^{-1}, i.e., $\phi_h = \phi_{ho} + M^{-1}\phi_{h1}$. Terms of $O(M^{-2})$ or smaller are truncated since at the Hartmann layer/core interface solution in the Hartmann layer must match the solution in the core where terms of $O(M^{-2})$ are neglected. At the duct wall the normal velocity and normal current must vanish. The final solutions for the non-zero leading terms in the asymptotic expansions are as follows (a complete presentation of the lengthy and complex analysis will be given elsewhere).

$$v_{xho} = [\beta\sec\theta\frac{\partial\phi_{ho}}{\partial\theta} - \beta^2\sec^2\theta\,\frac{\partial p_{ho}}{\partial x}]$$

$$\times[1 - e^{(B\cos\theta\zeta)}] \quad (4a)$$

$$v_{\theta ho} = -[\beta\sec\theta\frac{\partial\phi_{ho}}{\partial x} + \beta^2\sec^2\theta\,\frac{\partial p_{ho}}{\partial\theta}]$$

$$\times[1 - e^{(B\cos\theta\zeta)}] \quad (4b)$$

$$j_{xho} = [\beta\sec\theta\,\frac{\partial p_{ho}}{\partial\theta}]\,[1 - e^{(B\cos\theta\zeta)}]$$

$$\frac{\partial\phi_{ho}}{\partial x}e^{(B\cos\theta\zeta)} \quad (4c)$$

$$j_{\theta ho} = -[\beta\sec\theta\frac{\partial p_{ho}}{\partial x}]\,[1 - e^{(B\cos\theta\zeta)}]$$

$$\frac{\partial\phi_{ho}}{\partial\theta}\,e^{(B\cos\theta\zeta)} \quad (4d)$$

$$v_{rh1} = A_1(\theta,x)\zeta + A_2(\theta,x) + [A_3(\theta,x)\zeta$$

$$- A_2(\theta,x)]\,e^{(B\cos\theta\zeta)} \quad (4e)$$

$$j_{rh1} = C_1(\theta,x)\zeta + C_2(\theta,x) + [C_3(\theta,x)\zeta$$

$$- C_2(\theta,x)]e^{(B\cos\theta\zeta)} \quad (4f)$$

where ζ is a stretched coordinate in the Hartmann layer with the duct wall at $\zeta = 0$ and the layer/core interface at $\zeta \to -\infty$. The lengthy expressions for the functions A_1, A_2, A_3, C_1, C_2, C_3 will be omitted here.

D. Matching Between Hartmann and Core Solutions

The formal matching procedure is quite simple: it reduces to equating a core variable evaluated at $r = 1$ to the corresponding Hartmann variable as $\zeta \to -\infty$. Matching also provides boundary conditions for the core variables to guarantee continuity of mass and current. This matching procedure leads to two partial differential equations governing the pressure in the liquid metal and the electric potential at the inside wall.

$$\frac{\partial}{\partial x}\{\beta^2(x)\,[M\cos^2\theta - \beta(x)\sec^3\theta]\,\frac{\partial P}{\partial x}\,(x,\theta)\} +$$

$$\frac{\partial}{\partial\theta}\{\,[M\beta^2(x) + \frac{1}{3}M\beta'^2(x)\cos^2\theta - \beta^3(x)\sec^3\theta]\frac{\partial P}{\partial\theta}\,(x,\theta)\}$$

$$= \beta'(x)\cos\theta\,[M - 2\beta(x)\sec^3\theta]\,\frac{\partial\phi_w}{\partial\theta}(x,\theta) +$$

$$\beta(x)\sin\theta\,[M + 2\beta(x)\sec^3\theta]\,\frac{\partial\phi_w}{\partial x}\,(x,\theta) \quad (5)$$

$$\beta(x)\,\frac{\partial}{\partial\theta}\,[\sec\theta\,\frac{\partial\phi_w}{\partial\theta}(\theta,x)] + \sec\theta\,\frac{\partial}{\partial x}\,[\beta(x)\,\frac{\partial\phi_w}{\partial x}(x,\theta)]$$

$$= \cos\theta\,\beta'(x)\,[M - 2\beta(x)\sec^3\theta]\,\frac{\partial P}{\partial\theta}\,(x,\theta) +$$

$$\sin\theta\,\beta'(x)\,[M + 2\beta(x)\sec^3\theta]\,\frac{\partial P}{\partial x}\,(x,\theta) \quad (6)$$

E. Boundary Conditions

At each cross section, the boundary conditions are provided by symmetry at $\theta = 0$, namely:

$$\frac{\partial P}{\partial\theta} = 0$$

$$\phi_w = 0 \qquad \text{at } \theta = 0 \quad (7a,b)$$

As $\theta \to \frac{\pi}{2}$ and in particular, as $2\beta(x)\sec^3\theta \to M$ Eqs. (5) and (6) fail, because the Hartmann layer grows into a side region with dimensions $\Delta r = O(M_L^{-2/3})$ and $\Delta\theta = \cos\theta = O(M_L^{-1/3})$, where M_L is the local Hartmann number, $M_L = B_y(x)M$. Such a side region or singularity of the Hartmann layer has been treated by Roberts[3] for fully developed flow in uniform magnetic field. Here the flow is disturbed by the nonuniformity of magnetic field and the dimensions of the side region increase as the field decreases. A detailed and complicated analysis of the side region is necessary to completely define the problem. However, for practical purposes, it is sufficient to apply an approximate integral approach to treat the side region.

In our computation, Eqs. (5) and (6) are solved for $0 \le \theta \le \theta_m$ where θ_m is smaller than but close to $\frac{\pi}{2}$.

Consider the area S bounded by $0 \le y \le (1 - z^2)^{1/2}$, with $z_m = \sin\theta_m \le z \le 1$. If S is a very small area then the rate of change of mass flow and current over a volume with cross section S and length Δx are negligible. Thus the mass flow or current entering the volume $S\Delta x$ in a particular cross section must leave it in the same cross section. These conservation considerations lead to two equations determining the boundary conditions for $\frac{\partial \phi_w}{\partial \theta}(x, \theta_m)$ and $\frac{\partial P}{\partial \theta}(x, \theta_m)$:

$$[M - \beta(x)\sec^3\theta_m]\left[\frac{\partial \phi_w}{\partial x}(x,\theta_m) + \beta(x)\sec\theta_m \frac{\partial P}{\partial \theta}(x,\theta_m)\right] =$$

$$\frac{1}{3} M\cos\theta_m \frac{\partial}{\partial x}\left[\beta'(x)\frac{\partial P}{\partial \theta}(x,\theta_m)\right] \quad (8)$$

$$\sec^2\theta_m \frac{\partial \phi_w}{\partial \theta}(x,\theta_m) + [M - \beta(x)\sec^3\theta_m]\frac{\partial P}{\partial x}(x,\theta_m) = 0 \quad (9)$$

Sufficiently upstream and downstream of the region where the magnetic field is changing, the flow will be fully developed. For fully developed flow, there are no axial currents in the fluid. The appropriate boundary conditions at the upstream cross section, x_1, and at the downstream cross section, x_2, are:

$$P = P_1$$

$$\frac{\partial \phi_w}{\partial x} = 0 \qquad \text{at } x = x_1 \qquad (10a,b)$$

and

$$P = P_2$$

$$\frac{\partial \phi_w}{\partial x} = 0 \qquad \text{at } x = x_2 \qquad (10c,d)$$

The constants P_1, and P_2 ($P_1 \neq P_2$) can be arbitrarily chosen. After the solution is found, every variable is multiplied by a scaling factor to get the desired volumetric flux. The dimensionless axial velocity must satisfy the total volumetric condition

$$\int_0^{z_m} \int_0^{(1-z^2)^{1/2}} u_c(x,y,z)\,dy\,dz +$$

$$\int_{z_m}^{1} \int_0^{(1-z^2)^{1/2}} u_c(x,y,z_m)\,dy\,dz = \frac{\pi}{4} \quad (11)$$

in which the axial velocity inside the very small area S is approximated by its value at z_m.

Equations (5) and (6) constitute a set of coupled partial differential equations to be solved simultaneously in the X-θ domain. A staggered grid network is used to formulate the finte difference equations. Previous studies for circular conducting duct[2] and rectangular duct[3] have shown that the use of staggered grid is essential to provide efficient and stable numerical solutions.

III. RESULTS AND DISCUSSION

A. Pressure Gradient for Fully Developed Flow

One of the important parameters pertinent to the overall performance of a fusion reactor blanket is the pressure gradient, which, when integrated over a duct length of interest, yields the MHD pressure drop. As mentioned earlier, the use of electrically-insulated duct could result in a much lesser MHD pressure drop than that for a conducting duct.

The pressure gradient in a round thin-walled conducting duct is

$$\frac{\partial P}{\partial x} = -\frac{c}{1 + c} \quad (12)$$

where $c = \sigma_w t/\sigma L$ is the wall conductance ratio, t is the wall thickness, L is the duct's radius, σ_w and σ are the electrical conductivities of the wall and the liquid metal, respectively.

In an insulating duct the transverse current density is of $O(M^{-1})$, then by virtue of Eq. (2e), the pressure gradient is expected to be proportional to M^{-1} and is given by

$$\frac{\partial P}{\partial x} = -\frac{3\pi}{8} M^{-1} \quad (13)$$

Equation (14) is derived from Eqs. (5, 8, 11) for fully developed flow conditions and for $M >> 1$, θ_m close to $\pi/2$.

In a tokamak reactor, the conductance ratio, c, of the coolant duct is typically $10^{-3} - 10^{-2}$ while the Hartmann number is of the order of $10^4 - 10^5$. Consequently, the pressure drop in an insulating duct is at least an order of magnitude smaller than that in a conducting duct.

B. Flows in Fringing Magnetic Field

The nonuniform magnetic field distribution used here are the values actually measured in the ALEX experimental facility at Argonne National Laboratory. The tail of the normalized $B_y(x)$ is smoothly leveled to a value of 0.2 (Figure 2) since the inertialess, inviscid assumptions for the core solutions are no longer valid once B_y becomes too small. The results presented in Figs. 3 to 5 correspond to M= 7000 and θ_m = 1.45 radians. Thus, the area of the "side region", S, amounts to only 0.07% of the total cross sectional area.

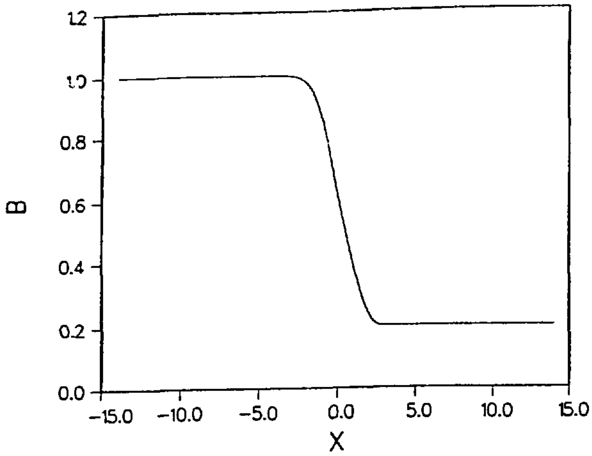

FIG. 2 THE NORMALIZED MAGNETIC FIELD DISTRIBUTION

Figure 3 shows the electrical potential profiles at the inside surface of the duct divided by the local magnetic field at various axial locations. If the flow were locally fully developed everywhere, all these curves would coincide with the upstream curve at x = -13.7. The axial potential differences drive axial electric currents in the liquid metal in the ± x direction for z $\lessgtr$ 0. These currents must eventually turn in the z direction to close through the Hartmann layer or the core. A large distance, of the order O($M^{1/2}$) is required to accommodate the complete current paths.

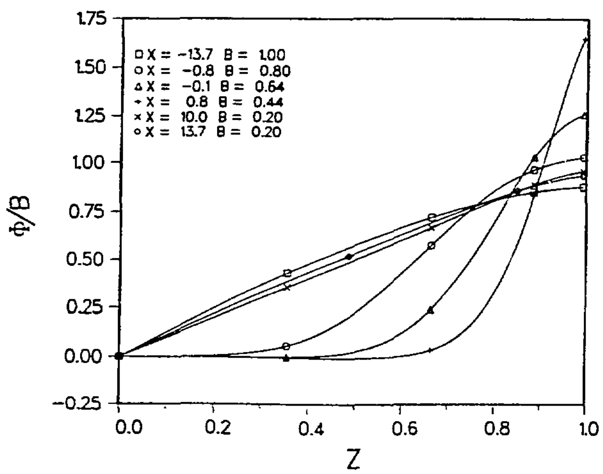

FIG. 3 ELECTRICAL POTENTIAL PROFILES AT THE INSIDE SURFACE OF THE DUCT DIVIDED BY THE LOCAL MAGNETIC FIELD.

The motion of the flow on the y = 0 plane passing through the nonuniform field region is shown in Figure 4a, b. Upstream, where the field is uniform and the flow is locally fully developed, the axial velocity peaks at the center and vanishes at the wall. As the flow evolves in the downstream direction, 3-D effects due to nonuniformity of the field causes the velocity to increase dramatically near the wall, leaving a stagnant fluid region in the center. The effect persists for a large distance before the flow slowly resumes its fully-developed flow profile (Figure 4b). A companion plot of the velocity variation along x at various z locations is shown in Figure 5. It is interesting to note a slight increase in velocity at the center as the flow approaches the nonuniform field region from upstream before a rapid decrease to zero. The evolution of the flow motion can also be visualized from the electrical potential profiles in Figure 3 in which the slopes of the curves are partially proportional to the local axial velocities.

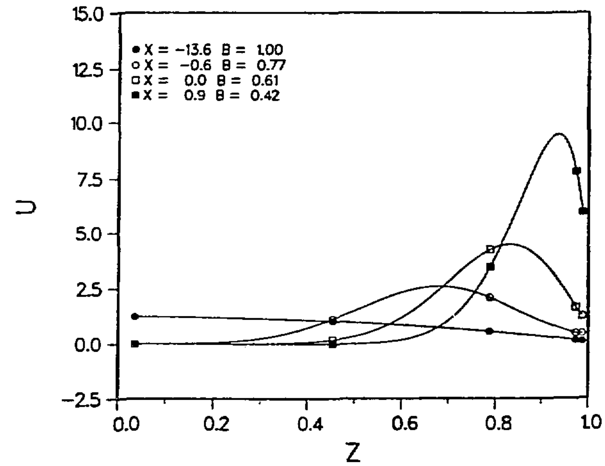

FIG. 4a VELOCITY PROFILES ON THE y = 0 PLANE AT VARIOUS AXIAL LOCATIONS.

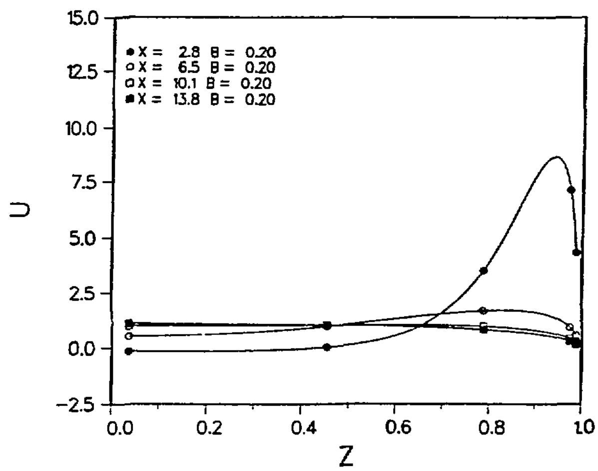

FIG. 4b VELOCITY PROFILES ON THE y = 0 PLANE AT VARIOUS AXIAL LOCATIONS.

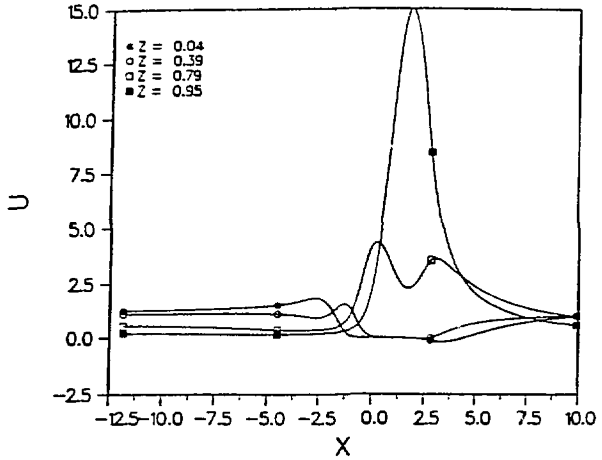

FIG. 5 AXIAL VELOCITY VARIATION AT VARIOUS RADIAL LOCATIONS.

SUMMARY

An analysis for liquid-metal flow in circular ducts with insulating walls in varying transverse magnetic fields has been carried out. The present solution involves M as an arbitrary parameter, but neglects terms which are $O(M^{-2})$ or smaller in the core. The matching of the Hartmann layer variables evaluated at $r = 1$ is related to the well-known Hartmann conditions at an insulating wall duct in a uniform magnetic field.[5] This matching procedure yields a pair of coupled partial differential equations governing p and ϕ_w which are solved simultaneously using numerical methods. The remaining variables in the core are then easily derived to provide a fully three-dimensional solution for the flow. An approximate integral method for the treatment of the side region, valid under the conditions of fusion reactor blankets, simplifies the analysis.

ACKNOWLEDGMENTS

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