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DETERMINATION OF THE SLOPE AND INTERCEPT OF THE UNIVERSAL VELOCITY PROFILE FROM PRESSURE LOSS MEASUREMENTS

by

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ABSTRACT

A method is demonstrated for determination of the slope and intercept of the universal velocity distribution law for flow past roughened surfaces without the need for measurement of the velocity profile. The slope is shown to vary with the nature of the roughened surface and in some cases to deviate considerably from that for turbulent flow past smooth walls. It is further shown that the intercept, commonly known as the roughness parameter $R(h^+)$, is independent of the width of the velocity profile. The dependence noted by previous investigators was due to an assumption that the slope of the law of the wall for roughened surfaces is constant and equal to that for smooth surfaces.

INTRODUCTION

Helium has advantages as a nuclear reactor coolant due to its chemical inertness, absence of phase transition, and very low cross section for neutron interaction. The latter attribute is particularly desirable in a breeder reactor, where neutron economy is of primary importance. However, because of its low density and poor thermal conductivity, helium is not a good heat transfer medium. Its use as a reactor coolant is characterized by high system pressure and large volumetric flow.

Since the maximum allowable fuel element cladding temperature for the reactor is limited, an increased surface heat transfer coefficient is needed to obtain an adequately high power density and coolant temperature with the associated improved cycle thermal efficiency. Because convective heat transfer is enhanced when surfaces are roughened, much study has been given to the evaluation of artificial roughening of the heat transfer surfaces of helium and other gas-cooled reactors.

Surface roughening promotes mixing by generating turbulent eddies in the high-thermal-resistance coolant sublayers adjacent to the heat transfer surface. The increased transport of heat from the wall into the fluid main stream is manifested as a higher Stanton number for the artificially roughened surface. Concomitantly, the resistance to flow is increased due to the form drag of the roughness element portions that protrude through the viscous sublayers at the wall. The increased energy dissipation over that of purely viscous shear on a smooth surface results in a higher friction factor. Thus any advantage gained by the use of artificial roughening in the reduction of required heat transfer surface area or the improvement of thermal performance is partially offset by an increase in pumping power due to the increased frictional loss.

The Stanton number and friction factor, which describe the performance of a given artificial roughness, are functions of pitch, height, width, angle of attack, and shape of the roughness elements on the heat transfer surface; the Reynolds and Prandtl numbers of the coolant fluid; and the geometry of the coolant passage. These functional relations must be determined empirically by experiments, which are most economically performed in simple test geometries.

Experiments performed with artificially roughened surfaces within circular tubes or rectangular ducts have been limited by the cost and difficulty of fabricating machined roughening in these geometries. Most of the experiments have been performed in annular geometries with the roughness machined on the surface of a rod placed centrally within a smooth tube. In addition to the relative ease of fabrication of the roughened surface, testing in annuli has the advantage that the convex surface of the inner rod is geometrically similar to that of the reactor fuel rods for which the surface roughening is being considered.

The friction factor experimentally determined in annular geometry is applicable to the entire channel area, which is bounded by one rough and one smooth surface. Accordingly, some procedure, generally termed a transformation, must subsequently be used to calculate the friction factor applicable to the inner zone of flow, that is, that bounded by the surface of zero shear and the rough surface being tested. If it is assumed that the flow on one side of the surface of zero shear is independent of the nature of the flow on the other side, then it follows that the inner zone friction factor (f_1) is determined by the location of the surface of zero shear, the Reynolds number of the inner zone, and the surface characteristics of the roughness. A successful transformation method based on the theory of universal dimensionless velocity profiles was developed by Maubach[1] and subsequently refined by Dalle Donne and Meyer[2].

THE ROUGHNESS PARAMETER $R(h^+)$

The inner zone friction factor (f_1), which is determined by a transformation procedure, is applicable to the flow between the surface of zero shear and the roughened surface of the inner rod. It is a function of the Reynolds number, the surface characteristics of the roughness, the shape of the flow channel, and the width Y_L of the velocity profile, that is, the distance from the surface of zero shear to the surface of the inner rod. The functional relationship is given for an annulus with an inner roughened rod of radius r_1 and a roughness height h by the equation[1]

$$(2/f_1)^{1/2} = A \ln(Y_L/h) - G_1 + R(h^+) , \quad (1)$$

where

$$G_1 = [1/2 + 1/(2 + Y_L/r_1)]\ A\ . \tag{1a}$$

Equation (1) is derived by use of

$$(2/f_1)^{1/2} \equiv \bar{u}^{+} = \frac{1}{A_c} \int_{A_c} u^{+}\, dA_c\ . \tag{2}$$

The average dimensionless velocity $\bar{u}^{+}$ is obtained by integration of the universal velocity distribution law for rough surfaces[3], expressed as

$$u^{+} = A\ \ln(y/h) + R(h^{+}) \tag{3}$$

across the portion of the annular gap bounded by the rough wall and the surface of zero shear. Equation (3) is commonly termed the "law of the wall for rough surfaces." The roughness parameter $R(h^{+})$ of this equation is a boundary condition representing the dimensionless velocity u^{+} at a distance h from the surface of the inner rod.

The first term following the equal sign in Eq. (1) is a function of the width Y_L of the velocity profile. The form of the term G_1 is determined by the channel type; the expression given in Eq. (1a) is for an annulus. As previously noted there is an additional dependence of the friction factor on the surface characteristics of the roughened wall. By the process of elimination, this dependence should be contained in the roughness parameter $R(h^{+})$ of Eq. (1).

In 1933, Nikuradse[4] experimentally determined the roughness parameter for flow in tubes roughened by sand grains glued to the wall. He found $R(h^{+})$ for a given surface to be a constant for fully rough flow, that is, for values of the roughness Reynolds number h^{+} sufficiently large that the roughness elements are not masked by viscous flow. For lower values of h^{+} in turbulent flow, $R(h^{+})$ and consequently f_1 vary with h^{+}, and the flow is said to be in the hydraulically smooth region or in the transition region between hydraulically smooth and fully rough flow.

For flow past two-dimensional roughness ribs, the friction factor f_1 will depend in part on the channel type (annulus, tube, etc.) and the basic rib shape (triangular, rectangular, etc.). Hudina[5] found from dimensional analysis, for the case of fully rough isothermal flow where compressibility effects are unimportant, that the additional dependence could be expressed in terms of three dimensionless groups:

$$f_1 = F[h/D_1,\ (P - b)/h,\ b/h] \ , \tag{4}$$

where P is the pitch and b is the width of the roughness ribs.

The equivalent diameter (D_1) for flow between the surface of zero shear and the rough inner wall of an annulus may be expressed as

$$D_1 = 2(2 + Y_L/r_1)Y_L \ , \tag{5}$$

where r_1 is the radius of the inner rod. Therefore, Eq. (4) can be written as

$$f_1 = F[Y_L/h,\ (P - b)/h,\ b/h] \ . \tag{6}$$

A comparison of Eqs. (6) and (1) shows that $R(h^+)$ for fully rough flow over two-dimensional transverse ribs of a given shape should be determined by the dimensionless groups $(P - b)/h$ and b/h alone. This conclusion depends on the assumption that the influence of the channel type and the parameter Y_L/h is confined to the first two terms on the right side of Eq. (1).

EXPERIMENTAL DETERMINATION OF $R(h^+)$

The slope A and intercept $R(h^+)$ of Eq. (1) can be obtained graphically using the law of the wall, Eq. (3), if the dimensionless velocity profile is experimentally determined. Alternately, if a value is assumed for the slope A, $R(h^+)$ can be calculated using Eq. (1) since Y_L and f_1 are computed by the transformation procedure. The latter method has the advantage that the velocity profile need not be measured.

Nikuradse[4] found the slope A for fully rough flow in sand-grain-roughened tubes to be the same as that for turbulent flow in smooth tubes. The common practice for subsequent testing has been to assume that the Nikuradse value of 2.50 for this slope is applicable to flow past all roughened surfaces. With this assumption, and with Y_L replaced by the equivalent $(r_0 - r_1)$ in the term G_1, Eq. (1) may be rewritten in the form

$$R(h^+) = (2/f_1)^{1/2} - 2.5 \ln(Y_L/h) + G_1 , \tag{7}$$

where, for an annulus,

$$G_1 = \frac{3.75 + 1.25 r_0/r_1}{1 + r_0/r_1} . \tag{7a}$$

Equation (7) has been used for testing many different roughened surfaces in annuli, including two-dimensional transverse rectangular ribs of varying pitch P, rib width b, and rib height h. The development of an empirical correlation for $R(h^+)$ with these parameters using all available data for rectangular ribs was attempted by Baumann and Rehme[6]. In an effort to include only data taken in fully rough flow, where $R(h^+)$ is not a function of h^+, data for flow with h^+ less than 100 were excluded. The transformed friction factors and Reynolds numbers applicable to the roughened surface were calculated by the Maubach[1] method in all cases.

Although $R(h^+)$ should be a property of the surface roughness in fully rough flow, Baumann and Rehme found that the values of this parameter calculated using Eq. (7) showed a dependence on the ratio h/Y_L. They concluded, as had other authors[7], that the effect of this ratio on the friction factor is not confined to the first term on the right side of Eq. (1) but affects the roughness parameter as well. Their approach was to seek a two-step correlation. They first developed an expression for the conversion of all $R(h^+)$ values to those applicable to a standard value of h/Y_L, that is, to those which would have been obtained had the original tests been conducted in geometries constructed with this hypothetical value of the ratio h/Y_L. A second expression was then developed for the correlation of these converted values of $R(h^+)$ with the roughness surface geometry ratios P/h and h/b.

In 1976, Dalle Donne and Meyer[2] reported the results of a series of tests in annuli which had been designed for the purpose, inter alia, of investigation of the effects of the ratio h/Y_L on the parameter $R(h^+)$. The series consisted of ten inner rods of approximately equal radius roughened with machined rectangular ribs of differing surface characteristics. Each rod was tested consecutively in four smooth outer shrouds of varying diameter. The friction factors and Reynolds numbers applicable to the inner zone were computed with the Dalle Donne–Meyer transformation[2] and Eq. (7) was used for the calculation of $R(h^+)$.

Dalle Donne and Meyer found a more pronounced effect of the ratio h/Y_L on the roughness parameter than had Baumann and Rehme and also developed a two-step correlation procedure. They assumed that a value for h^+ of 150 was high enough to ensure fully rough flow for all the rod surfaces tested. The values of $R(h^+)$ for a given surface calculated at this flow within each of the four outer shrouds were plotted against the corresponding values of the parameter $\ln(h/Y_L)$. This resulted in the correlation

$$R(h^+)_{01} = R(h^+) - 0.4 \ln\left(\frac{h/Y_L}{0.01}\right) , \tag{8}$$

where $R(h^+)_{01}$ is defined as the value of $R(h^+)$ that would apply to the tested surface in a hypothetical annular geometry for which the ratio h/Y_L was equal to 0.01. The parameter $R(h^+)_{01}$ was then correlated with the surface geometric characteristic ratios $(P - b)/h$ and h/b.

The value of $R(h^+)_{01}$ calculated by Eq. (8) is presumably independent of the ratio h/Y_L. Substitution for $R(h^+)$ in Eq. (8) using Eq. (7) and rearrangement shows that the Dalle Donne–Meyer correlation for $R(h^+)$ in effect adjusts the value of the slope A in Eq. (1) to 2.1 and replaces $R(h^+)$ in Eq. (1) by the two terms

$$R(h^+)_{01} + \left[1.84 - 0.4\left(\frac{1.5 + 0.5r_0/r_1}{1 + r_0/r_1}\right)\right] . \tag{9}$$

The second term is a weak function of the radius r_0 of the surface of zero shear and is in the range 1.45 to 1.55 for the radius ratios employed by Dalle Donne and Meyer.

DETERMINATION OF SLOPE WITHOUT VELOCITY TRANSVERSES

A plot of experimental data in the form of Eq. (1) is an alternate method for determining the slope A and intercept $R(h^+)$ which has not been previously used. This requires experimental determination of the friction factors for the same roughened surface for more than one value of the velocity profile width (Y_L). In annular geometry, this can be done by testing the inner rod in several smooth outer shrouds of varying diameter, as was done by Dalle Donne and Meyer[2].

Figures 1 and 2 show the experimental results of Dalle Donne and Meyer plotted in the manner of Eq. (1). The slope A and intercept $R(h^+)$, as determined by a least-squares fit for each of the ten rods tested, are given in Table 1. These results show that the adjustment of this slope to 2.10, which is inherent in the empirical correlation of Dalle Donne and Meyer, is only accurate for the case of rods 5 and 8, where the slope of the dimensionless velocity profile actually is 2.10. The adjusted slope is, however, closer to the true slope than is the value 2.5 for most of the other rods.

There are four clusters of points for each of the rods represented in Figs. 1 and 2, each of which represents data taken in one of the four different smooth outer shrouds; each point in the cluster represents data taken at a different value of the roughness Reynolds number h^+. A significant vertical spread of the points indicates that the flow was not fully rough, so that the intercept $R(h^+)$ was not a constant for the lower values.

Other investigators[8] found that the value of h^+ at which the flow becomes fully rough increases with roughness height h. This implies that the Reynolds number at which this transition occurs varies little, if at all, with the rib height. The rods represented in Fig. 1 all have values of h less than 0.52 mm; the lowest values are ∿0.3 mm for rods 1 and 2. All points plotted in this figure represent flow with h^+ greater than 100. The vertical spread in the plotted points for rods 3, 4, and 5 indicates flow in the transition region at values of h^+ above 100. Rods 6 through 10 (Fig. 2) all had a roughness height h of approximately 0.8 mm. All points plotted in this figure were computed for values of h^+ greater

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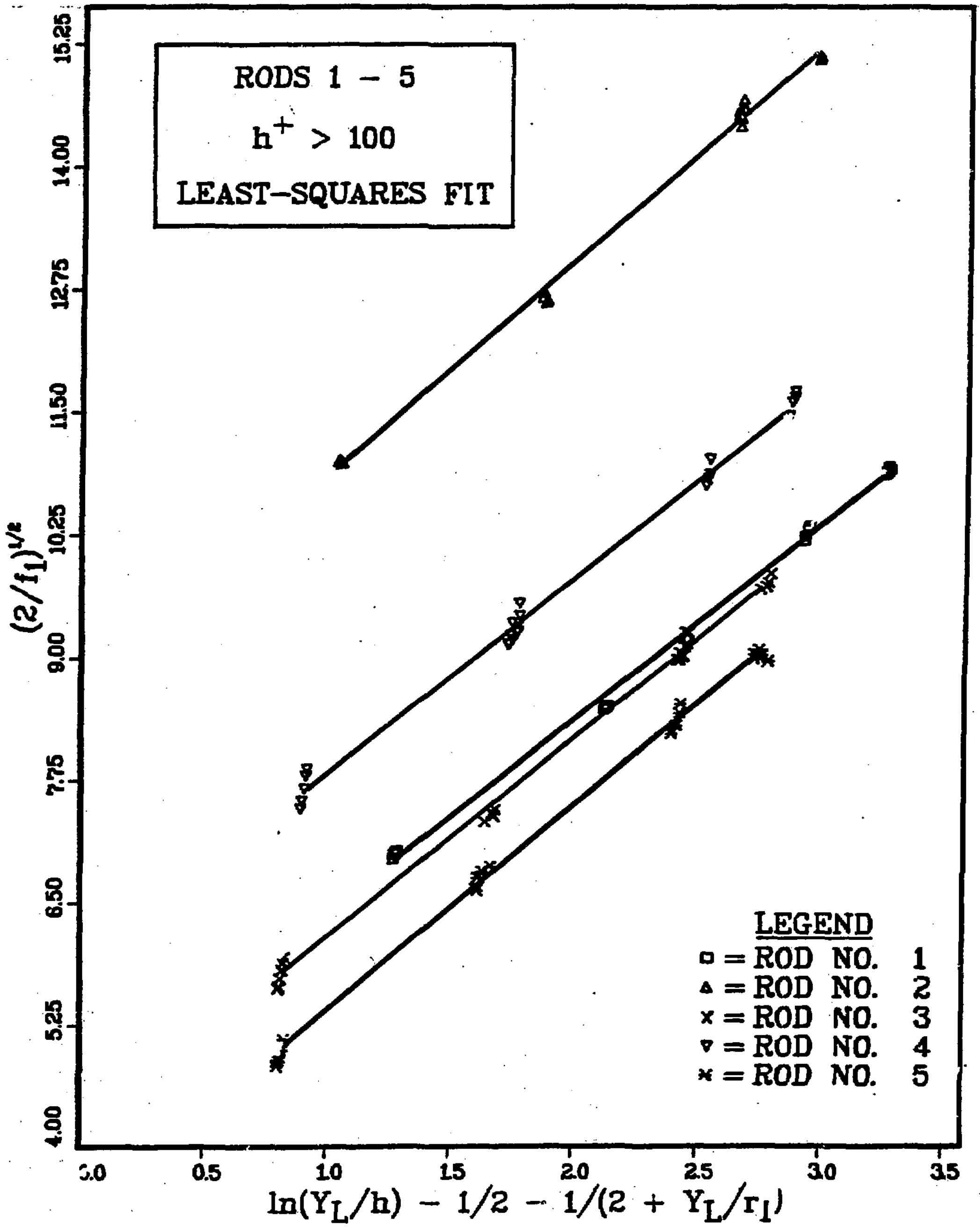

Fig. 1. The universal law of friction for rods 1–5 tested by Dalle Donne and Meyer.

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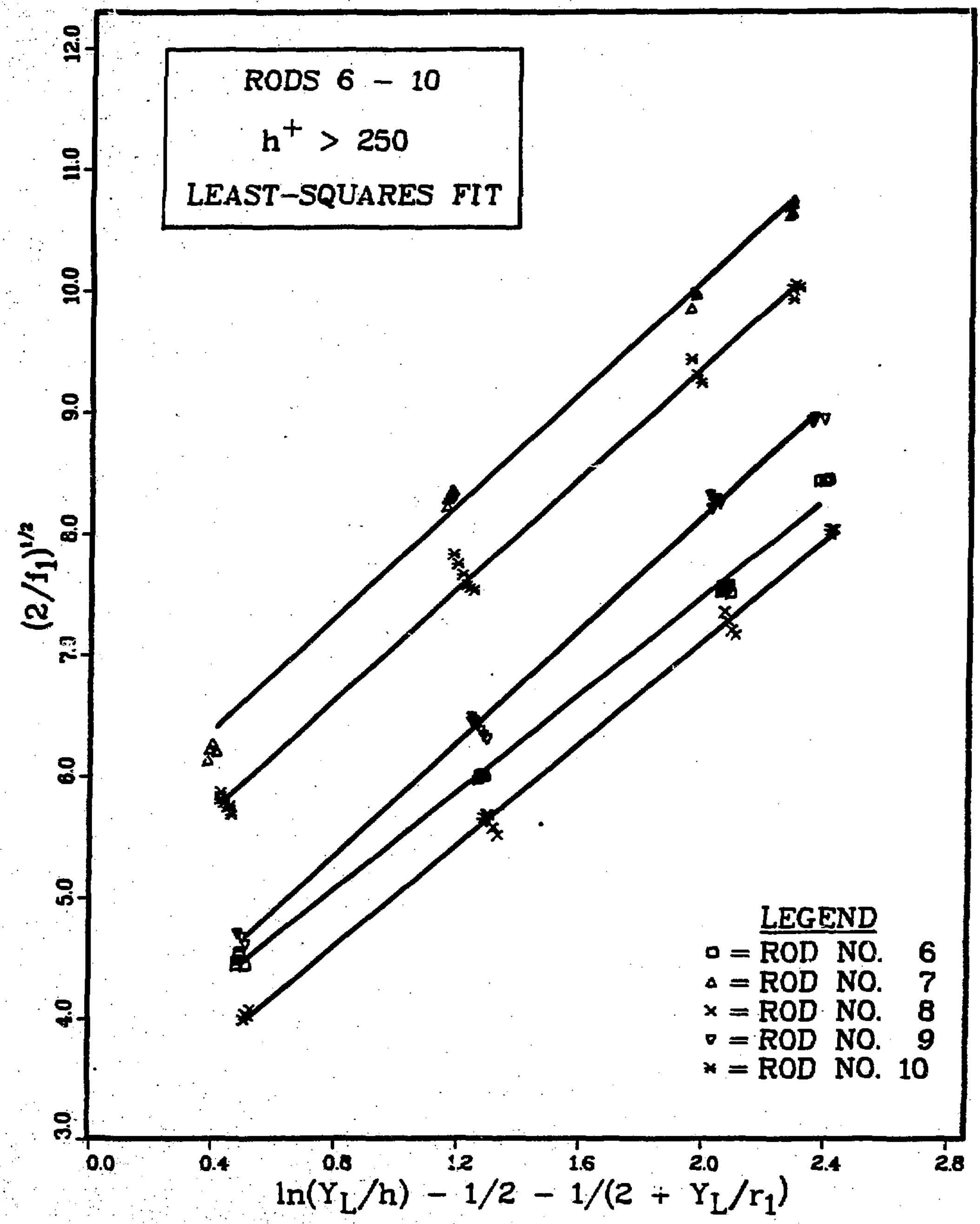

Fig. 2. The universal law of friction for rods 6–10 tested by Dalle Donne and Meyer.

Table 1. Slope and intercept of the universal velocity distribution law determined in accordance with Eq. (1) for the ten rods of Ref. 2

Rod	A	$R(h^{+})$
1	2.00	4.37
2	2.20	8.67
3	2.05	4.09
4	2.00	5.83
5	2.10	3.29
6	2.01	3.45
7	2.33	5.44
8	2.10	2.91
9	2.34	3.48
10	2.30	4.79

than 250, but some vertical spread is evident; this again indicates that the lower points represent flow in the transition range.

The data points for each rod plotted in Figs. 1 and 2 clearly obey a linear logarithmic relation; thus it may be concluded that the ratio h/Y_L will not affect $R(h^+)$ if the latter is calculated using the actual value of the slope A in Eq. (1). Figure 3 illustrates the case where a slope of 2.5, as determined by Nikuradse for sand-grain-roughened tubes, is assumed. The points *a* and *b* are taken from the experimental results of Dalle Donne and Meyer for rod 1. Point *a* represents the smallest of the four smooth outer shrouds and point *b* corresponds to the largest. Calculation of respective roughness parameters $R(h^+)$ using Eq. (7) results in the two differing values $R(h^+)_a$ and $R(h^+)_b$, as shown on the figure. There is an apparent decrease in the value of $R(h^+)$ as the diameter of the outer shroud (and therefore the ratio Y_L/h) is increased, as has been reported by previous investigators. However, if this parameter is computed with Eq. (1) using the actual value of 2.00 for the slope A, the same value will be calculated for both points. This value, $R(h^+)$ on the figure, may be considered to be the physically meaningful value of the roughness parameter in that it is a function of the characteristics of the wall surface alone.

Previous attempts[6] to develop an empirical correlation for $R(h^+)$ have encountered considerable scatter in the experimentally determined values of this parameter. Much of this scatter can be attributed to a deviation that varies with the magnitude of the difference between the value of the actual slope A for the dimensionless velocity profile in a particular experiment and the assumed value of 2.5. Attempts to remove this variable deviation by the isolation of what was observed to be an effect of the ratio h/Y_L have been only partially successful. For example, Dalle Donne and Meyer introduced a correlation that, in effect, changed the assumed value for the slope A from 2.5 to 2.1. Since this value is closer to the actual slope for most cases, the magnitude of the variable deviation is decreased but the effect is not eliminated.

The development of an accurate empirical correlation for the physically correct values of the roughness parameter $R(h^+)$ should not be difficult once sufficient data are available. A second empirical correlation is required, however, since the value of the slope A, as well as $R(h^+)$,

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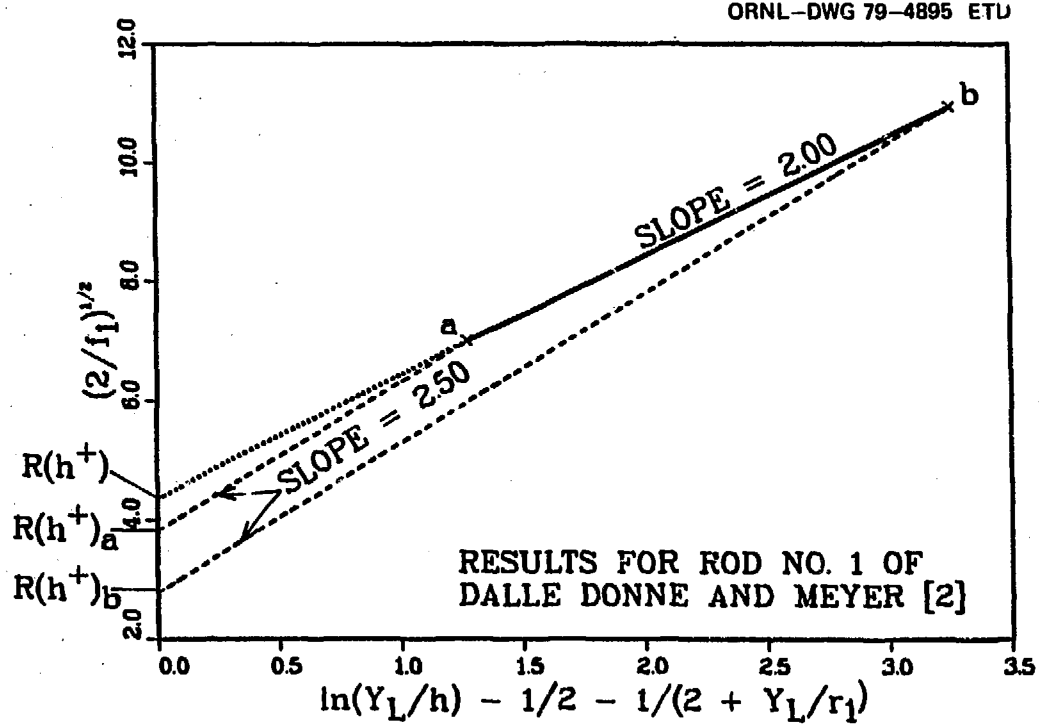

Fig. 3. Apparent effect of Y_L/h on $R(h^+)$ due to assumed value of 2.50 for slope of dimensionless velocity profile.

must be determined if Eq. (1) is to be used for the determination of the friction factor in untested channels. Unfortunately, the majority of available values for $R(h^+)$ were calculated using an assumed slope of 2.5 for flow in only one outer shroud. Thus, the correct slope for the flow conditions of these previous tests cannot be ascertained from the data.

The conclusion of this paper regarding the slope of the dimensionless velocity profile is substantiated by recent experimental results. Bauman[8], for bisurface parallel plate geometry, and Berger and Whitehead[9], for circular tubes, measured the velocity profile in the vicinity of walls roughened with transverse ribs and reported values significantly different from 2.5 for the slopes of the resulting dimensionless velocity profiles.

SUMMARY

This paper demonstrates that the slope of the linear portion of the dimensionless velocity profile as well as the roughness parameter $R(h^+)$ can be determined by plotting experimental results according to Eq. (1). The method is more complex for the testing of roughened surfaces in annular geometry because results must be obtained for at least one additional smooth outer shroud. However, the velocity profile need not be measured.

Values of the slope A computed in accordance with the method presented here are probably more accurate than are those taken from experimentally determined dimensionless velocity profiles. Although the method presented requires a transformation, this procedure is highly developed and the friction factor can be determined more accurately than can the dimensionless velocity profile. Furthermore, only a portion of this profile obeys the law of the wall, Eq. (3), and inaccuracies in the determined values of the slope and intercept will result if the range of applicability is not carefully defined.

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NOMENCLATURE

A	slope of linear portion of dimensionless velocity profile
A_c	cross-sectional area
b	width of rectangular rib
D	equivalent diameter
f	friction factor
F	functional dependence
G	geometric term that arises when the average dimensionless velocity is calculated by the integration of Eq. (3)
h	height of rectangular rib
h^+	roughness Reynolds number ($= hu^*/\nu$)
P	pitch of rectangular ribs
r	radius (measured from center of inner rough rod)
$R(h^+)$	roughness parameter
u	velocity
u^*	friction velocity [$= (\tau/\rho)^{1/2}$]
u^+	dimensionless velocity ($= u/u^*$)
$\bar{u}^+$	average dimensionless velocity
y	distance perpendicular to the rough wall
Y_L	width of velocity profile ($= r_0 - r_1$)
ν	kinematic viscosity
ρ	density
τ	wall shear stress

Subscripts

0	surface of zero shear
01	hypothetical annular geometry for which the ratio h/Y_L is equal to 0.01
1	roughened wall or flow passage between the surface of zero shear and the roughened wall