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By

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PARABOLIC APPROXIMATION METHOD FOR THE MODE CONVERSION-TUNNELING EQUATION

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ABSTRACT

The derivation of the wave equation which governs ICRF wave propagation, absorption, and mode conversion within the kinetic layer in tokamaks has been extended to include diffraction and focussing effects associated with the finite transverse dimensions of the incident wavefronts. The kinetic layer considered consists of a uniform density, uniform temperature slab model in which the equilibrium magnetic field is oriented in the z-direction and varies linearly in the x-direction. An equivalent dielectric tensor as well as a two-dimensional energy conservation equation are derived from the linearized Vlasov-Maxwell system of equations. The generalized form of the mode conversion-tunneling equation is then extracted from the Maxwell equations, using the parabolic approximation method in which transverse variations of the wave fields are assumed to be weak in comparison to the variations in the primary direction of propagation. Methods of solving the generalized wave equation are discussed.

I. INTRODUCTION

Recently, the parabolic approximation method has been applied to the cold plasma wave equation in a cylindrical tokamak geometry in order to analyze fast magnetosonic wave propagation in the region bounded by the plasma-antenna interaction zone at the edge of the discharge and the mode conversion-absorption layer located about the magnetic axis of the discharge.[1,2] The physical motivation for this approximation is that the fast wave propagates primarily in the radial direction within this region, if surface mode excitation, which is characterized by large poloidal mode numbers, is ignored. By treating poloidal derivatives as small perturbations and factoring the poloidal component of the wave electric field, E_θ, into a rapidly varying radial waveform, u(r), multiplied by a slowly varying amplitude function, $a(r,\theta)$, the Maxwell equations which determine the structure of the wave fields simplify considerably. In particular, the radial component of the wave electric field, E_r, is determined from an algebraic equation, u(r) is determined by a second-order ordinary differential equation, and $a(r,\theta)$ is determined by a parabolic diffusion-type equation. Since continuous solutions for the wave fields are constructed directly from an approximate form of the wave equation, diffraction, refraction, and focussing effects are treated self-consistently.

The parabolic approximation method may also be applied within the mode conversion-absorption layer in order to gain some insight into the diffraction effects associated with a finite size wavefront propagating at near normal incidence onto the layer. The finite transverse dimension of the incident wavefront is limited by focussing induced by the launcher geometry and the refractive properties of the equilibrium. In the simplest case, the layer is modelled as a slab of uniform density and temperature in which the $\hat{x}$ direction corresponds to the major radial direction of the tokamak, the $\hat{y}$ direction corresponds to the vertical direction, and the $\hat{z}$ axis corresponds to the toroidal direction. The equilibrium magnetic field is aligned solely in the $\hat{z}$ direction but varies linearly in the $\hat{x}$ direction. For this case, the problem of an incident wavefront, which is of finite size in the $\hat{y}$ and $\hat{z}$ directions, may be treated exactly by Fourier decomposing the wavefront into a set of plane waves characterized by k_y and k_z, the wave numbers in the $\hat{y}$ and $\hat{z}$ directions, respectively. Each incident plane wave may then be treated with the standard linear fourth-order mode conversion-tunneling equation[3-10] and the results summed over k_y and k_z to obtain the overall wave structure and the net absorption profile for the wavefront within the layer. Numerically, this exact procedure may get expensive if it becomes necessary to include many different values of k_y and/or k_z. Furthermore, it is difficult to extend the exact analysis to the case in which equilibrium gradients exist in the $\hat{y}$ or $\hat{z}$ directions.[11-14] For incident wavefronts which vary slowly in the $\hat{y}$ direction relative to the $\hat{x}$ direction and hence are composed primarily of plane waves which satisfy the paraxial propagation constraint, i.e., $k_y \ll k_x$, the parabolic approximation can be used to avoid solving the fourth-order equation for each value of k_y present in the wavefront.

In this report, a derivation of the linear fourth-order mode conversion-tunneling equation appropriate for wavefronts with a finite transverse extent is presented, starting from the linearized Vlasov-Maxwell system of equations and using the parabolic approximation. The basic model and assumptions are described in Sec. II. An equivalent dielectric tensor is developed in Sec. III in a manner closely resembling the derivation by Swanson,[5] except that derivatives of the wave fields with respect to vertical distance in the layer are retained. In Sec. IV, the appropriate form of the mode conversion-tunneling equation in the paraxial propagation limit is derived from the Maxwell equations using the parabolic approximation method. Power conservation is discussed in Sec. V, and the concluding remarks are summarized in Sec. VI.

II. THE BASIC MODEL

During RF heating of tokamak plasmas, mode conversion and absorption processes are generally important only in a thin layer located about the cyclotron resonance layer for the waves. As a first approximation, the mode conversion-absorption layer may be treated using a uniform density, uniform temperature slab model in which the $\hat{x}$, $\hat{y}$, and $\hat{z}$ directions are directed along the major radial, the vertical, and the toroidal directions of the torus. Neoclassical effects, magnetic field curvature effects, and rotational transform effects are neglected by assuming the equilibrium magnetic field is of the form:

$$\bar{B} = B_o(1 + x/L)\,\hat{z} \quad , \tag{1}$$

where $|L|$ is equal to R_o, the major radius of the torus.

For notational ease, only second harmonic heating of a single ion species plasma will be considered explicitly, with the wave frequency, ω, equal to twice the fundamental ion cyclotron frequency, ω_{co}, evaluated at $x = 0$. The generalization to multiple ion species plasmas and minority heating schemes is self-evident.

A Fourier decomposition of the incident wavefront in time and in the $\hat{z}$ direction is taken, but only a single value of $k \equiv k_z$ will be treated explicitly in the remaining sections. Furthermore, by neglecting effects related to finite electron inertia, the parallel component of the wave electric field, E_z, can be neglected to lowest order in m_e/m_i, where m_e and m_i are the electron and ion masses, respectively. Under these assumptions, the total electric and magnetic fields in the plasma may be written as:

$$\bar{E} = [E_x(x,y)\hat{x} + E_y(x,y)\hat{y}] \exp[i(kz - \omega t)] \ , \tag{2}$$

$$\bar{B} = B_o(1 + x/L)\hat{z} + \left[B_x(x,y)\hat{x} + B_y(x,y)\hat{y} + B_z(x,y)\hat{z}\right] \exp[i(kz - \omega t)] \ . \tag{3}$$

Similarly, the particle distribution functions may be written in the form,

$$f = f_o(x,y,v_x,v_y,v_z) + f_1(x,y,k,\omega,v_x,v_y,v_z) \exp[i(kz - \omega t)] \ , \tag{4}$$

where f_1 represents the perturbed component caused by the presence of the waves in the plasma and f_o is normalized such that

$$n = \int f_o \, d^3v \ , \tag{5}$$

with n denoting the species density.

Using Eqs. (1-4), the Vlasov-Maxwell equations for the plasma-wave system may be written as:

$$\left[k^2 - \omega^2/c^2\right] E_x - \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_y}{\partial x \partial y} = \frac{4\pi i \omega}{c^2} J_x \tag{6}$$

and

$$\left[k^2 - \omega^2/c^2\right] E_y - \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_x}{\partial x \partial y} = \frac{4\pi i \omega}{c^2} J_y \ , \tag{7}$$

where the currents J_x and J_y arising from the plasma response are given by

$$J_x = \sum_j q_j \int v_x f_{1j} d^3v \quad , \tag{8}$$

$$J_y = \sum_j q_j \int v_y f_{1j} d^3v \quad , \tag{9}$$

with a sum over species denoted by the subscript j.

The corresponding kinetic equation for each species, which describes the response of the plasma to the applied fields, is given by

$$\frac{\partial f}{\partial t} + \bar{v}\cdot\nabla f + \frac{\vec{F}}{m} \cdot\nabla_v f = 0 \tag{10}$$

where

$$\vec{F} = q \vec{E} + \frac{q}{mc} \vec{v} \times \vec{B} \quad . \tag{11}$$

Using Eq. (4), the kinetic equation may be separated into an equation for the unperturbed particle distribution function,

$$\bar{v}\cdot\nabla f_o + \omega_c \bar{v} \times \hat{z}\cdot\nabla_v f_o = 0 \quad , \tag{12}$$

and an equation governing the development of the perturbed distribution function,

$$-i\omega f_1 + \bar{v}\cdot\nabla f_1 + \omega_c \bar{v} \times \hat{z}\cdot\nabla_v f_1 = -\frac{q}{m} \vec{E}\cdot\nabla_v f_o \quad , \tag{13}$$

where

$$\omega_c = \frac{q\ B_o(1 + x/L)}{mc} \quad . \tag{14}$$

Ignoring the ∇B drifts, which have been shown to be unimportant to lowest order in the small parameter (x/L) by Swanson,[5] the solution for the zeroth-order unperturbed particle distribution function becomes:

$$f_o = \frac{n}{\pi^{3/2} v_o^{\ 3}} \exp[-(v_x^{\ 2} + v_y^{\ 2} + v_z^{\ 2})/v_o^{\ 2}] \quad , \tag{15}$$

where

$$v_o^{\ 2} = \frac{2k_B T}{m} \quad , \tag{16}$$

with k_B denoting Boltzmann's constant.

By substituting Eq. (15) for f_o into Eq. (13) for f_1, and changing velocity space variables from v_x, v_y, v_z to $v_\perp$, $v_\parallel$, and ϕ, where $v_x = v_\perp \cos\phi$, $v_y = v_\perp \sin\phi$, and $v_\parallel = v_z$, the equation determining f_1 is transformed into:

$$\frac{\partial f_1}{\partial \phi} + i\Omega f_1 = \frac{v_\perp \cos\phi}{\omega_c} \frac{\partial f_1}{\partial x} + \frac{v_\perp \sin\phi}{\omega_c} \frac{\partial f_1}{\partial y}$$

$$- \frac{2 f_o v_\perp c}{v_o^{\ 2}} \left[\frac{E_x \cos\phi + E_y \sin\phi}{B}\right] \quad , \tag{17}$$

where $\Omega \equiv \omega - k v_z/\omega_c$. This equation may be integrated once to yield:

$$f_1 = \exp(-i\Omega\phi) \int^{\phi} d\phi' \exp(i\Omega\phi') \left[\frac{v_\perp \cos\phi'}{\omega_c} \frac{\partial f_1}{\partial x} + \frac{v_\perp \sin\phi'}{\omega_c} \frac{\partial f_1}{\partial y}\right.$$

$$- \frac{2f_o v_\perp c}{v_c^2} \left[\frac{E_x \cos\phi' + E_y \sin\phi'}{B}\right]\right] \quad . \tag{18}$$

Solutions of Eq. (18) may be developed in terms of the perturbation expansion,

$$f_1 = f_1^{(0)} + \left(\frac{v_\perp}{\omega_c}\right) f_1^{(1)} + \left(\frac{v_\perp}{\omega_c}\right)^2 f_1^{(2)} + \ldots \quad , \tag{19}$$

as will be discussed in the next section. The procedure is identical to the one used by Swanson,[5] with the exception of the presence of the extra term proportional to $\partial f_1/\partial y$ in Eq. (18).

Once solutions to the integral equation for the perturbed distribution function have been found, the results may be used to compute the perturbed plasma currents for use in the Maxwell equations governing the wave field. The parabolic approximation may then be applied to determine the wave structure and power deposition profiles within the mode conversion/absorption layer.

III. DERIVATION OF THE EQUIVALENT DIELECTRIC TENSOR

The coupled, linearized Vlasov-Maxwell system self-consistently determines the structure of small amplitude waves propagating within the mode conversion-absorption layer. In order to solve these equations, it is customary to solve for f_1 in terms of the wave electric field, $\bar{E}$, and to use the result to define an equivalent dielectric tensor, $\bar{\bar{K}}$, using Eqs. (8) and (9) in the form:

$$\bar{\bar{E}} + \frac{4\pi i}{\omega} \bar{J} = \bar{\bar{K}}\cdot\bar{E} \ , \tag{20}$$

where $\overline{\overline{K}}$ may include differential operators with respect to x and y. Substitution of Eq. (20) into the linearized Maxwell equations, Eqs. (6) and (7), then yields a pair of coupled partial differential equations for E_x and E_y alone. In this section, the dielectric tensor, $\overline{\overline{K}}$, will be derived by solving for f_1 through second order in the expansion parameter $(v_\perp/\omega_c)$.

Returning to the integral equation for f_1, Eq. (18), since the terms in the integrand which are proportional to $\partial f_1/\partial x$ and $\partial f_1/\partial y$ are already of first order in $(v_\perp/\omega_c)$, only the zeroth- and first-order components of these terms need be retained so that:

$$\frac{\partial f_1}{\partial x} = \frac{\partial f_1^{(0)}}{\partial x} + v_\perp \frac{\partial}{\partial x} \left(\frac{f_1^{(1)}}{\omega_c}\right) , \tag{21}$$

and

$$\frac{\partial f_1}{\partial y} = \frac{\partial f_1^{(0)}}{\partial y} + v_\perp \frac{\partial}{\partial y} \left(\frac{f_1^{(1)}}{\omega_c}\right) . \tag{22}$$

Substitution of the expansion, Eq. (19), into the integral equation, Eq. (18), and using Eqs. (21) and (22), then leads to the following expressions for the zeroth- and first-order components of f_1:

$$f_1^{(0)} = \frac{i f_o q v_\perp}{m v_o^2} \left\{\frac{(E_x - iE_y)\exp(i\phi)}{\omega + \omega_c - kv_z} + \frac{(E_x + iE_y)\exp(-i\phi)}{\omega - \omega_c - kv_z}\right\} , \tag{23}$$

$$f_1^{(1)} = \frac{\omega_c f_o q v_\perp}{2 m v_o^2} \left\{\left[\frac{\exp(2i\phi)}{\omega + 2\omega_c - kv_z} + \frac{1}{\omega - kv_z}\right] \frac{\partial}{\partial x} \left(\frac{E_x - iE_y}{\omega + \omega_c - kv_z}\right)\right.$$

$$+ \left[\frac{1}{\omega - kv_z} + \frac{\exp(-2i\phi)}{\omega - 2\omega_c - kv_z}\right] \frac{\partial}{\partial x} \left(\frac{E_x + iE_y}{\omega - \omega_c - kv_z}\right)$$

$$+ i\left[\frac{1}{\omega - kv_z} - \frac{\exp(2i\phi)}{\omega + 2\omega_c - kv_z}\right] \frac{\partial}{\partial y} \left(\frac{E_x - iE_y}{\omega + \omega_c - kv_z}\right)$$

$$+ i \left[\frac{\exp(-2i\phi)}{\omega - 2\omega_c - kv_z} - \frac{1}{\omega - kv_z}\right] \frac{\partial}{\partial y} \left(\frac{E_x + iE_y}{\omega - \omega_c - kv_z}\right)\} \ . \qquad (24)$$

Since the second-order contributions to f_1 will be small except for the resonant ion terms, in which the denominator $\omega - 2\omega_c - kv_z$ becomes small, only the terms containing the resonant ion contributions will be retained in the second-order component of f_1, leading to

$$f_1^{(2)} = \frac{\omega_c^{\ 2} f_0 q v_\perp}{4imv_0^{\ 2}} \left\{\frac{\exp(-i\phi)}{\omega - \omega_c - kv_z} \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y}\right) + \frac{\exp(-3i\phi)}{\omega - 3\omega_c - kv_z} \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y}\right)\right\}$$

$$\times \left\{\frac{1}{\omega - 2\omega_c - kv_z} \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y}\right)\right\} \left\{\frac{E_x + iE_y}{\omega - \omega_c - kv_z}\right\} \ . \qquad (25)$$

The elements of the equivalent dielectric tensor, $\overline{\overline{K}}$, may now be identified by computing the average perturbed particle velocities, v_x and v_y, and using the results to form the perturbed particle currents, J_x and J_y, and the dielectric tensor, $\overline{\overline{K}}$. The average perturbed particle velocities are obtained by integrating over ϕ, $v_\perp$, and v_z so that

$$\langle v_x \rangle = \int_0^{2\pi} d\phi \int_0^\infty v_\perp \, dv_\perp \int_{-\infty}^\infty dv_z \, f_1 v_\perp \cos\phi \ , \qquad (26)$$

and

$$\langle v_y \rangle = \int_0^{2\pi} d\phi \int_0^\infty v_\perp \, dv_\perp \int_{-\infty}^\infty dv_z \, f_1 v_\perp \sin\phi \ . \qquad (27)$$

Corresponding to the perturbation expansion, Eq. (19), for f_1, the average velocities may also be expanded as:

$$\langle v\rangle = \langle v^{(0)}\rangle + \rho_L \langle v^{(1)}\rangle + \rho_L^2 \langle v^{(2)}\rangle + \ldots \tag{28}$$

with $\rho_L \equiv v_o\omega_{co}$. The zeroth-order components of $\langle v_x\rangle$ and $\langle v_y\rangle$ are given by:

$$\langle v_x^{(0)}\rangle = \frac{-inq}{2mkv_o}\left[(E_x - iE_y)\, Z(\zeta_1) + (E_x + iE_y)\, Z(\zeta_{-1})\right] \quad , \tag{29}$$

$$\langle v_y^{(0)}\rangle = \frac{nq}{2mkv_o}\left[(E_x - iE_y)\, Z(\zeta_1) - (E_x + iE_y)\, Z(\zeta_{-1})\right] \quad , \tag{30}$$

where the plasma dispersion function, $Z(\zeta_n)$, is defined by:

$$Z(\zeta_n) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{dt\, \exp(-t^2)}{t - \zeta_n} \quad , \tag{31}$$

and the argument, ζ_n, is equal to:

$$\zeta_n = \frac{\omega + n\omega_c}{kv_o} \quad . \tag{32}$$

The first-order components of $\langle v_x\rangle$ and $\langle v_y\rangle$ vanish because of the integration over the angle ϕ of orthogonal functions, so

$$\langle v_x^{(1)}\rangle = \langle v_y^{(1)}\rangle = 0 \quad . \tag{33}$$

In evaluating the second-order terms, the contribution of kv_z to the nonresonant denominator, $\omega - \omega_c - kv_z$, will be neglected, since $\omega - \omega_c >> kv_z$ and this term does not contribute to the absorption processes. Furthermore, using the variation in the equilibrium magnetic field such that $\omega_c = \omega_{co}(1 + x/L)$ and $\omega = 2\omega_{co}$, so that $\zeta_{-2} = -\omega x/kLv_o$, the second-order components of $\langle v_x\rangle$ and $\langle v_y\rangle$ may be written as:

$$\langle v_x^{(2)}\rangle = \frac{inq}{4mkv_o}\ \{Z(\zeta_{-2})\ [(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\ (E_x + iE_y) + 2\ \frac{\partial}{\partial x}\ (\frac{E_x + iE_y}{L})$$

$$- (\frac{\omega}{kv_o})\ Z'(\zeta_{-2})\ [(\frac{\partial}{\partial x} + i\ \frac{\partial}{\partial y})\ (\frac{E_x + iE_y}{L}) + (\frac{E_x + iE_y}{L^2})]\} \ , \qquad (34)$$

and

$$\langle v_y^{(2)}\rangle = -i\ \langle v_x^{(2)}\rangle \quad . \qquad (35)$$

Examination of the type of terms present in $\langle v_x\rangle$ and $\langle v_y\rangle$ indicates that the product, $\bar{\bar{K}}\cdot\bar{E}$, may be written as

$$E_x + \frac{4\pi i}{\omega}\ J_x = K_{xxo}\ E_x + K_{xyo}\ E_y + K_{xx1}\ \frac{\partial E_x}{\partial x} + K_{xy1}\ \frac{\partial E_y}{\partial x}$$

$$+ K_{xx2}\ \frac{\partial^2 E_x}{\partial x^2} + K_{xy2}\ \frac{\partial^2 E_y}{\partial x^2} + M_{xx1}\ \frac{\partial E_x}{\partial y}$$

$$+ M_{xy1}\ \frac{\partial E_y}{\partial y} + M_{xx2}\ \frac{\partial^2 E_y}{\partial y^2} + M_{xy2}\ \frac{\partial^2 E_y}{\partial y^2} \qquad (36)$$

and

$$E_y + \frac{4\pi i}{\omega}\ J_y = K_{yxo}\ E_x + K_{yyo}\ E_y + K_{yx1}\ \frac{\partial E_x}{\partial x} + K_{yy1}\ \frac{\partial E_y}{\partial x} + K_{yx2}\ \frac{\partial^2 E_x}{\partial x^2}$$

$$+ K_{yy2}\ \frac{\partial^2 E_y}{\partial x^2} + M_{yx1}\ \frac{\partial E_x}{\partial y} + M_{yy1}\ \frac{\partial E_y}{\partial y} + M_{yx2}\ \frac{\partial^2 E_x}{\partial y^2}$$

$$+ M_{yy2}\ \frac{\partial^2 E_y}{\partial y^2} \quad . \qquad (37)$$

Electrons contribute only to K_{xxo} and K_{xyo}, by virtue of the expansion of f_1 in powers of $(v_\perp/\omega_c)$ and because $\omega \sim 2\,\omega_{ci} << \omega_{ce}$, so there are no resonant contributions from electrons to the higher order terms in the expansion of $<v_x>$ and $<v_y>$. Finally, performing the sum over electrons and ions, as indicated in Eqs. (8)-(9), and noting that the large argument expansion for $Z(\zeta_1)$ and $Z(\zeta_{-1})$ may be used, the elements of the equivalent dielectric tensor, as denoted in Eqs. (36)-(37) may be written as follows:

$$K_{xxo} = K_{yyo} = 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega^2} + \frac{\omega_{pi}^2 \rho_L^2 F'}{4\omega^2 X^2} \quad , \tag{38}$$

$$K_{xyo} = -K_{yxo} = i\,\frac{\omega}{\omega_{ci}}\,\frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega^2} + i\,\frac{\omega_{pi}^2 \rho_L^2 F'}{4\omega^2 X^2} \quad , \tag{39}$$

$$K_{xx1} = K_{yy1} = \frac{\omega_{pi}^2 \rho_L^2 L F'}{4\omega^2 X^2} = \frac{\partial K_{xx2}}{\partial x} \quad , \tag{40}$$

$$K_{xy1} = -K_{yx1} = i\,K_{xx1} \quad , \tag{41}$$

$$K_{xx2} = K_{yy2} = \frac{-\omega_{pi}^2 \rho_L^2 L F}{4\omega^2 X} \quad , \tag{42}$$

$$K_{xy2} = -\,K_{yx2} = i\,K_{xx2} \quad , \tag{43}$$

$$M_{xx1} = M_{yy1} = -i\,M_{xy1} = i\,M_{yx1} = K_{xy1} \quad , \tag{44}$$

$$M_{xx2} = M_{yy2} = K_{xx2} \quad , \tag{45}$$

$$M_{xy2} = -M_{yx2} = iM_{xx2} \quad , \tag{46}$$

where

$$F = -\zeta_{-2}\, Z(\zeta_{-2}) \ , \tag{47}$$

and

$$F' = \zeta_{-2}^{\ 2}\, Z'(\zeta_{-2}) \ . \tag{48}$$

The elements as defined in Eqs. (38-43) are identical to those found by Swanson[5] for a plane wave incident normally onto the mode conversion-absorption layer. The additional elements defined in Eqs. (44-46) arise because of the finite transverse extent assumed for the incident wavefront. However, as will be discussed in the next section, these latter elements have a negligible effect on the wave structure for wavefronts which satisfy the paraxial propagation limit.

IV. APPLICATION OF THE PARABOLIC APPROXIMATION METHOD TO THE WAVE EQUATION

The structure of a wavefront as it propagates through the mode conversion absorption layer is governed by the Maxwell equations, Eqs. (6) and (7), combined with the equivalent dielectric tensor, derived in the preceding section, which describes the response of the plasma to the applied wave fields. Using the notation specified in Eqs. (36)-(48), the Maxwell equations may be rewritten in the form:

$$\left[\gamma_1 - L_1 - \frac{\partial^2}{\partial y^2} - L_2\right]E_x + \left[\frac{\partial^2}{\partial x \partial y} - i\gamma_2 - iL_1 - iL_2\right]E_y = 0 \tag{49}$$

and

$$\left[\frac{\partial^2}{\partial x \partial y} + i\gamma_2 + iL_1 + iL_2\right]E_x + \left[\gamma_1 - \frac{\partial^2}{\partial x^2} - L_1 - L_2\right]E_y = 0 \qquad (50)$$

where

$$\gamma_1 = k^2 - \frac{\omega^2}{c^2} K_{xx0} \quad , \qquad (51)$$

$$\gamma_2 = -i\frac{\omega^2}{c^2} K_{xyo} \quad , \qquad (52)$$

$$L_1 = \frac{\omega^2}{c^2} K_{xx1} \frac{\partial}{\partial x} + \frac{\omega^2}{c^2} K_{xx2} \frac{\partial^2}{\partial x^2} = \frac{\omega^2}{c^2} \frac{\partial}{\partial x} K_{xx2} \frac{\partial}{\partial x} \, , \qquad (53)$$

and

$$L_2 = i\frac{\omega^2}{c^2} K_{xx1} \frac{\partial}{\partial y} + \frac{\omega^2}{c^2} K_{xx2} \frac{\partial^2}{\partial y^2} \quad . \qquad (54)$$

In the single mode limit, in which $\partial E/\partial y \to k_y E$, these equations reduce to those derived previously by Colestock and Kashuba,[6] when only the second harmonic heating terms are considered.

For incident wavefronts in which the transverse variations are weaker than the variations in the direction of propagation, the paraxial propagation constraint, that is, $k_y \sim 1/E\ (\partial E/\partial y) \ll k_x \sim 1/E\ (\partial E/\partial x)$, may be used to solve Eqs. (49) and (50) iteratively for E_x and E_y. The procedure is more complicated than in the cold plasma limit[1] because of the presence of the finite Larmor radius terms, L_1 and L_2. In the expansion procedures to follow, all terms through order $(k_x\rho_L)^2$ or $(k_y/k_x)^2$ will be retained, while higher order terms, involving $(k_y/k_x)\ (1/L\ k_x)$, $(k_y\rho_L)^2$, etc. will be neglected.

Hence, because the operator L_2 involves products of two of the small expansion parameters, $(k_x\rho_L)^2$ and (k_y/k_x), it may be neglected immediately, thereby reducing Eqs. (49) and (50) to:

$$[\gamma_1 - L_1 - \frac{\partial^2}{\partial y^2}]E_x + [\frac{\partial^2}{\partial x \partial y} - i\gamma_2 - iL_1]E_y = 0 \qquad (55)$$

and

$$[\frac{\partial^2}{\partial x \partial y} + i\gamma_2 + iL_1]E_x + [\gamma_1 - \frac{\partial^2}{\partial x^2} - L_1]E_y = 0 \quad . \qquad (56)$$

Turning to Eq. (55), since $\partial^2 E_x/\partial y^2$ is small in comparison to the remaining terms, it is desirable to eliminate it in favor of an expression involving E_y alone. This is accomplished by operating on Eq. (55) with $\partial^2/\partial y^2$ and finding, to lowest order in the expansion parameters, that:

$$\frac{\partial^2 E_x}{\partial y^2} = \frac{i\gamma_2}{\gamma_1}\frac{\partial^2 E_y}{\partial y^2} \quad . \qquad (57)$$

Substitution of this expression back into Eq. (55) reduces Eq. (55) to the form:

$$[\gamma_1 - L_1]E_x = [\frac{i\gamma_2}{\gamma_1}\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x \partial y} + i(\gamma_2 + L_1)]E_y \quad . \qquad (58)$$

The finite Larmor radius term, $L_1 E_x$, may be eliminated from Eq. (58) by combining Eq. (56) with Eq. (58) multiplied by i, leading to:

$$[i(\gamma_1 + \gamma_2) + \frac{\partial^2}{\partial x \partial y}]E_x = -[\frac{\gamma_2}{\gamma_1}\frac{\partial^2}{\partial y^2} + i\,\frac{\partial^2}{\partial x \partial y} + (\gamma_1 + \gamma_2) - \frac{\partial^2}{\partial x^2}]E_y \quad . \qquad (59)$$

In the limit $\partial E/\partial y \to 0$, which is treated by Swanson,[5] Eq. (59) could be solved algebraically for E_x in terms of E_y and the resulting expression could then be used in Eq. (56) to develop the mode conversion-tunneling equation. Here, the paraxial propagation constraint will be used to reduce Eq. (59) to an algebraic equation for E_x in terms of E_y which includes corrections up to order $(k_y/k_x)^2$.

The first step in this process is to apply the operator $[i(\gamma_1 + \gamma_2) - \partial^2/\partial x\partial y]$ to Eq. (59). This reduces Eq. (59) to an algebraic relationship for E_x in terms of E_y, given as:

$$E_x = i\left[1 - \frac{1}{\gamma_1+\gamma_2}\frac{\partial^2}{\partial x^2}\right]E_y - \frac{2}{\gamma_1+\gamma_2}\frac{\partial^2 E_y}{\partial x\partial y} + \frac{i\;\gamma_2/\gamma_1}{\gamma_1+\gamma_2}\frac{\partial^2 E_y}{\partial y^2}$$
$$- \frac{1}{(\gamma_1+\gamma_2)^2}\frac{\partial \ln(\gamma_1+\gamma_2)}{\partial x}\frac{\partial^3 E_y}{\partial x^2\partial y} + \frac{1}{(\gamma_1+\gamma_2)^2}\frac{\partial^4 E_y}{\partial x^3\partial y} - \frac{i}{\gamma_1(\gamma_1+\gamma_2)}\frac{\partial^4 E_y}{\partial x^2\partial y^2} \quad , \qquad (60)$$

where Eq. (57) has been used and higher order terms have been discarded. The first term in the square brackets represents the contribution of a plane wave propagating at normal incidence through the layer. The remaining terms represent the diffractive modifications which arise due to the oblique incidence and finite size of the incident wavefront. Because the final three terms are each of order (k_y/k_x) or higher relative to the dominant contribution, the cold plasma wave equation in the limit of normal incidence,

$$\frac{\partial^2 E_y}{\partial x^2} = \left[\frac{\gamma_1^2 - \gamma_2^2}{\gamma_1}\right]E_y \quad , \qquad (61)$$

may be used to reduce the order of the derivatives in the terms. In particular, the fourth term is reduced as:

$$\frac{-1}{(\gamma_1+\gamma_2)^2}\,\frac{\partial \ln(\gamma_1+\gamma_2)}{\partial x}\,\frac{\partial}{\partial y}\left(\frac{\partial^2 E_y}{\partial x^2}\right) \approx -\,\frac{(\gamma_1-\gamma_2)}{\gamma_1(\gamma_1+\gamma_2)}\,\frac{\partial \ln(\gamma_1+\gamma_2)}{\partial x}\,\frac{\partial E_y}{\partial y}\,, \tag{62}$$

the fifth term may be approximated as:

$$\frac{1}{(\gamma_1+\gamma_2)^2}\,\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial^2 E_y}{\partial x^2}\right) \approx \frac{(\gamma_1-\gamma_2)}{\gamma_1(\gamma_1+\gamma_2)}\left[\frac{\partial^2 E_y}{\partial x\partial y} + \frac{\partial \ln[(\gamma_1^2-\gamma_2^2)/\gamma_1]}{\partial x}\,\frac{\partial E_y}{\partial y}\right], \tag{63}$$

whereas the last term may be simplified as:

$$\frac{-i}{\gamma_1(\gamma_1+\gamma_2)}\,\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 E_y}{\partial x^2}\right) \approx \frac{i\,(\gamma_2-\gamma_1)}{\gamma_1^{\,2}}\,\frac{\partial^2 E_y}{\partial y^2}\;. \tag{64}$$

Using Eqs. (62-64), the algebraic equation for E_x may finally be reduced to:

$$E_x = i\left[1 - \frac{1}{\gamma_1+\gamma_2}\,\frac{\partial^2}{\partial x^2}\right]E_y - \frac{1}{\gamma_1}\,\frac{\partial^2 E_y}{\partial x\partial y} + i\left[\frac{\gamma_1\gamma_2+\gamma_2^2-\gamma_1^2}{\gamma_1^2\,(\gamma_1+\gamma_2)}\right]\frac{\partial^2 E_y}{\partial y^2}$$

$$-\left(\frac{1}{\gamma_1+\gamma_2}\right)\left[\frac{\partial}{\partial x}\,\frac{\gamma_2}{\gamma_1}\right]\frac{\partial E_y}{\partial y}\,. \tag{65}$$

The appropriate mode conversion-tunneling equation may now be derived by substituting Eq. (65) into Eq. (56), leading to:

$$\frac{1}{\gamma_1}\,L_1\,\frac{\partial^2 E_y}{\partial x^2} - 2\left(\frac{\gamma_1+\gamma_2}{\gamma_1}\right)L_1 E_y - \frac{\partial^2 E_y}{\partial x^2} - \left[\frac{\gamma_2^{\,2}-\gamma_1^{\,2}}{\gamma_1}\right]E_y - \frac{\partial^2 E_y}{\partial y^2}$$

$$+\, i\left[\frac{\partial}{\partial x}\,\frac{\gamma_2}{\gamma_1}\right]\frac{\partial E_y}{\partial y} = 0\;. \tag{66}$$

This equation reduces to the mode conversion-tunneling equation considered by Swanson[5] in the limit $\partial E_y/\partial y \to 0$. Furthermore, it reduces to the appropriate cold plasma equations, with and without the y-derivative terms, when $k_x\rho_L \to 0$.

The final step of the parabolic approximation involves the separation of the rapidly varying wavelike component, $u(x)$, of E_y from the more slowly varying amplitude modulation, $a(x,y)$, which contains the diffractive effects associated with the finite transverse extent of the wavefront. This is accomplished by substituting the expression,

$$E_y(x,y) = a(x,y)\; u(x) \quad , \tag{67}$$

into Eq. (66) and factoring the resulting terms appropriately. Neglecting the term $\partial^2 a/\partial x^2$ in comparison to $\partial^2 u/\partial x^2$, Eq. (66) may be factored in the form:

$$\begin{aligned} a\; &\Big\{\frac{1}{\gamma_1} L_1 \frac{\partial^2 u}{\partial x^2} - 2\left(\frac{\gamma_1+\gamma_2}{\gamma_1}\right) L_1 u - \frac{\partial^2 u}{\partial x^2} - k_\perp^2\, u\Big\} \\ &+ u\; \Big\{ i\left(\frac{\partial}{\partial x}\frac{\gamma_2}{\gamma_1}\right)\frac{\partial a}{\partial y} - \frac{\partial^2 a}{\partial y^2} + \frac{3}{\gamma_1}\frac{1}{u} L_1 \frac{\partial u}{\partial x} - 2\left(\frac{\gamma_1+\gamma_2}{\gamma_1}\right)\Big[\Big(\frac{\omega^2}{c^2} K_{xx1} \\ &+ 2\,\frac{\omega^2}{c^2} K_{xx2}\,\frac{1}{u}\frac{\partial u}{\partial x}\Big) - \frac{2}{u}\frac{\partial u}{\partial x}\Big]\frac{\partial a}{\partial x}\Big\} = 0 \;, \end{aligned} \tag{68}$$

where

$$k_\perp^{\,2} = \frac{\gamma_2^{\,2} - \gamma_1^{\,2}}{\gamma_1} \quad . \tag{69}$$

The dominant wavelike component of $E_y(x,y)$ is determined by equating the terms enclosed by the first set of brackets in Eq. (68) to zero, yielding

$$\frac{1}{\gamma_1} L_1 \frac{\partial^2 u}{\partial x^2} - 2\left(\frac{\gamma_1 + \gamma_2}{\gamma_1}\right) L_1 u - \frac{\partial^2 u}{\partial x^2} - k_\perp^{\,2} u = 0 \quad . \tag{70}$$

The resulting equation is identical to the mode conversion-tunneling equation derived by Swanson,[5] appropriate for a plane wave incident normally onto the resonance layer. Equating the terms enclosed by the second pair of brackets in Eq. (68) to zero reveals that the amplitude modulation function, a(x,y), is governed by a parabolic diffusion-type equation which may be written in the form:

$$g(x)\,\frac{\partial a}{\partial x} + ih(x)\,\frac{\partial a}{\partial y} - \frac{\partial^2 a}{\partial y^2} = 0. \tag{71}$$

The general solution for a(x,y) is easily obtained using the method of separation of variables, yielding the result:

$$a(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dm\; a(x_o,y')\exp[im(y-y')]\exp[-m^2F_2(x)]\exp[mF_1(x)], \tag{72}$$

where

$$F_1(x) = \int_{x_o}^{x} \frac{dx'\,h(x')}{g(x')}\;, \tag{73}$$

$$F_2(x) = \int_{x_o}^{x} \frac{dx'}{g(x')}\;, \tag{74}$$

and $a(x_o,y')$ specifies the transverse structure of the wavefront on the plane $x = x_o$. In the particular case of an incident Gaussian-shaped wavefront at $x = x_o$, the amplitude function for $x \geq x_o$ reduces to:

$$a(x,y) = \frac{A_o b}{[b^2 + 4F_2(x)]^{1/2}} \exp\{-\frac{[y-iF_1(x)]^2}{[b^2 + 4F_2(x)]}\}, \tag{75}$$

where the effective width and amplitude of the Gaussian depend on $F_2(x)$.

The two-dimensional structure of the wavefront as it propagates through the mode conversion absorption region is completely specified in the paraxial propagation limit by Eqs. (66), (67), (70), and (72-74). In this limit, the dominant wave structure, u(x), in the primary direction of propagation is governed by the linear fourth-order mode conversion-tunneling equation, appropriate for a plane wave incident normally onto the kinetic layer. Diffraction effects on the transverse structure are determined by a parabolic diffusion-type equation, in which the coefficients depend on u(x) and on equilibrium quantities. The wave fields determined in this manner may now be utilized to construct the two-dimensional power deposition profile, as will be shown in the next section.

V. POWER CONSERVATION EQUATION

Within the kinetic layer, the applied ICRF waves resonantly interact with the particles, yielding a net transfer of power from the wave fields to the plasma. The two-dimensional form of the local power conservation equation can be constructed directly from the linearized Vlasov-Maxwell equations. The resulting equation is a generalization of earlier formulations[6,7,10,15] derived for wave components characterized by a single value of k_y.

Real power flow within the layer is governed by a generalized Poynting's theorem in the form:

$$\text{Real } \nabla\cdot\bar{S} = -\frac{1}{2}\text{ Real }\left(\bar{E}\cdot\bar{J}^*\right) , \tag{76}$$

where

$$\vec{S} = \frac{c}{8\pi} \left(\bar{E}\times\bar{B}^*\right) \tag{77}$$

is the complex Poynting vector, $\bar{J} = \bar{\bar{\sigma}}\cdot\bar{E}$, and the conductivity, $\bar{\bar{\sigma}}$, is related to the effective dielectric tensor, $\bar{\bar{K}}$, through Eq. (20).

The left-hand side of Eq. (76) represents real power flow into the kinetic layer, carried by the incident waves. In general, the right-hand side of Eq. (76) consists of two parts:

$$Q = \frac{-1}{2}\,\mathrm{Real}\,\left(\bar{E}\cdot J^*\right) = -\mathrm{Real}\ P(x,y) - \mathrm{Real}\ \nabla\cdot\bar{T}(x,y)\ , \tag{78}$$

where $P(x,y)$ is the local power deposition and $\bar{T}(x,y)$ is the kinetic flux associated with the coherent motion of particles in the wave fields. The separation of Q into the local power deposition and a kinetic flux is motivated by earlier studies.[6,7,10,15] To derive the appropriate expression for the local power deposition, $P(x,y)$, one may first calculate the left-hand side of Eq. (78) and then extract from it those components, identified as the kinetic flux, which can be written in divergence form. The remaining components then correspond to the local power deposition, which must vanish when the dissipative terms vanish. In essence, this procedure should be the reverse of the procedure used in Ref. 15 and is equivalent to that followed in Ref. 6.

Using Eq. (20) in Eq. (76), Q can be written in terms of the effective dielectric tensor as:

$$Q = \frac{-i\omega}{16\pi}\,\tilde{Q} = \frac{-i\omega}{16\pi}\left[\bar{E}\cdot\bar{\bar{K}}^*\cdot E^{-*} - \bar{E}^*\cdot\bar{\bar{K}}\cdot\bar{E}\right]\ , \tag{79}$$

where $*$ denotes the complex conjugate of the term. For the case considered in this report, in which equilibrium quantities are independent of the vertical coordinate, y, but the wave fields may depend on y, $\bar{E}\cdot\bar{\bar{K}}^*\cdot\bar{E}^*$ is given by

$$\bar{E}\cdot\bar{\bar{K}}^*\cdot\bar{E} = \bar{E}\cdot\bar{\bar{K}}_0^*\cdot\bar{E}^* + \bar{E}\cdot\bar{\bar{K}}_1^*\cdot \frac{\partial\bar{E}^*}{\partial x} + \bar{E}\cdot\bar{\bar{K}}_2^*\cdot \frac{\partial^2\bar{E}^*}{\partial x^2}$$

$$+ \bar{E}\cdot\bar{\bar{M}}_1^*\cdot \frac{\partial\bar{E}^*}{\partial y} + \bar{E}\cdot\bar{\bar{M}}_2^*\cdot \frac{\partial^2\bar{E}^*}{\partial y^2} , \qquad (80)$$

where $\bar{\bar{K}}_0$, $\bar{\bar{K}}_1$, $\bar{\bar{K}}_2$, $\bar{\bar{M}}_1$, and $\bar{\bar{M}}_2$ are defined in Eqs. (38-48). To proceed with the construction of an expression for the local power deposition, P(x,y), two relationships are useful. From Lagrange's identity for linear operators,[6,16] one can show that for any two vectors $\bar{u},\bar{v}$ of dimension n and an n X n matrix A

$$\bar{v}^*\cdot\left(\bar{\bar{A}}\,\frac{\partial}{\partial x}\right)\cdot\bar{u} - \bar{u}\cdot\left[\left(\bar{\bar{A}}\,\frac{\partial}{\partial x}\right)^+\cdot\bar{v}\right]^* = \frac{\partial}{\partial x}\left(\bar{v}^*\cdot\bar{\bar{A}}\cdot\bar{u}\right) , \qquad (81)$$

where the superscript (+) denotes the adjoint of the quantity. When no linear operators are involved, the following identity is also valid

$$\bar{u}\cdot\bar{\bar{A}}^*\cdot\bar{v}^* = \bar{v}^*\cdot\bar{\bar{A}}^+\cdot\bar{u} . \qquad (82)$$

Using Eqs. (80-84) and Eqs. (38-48), the quantity $\tilde{Q}$ can be written explicitly in terms of the effective dielectric tensor, $\bar{\bar{K}}$, as

$$\tilde{Q} = \frac{\partial}{\partial x}\left\{\left[\frac{\partial\bar{E}^*}{\partial x}\cdot\bar{\bar{K}}_2^+\cdot\bar{E} - \bar{E}^*\cdot\bar{\bar{K}}_2\cdot\frac{\partial\bar{E}}{\partial x}\right] - i\left[\frac{\partial\bar{E}^*}{\partial y}\cdot\bar{\bar{K}}_2^+\cdot\bar{E} + \bar{E}^*\cdot\bar{\bar{K}}_2\cdot\frac{\partial\bar{E}}{\partial y}\right]\right\}$$

$$+ \frac{\partial}{\partial y}\left\{\left[\frac{\partial\bar{E}^*}{\partial y}\cdot\bar{\bar{K}}_2^+\cdot\bar{E} - \bar{E}^*\cdot\bar{\bar{K}}_2\cdot\frac{\partial\bar{E}}{\partial y}\right] + i\left[\frac{\partial\bar{E}^*}{\partial x}\cdot\bar{\bar{K}}_2^+\cdot\bar{E} + \bar{E}^*\cdot\bar{\bar{K}}_2\cdot\frac{\partial\bar{E}}{\partial x}\right]\right\}$$

$$+ \bar{E}^* \cdot [\bar{\bar{K}}_o^+ - \bar{\bar{K}}_o] \cdot \bar{E} + \frac{\partial \bar{E}^*}{\partial x} \cdot [\bar{\bar{K}}_2^+ - \bar{\bar{K}}_2] \cdot \frac{\partial \bar{E}}{\partial x} + \frac{\partial \bar{E}^*}{\partial y} \cdot [\bar{\bar{K}}_2 - \bar{\bar{K}}_2^+] \cdot \frac{\partial \bar{E}}{\partial y}$$

$$- i \frac{\partial \bar{E}}{\partial x} \cdot [\bar{\bar{K}}_2^+ - \bar{\bar{K}}_2] \cdot \frac{\partial \bar{E}}{\partial y} + i \frac{\partial \bar{E}^*}{\partial y} \cdot [\bar{\bar{K}}_2^+ - \bar{\bar{K}}_2] \cdot \frac{\partial \bar{E}}{\partial x} \quad . \tag{83}$$

The first two lines contain the generalized form of the kinetic flux, $\bar{T}$, while the remaining terms comprise the two-dimensional form of the localized power deposition. When no dissipation is present in the system, so that the $\bar{\bar{K}}_i$'s are all Hermitian, the terms corresponding to the local power deposition vanish identically. In the limit that the y-dependence of the electric field depends on only a single value of k_y, the power conservation equation reduces in form to that derived earlier by Colestock and Kashuba.[6] The power conservation equation derived here is valid for a wavefront with a finite transverse extent which is incident onto a mode conversion - tunneling layer characterized to lowest order by one-dimensional equilibrium inhomogeneities. Two-dimensional power deposition profiles can be constructed using the exact field solutions obtained numerically from Eqs. (49) and (50) or the parabolic field solutions, obtained semi-analytically from Eqs. (65-74).

VI. CONCLUSIONS

In this paper, the wave equation which governs ICRF wave propagation, absorption, and mode conversion within the kinetic layer in tokamaks has been extended to include diffraction and focussing effects associated with the finite transverse dimensions of the incident wavefronts. Though the equilibrium within the kinetic layer has been assumed to vary only with the major radius, the incident wavefronts have a finite structure, transverse to the direction of propagation through the layer, which is caused by focussing related to the launcher geometry and the refractive properties of the medium. A two-dimensional energy conservation equation, including explicit forms for the local power deposition and the kinetic flux vector, has been derived which generalizes earlier results[6] that were appropriate for obliquely incident plane waves characterized by a single value of k_y.

Using the parabolic approximation method, a generalized two-dimensional form of the mode conversion-tunneling equation for the vertical component of the wave electric field, E_y, has been extracted from the wave equation. Solutions to this equation may be obtained using the ansatz $E_y(x,y) = a(x,y)\, u(x)$, where $a(x,y)$ is a slowly varying amplitude function and $u(x)$ is a rapidly varying waveform. The waveform, $u(x)$, is determined by the usual fourth-order mode conversion-tunneling equation for plane waves which are incident normally onto the kinetic layer, while the amplitude function, $a(x,y)$, is determined by a second order parabolic diffusion-type equation whose coefficients depend on equilibrium quantities and on $u(x)$. Work is currently underway to incorporate this method into an existing cold plasma code which solves for the wave propagation between the launcher and the kinetic layer in tokamak geometry.[1] Previous methods of constructing the two-dimensional structure of the wavefronts and the power deposition within the kinetic layer have proposed to utilize a Fourier decomposition of the wavefront in the vertical direction and, subsequently, solve a corresponding fourth-order equation for the amplitude for each of the harmonics present in the wave. The method described in this report is numerically more efficient since only one set of solutions to the fourth-order equation and the associated second-order equation needs to be generated for each pass of the wavefront through the kinetic layer.

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