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RADON GENERATION, ADSORPTION, ABSORPTION, AND TRANSPORT IN POROUS MEDIA

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ABSTRACT

A unified model of radon emanation, adsorption, absorption, and transport has been developed and incorporated into the RAETRAN code that uses new mathematical models of advective transport, moisture effects, and radon emanation. The model accounts for advective depletion in radon source regions, and for the effects of varying moistures on radon emanation, diffusion, absorption, adsorption, and advective transport rates. Radon transport in gas- and water-filled pore space is characterized, and exchange between the phases is considered. Correlations are also given for diffusion and permeability coefficients. The model provides a comprehensive assessment of source potentials for indoor radon accumulation based on soil moistures, radium, emanation, and advection of soil gas.

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1. INTRODUCTION

Radon emanation and transport in soils is illustrated schematically in Figure 1-1. Radon is emanated from radium-bearing minerals into the soil pore space, followed by diffusive and advective transport of the radon gas in both liquid and gas phases into a home, entering via cracks, sumps, porous building materials, water, and other routes. Advection becomes dominant with proximity to many home entry points, and exchange between air and water phases is continual.

Rogers and Associates Engineering Corporation (RAE) is performing a project for the Department of Energy to coordinate the eight key components of radon emanation and transport shown in Figure 1-1 into a unified, simplified model.

Previous RAE theoretical developments on radon emanation, diffusion and advection in porous soils include a radon emanation computer model (1), diffusion coefficient model (2,3), and radon diffusion equations with analytical solutions for two-phase diffusion through porous materials (4). Although the two-phase diffusion model was restrictive and only treated the radon interactions in the water-air pore spaces, it has been used extensively in predicting radon migration through earthen materials (5,6) and for reducing diffusion and emanation coefficient data from laboratory measurements (7,8). The model was also extended to include both constant and cyclical advection (9).

In addition to the theoretical developments, RAE has developed several experimental procedures for determining the values of key input parameters (7,8) and has amassed an extensive comprehensive data base for a wide range of soils. This data base has been augmented and used extensively in the work associated with this project.

Theoretical developments for radon transport are presented in the next section, followed by the results of efforts to predict appropriate values for the diffusion, permeability, and emanation coefficients used in the model.

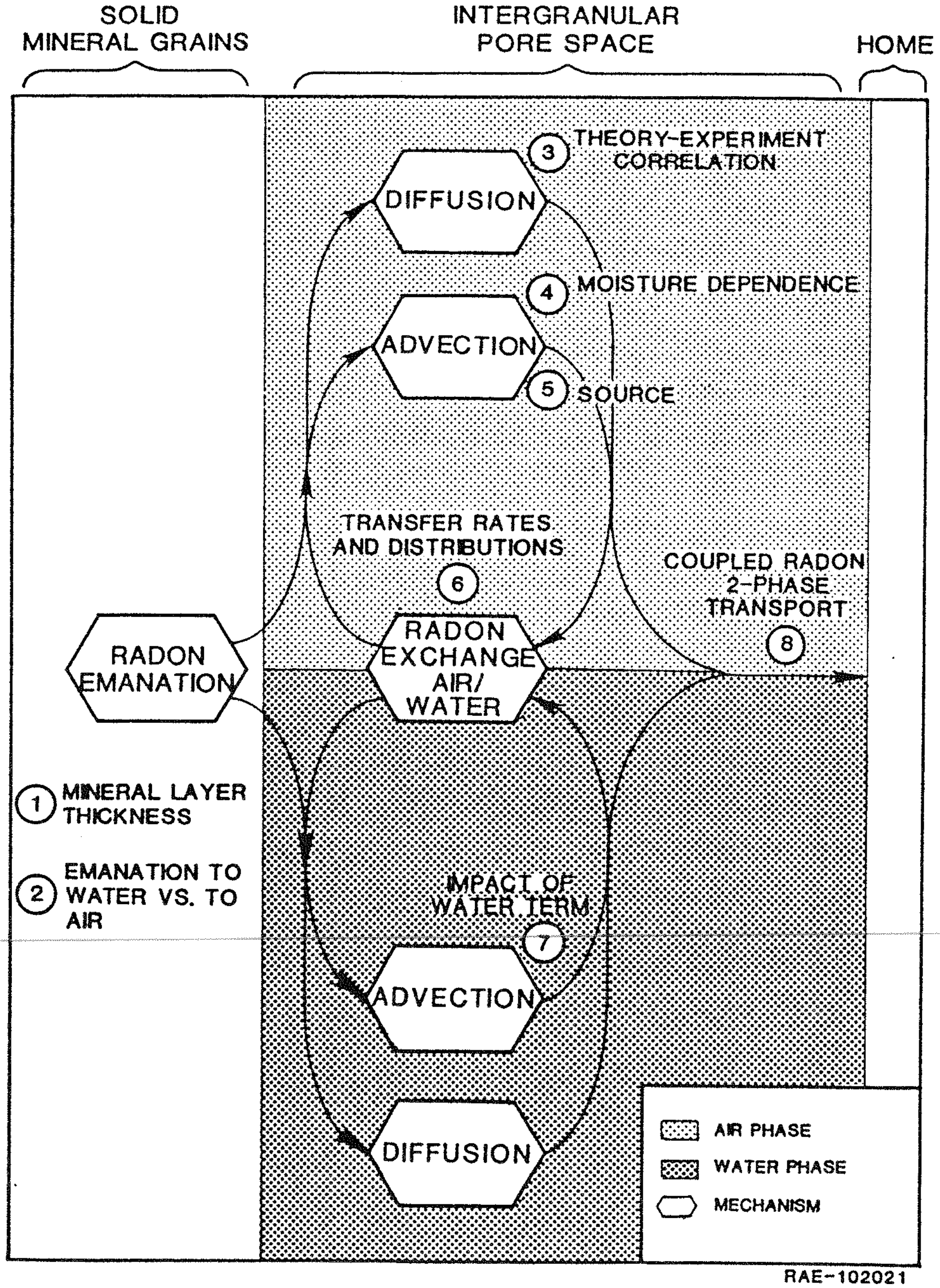

FIGURE 1-1. PHYSICAL MECHANISMS OF RADON GENERATION AND TRANSPORT.

2. RADON EMANATION, ADSORPTION, AND TRANSPORT MODELING IN A TWO-PHASE MEDIUM

Radon emanation, adsorption, and transport through a two-phase medium in the pore spaces of earthen materials involves several complex processes. There is advection and diffusion through the air in the presence of a water layer. As shown in Figure 2-1, there is radon diffusion through the water in water-blocked pore regions, there is radium and radon absorption in the water, and radon adsorption on some solid subsurfaces, as well as other mechanisms. The general objective of the present effort is to characterize these mechanisms which occur on a microscopic scale by a simple advection, diffusion-type equation which can be used to describe these processes as if occurring through a simple homogeneous medium characteristic of the porous material.

It has been shown previously that radon diffusion through a porous material with moisture can be characterized by the standard diffusion equation in which the radon concentration is a pore average, and the emanation and diffusion coefficients are also appropriate pore averages (4). In order to examine the distinct mechanisms in more detail, rate balances have been constructed separately for the radon in the gas and liquid components of the pore space. These coupled equations, for the one-dimensional, steady-state condition, are:

$$D_a \frac{d^2C_a}{dx^2} - \lambda C_a - \frac{k_a\rho\lambda}{(1-m)} C_a + \frac{K}{p(1-m)}\left(\frac{dP}{dx}\right)C_a + \frac{R\rho\lambda E_{air}}{p(1-m)} - R\frac{m\lambda}{(1-m)k_d} + T_{wa} = 0 \tag{2-1}$$

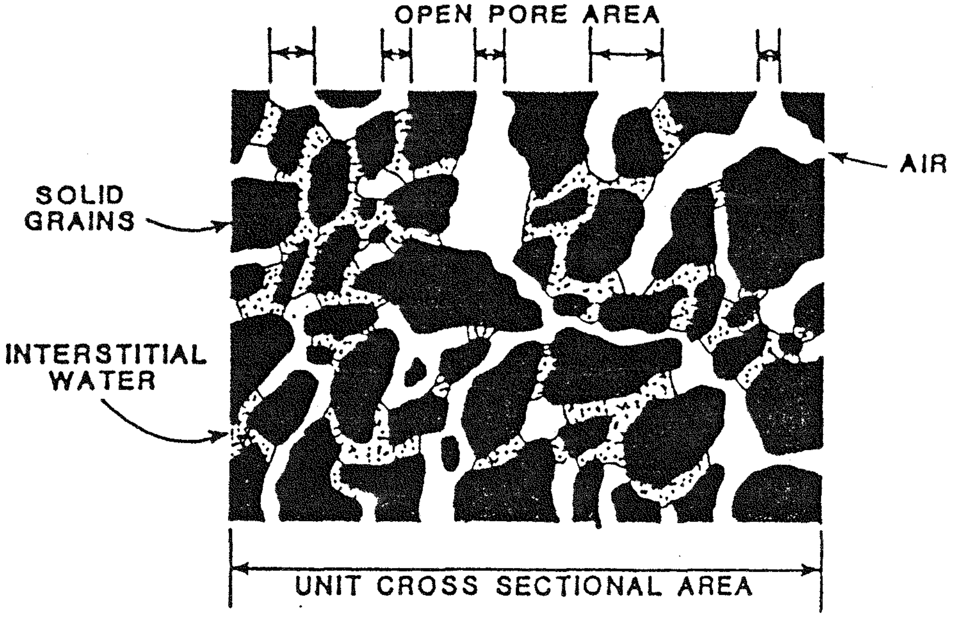

FIGURE 2-1. VOLUME OF UNSATURATED POROUS MATERIAL. RADIUM IS IN THE SOIL GRAINS AND INTERSTITIAL WATER.

$$D_w \frac{d^2C_w}{dx^2} - \lambda C_w - \frac{K_w \rho \lambda}{m} C_w + \frac{R \rho \lambda E_{water}}{pm}$$

$$- R\lambda \frac{(1-p)}{k_d} + T_{aw} = 0 \qquad (2\text{-}2)$$

where

D_a = radon diffusion coefficient in air, including the tortuosity factor

D_w = radon diffusion coefficient in water

C_a = radon concentration in the air-filled pore space

C_w = radon concentration in the water-filled pore space

λ = radon decay constant

k_a = air-surface adsorption coefficient for radon

R = radium concentration in the solid matrix

k_w = water-surface adsorption coefficient for radon

ρ = bulk dry density

p = total porosity

m = fraction of moisture saturation

E_{air} = component of emanation coefficient that is a direct pore air source of radon

E_{water} = component of emanation coefficient that is a direct pore water source of radon

k_d = equilibrium distribution coefficient for radium in solid-to-pore-liquid

T_{wa} = transfer factor of radon from pore liquid to pore air

T_{aw} = transfer factor of radon from pore air to pore liquid

K = pore gas permeability

P = pore gas pressure

For Equation (2-1), the first term accounts for diffusion; the second term accounts for radon decay in the gas-filled pore space. The third term is radon adsorption onto a solid surface from an air-solid interface; the fourth term is radon advection. The fifth term is radon emanation into the air-filled pore space and the sixth term is a correction to the emanation from radon production due to the radium dissolved in the pore water. The last term represents the transfer of radon from water to air in the pore. The sixth term in Equation (2-1) accounts for the present method that radium concentration measurements are made. If only R in the solid material is measured, then the sixth term is multiplied by

$$\frac{\rho E_{air}}{p} - 1 .$$

The Equation (2-2) terms are similar to those for Equation (2-1) except there is no advection term.

It follows, for negligible radon decay in the pore liquid,

$$T_{wa} = \frac{m}{1-m} T_{aw} \tag{2-3}$$

and

$$T_{aw} = - \frac{R\rho\lambda}{pm} [E_{water} - \frac{m(1-p)}{K_d\rho}] \tag{2-4}$$

The E_{air} and E_{water} components of the emanation coefficient are shown in Figure 2-2 and are given by

$$\begin{aligned} E_{water} &= \frac{m}{m^*} E_w & \text{for } m < m^* \qquad (2\text{-}5) \\ &= E_w & \text{for } m \geq m^* \end{aligned}$$

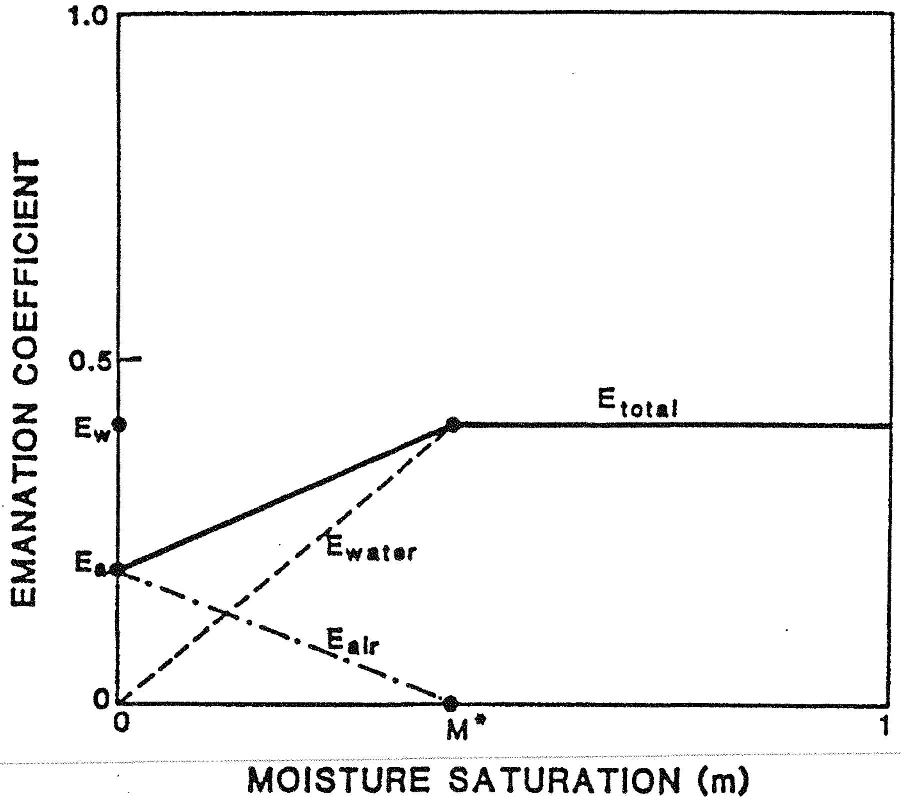

FIGURE 2-2. COMPONENTS OF THE EMANATION COEFFICIENT AS A FUNCTION OF MOISTURE.

$$E_{air} = E_a\,(1 - m/m^*) \qquad \text{for } m < m^* \qquad (2\text{-}6)$$
$$= 0 \qquad \text{for } m \geq m^*$$

where, m^*, E_a and E_w are constants of the emanation coefficient model (2). The term E_a is the emanation coefficient for no moisture, E_w is the emanation coefficient at saturation, and the plateau in the E curve starts at m^*.

Combining Equations (2-1) and (2-2), using Equations (2-3) and (2-4) yields:

$$D\,\frac{d^2C_a}{dx^2} - K'\left(\frac{dP}{dx}\right)\frac{dC_a}{dx} - \lambda C_a + \frac{R\rho\lambda E'}{p} = 0 \qquad (2\text{-}7)$$

where

D = pore average diffusion coefficient with adsorption
= $D'(1-m+mk)/f$ (2-8)

D' = pore diffusion coefficient with no adsorption

f = $1 - m + mk + K_a\rho$

K' = $\dfrac{K}{p\ f}$ (2-9)

k = radon equilibrium distribution coefficient, water-to-air

E' = $[E - m(1-p)/k_d\rho]\ /\ f$ (2-10)

k_a = effective surface adsorption coefficient (cm^3/g).

Henry's Law constant, k, is used to eliminate C_w by the relation $C_w = kC_a$. The degree of surface adsorption is a function of pore moisture, and generally greatly decreases with increasing moisture. Consequently, in combining Equations (2-1) and (2-2) the term $(k_a + k_w k)$ has been replaced by an effective surface adsorption coefficient, K_a. Also, D is written explicitly to show the dependence of the diffusion coefficient term on surface adsorption effects.

For no advection, Equation (2-7) is equal to the standard radon diffusion equation, using the pore average D and an adjusted emanation coefficient that decouples Equations (2-1) and (2-2). Consequently, standard measurements (7,8) and theoretical models (1,2) of D and E can be used with the new formulation, and previous analytical and numerical radon diffusion solutions are applicable. The computer code RAECOM (5,6) has been adapted to solve Equation (2-7) including advection, where the layer thicknesses are selected for constant (dP/dx). The development is readily extended to multi-dimensional and time-dependent formalisms. The new code is called RAETRAN.

3. THE RAETRAN CODE

The radon transport equation modeled in RAETRAN is

$$D \frac{d^2C}{dx^2} - V \frac{dC}{dx} - \lambda C + Q = 0 \qquad (3\text{-}1)$$

where

C = radon concentration in the air-filled pore space (pCi/cm^3)

λ = radon decay constant (2.1×10^{-6} s^{-1})

Q = effective radon source term (pCi/cm^3-s)

V = effective pore air velocity = $K' \frac{dP}{dx}$

p = total porosity of porous medium.

The radon flux, J, is related to the radon concentration by

$$\frac{J}{pf} = -D \frac{dC}{dx} + V(1-m)\ C\ J \qquad (3\text{-}2)$$

where

J = radon flux (pCi cm^{-2} s^{-1})

Additional insight can be gained into the role of advection in radon transport by noting that the radon concentration is very small or zero, at the soil-air interface. Therefore, the last term in Equation (3-2), the advective component of the flux, is generally negligibly small compared to the diffusive term. The main contribution of advection is to change the shape of the radon concentration profile, thus influencing the diffusive radon flux at the soil-air interface. The radon flux crossing this interface is thus nearly entirely moving by a diffusive process.

The general solution to this equation is

$$C(x) = A \exp\left[\left(\frac{V}{2D} + \sqrt{\frac{V^2}{4D^2} + \frac{\lambda}{D}}\right) x\right] + B \exp\left[\left(\frac{V}{2D} - \sqrt{\frac{V^2}{4D^2} + \frac{\lambda}{D}}\right) x\right] + \frac{Q}{\lambda} \tag{3-3}$$

This general solution must be fitted to the boundary conditions of the particular problem of interest by solving for the coefficients A and B.

3.1 APPLICATION TO A MULTILAYER SYSTEM

The configuration and coordinate system used in the present model is given in Figure 3-1. Each of the n layers in the system is associated with a thickness, d_i, a diffusion coefficient, D_i, a porosity, p_i, and a radon source, Q_i. Additional data needed to characterize the system include the radium content, density, radon emanation coefficient, and radium distribution coefficient for each layer. Each layer has its own coordinate system whose origin is at the center of the layer. The coordinate system in the subsoil layer has its origin a distance $d_1/2$ below the lower boundary of layer 1. All coordinates are positive in the upward direction.

The choice of a separate coordinate system centered on each layer greatly improves the numerical accuracy of the solution algorithms. In Equation (3-3) a large value for the x-coordinate can cause the exponential terms to have very large or very small values. This can complicate the solution process when implemented on a computer because of numerical overflow and underflow errors. The coordinate system with origin at the center of each layer restricts the maximum value of the x-coordinate to half of the layer thickness. This alleviates many of the numerical difficulties with the solution and minimizes the possibility of numerical errors.

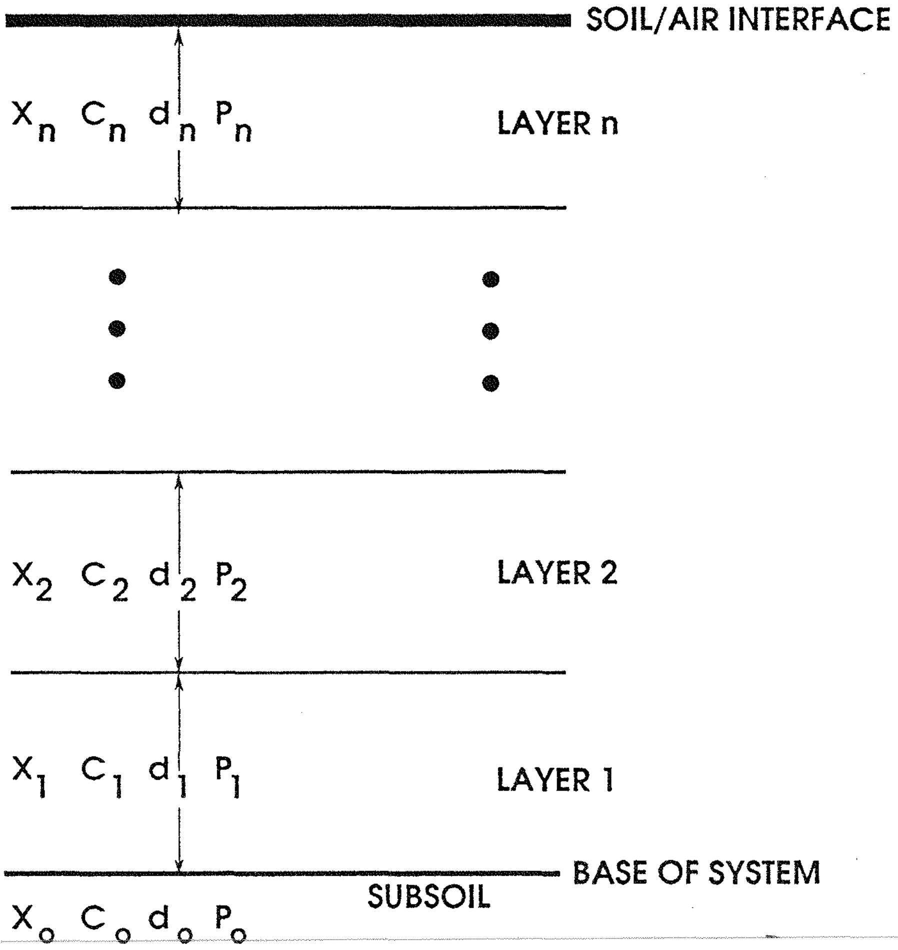

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FIGURE 3-1. COORDINATES FOR MULTILAYER SYSTEM.

3.2 BOUNDARY CONDITIONS

As boundary conditions for each layer the concentration, $C_i(x)$, and the flux, $J_i(x)$, are continuous across layer boundaries. The radon concentration in the subsoil is assumed to approach a constant finite value, C_s, as depth increases. The radon concentration in the air above the top layer of the system is a constant, C_a, usually equal to zero. These boundary conditions are expressed as

$$\lim_{x \to -\infty} C_o(x) = C_s \tag{3-4}$$

$$C_i\left(\frac{d_i}{2}\right) = C_{i+1} \ -\left(\frac{d_{i+1}}{2}\right) \qquad i = 0,1,\ldots,n-1 \tag{3-5}$$

$$J_i\left(\frac{d_i}{2}\right) = J_{i+1} \ -\left(\frac{d_{i+1}}{2}\right) \qquad i = 0,1,\ldots,n-1 \tag{3-6}$$

$$C_n\left(\frac{d_n}{2}\right) = C_a \tag{3-7}$$

The general solution of Equation (3-1) in layer i is

$$C_i(x_i) = A_i \exp(a_i x_i) + B_i \exp(b_i x_i) + \frac{Q_i}{\lambda} \tag{3-8}$$

where

$$a_i = \frac{V_i}{2D_i} + \sqrt{\left(\frac{V_i}{2D_i}\right)^2 + \frac{\lambda}{D_i}}$$

$$bi = \frac{V_i}{2D_i} - \sqrt{\left(\frac{V_i}{2D_i}\right)^2 + \frac{\lambda}{D_i}}$$

The corresponding radon flux, calculated from Equation (3-2), is

$$J = r_i A_i e^{a_i x_i} + s_i B_i e^{b_i x_i} + v_i Q_i/\lambda \tag{3-9}$$

where

$$r_i = v_i - a_i D_i f_i P_i$$

$$s_i = v_i - b_i D_i f_i P_i$$

$$v_i = V_i f_i P_i (1 - m_i)$$

In Equation (3-8), note that whether the advection velocity, V_i, is positive (upward), negative (downward), or zero, it is always true that a_i is positive and b_i is negative. Using this information with Equation (3-4), the result is

$$B_0 = 0 \tag{3-10}$$

Applying Equations (3-5) through (3-7) to Equation (3-8), the following system of equations results:

$$A_i e^{a_i d_i/2} + B_i e^{b_i d_i/2} - A_{i+1} e^{-a_{i+1} d_{i+1}/2} - B_{i+1} e^{-b_{i+1} d_{i+1}/2} = (Q_{i+1} - Q_i)/\lambda \qquad i = 0,1,\ldots,n-1 \tag{3-11}$$

$$r_i A_i e^{a_i d_i/2} + s_i B_i e^{b_i d_i/2} - r_{i+1} B_{i+1} e^{-a_{i+1} d_{i+1}/2} - s_{i+1} B_{i+1} e^{-b_{i+1} d_{i+1}/2} = (v_{i+1} Q_{i+1} - v_i Q_i)/\lambda \qquad i = 0,1,\ldots,n-1 \tag{3-12}$$

$$A_n e^{a_n d_n/2} + B_n e^{b_n d_n/2} = C_a - \frac{Q_n}{\lambda} \tag{3-13}$$

Equations (3-11) through (3-13) can be solved to yield the coefficients A_i and B_i which, when used in Equation (3-8), give the radon concentration in each layer of the system. The radon flux at any point can be determined using Equation (3-9).

Equations (3-11) through (3-13) can be more clearly expressed in matrix form:

$$M\,C = R \tag{3-14}$$

where

M = coefficient matrix

C = column matrix composed of the A_i and B_i

R = column matrix representing the right sides of Equations (3-11) through (3-13).

For a system of n layers above the subsoil the coefficient matrix is square with dimensions of 2n + 1 by 2n + 1. The explicit form of Equation (3-14) is shown in Figure 3-2. This matrix is solved by using Gaussian elimination to transform the matrix to upper triangular form. This removes all non-zero elements below the main diagonal of the matrix. The values for A_i and B_i are then obtained by backward substitution. Once the coefficients are known, the radon concentrations and fluxes are evaluated using Equations (3-8) and (3-9).

3.3 RAETRAN COMPUTER CODE

The equations describing radon transport in a multilayer system are solved by the RAETRAN computer code. The logic used in the code is illustrated in Figure 3-3. The equations are solved in the matrix form shown in Equation (3-14). The matrix is converted to upper triangular form by Gaussian elimination and row interchanges. The coefficients are then evaluated by backward substitution. The radon concentrations and fluxes at each of the layer boundaries are calculated and printed.

$$
\begin{bmatrix}
e^{a_0 d_0/2} & -e^{-a_1 d_1/2} & -e^{-b_1 d_1/2} & 0 & 0 & 0 \cdots \\
r_0 e^{a_0 d_0/2} & -r_1 e^{-a_1 d_1/2} & -s_1 e^{-b_1 d_1/2} & 0 & 0 & 0 \cdots \\
0 & e^{a_1 d_1/2} & e^{b_1 d_1/2} & -e^{-a_2 d_2/2} & -e^{-b_2 d_2/2} & 0 \cdots \\
0 & r_1 e^{a_1 d_1/2} & s_1 e^{b_1 d_1/2} & -r_2 e^{-a_2 d_2/2} & -s_2 e^{-b_2 d_2/2} & 0 \cdots \\
0 & 0 & 0 & e^{a_2 d_2/2} & e^{b_2 d_2/2} & \cdots \\
0 & 0 & 0 & r_2 e^{a_2 d_2/2} & s_2 e^{b_2 d_2/2} & \cdots \\
\vdots & \vdots & \vdots & 0 & 0 & \vdots \\
 & & & \vdots & \vdots & \\
 & & & 0 \cdots & e^{a_n d_n/2} & e^{b_n d_n/2}
\end{bmatrix}
\begin{bmatrix}
A_0 \\ A_1 \\ B_1 \\ A_2 \\ B_2 \\ \vdots \\ \\ A_n \\ B_n
\end{bmatrix}
=
\begin{bmatrix}
(Q_1 - Q_0)/\lambda \\
(v_1 Q_1 - v_0 Q_0)/\lambda \\
(Q_2 - Q_1)/\lambda \\
(v_2 Q_2 - v_1 Q_1)/\lambda \\
\vdots \\
\\
(v_n Q_n - v_{n-1} Q_{n-1})/\lambda \\
C_a - Q_n/\lambda
\end{bmatrix}
$$

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FIGURE 3-2. MATRIX EQUATION FOR RADON TRANSPORT.

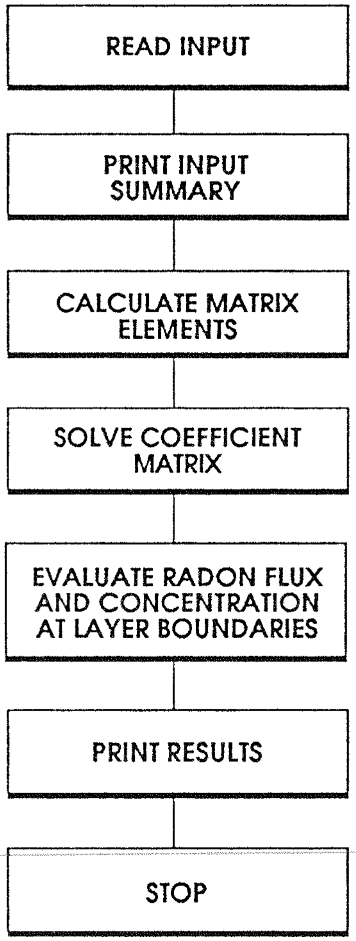

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FIGURE 3-3. LOGIC FLOW FOR RAETRAN CODE.

Input variables for the RAETRAN code are described in Table 3-1. The data can be read from an existing file or entered interactively by the user. In the interactive mode, the code prompts the user for all variables. If the input data are prepared in a file, the variables can be entered in free format separated by commas. The code prompts the user for the name of the input data file. The results of the calculations are written to the terminal screen and also to a disk file.

The RAETRAN code has been used to generate sample results for a three-layer soil system. The three layers range in thickness from 30 to 100 cm. The layers have varying moisture contents, porosities, densities, and air permeabilities. All three layers have a radon emanation coefficient of 0.22 and a radium content of 5 pCi/g. The diffusion coefficients are entered as zeros in the data file and are therefore calculated in the code using a correlation formula. The complete input data set is given in the Appendix. The results of the analysis show a radon concentration of over 10,000 pCi/L at the base of the system and a ground surface radon flux of about 6.8 pCi/m^2-s. The computer output for the sample problem and a listing of the computer code are also included in the appendix.

TABLE 3-1

RAETRAN INPUT DATA

Record	Variable	Description
1	TITLE	Title of run. Up to 80 characters are allowed.
2	N	Number of layers in the system.
3	CN1	Radon concentration in air above the top layer of the system (pCi/L).
	DPDX1	Average atmospheric potential (pressure) gradient (Pa/m). Used to calculate advection velocity.
4	DX(I)	Thickness of layer I (cm). Layer 1 is at the bottom of the system, layer N is at the top.
	RAD(I)	Total radium content of layer I (pCi/g).
	RHO(I)	Dry bulk density of layer I (g/cm^3).
	POR(I)	Total porosity of layer I.
	EMAN(I)	Radon emanation coefficient for layer I.
	SAT(I)	Fraction of moisture saturation for layer I.
	RNKA(I)	Radon soil/air partition (adsorption) coefficient for layer I (ml/g).
	RAKD(I)	Radium soil/water distribution coefficient for layer I (ml/g).
	AIRPI(I)	Intrinsic permeability of layer I (cm^2).
	D(I)	Radon diffusion coefficient for layer I (cm^2/s). Calculated internally if a value of zero is used.

Note: Record 4 is repeated for each layer in the system.

4. DIFFUSION, PERMEABILITY, AND EMANATION COEFFICIENTS

Site-specific measurements of D and K are most desirable for obtaining input data for the radon transport models. RAE has accumulated an extensive soil data base including D and K measured values as well as values of many other nuclear, physical, and geological parameters. In addition, models have been developed to calculate D and K from basic data and first principles. The random-pore combination model that was developed earlier (2,3) was modified and extended to also include the calculation of soil permeabilities in a similar manner.

4.1 PERMEABILITY COEFFICIENT

In the K calculations, soils were modeled to contain log-normal size distributions of cylindrical pores with uniform surface water films that could vary from dryness to saturation. Contiguous pore space contained all possible random combinations of the pore sizes, both in series and parallel configurations. Individual cylindrical pore permeabilities were computed from the radii (r) of the air-filled pore spaces as $r^2/8$, following the functional size-dependence used by Youngquist (10). The resulting permeabilities of individual pore segments were combined for the entire soil in the same way as pore diffusion coefficients were previously combined to estimate bulk soil radon diffusion coefficients. The resulting permeability calculations are illustrated by the black dots in Figure 4-1. Gas permeabilities varied slowly with moisture, increased rapidly with the geometric standard deviation of the grain-size distribution, and increased as the square of the average particle diameter.

In addition to field measurements, laboratory measurements, or complex model calculations of K, an empirical correlation has also been developed. This correlation has the advantage of being simple and easy to use, with a minimum amount of information needed.

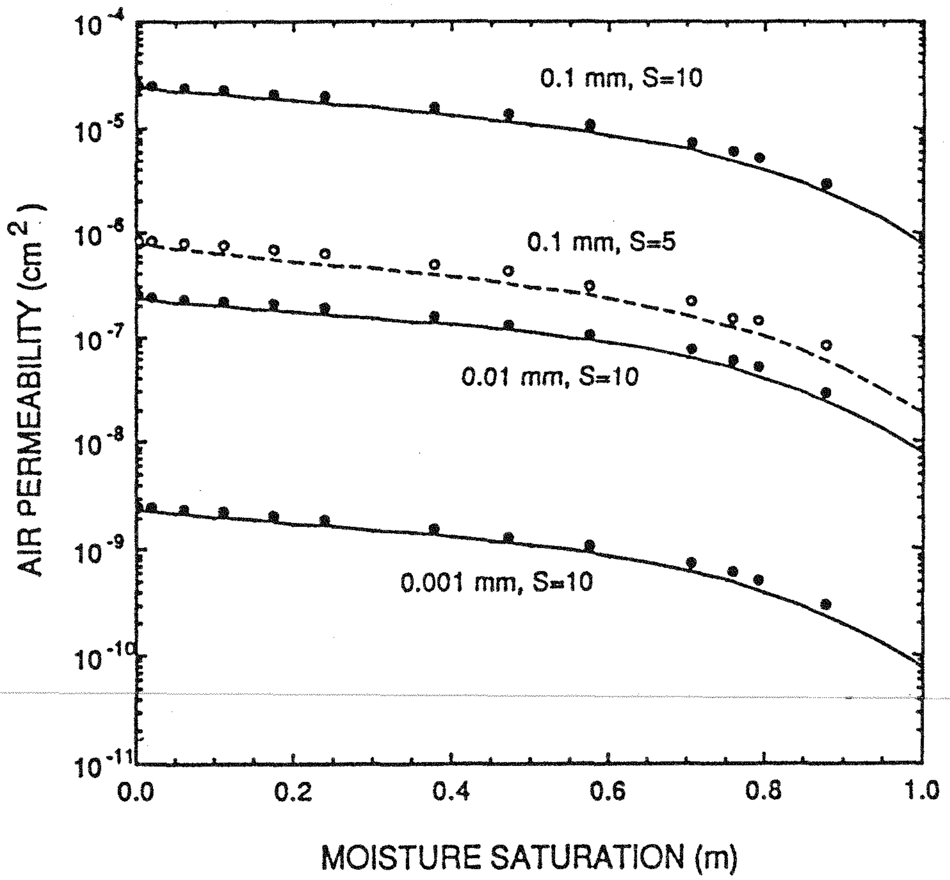


FIGURE 4-1. AIR PERMEABILITY FOR VARIOUS SOIL TYPES. DATA POINTS ARE MODEL CALCULATIONS. CURVES ARE CORRELATION VALUES. GEOMETRIC MEAN PARTICLE DIAMETERS, AND GEOMETRIC STANDARD DEVIATIONS, S, ARE GIVEN FOR EACH SOIL TYPE.

The correlation for soil permeabilities is

$$K = p\,(1+p)(d'/3)^2 \exp[-\,1.4\ m - 2\ m^5] \qquad (4\text{-}1)$$

where

d' = geometric mean particle diameter (cm)

Equation (4-1) applies to soils whose particle distribution can be approximated by a log-normal distribution with a geometric standard deviation, S, of ten.

For other values of S, Equation (4-1) must be multiplied by

$$0.0034 \exp[0.264\ S^{4/3} - 0.6\ m\ (1 - 0.1\ S)] \qquad (4\text{-}2)$$

The correlation values for K are also plotted in Figure 4-1 for several values of d′. Curves are given for an S of ten and of five.

4.2 DIFFUSION COEFFICIENTS

One correlation frequently used for D is (5)

$$D = 0.07 \exp[-4\ (m - mp^2 + m^5)] \qquad (4\text{-}3)$$

Since that correlation was developed, RAE has measured over 1500 additional D measurements for the Department of Energy's Uranium Mill Tailings Remedial Action Program. Most of these D's were measured at high compactions. Consequently, Equation (4-3) has been modified as follows to include the effects of high compactions:

$$D = \frac{3p(1+p)d}{8(2+\pi d)} \exp\left[-\ \frac{7p(1+p)d}{2+d}\ m - 7\ m^5\right] \qquad (4\text{-}4)$$

where

d = geometric mean particle diameter (um).
= 10,000 d'.

For most natural soils near Proctor compaction, Equation (4-4) can be approximated by

$$D = 0.035 \exp\left(\frac{5}{4} m - 7 m^5\right) \tag{4-5}$$

A portion of the D values in the data base are plotted in Figure 4-2, along with correlation curves for a low compaction, large-grained material and a high-compaction, small-grained material. The higher compaction reduces the value of D at low moistures.

4.3 ANISOTROPY IN DIFFUSION AND PERMEABILITY COEFFICIENTS

Possible anisotropy of radon diffusion coefficients of soils was examined by conducting laboratory diffusion measurements on soils in both the direction of the compactive force and perpendicular to the compactive force. Six different soils were prepared by compacting to 95 percent of standard Proctor density, and then pressing a 10-cm diameter thin-walled diffusion tube into the soil either parallel or perpendicular to the direction of compaction. Eighteen time-dependent radon diffusion measurements (7) on the resulting soil cylinders gave an average diffusion coefficient ratio (perpendicular/parallel) of 1.2 ± 0.3, indicating only a marginal increase in diffusion along the bedding plane. This could still be greater in some undisturbed soils, but is probably insignificant in re-compacted soils in construction excavations.

Over 50 similar laboratory air-permeability measurements were conducted on five of the same soil samples. The average ratio of air-permeabilities (perpendicular/parallel to compaction) was 6 ± 6, indicating a somewhat more significant increase in permeability along the

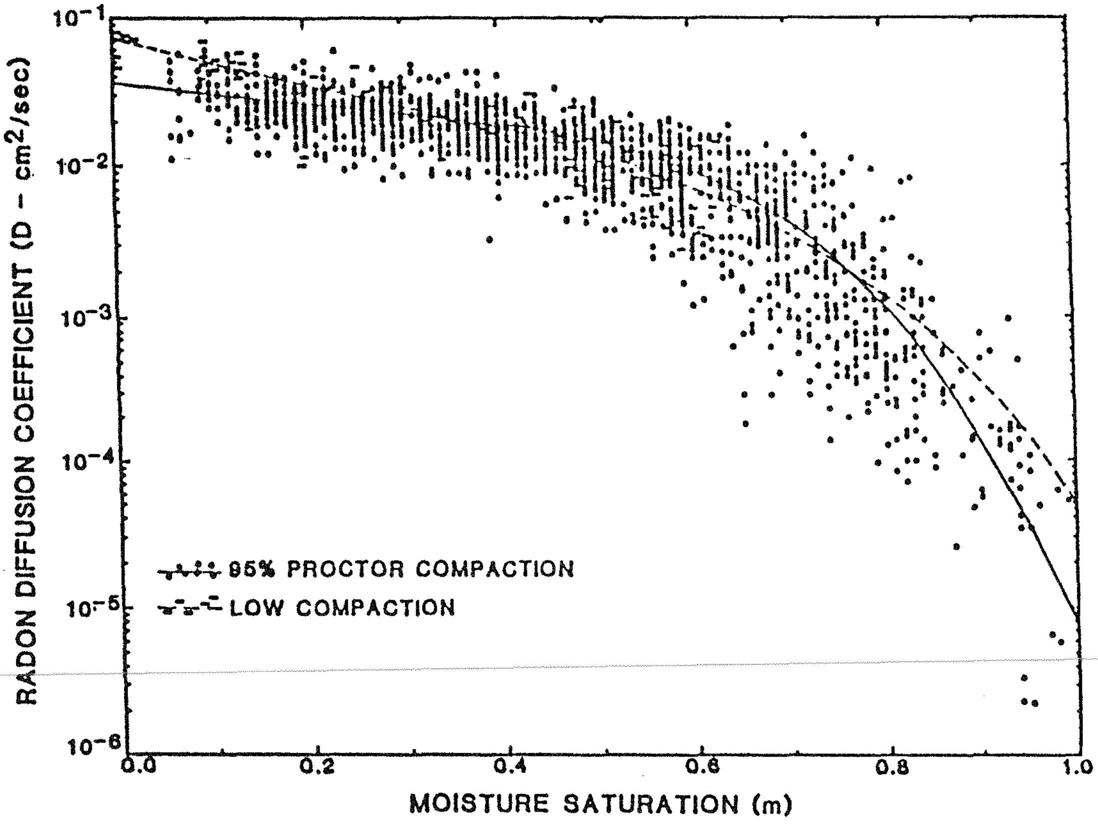


FIGURE 4-2. MEASURED RADON DIFFUSION COEFFICIENT AND CORRELATION PREDICTIONS.

bedding plane. This ratio also may be greater in undisturbed soils, but may be important even in some re-compacted soils.

4.4 RADON EMANATION COEFFICIENTS

Emanation coefficients have been measured for several different soil types, because the RAE data base for E consisted mainly of values for uranium ores, uranium mill tailings, and phosphogypsum. The new E values were measured on soils documented in the data base to facilitate further development of the emanation coefficient model (1). The new values are shown in Figure 4-3. The fit to a relatively narrow normal distribution is encouraging, especially considering the widely diverse soils used for the measurements. These data indicate that a best-estimate default value of E for normal soils is 0.22, if site-specific data are not available.

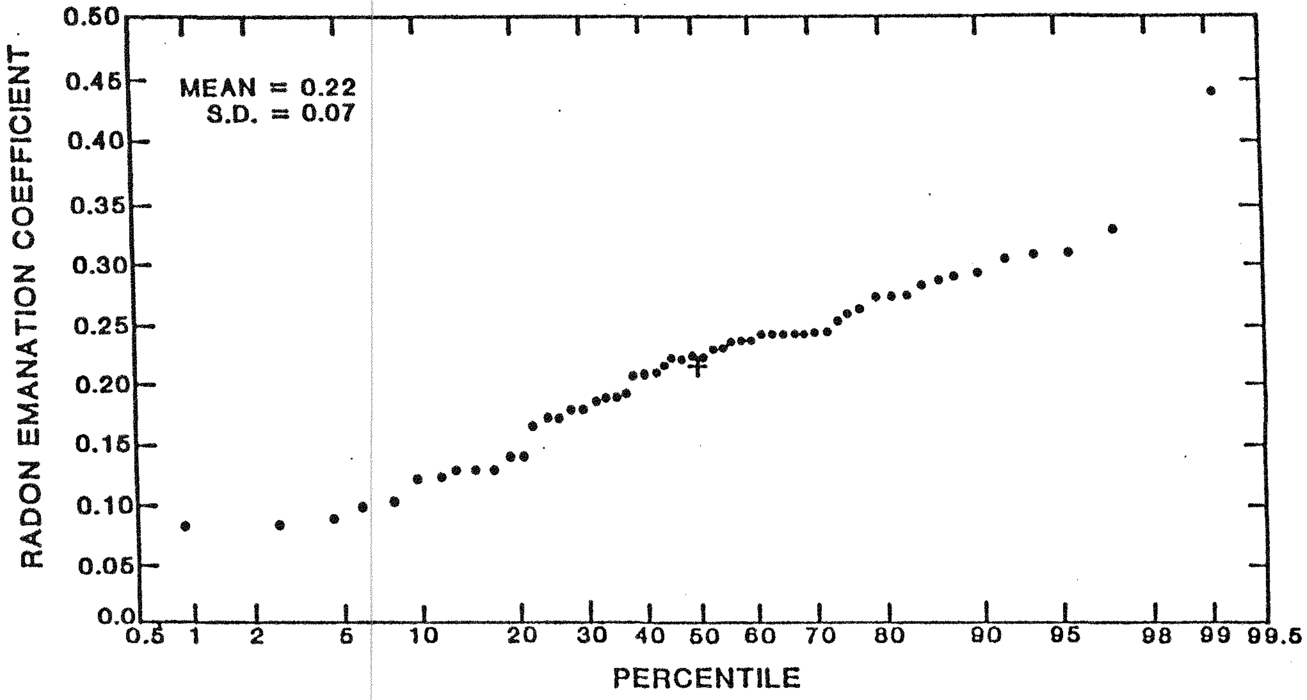


FIGURE 4-3. PROBABILITY PLOT OF EMANATION COEFFICIENTS FOR 56 SOILS.

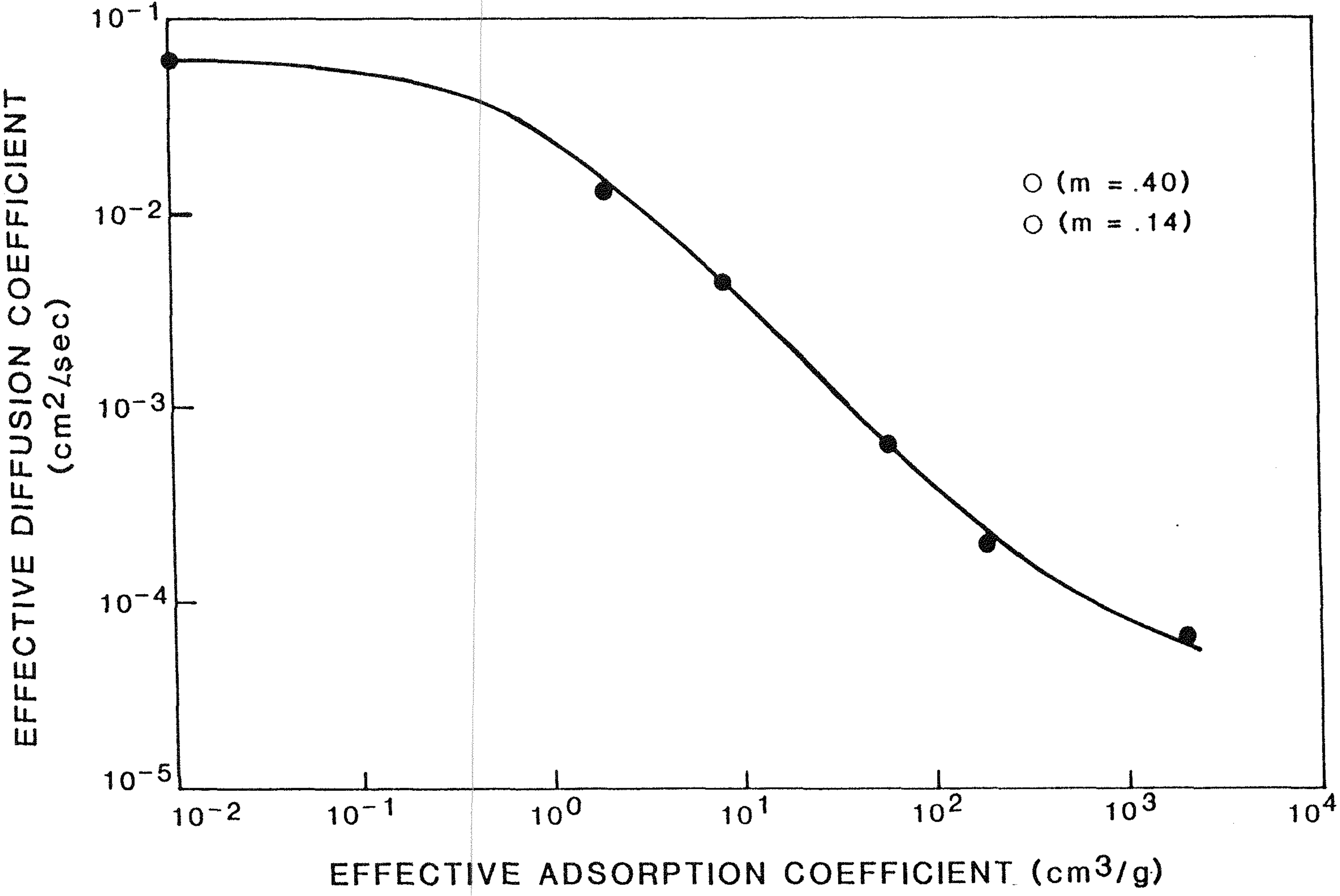
5. SURFACE ADSORPTION OF RADON

The adsorption terms in Equations (2-1) and (2-2) combine to form an effective diffusion coefficient, permeability coefficient, and emanation coefficient. The validity of this model was tested by measuring effective diffusion coefficients including adsorption in the laboratory and comparing the measured results with Equation (2-8) for various values of f.

In the measurements, clean coarse sand was mixed with activated charcoal. The k_a of the dry charcoal was previously determined to be 2,000 cm^3/g, and the diffusion coefficient for the dry sand was previously measured to be 0.065 cm^2/sec. Measurements of the effective diffusion coefficient were made for the activated charcoal and for various mixtures of dry sand and activated charcoal. The weight fractions of charcoal in the mixtures were 0.1, 0.03, and 0.01. The measured diffusion coefficients are shown as solid circles in Figure 5-1. They compare very well with the theoretical curve obtained from applying Equation (2-8). for the calculations, a value of 0.065 cm^2/sec was used for the D without adsorption, and the varying fractions of activated charcoal with the sand gave an effective homogeneous K_a of 200, 60, and 20 cm^3/g.

The effects of moisture on K_a and D were studied by repeating the experiment for $K_a = 200$ with the overall system at 14 percent and 40 percent of saturation. The measured effective D values are shown as open circles in Figure 5-1.

The moisture preferentially accumulates with the activated charcoal and effectively blocks the adsorption sites for radon, greatly reducing the K_a with moisture. The dependence of K_a on moistures yields the interesting result that for highly adsorbing systems, the effective diffusion coefficient increases with increasing moisture up to a certain degree of saturation.



RAE-102828

FIGURE 5-1. DIFFUSION COEFFICIENT DEPENDENCE ON ADSORPTION.

6. SUMMARY

A simplified model has been developed for radon emanation and transport through a two-phase medium in the pore spaces of earthen materials. The model explicitly considers radon emanation into the water and the air pore regions, radon adsorption on solid surfaces, radon diffusion through the pore water and air regions, radon and radium absorption in the water, radon transfer between the pore water and air, and advection in the air region. The one-dimensional steady-state formalism is presented and has been encoded into the RAETRAN computer code.

The DIFPERM code for calculating diffusion and permeability coefficients from basic principles is described and new correlations for D and K are presented. It is also shown that horizontal permeabilities are slightly greater than vertical permeabilities, but the difference is not as evident in the D measurements. Several new E measurements have also been made for a wide variety of soils. The geometric mean of the E measurements is 0.22.

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APPENDIX

```
PUT FOR RAETRAN SAMPLE PROBLEM
------------------------------


ETRAN SAMPLE PROBLEM -- 3 LAYER SYSTEM

,100.
0., 5., 1.7, .20, .22, .55,   0., 200., 1.E-8, 0.
0., 5., 1.6, .25, .22, .60,   0., 200., 3.E-8, 0.
0., 5., 1.6, .35, .22, .40, 100., 500., 2.E-7, 0.
```

UTPUT FOR RAETRAN SAMPLE PROBLEM

```
RAETRAN SAMPLE PROBLEM -- 3 LAYER SYSTEM

**********  INPUT SUMMARY AND PRELIMINARY CALCULATIONS  **********

Number of layers =  3
Surface radon concentration in air      =  0.00E+00 pCi/L
Average atmospheric potential gradient =  1.00E+02 Pa/m

                                    Radium       Dry Bulk
                      Thickness     Content      Density                    Emanation      Saturation
    Layer               (cm)        (pCi/g)      (g/cc)        Porosity     Coefficient     Fraction
-------------      ------------  ------------  ------------  ------------  ------------  ------------
  3   Top              30.0        5.00E+00       1.600         0.350         0.220         0.400
  2                    50.0        5.00E+00       1.600         0.250         0.220         0.600
  1   Bottom          100.0        5.00E+00       1.700         0.200         0.220         0.550

                       Radon         Radium          Air        Diffusion     Advection
                     Adsorption        Kd       Permeability  Coefficient     Velocity
    Layer              (ml/g)        (ml/g)       (cm**2)      (cm**2/s)       (cm/s)
-------------      ------------  ------------  ------------  ------------  ------------
  3   Top            1.00E+02      5.00E+02      2.00E-07      1.65E-02      4.03E-03
  2                  0.00E+00      2.00E+02      3.00E-08      5.41E-03      8.46E-03
  1   Bottom         0.00E+00      2.00E+02      1.00E-08      6.93E-03      9.40E-03


**********  RESULTS OF RADON TRANSPORT CALCULATION  **********

                       Thickness      Exit Conc.      Exit Flux
    Layer                (cm)          (pCi/L)      (pCi/m**2-s)
--------------      ------------   ------------   ------------
  3   Top                30.0        0.00E+00       6.78E+00
  2                      50.0        6.97E+03       6.23E+01
  1   Bottom            100.0        1.15E+04       3.90E+01
  0   Sub-soil            0.0        1.08E+04       4.12E+01
```

```
      PROGRAM RAETRAN
      IMPLICIT REAL*8 (A-H,O-Z)
      CHARACTER*8 LAYER
      CHARACTER*80 TITLE
      COMMON AIRPI(50),CN1,D(0:50),DPDX1,DX(0:50),EMAN(50),LAYER,N,
     &       POR(0:50),RAD(50),RAKD(50),RHO(50),RNKA(50),SAT(50),TITLE
      DIMENSION A(0:50),AIRP(50),ALPHA(0:50),B(0:50),BETA(0:50),CM(625),
     &          DM(25,25),D2(0:50),EE(4),EF(4),F(0:50),RHS(50),RC(0:50),
     &          RF(0:50),S(0:50),V(0:50),VAD(0:50),VBD(0:50),VLIT(0:50)
      DATA EPS,RNWA,XL / 1.00D-5, 2.6D-1, 2.10D-6 /
C
C***********************************************************************
C
C    AIRP(I)  = Dry Air permeability of layer I (cm/s)
C    AIRPI(I) = Intrinsic dry air permeability of layer I (cm**2)
C    CN1      = Radon concentration above top layer (pCi/l)
C    D(I)     = Pore diffusion coefficient of layer I (cm**2/s)
C    DM       = Diagonal matrix of coefficients
C    DPDX     = Dimensionless atmospheric potential gradient (cm air/cm)
C    DPDX1    = Ave. atmospheric potential (pressure) gradient (Pa/m)
C    DX(I)    = Thickness of layer I (cm)
C    EMAN(I)  = Radon emanation coefficient of layer I
C    EPS      = Error bound for matrix solution
C    POR(I)   = Porosity of layer I
C    RAD(I)   = Radium content of layer I (pCi/g)
C    RAKD(I)  = Radium distribution coefficient for layer I (ml/g)
C    RC(I)    = Radon concentration at the top of layer I (pCi/cm**3)
C    RF(I)    = Radon flux at the top of layer I (pCi/cm**2-s)
C    RHO(I)   = Dry bulk density of layer I (g/cm**3)
C    RHS      = Right hand side column vector in matrix equation
C    RNKA(I)  = Radon soil/air partition coefficient for layer I (ml/g)
C    RNWA     = Radon water/air partition coefficient
C    S(I)     = Pore space radon source (RAD*RHO*EMAN/POR), layer I (pCi/cc)
C    SAT(I)   = Fraction of moisture saturation for layer I
C    V(I)     = Advective pore velocity in layer I (cm/s)
C    XL       = Radon decay constant (1/s)
C
C    Radon concentration is calculated from the following:
C
C       RC(X) = A*exp(ALPHA*X) + B*exp(BETA*X) + S
C
      OPEN(UNIT=6,FILE='RAETRAN.OUT',STATUS='NEW')

C---  CALL SUBROUTINE TO READ INPUT DATA ------------------------------

      CALL READ

C---  PRELIMINARY CALCULATIONS ----------------------------------------

      CN = CN1/1000.
      AIRDNS = 1.29            ! Air density (kg/m**3)
      GRAV = 9.8               ! Gravitational acc. (m/s**2)
      DPDX = DPDX1/(AIRDNS*GRAV)
      AIRDNS = 1.29E-3         ! Air denstiy (g/cm**3)
      GRAV = 980.              ! Grav. acc. (cm/s**2)
      AIRVIS = 1.80E-4         ! Air viscosity (poise)

C**   Define air permeabilities by layer and the average permeability
      DXT = 0.
```

```
      SUM = 0.
      DO 20 I=1,N
        DXT = DXT + DX(I)
        AIRP(I) = AIRPI(I)*AIRDNS*GRAV/AIRVIS
        SUM = SUM + DX(I)/AIRP(I)
20    CONTINUE
      AVPERM = DXT/SUM

**    Calculate diffusion coeffs., radon sources, advection velocities
      DO 30 I=1,N
        XM = SAT(I)
        IF (D(I) .EQ. 0.)
     &     D(I) = 0.07*DEXP(-4.*XM*(1.-POR(I)**2+XM**4))
        V(I) = DPDX*AVPERM/(POR(I)*(1.-XM))
30    CONTINUE


---   WRITE INPUT DATA -------------------------------------------------

      WRITE(5,70) TITLE,N,CN1,DPDX1
      WRITE(6,70) TITLE,N,CN1,DPDX1
70    FORMAT('1',/,1X,A80,///,' **********  INPUT SUMMARY AND ',
     &  'PRELIMINARY CALCULATIONS  **********',//,
     &  ' Number of layers = ',I2,/,
     &  ' Surface radon concentration in air      = ',1PE9.2,' pCi/L',/,
     &  ' Average atmospheric potential gradient = ',E9.2,' Pa/m')

      WRITE(5,72)
      WRITE(6,72)
72    FORMAT(//,35X,'Radium',7X,'Dry Bulk',/,20X,'Thickness',5X,
     &  'Content',7X,'Density',20X,'Emanation',5X,'Saturation',/,5X,
     &  'Layer',12X,'(cm)',8X,'(pCi/g)',7X,'(g/cc)',8X,'Porosity',4X,
     &  'Coefficient',5X,'Fraction',/,' -------------   ',
     &  6('  -----------'))
      LAYER = 'Top      '
      DO 74 I=N,1,-1
        IF (I .EQ. 1) LAYER = 'Bottom   '
        WRITE(5,73) I,LAYER,DX(I),RAD(I),RHO(I),POR(I),EMAN(I),SAT(I)
        WRITE(6,73) I,LAYER,DX(I),RAD(I),RHO(I),POR(I),EMAN(I),SAT(I)
73      FORMAT(2X,I2,2X,A8,7X,F6.1,6X,1PE9.2,6X,0PF6.3,3(8X,F6.3))
        LAYER = '         '
74    CONTINUE

      WRITE(5,76)
      WRITE(6,76)
76    FORMAT(//,21X,'Radon',9X,'Radium',9X,'Air',8X,'Diffusion',5X,
     &  'Advection',/,19X,'Adsorption',8X,'Kd',7X,'Permeability',2X,
     &  'Coefficient',4X,'Velocity',/,5X,'Layer',11X,'(ml/g)',8X,
     &  '(ml/g)',7X,'(cm**2)',6X,'(cm**2/s)',6X,'(cm/s)',/,
     &  ' -------------   ',5('  -----------'))
      LAYER = 'Top      '
      DO 78 I=N,1,-1
        IF (I .EQ. 1) LAYER = 'Bottom   '
        WRITE(5,77) I,LAYER,RNKA(I),RAKD(I),AIRPI(I),D(I),V(I)
        WRITE(6,77) I,LAYER,RNKA(I),RAKD(I),AIRPI(I),D(I),V(I)
77      FORMAT(2X,I2,2X,A8,5(5X,1PE9.2))
        LAYER = '         '
78    CONTINUE
```

```
---   ADJUST PARAMETERS  ---------------------------------------------------

      DO 80 I=1,N
        XM = SAT(I)
        G = 1. - XM + XM*RNWA
        F(I) = G + RNKA(I)*RHO(I)
        D(I) = D(I)*G/F(I)
        V(I) = DPDX*AVPERM/(POR(I)*F(I))
        VLIT(I) = V(I)*F(I)*POR(I)*(1.-XM)
        EMAN(I) = (EMAN(I)-XM*(1.-POR(I))/(RAKD(I)*RHO(I)))/F(I)
        S(I) = RAD(I)*RHO(I)*EMAN(I)/POR(I)
 80   CONTINUE
      POR(0) = POR(1)
      DX(0) = DX(1)
      D(0) = D(1)
      F(0) = F(1)
      S(0) = S(1)
      V(0) = V(1)

C---  CALCULATE PARAMETERS FOR MATRIX  ----------------------------------

      DO 105 I=0,N
        D2(I) = 0.5*DX(I)
        VR2D = 0.5*V(I)/D(I)
        ROOT = DSQRT(VR2D*VR2D+XL/D(I))
        ALPHA(I) = VR2D+ROOT
        BETA(I) = VR2D-ROOT
        VAD(I) = VLIT(I)-ALPHA(I)*D(I)*F(I)*POR(I)
        VBD(I) = VLIT(I)-BETA(I)*D(I)*F(I)*POR(I)
105   CONTINUE

C---  SPECIFY MATRIX ELEMENTS  ------------------------------------------

      NN = N+N
      NN1 = NN+1
      DO 110 I=0,N-1
        II = I+I
        EE(1) = DEXP(ALPHA(I)*D2(I))
        EE(2) = DEXP(BETA(I)*D2(I))
        EE(3) = -DEXP(-ALPHA(I+1)*D2(I+1))
        EE(4) = -DEXP(-BETA(I+1)*D2(I+1))
        RHS(II+1) = S(I+1)-S(I)
        EF(1) = VAD(I)*EE(1)
        EF(2) = VBD(I)*EE(2)
        EF(3) = VAD(I+1)*EE(3)
        EF(4) = VBD(I+1)*EE(4)
        RHS(II+2) = VLIT(I+1)*S(I+1)-VLIT(I)*S(I)
        IF (I .EQ. 0) THEN
          EE(2) = EE(1)
          EF(2) = EF(1)
        ENDIF
        DO 108 J=1,4
          IF (I.EQ.0 .AND. J.EQ.1) GOTO 108
          DM(II+1,II+J-1) = EE(J)
          DM(II+2,II+J-1) = EF(J)
108     CONTINUE
110   CONTINUE
      DM(NN+1,NN) = DEXP(ALPHA(N)*D2(N))
```

```
      DM(NN+1,NN+1) = DEXP(BETA(N)*D2(N))
      RHS(NN+1) = CN-S(N)

 ---  STORE DIAGONAL MATRIX COLUMNWISE IN CM -------------------------

      K = 0
      DO 130 J=1,NN1
      DO 130 I=1,NN1
        K = K+1
        CM(K) = DM(I,J)
 30   CONTINUE

:---  CALL SUBROUTINE TO SOLVE COEFFICIENT MATRIX --------------------

      CALL GELG(RHS,CM,NN1,EPS,IER)
      IF (IER .NE. 0) THEN
        WRITE(5,132) IER
      ENDIF
 32   FORMAT(/,' IER=',I2,4X,'POSSIBLE LOSS OF SIGNIFICANCE IN ',
     &  'MATRIX SOLUTION.')

:---  SUBSTITUTE COEFFICIENTS ----------------------------------------

      A(0) = RHS(1)
      B(0) = 0.
      DO 140 I=1,N
        II = I+I
        A(I) = RHS(II)
        B(I) = RHS(II+1)
140   CONTINUE

:---  MATRIX SOLUTION COMPLETE.  EVALUATE RADON CONCENTRATION AND FLUX.

      DO 200 I=0,N
        ASI = A(I)*DEXP(ALPHA(I)*D2(I))
        BSI = B(I)*DEXP(BETA(I)*D2(I))
        RC(I) = ASI + BSI + S(I)
        RF(I) = VAD(I)*ASI + VBD(I)*BSI + VLIT(I)*S(I)
200   CONTINUE
      RC(N) = CN
      DX(0) = 0.

C---  OUTPUT RESULTS -------------------------------------------------

      WRITE(5,300)
      WRITE(6,300)
300   FORMAT(////,' **********  RESULTS OF RADON TRANSPORT ',
     &  'CALCULATION  **********',//,21X,'Thickness',6X,'Exit Conc.',
     &  6X,'Exit Flux',/,5X,'Layer',13X,'(cm)',10X,'(pCi/L)',7X,
     &  '(pCi/m**2-s)',/,' --------------',3(4X,'------------'))
      LAYER = 'Top      '
      DO 320 I=N,0,-1
        IF (I .EQ. 1) LAYER = 'Bottom   '
        IF (I .EQ. 0) LAYER = 'Sub-soil'
        RC(I) = RC(I)*1000.
        RF(I) = RF(I)*10000.
```

```
          WRITE(5,310) I,LAYER,DX(I),RC(I),RF(I)
          WRITE(6,310) I,LAYER,DX(I),RC(I),RF(I)
 10       FORMAT(2X,I2,2X,A8,7X,F6.1,9X,1PE9.2,7X,E9.2)
          LAYER = '        '
 20     CONTINUE
        CLOSE(UNIT=6)
        STOP
        END



        SUBROUTINE READ
        IMPLICIT REAL*8 (A-H,O-Z)
        CHARACTER*8 LAYER
        CHARACTER*16 BLANK,FILNAM
        CHARACTER*80 TITLE
        COMMON AIRPI(50),CN1,D(0:50),DPDX1,DX(0:50).EMAN(50),LAYER,N,
     &         POR(0:50),RAD(50),RAKD(50),RHO(50),RNKA(50),SAT(50),TITLE
        DATA BLANK/'                '/
        WRITE(5,10)
 10     FORMAT(' ENTER INPUT DATA FILE NAME OR',/,' <RETURN> FOR ',
     &  'INTERACTIVE INPUT >',$)
        READ(5,20) FILNAM
 20     FORMAT(A16)
        IF (FILNAM .NE. BLANK) THEN
          OPEN(UNIT=3,FILE=FILNAM,STATUS='OLD')
          READ(3,500) TITLE
          READ(3,505) N
          READ(3,510) CN1,DPDX1
          DO 100 I=1,N
            READ(3,510) DX(I),RAD(I),RHO(I),POR(I),EMAN(I),SAT(I),
     &                  RNKA(I),RAKD(I),AIRPI(I),D(I)
100       CONTINUE
          CLOSE(UNIT=3)
        ELSE
          WRITE(5,200)
200       FORMAT(' Enter title of run',/,' >',$)
          READ(5,500) TITLE
          WRITE(5,205)
205       FORMAT(' Number of layers in system >',$)
          READ(5,505) N
          WRITE(5,210)
210       FORMAT(' Radon concentration in air above top layer ',
     &      '(pCi/L) >',$)
          READ(5,510) CN1
          WRITE(5,215)
215       FORMAT(' Average atmospheric potential gradient (Pa/m)',
     &      6X,'>',$)
          READ(5,510) DPDX1
          WRITE(5,220) N
220       FORMAT(/,' FOR EACH OF THE ',I2,' LAYERS ENTER:',/,5X,
     &      'Layer Thickness (cm)',/,5X,'Radium Content (pCi/g)',/,5X,
     &      'Dry Bulk Density (g/cc)',/,5X,'Total Porosity',/,5X,
     &      'Emanation Coefficient',/,5X,
     &      'Fraction of Moisture Saturation',/,5X,
     &      'Radon Soil/Air Partition Coeff. (ml/g)',/,5X,
     &      'Radium Soil/Water Distribution Coeff. (ml/g)',/,5X,
     &      'Intrinsic Dry Air Permeability (cm**2)',/,5X;
     &      'Optional Radon Diffusion Coeff. (cm**2/s)')
```

```
        LAYER = '(Bottom)'
        DO 280 I=1,N
          IF (I .EQ. N) LAYER = '(Top)   '
          WRITE(5,230) I,LAYER
230       FORMAT(//,' LAYER ',I2,4X,A8,/,5X,'Layer Thickness (cm)',
     &      25X,'>',$)
          READ(5,510) DX(I)
          WRITE(5,235)
235       FORMAT(5X,'Radium Content (pCi/g)',23X,'>',$)
          READ(5,510) RAD(I)
          WRITE(5,240)
240       FORMAT(5X,'Dry Bulk Density (g/cc)',22X,'>',$)
          READ(5,510) RHO(I)
          WRITE(5,245)
245       FORMAT(5X,'Total Porosity',31X,'>',$)
          READ(5,510) POR(I)
          WRITE(5,250)
250       FORMAT(5X,'Emanation Coefficient',24X,'>',$)
          READ(5,510) EMAN(I)
          WRITE(5,255)
255       FORMAT(5X,'Fraction of Moisture Saturation',14X,'>',$)
          READ(5,510) SAT(I)
          WRITE(5,260)
260       FORMAT(5X,'Radon Soil/Air Partition Coeff. (ml/g)',7X,'>',$)
          READ(5,510) RNKA(I)
          WRITE(5,265)
265       FORMAT(5X,'Radium Soil/Water Distribution Coeff. (ml/g) >',$)
          READ(5,510) RAKD(I)
          WRITE(5,270)
270       FORMAT(5X,'Intrinsic Dry Air Permeability (cm**2)',7X,'>',$)
          READ(5,510) AIRPI(I)
          WRITE(5,275)
275       FORMAT(5X,'Optional Radon Diffusion Coeff. (cm**2/s)    >',$)
          READ(5,510) D(I)
          LAYER = '        '
280     CONTINUE
      ENDIF
500   FORMAT(A80)
505   FORMAT(I6)
510   FORMAT(10F12.6)
      RETURN
      END
```

```
      SUBROUTINE GELG(RHS,CM,M,EPS,IER)
      IMPLICIT REAL*8 (A-H,O-Z)
C
C        PURPOSE
C           TO SOLVE A GENERAL SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS.
C
C        USAGE
C           CALL GELG(RHS,CM,M,EPS,IER)
C
C        DESCRIPTION OF PARAMETERS
C           RHS    - ONE DIMENSIONAL MATRIX CONTAINING THE M ELEMENTS OF
C                    THE RIGHT HAND SIDE COLUMN VECTOR. ON RETURN, RHS
C                    CONTAINS THE SOLUTION OF THE EQUATIONS.
```

```
C           CM     - ONE DIMENSIONAL MATRIX CONTAINING THE ELEMENTS OF
C                    THE M BY M COEFFICIENT MATRIX.  (DESTROYED)
C           M      - THE NUMBER OF EQUATIONS IN THE SYSTEM.
C           EPS    - AN INPUT CONSTANT WHICH IS USED AS RELATIVE
C                    TOLERANCE FOR TEST ON LOSS OF SIGNIFICANCE.
C           IER    - RESULTING ERROR PARAMETER CODED AS FOLLOWS
C                    IER= 0 - NO ERROR.
C                    IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR
C                             PIVOT ELEMENT AT ANY ELIMINATION STEP
C                             EQUAL TO 0.
C                    IER= K - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-
C                             CANCE INDICATED AT ELIMINATION STEP K+1,
C                             WHERE PIVOT ELEMENT WAS LESS THAN OR
C                             EQUAL TO THE INTERNAL TOLERANCE EPS TIMES
C                             ABSOLUTELY GREATEST ELEMENT OF MATRIX CM.
C           MM     - M*M
C
C        REMARKS
C           INPUT MATRICES RHS & CM ARE ASSUMED TO BE STORED COLUMNWISE
C           IN M AND M*M SUCCESSIVE STORAGE LOCATIONS. ON RETURN, RHS
C           IS THE SOLUTION MATRIX. THE PROCEDURE GIVES RESULTS IF THE
C           NUMBER OF EQUATIONS M IS GREATER THAN 0 AND PIVOT ELEMENTS
C           AT ALL ELIMINATION STEPS ARE DIFFERENT FROM 0. HOWEVER,
C           THE WARNING IER=K (IF GIVEN) INDICATES POSSIBLE LOSS OF
C           SIGNIFICANCE. IN CASE OF A WELL SCALED MATRIX CM AND
C           APPROPRIATE TOLERANCE EPS, IER=K MAY BE INTERPRETED THAT
C           MATRIX CM HAS THE RANK K. NO WARNING IS GIVEN IN CASE M=1.
C
C        SUBROUTINES AND FUNCTION SUBROUTINES REQUIRED
C           NONE
C
C        METHOD
C           SOLUTION IS DONE BY MEANS OF GAUSS ELIMINATION WITH
C           COMPLETE PIVOTING
C
C     -----------------------------------------------------------------
C
      DIMENSION CM(625),RHS(50)
      MM = M*M
C
C     SEARCH FOR GREATEST ELEMENT IN MATRIX CM
C
      IER=0
      PIV=0.
      DO 3 L=1,MM
        TB=DABS(CM(L))
        IF(TB-PIV) 3,3,2
 2      PIV=TB
        I=L
 3    CONTINUE
      TOL=EPS*PIV

C**   CM(I) IS PIVOT ELEMENT.  PIV CONTAINS THE ABSOLUTE
C**   VALUE OF CM(I).  START ELIMINATION LOOP.

      LST=1
      DO 17 K=1,M
C**     TEST ON SINGULARITY
        IF(PIV) 23,23,4
 4      IF(IER) 7,5,7
```

```
 5      IF(PIV-TOL) 6,6,7
 6      IER=K-1
 7      PIVI=1./CM(I)
        J=(I-1)/M
        I=I-J*M-K
        J=J+1-K

C**     I+K IS ROW-INDEX, J+K IS COLUMN INDEX OF PIVOT ELEMENT.
C**     PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE.

        DO 8 L=K,M,M
          LL=L+I
          TB=PIVI*RHS(LL)
          RHS(LL)=RHS(L)
          RHS(L)=TB
 8      CONTINUE

C**     IS ELIMINATION TERMINATED?

        IF(K-M) 9,18,18

C**     COLUMN INTERCHANGE IN MATRIX CM

 9      LEND=LST+M-K
        IF(J) 12,12,10
10      II=J*M
        DO 11 L=LST,LEND
          TB=CM(L)
          LL=L+II
          CM(L)=CM(LL)
          CM(LL)=TB
11      CONTINUE

C**     ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX CM

12      DO 13 L=LST,MM,M
          LL=L+I
          TB=PIVI*CM(LL)
          CM(LL)=CM(L)
          CM(L)=TB
13      CONTINUE

C**     SAVE COLUMN INTERCHANGE INFORMATION

        CM(LST)=J

C**     ELEMENT REDUCTION AND NEXT PIVOT SEARCH

        PIV=0.
        LST=LST+1
        J=0
        DO 16 II=LST,LEND
          PIVI=-CM(II)
          IST=II+M
          J=J+1
          DO 15 L=IST,MM,M
            LL=L-J
            CM(L)=CM(L)+PIVI*CM(LL)
            TB=DABS(CM(L))
            IF(TB-PIV) 15,15,14
```

```
14          PIV=TB
            I=L
15        CONTINUE
        DO 16 L=K,M,M
          LL=L+J
          RHS(LL)=RHS(LL)+PIVI*RHS(L)
16      CONTINUE
        LST=LST+M
17    CONTINUE

C**   END OF ELIMINATION LOOP.  BEGIN
C**   BACK SUBSTITUTION AND BACK INTERCHANGE.

18    IF(M-1) 23,22,19
19    IST=MM+M
      LST=M+1
      DO 21 I=2,M
        II=LST-I
        IST=IST-LST
        L=IST-M
        L=CM(L)+0.5
      DO 21 J=II,M,M
        TB=RHS(J)
        LL=J
        DO 20 K=IST,MM,M
          LL=LL+1
          TB=TB-CM(K)*RHS(LL)
20      CONTINUE
        K=J+L
        RHS(J)=RHS(K)
        RHS(K)=TB
21    CONTINUE
22    RETURN

C**   ERROR RETURN

23    IER=-1
      RETURN
      END
```

```
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lection? er60664
02-88ER60664             ROGERS & ASSOC ENG CORP                 CO 03/31/1988
                  BV     SALT LAKE CITY                          UT 01/31/1991
                  BV     SWIERCZEK, CHRIS                        90 02/07/1991
55,968                   MCCAMMON HELEN                          CH HA0203010   Y
4,000                    Nielson, K.K.; Rogers, V                D  KP0203000
UNIFYING THEORY OF RADON GENERATION & TRANSPORT IN POROUS MEDIA

RHF SEARCH:ER60664 ENTRIES:6

try     Report Number                                  Date   Type   UC Cat  Order No.
    DOE/ER/60664--T2                                  890500 C/10/88  407   DE89013601
    DOE/ER/60664--3                                   890000 C/02/89  402   DE90011420
    DOE/ER/60664--T2                                  890500 T/05/89  407   DE89013601
    DOE/ER/60664--T2                                  890500 T/05/89  407   DE89013601
    DOE/ER/60664--5                                   900400 A/89/90  407   DE90011418
~~    DOE/ER/60664--4                                   900000 C/02/90  402   DE90011419~~
    DOE/ER/60664--6                                   910200 F/88/91  402   DE91009044

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er number 910425-140340-168     -001-001
age 1      set 9 with 2 of 2 items


 Item    1
IS ACCESSION NUMBER   1002800
IS DATE               890721
SSIFICATION           U
ILABLE AT GPO         Y
ORT NUMBER            DOE/ER/60664--T2
TRACT NUMBER          FG02-88ER60664
REV CONTRACT NO       ER60664
 AWARDING OFFICE CODE 02
ORT TYPE/FREQUENCY    C/10/88 ;T/05/89
D COPY DISTRIB.       0: No Stock Copies
IBILITY CODE          1: Portions Illegible
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ROFICHE DISTRIB.      4: Unlimited
DOR ID                131880-0001-4
TRACTOR               ROGERS & ASSOC ENG CORP
ER NUMBER             DE89013601
KED REPORT NUMBER     DE;DOE;DOEER;DOEER60664;DOEER60664T2
LICATION DATE         890500
 ENTRY DATE           890616
ILABILITY             NTIS, PC A03;OSTI
TRIBUTION CATEGORY    407
UMENT ORIGIN          DOE
ILABILITY CODE        1
IUM                   H
IENCE CODE            01
OUNCEMENT CODE        EDB;ERA;ETD;INS;NTS
IES RECEIVED          1

 Item    2
IS ACCESSION NUMBER   991519
IS DATE               890328
IS REPLACED DATE      890622
CHECK NUMBER          89000021382
UMENT TYPE            R
SSIFICATION           Uncl
ILABLE AT GPO         Y
ORT NUMBER            DOE/ER/60664--T2
ONDARY REPORT NO      RAE--8810-1
TRACT NUMBER          FG02-88ER60664
REV CONTRACT NO       ER60664
 AWARDING OFFICE CODE 02
ORT TYPE/FREQUENCY    T/05/89
D COPY DISTRIB.       0: No Stock Copies
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der number 910425-140340-168 -001-001
age 2 set 9 with 2 of 2 items

-INIT. OFFICE CODE	ER
ROFICHE DISTRIB.	4: Unlimited
DOR ID	131880-0001-4
TRACTOR	ROGERS & ASSOC ENG CORP
ER NUMBER	DE89013601
KED REPORT NUMBER	DOE;DOEER;DOEER60664;DOEER60664T2;RAE;RAE88101
LICATION DATE	890500
ENTRY DATE	890602
ROFICHE AVAIL DATE	890622
ITATION CODE	UNL
ILABILITY	NTIS, PC A03/MF A01 - OSTI
TRIBUTION CATEGORY	407
UMENT ORIGIN	DOE/ER
ILABILITY CODE	1
IUM	H
IENCE CODE	01
OUNCEMENT CODE	EDB;ERA;ETD;INS;NTS
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