A COMPARISON OF WEIBULL AND β_{Ic} ANALYSES OF TRANSITION RANGE DATA*

D. E. McCabe

Metals and Ceramics Division
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37831-6151

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ABSTRACT

Specimen size effects on K_{Jc} data scatter in the transition range of fracture toughness have been explained by extremal (weakest link) statistics. In this investigation, compact specimens of A 533 grade B steel were tested in sizes ranging from 1/2TC(T) to 4TC(T) with sufficient replication to obtain good three-parameter Weibull characterization of data distributions. The optimum fitting parameters for an assumed Weibull slope of 4 were calculated. Extremal statistics analysis was applied to the 1/2TC(T) data to predict median K_{Jc} values for 1TC(T), 2TC(T), and 4TC(T) specimens. The distributions from experimentally developed 1TC(T), 2TC(T), and 4TC(T) data tended to confirm the predictions. However, the extremal prediction model does not work well at lower-shelf toughness. At -150°C the extremal model predicts a specimen size effect where in reality there is no size effect.

Another model that has potential for dealing with data scatter effects in the transition range is the Irwin β_c - β_{Ic} relationship. This model uses breakdown in constraint as the argument for specimen size effects and suggests that data sets can be transposed from one size to another by operating on each individual datum with the following equation:

$$K_{Ic} = K_{Jc}\sqrt{\beta_{Ic} / \beta_c} \ .$$

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Both models predict about the same distributions for specimens larger than 1TC(T) and only the extremal statistical model can correctly predict the smaller specimen distribution. With the β_c - β_{Ic} relationship, the limitation appears to be that $\beta_c \leq \pi$ must not be exceeded. Therefore, both the statistical and β_{Ic} models have limitations for their use. This study explores these limitations and makes specimen size requirement recommendations on K_{Jc} data.

KEY WORDS

Transition temperature, Weibull, size effects, constraint, cleavage fracture, size requirements.

INTRODUCTION

The fact that section size has an effect on the transition temperature of ferritic steels has been known for several decades, but aside from empirical observations of constraint effects [1,2], no rationale in the form of analytically based models had been forthcoming until recently. Early application of statistical practices lacked a physical concept that could serve as the basis needed to contribute to an improved understanding of what had already been known empirically. Recently, Weibull fitting of data has been used to characterize data distributions and the principle of extremal statistics (weakest link theory) has been shown to provide the needed size effect model. The accuracy of determinations requires considerable replication of tests, however. In the current project, over 120 compact specimens of A 533B base metal in sizes ranging from 1/2TC(T) to 4TC(T) and A 533B weld metal ranging from 1TC(T) to 8TC(T) have been tested in the transition range with sufficient replication at some of the test temperatures for viable statistical analysis. Hence the new methods that are used to predict trends in median toughness values due to specimen size can be effectively tested. The toughness parameter to be used herein is K_{Jc} which is defined as K_J at onset of cleavage instability, and it is derived by conversion from J-integral at instability, J_c. This paper will evaluate Weibull fitting methods and extremal statistics that are used to predict specimen size effects. An alternative predictive model, the β_{Ic} fracture toughness factor, that is derived from measured values of K_{Jc} and that uses a constraint based argument, will also be reported.

Test Data

The test temperatures and numbers of specimens for the various specimen sizes of A 533B steel are given in Table 1. All specimens were proportionally dimensioned compacts with relative initial crack size, a/W, nominally at 0.5. Data scatter observed here is shown in Fig. 1. The dependence of data scatter on specimen size is most evident at -75°C. Specimens of small thickness tend to lose constraint earlier when entering the transition range because the volume of cross-slip type of plastic deformation relative to the material thickness controls the transition toughness development rate. Larger specimens require more ductility for proportional cross-slip, and essentially similar data scatter characteristics are delayed to higher temperatures. It can be noted also that specimen size effects do not exist on the lower shelf and tend to vanish again at high toughness levels on the transition curve. To add evidence for the data scatter characteristics of large specimens at high toughness, test data from the Fifth Irradiation Series at ORNL [3] were added herein. There were two weld metals of identical chemistries except for copper content, Table 2. Extremal statistics had been applied in that project because there was a need for making specimen size predictions.

Extremal Statistics

An application of extremal statistics to transition temperature behavior was developed in 1979 by Landes and Shaffer [4]. Using a two-parameter Weibull model, they demonstrated how data from

1T compact specimens, 1TC(T), could be used to characterize the fracture toughness distribution of larger 4T compact specimens, 4TC(T). The scatter in fracture toughness between replicate specimens was proposed to be governed by occasional weak points or sources for brittle cleavage crack initiation that are distributed randomly throughout the microstructure. Specimens with through-thickness cracks have zones of concentrated stress at the crack tip, the volumes of which are proportional to the specimen thickness. Therefore, the probability for imperfections of critical size to cause cleavage fracture is relatable to specimen thickness. The mean fracture toughness was projected to be lower and the standard deviation smaller for larger specimens. The fracture toughness was expressed in terms of J_c and the distribution for the baseline data was fitted to the following two-parameter Weibull model:

$$P_{f1} = 1 - \exp\left[-(J/\theta_1)^b\right] \tag{1}$$

Where P_{f1} is the probability that an arbitrarily chosen 1TC(T) specimen will have $J_c < J$, θ_1 is a scale parameter ($J_c = \theta_1$ when $P_{f1} = 0.632$), and b is the Weibull slope.

The fitting constants determined from the data are θ_1 and b. In using this model, it is assumed that the constraint is equal over all specimen sizes. Prior experience indicated that constraint does not vary sufficiently in compact specimens when the remaining ligament length is equal to or less than the specimen thickness [5]. Then if one were to test 4TC(T) specimens, the probability for J_c instability prior to reaching the toughness level J, is given by:

$$P_{f4}=1-\exp\left[-(J/\theta_4)^b\right] \qquad (2)$$

Where $\theta_4 = \theta_1/(N)^{1/b}$, and

$N = (4/1)$

The above two-parameter model had predicted mean J_c for 4TC(T) specimens of ASTM A 471 steel quite accurately at two of three test temperatures [4]. The Weibull slope (on J_c) was determined to be b = 5. Later experience suggests that they had an insufficient amount of data replication to obtain accurate Weibull slopes. Also a weakness not recognized was that the two-parameter extremal model will tend toward zero fracture toughness as the specimen size tends to infinity. Therefore, in a later publication [6] the weakness was corrected by introducing a three-parameter Weibull model. This has a lower-bound toughness value, J_{min}, which defines a lower limiting toughness for specimens of infinite thickness. The toughness parameter is expressed as (J_c - J_{min}) and the denominator in Eq. (1) becomes (θ_1 - J_{min}). Figure 2 was used to illustrate the trial and error procedure used to identify an optimum J_{min} value on 1/2TC(T) specimens of A 508 steel. The general form is:

$$P_{f1/2}=1-\exp\left\{-\frac{(J-J_{min})}{(\theta-J_{min})}\right\}^b \qquad (3)$$

Seven examples of three-parameter determinations gave four apparently reasonable J_{min} results for lower-bound toughness predictions. The three poor predictions were from data sets that had only four to seven datum, and these were far too few to expect a good measure of the nonlinearity of a data population.

In current publications, it is more common to see three-parameter Weibull fitting to K_{Jc} data, where J_c is first calculated and then converted to K_{Jc} using:

$$K_{Jc}=\sqrt{J_c E} \tag{4}$$

Weibull Constant Fitting Methods

Wallin [7] has performed Weibull analyses, using K_{Jc} data on numerous similar material data sets, large and small, and has concluded that toughness distributions generally show fixed Weibull slope of 4 and that K_{min} also tends to be constant at about 20 MPa√m, independent of test temperature. Implicit in this argument is that all J_c distributions should have a slope b = 2; noting that K is proportional to the square root of J-integral. Brought into question is the initial finding of Landes and Shaffer where slope, b, was 5 for their J_c data on A 471. The Wallin observation has been generally supported by the work of others [8,9] who have shown that a slope of 4 has a basis in micromechanics theory. The assertion that K_{min} is constant is less secure from a fundamental standpoint. Assuming K_{min} has physical meaning as a lower-bound toughness, some have suggested that lower bound K_{Ic} or K_{Ia} values obtained from ASME Code regulations could be used [10]. On the other hand, the best fits to the Weibull model are usually obtained with K_{min} values considerably lower than those indicated by the code curves. Figures 3 and 4 are representative of what results from seeking the best K_{min} values using the base metal data from the test matrix of Table 1. There are four specimen sizes and four test temperatures represented. The two fitting techniques used were (1) adjusting all three Weibull constants to get an optimum linear fit to the data and (2) Weibull slope set to 4 and then finding K_{min} for optimum fit. Table 3 lists the fitting constants and correlation coefficients of the two methods, and it appears that the fundamentally justified Weibull slope of 4 can provide a suitable representation of the distributions in most cases. One rule that was used, however, is that K_{min} was never allowed to be a negative value. Because of this, a few slopes were only near to 4. There were two cases where good linearity and a Weibull slope of 4 were not entirely compatible, and these are shown in Figs. 5 and 6. Both cases had some data at relatively high toughness conditions for the size of specimen used, and their Weibull plots suggest bilinearity with an apparent break point at 125 MPa√m for 1/2TC(T) and 192 MPa√m for 1TC(T).

Prediction of Size Effects

The density function for the 1/2T compact specimens was used to predict median K_{Jc} values for the Weibull fits to 1T, 2T, and 4T compact specimen data generated at the same test temperature (-75°C). See Fig. 7 and Table 4. There are two sets of determinations in Table 4. In both cases a fixed Weibull slope of 4 was used, with K_{min} variable in one case and fixed at 20 MPa√m in the other case. The magnitude of median shift predicted for increased specimen size was reasonable in both cases.

The same exercise applied to tests made at -150°C (Table 4) was not as satisfactory. A specimen size effect was expected, but the distributions fitted to real data indicated no effect. The scatter bands of data for all tests made on A 533B plate on all specimen sizes and for all test temperatures was shown in Fig. 1. Note that at -150°C, the smallest specimens tested [1/2TC(T)] had both the highest and lowest K_{Jc} toughness values. Extremal statistics erroneously predicted a size effect because of a breakdown in the weakest link model. This will happen when the size of the imperfection needed to cause cleavage initiation becomes very small such that many cleavage sources exist at all points along the crack tip. Hence there is a need to identify a lower toughness limit below which extremal statistics will not apply. A suggested approach will be addressed in the discussion section.

There can be some difficulty with the application of extremal statistics at the high toughness end of the material transition curve. This was experienced in the Heavy-Section Steel Irradiation Program in the Fifth Irradiation Series [3]. The objective of the experiment was to establish lower-bound K_{Ic} curves on two A 533B weld metals of different copper contents. Of special interest was the shift and potential change in shape of the lower bound due to irradiation damage. Four 8T compact specimens (two of each copper content) were to be tested at the highest possible toughness level that would be consistent with the ASTM validity requirements on K_{Ic}. It was determined that the maximum K_{Jc} would be a valid K_{Ic} at 150 MPa√m, and a temperature where this was likely to happen was chosen using smaller specimens. The sequence used was to first test four 2TC(T) specimens at the selected temperature to provide a baseline Weibull distribution for predicting the 8TC(T) distribution. One of the two plots made is shown in Fig. 8. Because median K_{Jc} was predicted to be 150 MPa√m, it was presumed that the chance of obtaining valid K_{Ic} should be 1 in 2 for each large specimen tested. Nevertheless, none of the four large specimens gave valid K_{Ic}. The trend indicated with 4TC(T) and 6TC(T) specimens gave no evidence that there might have been a breakdown in the extremal assumption, but the high toughness position on the transition curve evidently had broadened the scatter band width for large specimens enough to make it difficult to assure an aim value. Hence, the utility of these predictions of size effects may be limited to a transition temperature window in the lower transition range.

β_c - β_{Ic} Fracture Toughness Correlation

Another perspective on the K_{Jc} data scatter phenomenon is to consider that the early (lower temperature) increase in K_{Jc} data scatter of small specimens is due to the lower constraint. Smaller specimens tend to respond nonlinearly with less crack tip plastic deformation, readily losing constraint in the crack tip region. Larger specimens require proportionately more cross slip, and similar data scatter is delayed to higher temperatures. To relate high and low constraint toughness, Irwin [11] had developed a semiempirical relationship based on the behavior of high strength metallic materials. Merkle has investigated the potential of this relationship for use with the structural steels that are used in pressure vessels. It is as follows:

$$\beta_c = \beta_{Ic} + 1.4\beta_{Ic}^3 \tag{5}$$

Where $\beta_c = (1/B)(K_{Jc}/\sigma_{ys})^2$, and

$\beta_{Ic} = (1/B)(K_{Ic}/\sigma_{ys})^2$

The β_c value determined for each individual datum is picked out of a family of replicate tests. An estimate of K_{Ic} is made on each one, thereby establishing a family of K_{Ic} distributions. The procedure is to use β_c in Eq. (5) and determine the corresponding β_{Ic} either by iteration or by using a preformulated solution of the cubic equation. Then K_{Ic} is determined using:

$$K_{Ic} = K_{Jc}\sqrt{\frac{\beta_{Ic}}{\beta_c}} \tag{6}$$

Three-parameter Weibull can then be fitted to the K_{Ic} distribution or to interpolated values for intermediate specimen sizes. Equation (5) is used to interpolate in all cases. Figure 9 shows K_{Jc} data selected at three toughness levels from within the actual data sets for 1TC(T), 2TC(T), and 4TC(T) specimens (A 533 grade B base metal tested at -75°C). These are the solid data points in Fig. 9. The interpolation and extrapolation by Eq. (5) of the three specifically selected toughness levels are shown as open data points. The solid line represents the toughness trend over varied thickness that is implied by Eq. (5). Irwin had cautioned that the semiempirical relationship should not be used when β_c is greater than π and this limit is denoted in Fig. 9 as a dashed line. This limitation required that for the toughness of A 533B at -75°C, data from 1T or larger compact specimens must be used to develop the baseline Weibull plot. Figure 10 shows predictions of density functions from use of the 1TC(T) baseline data. Table 5 compares the predicted size effect on median K_{Jc} obtained from the density functions to those from the extremal statistical model. Again, this is using the 1TC(T) specimen data as baseline. Note that the beta method projects essentially the same result except for 1/2TC(T) where most of the projected values have β_c much greater than π.

DISCUSSION

The practical application for this work is to learn how data taken from small fracture mechanics type specimens can be used to infer the fracture toughness performance in full-scale structures. The general format of data development limitations is illustrated schematically in Fig. 11. This is for 1/2T compacts made of A 533B. From evidence in Figs. 5 and 6, it appears that constraint is controlled sufficiently for Weibull fitting and extremal statistics predictions for β_c up to 2π. The low toughness limitation for extremal statistics has not been determined, but a practical lower limit might be where $\beta_c = 0.4$. These limits would apply to distributions where a high percentage of the K_{Jc} values within the baseline distribution would satisfy the suggested criteria. Figure 11 indicates that the semiempirical β_c - β_{Ic} relationship might be suitable for toughness where β_c is equal to π or less. This model tends to plateau along with test data at the lower plateau of transition toughness, and median K_{Ic} can be more reasonably determined with this model.

If it can be established with reasonable confidence that Weibull slope is almost always 4 for most structural steels, and that K_{min} = 20 MPa$\sqrt{m}$ is a reasonable compromise value, then the number of small specimens needed to establish a reasonable baseline Weibull distribution is highly reduced because only the scale parameter need be determined. Perhaps only a half-dozen specimens would suffice. Such a practice would only be suitable for establishing trends in mean toughness, however, because the tails of the fitted distribution curves would be quite unreliable and not usable to estimate lower-bound values. The utility would be for the determination of median transition curve shift due to irradiation damage effects.

CONCLUSION

This paper has used selected data from two projects that were designed to study the fracture mechanics aspects of transition temperature behavior of structural steels. It is concluded that statistical methods and a constraint based model can be incorporated into an overall plan to deal with size effects. Transition temperature shifts can be predicted for materials that are used in large structures using small surveillance size specimens. The establishment of lower-bound K_{Ic} curves by testing just a few small specimens is not suggested at the present time.

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DISCLAIMER

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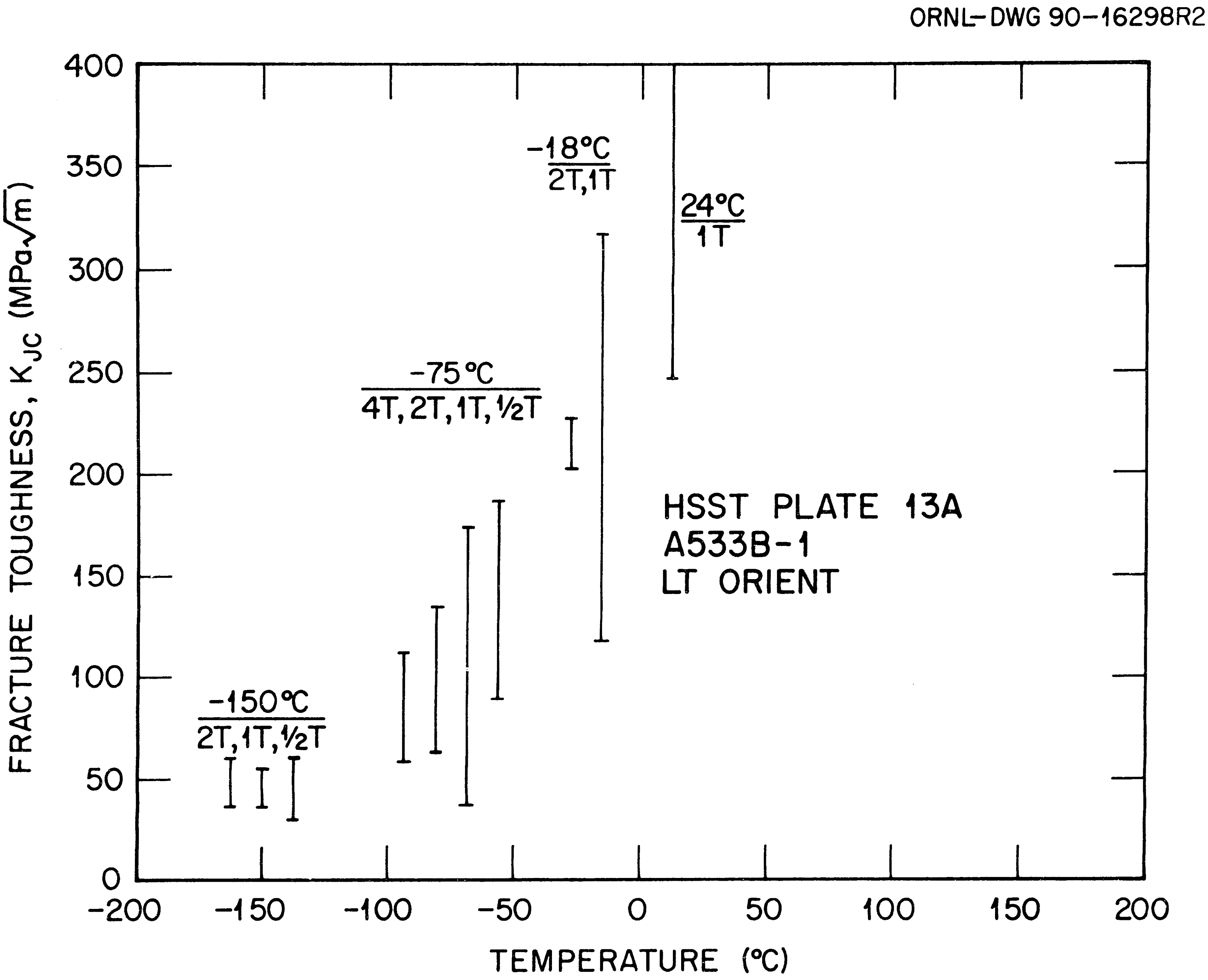
LIST OF FIGURES

Fig. 10. Distribution functions for 1/2TC(T), 2TC(T), and 4TC(T) specimens predicted from 1TC(T) K_{Jc} data and size effect prediction by constraint effects assumption.

Fig. 11. Zone of application for models that predict size effects in the lower transition range of fracture toughness.



ORNL-DWG 90-16298R2
FRACTURE TOUGHNESS, K_JC (MPa√m)
TEMPERATURE (°C)
400
350
300
250
200
150
100
50
0
-200
-150
-100
-50
0
50
100
150
200
-18°C / 2T,1T
24°C / 1T
-75°C / 4T, 2T, 1T, ½T
-150°C / 2T, 1T, ½T
HSST PLATE 13A
A533B-1
LT ORIENT

Fig. 1.

ORNL-DWG 90-16296

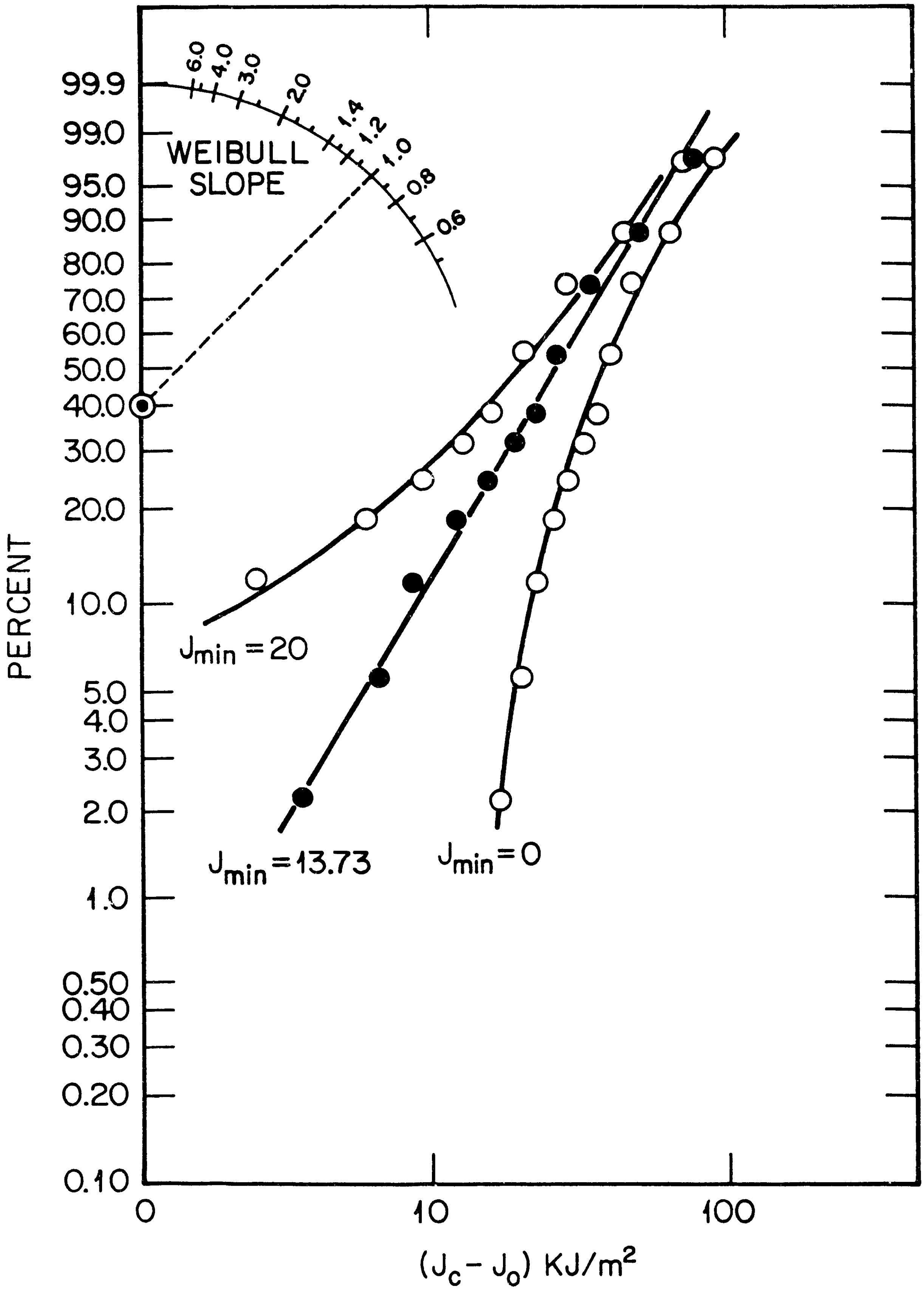


Fig. 2.

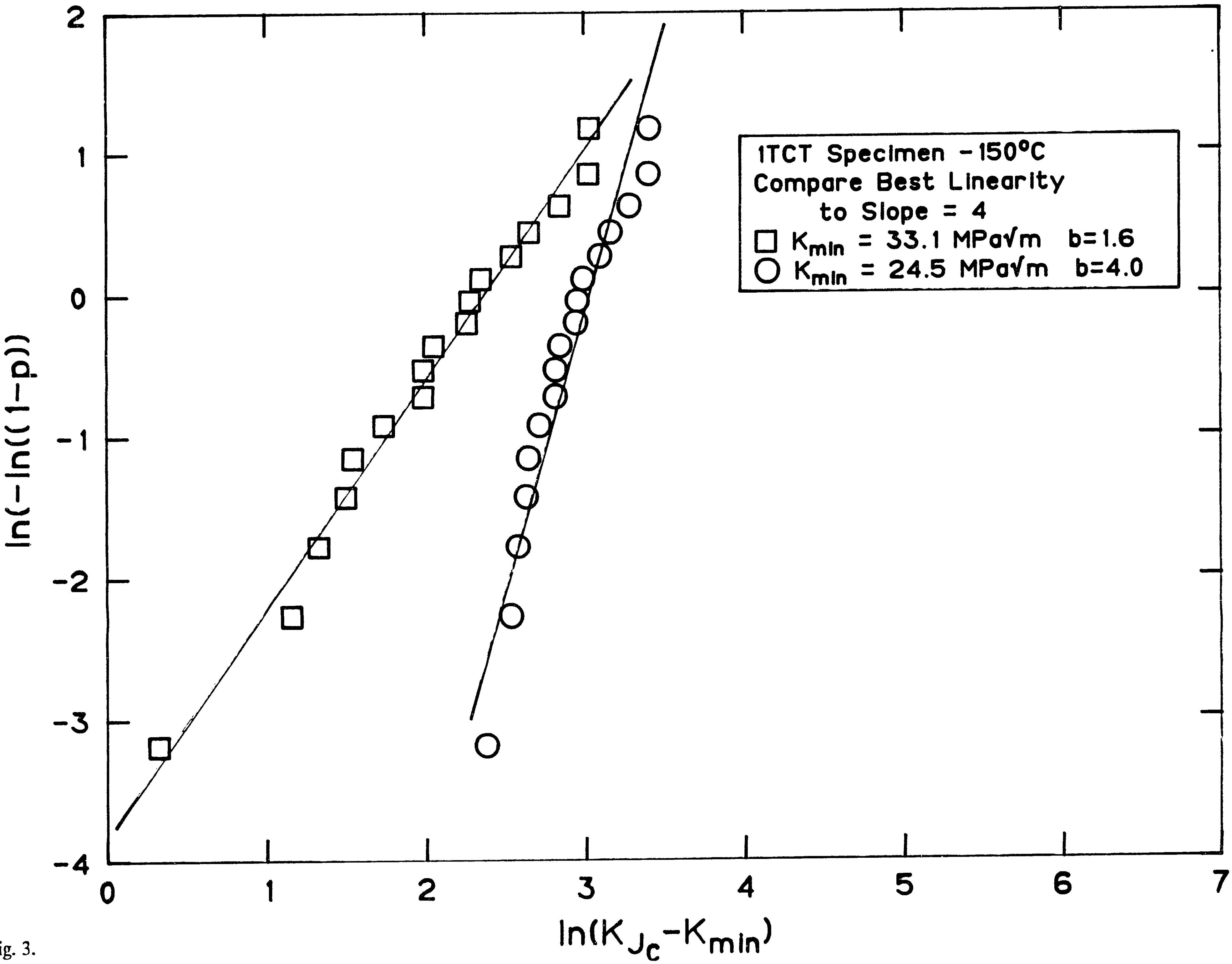

1TCT Specimen -150°C
Compare Best Linearity
to Slope = 4
K_{min} = 33.1 MPa√m b=1.6
K_{min} = 24.5 MPa√m b=4.0
ln(-ln((1-p))
ln(K_{Jc}-K_{min})

Fig. 3.

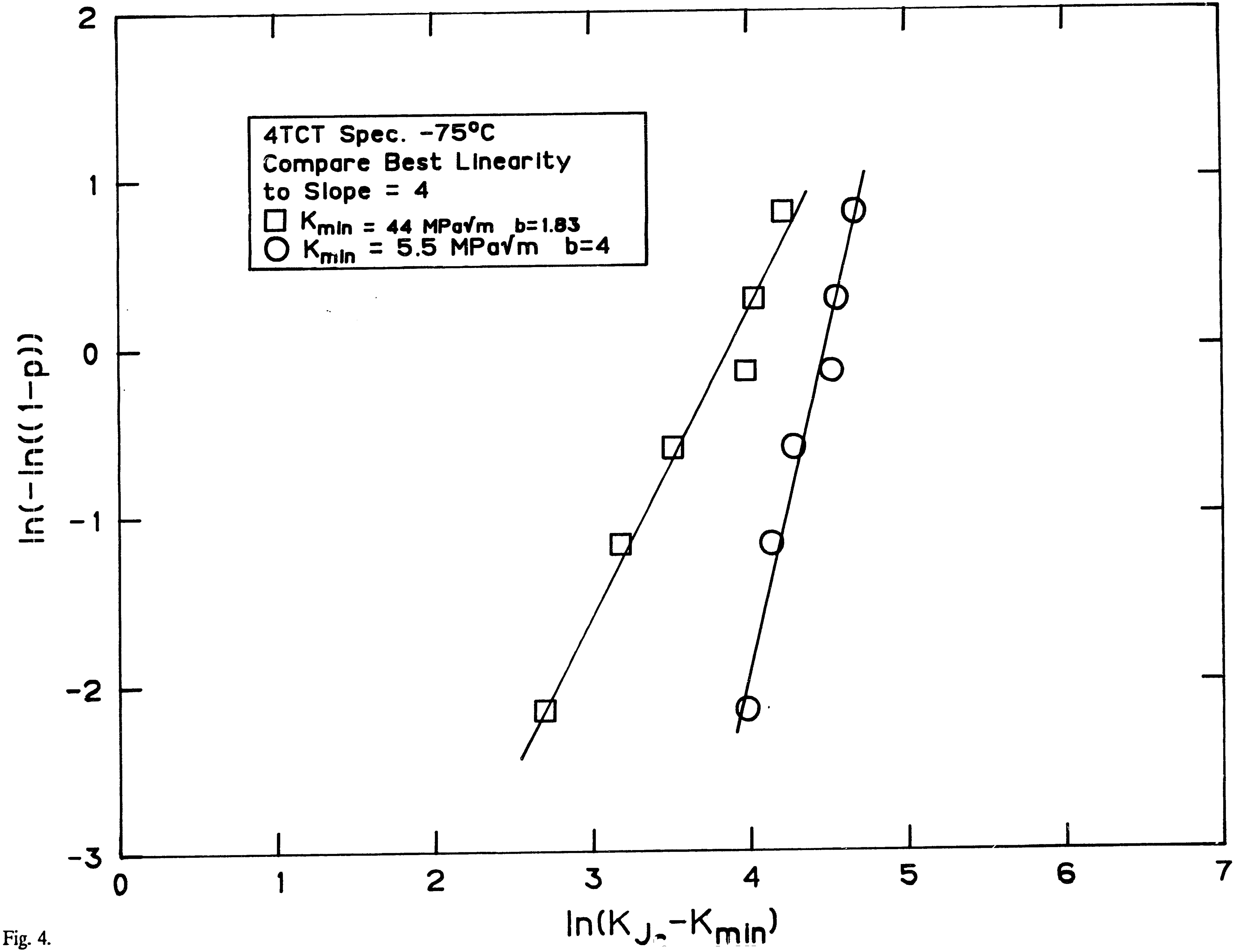
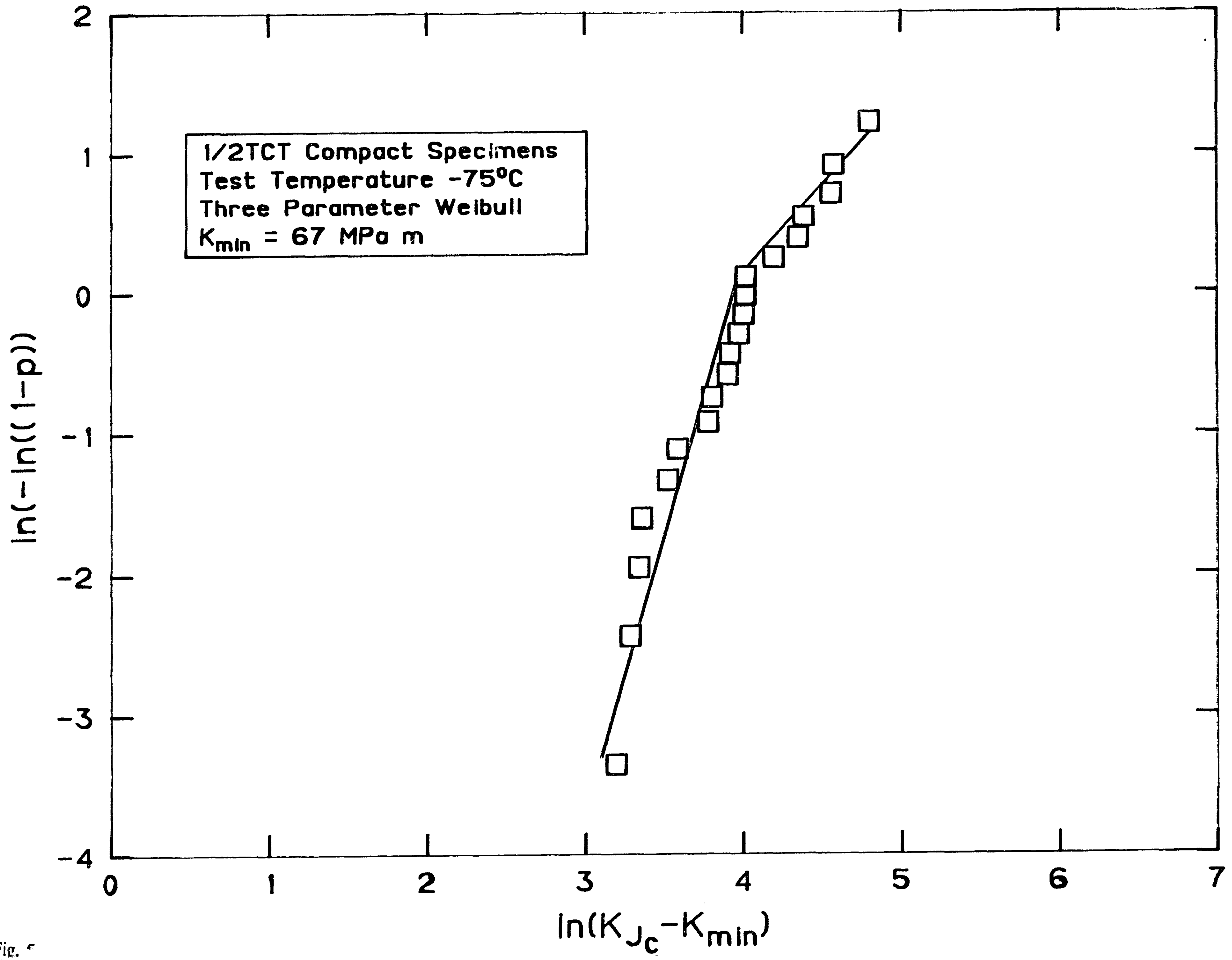

4TCT Spec. -75°C
Compare Best Linearity
to Slope = 4
□ Kmin = 44 MPa√m b=1.83
○ Kmin = 5.5 MPa√m b=4
ln(-ln((1-p))
ln(KJc-Kmin)
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7

Fig. 4.



1/2TCT Compact Specimens
Test Temperature -75°C
Three Parameter Weibull
Kmin = 67 MPa m
ln(-ln((1-p))
ln(KJc-Kmin)
2
1
0
-1
-2
-3
-4
0
1
2
3
4
5
6
7

Fig. 5

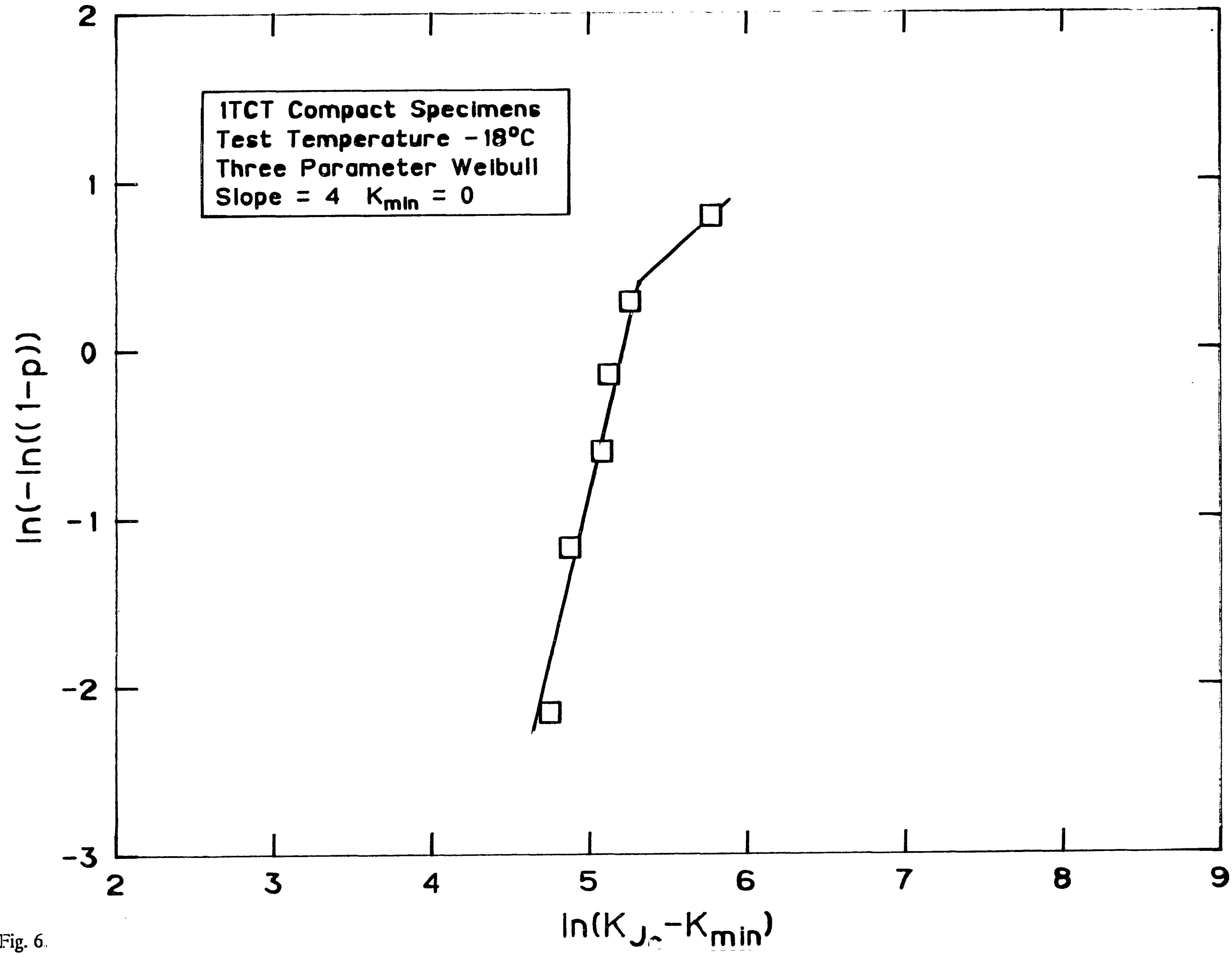
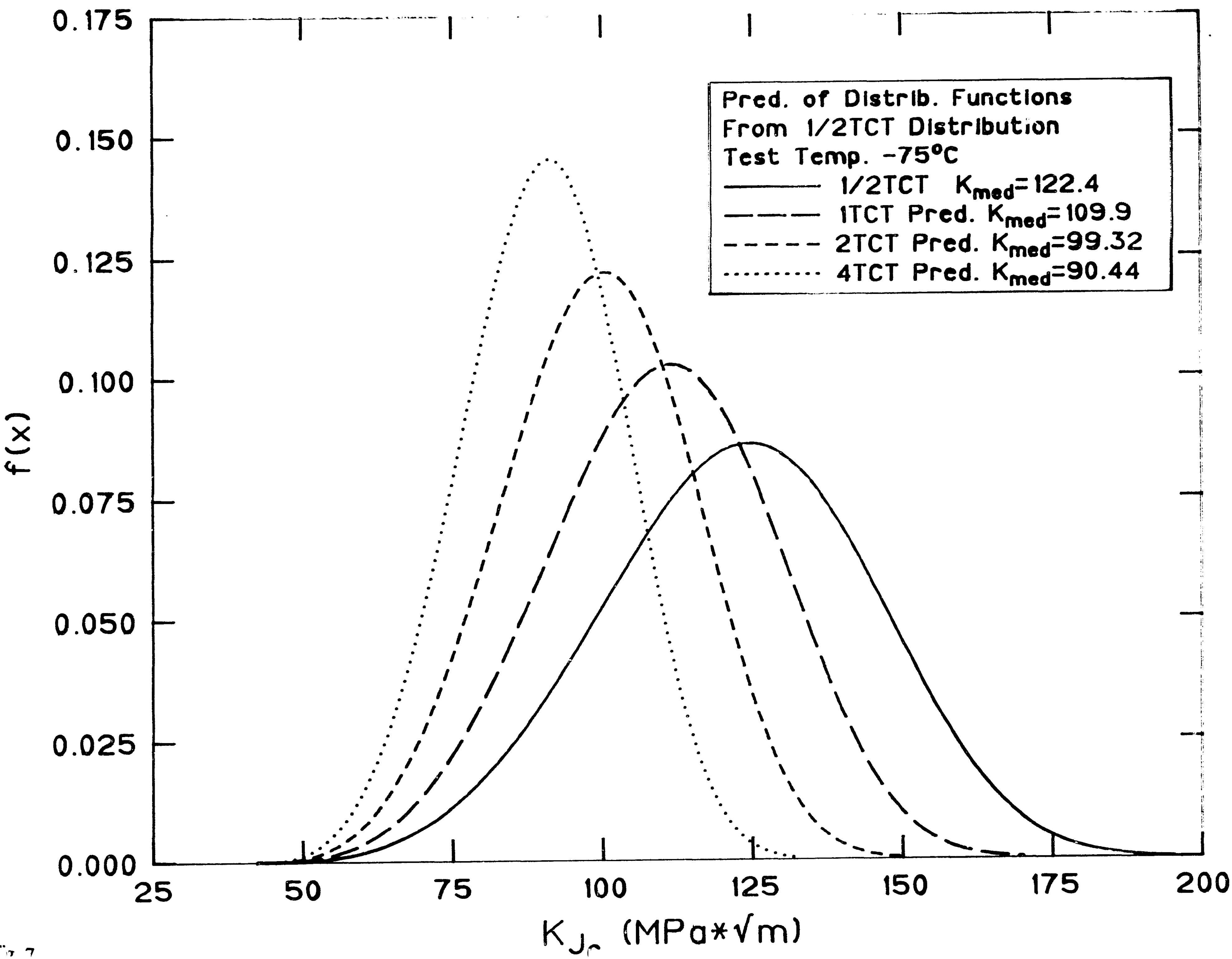

1TCT Compact Specimens
Test Temperature −18°C
Three Parameter Weibull
Slope = 4 Kmin = 0
ln(−ln((1−p))
ln(KJc−Kmin)
2
1
0
−1
−2
−3
2
3
4
5
6
7
8
9

Fig. 6.



Pred. of Distrib. Functions
From 1/2TCT Distribution
Test Temp. -75°C
1/2TCT K_med=122.4
1TCT Pred. K_med=109.9
2TCT Pred. K_med=99.32
4TCT Pred. K_med=90.44
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
f(x)
25
50
75
100
125
150
175
200
K_Jc (MPa*√m)

Fig. 7

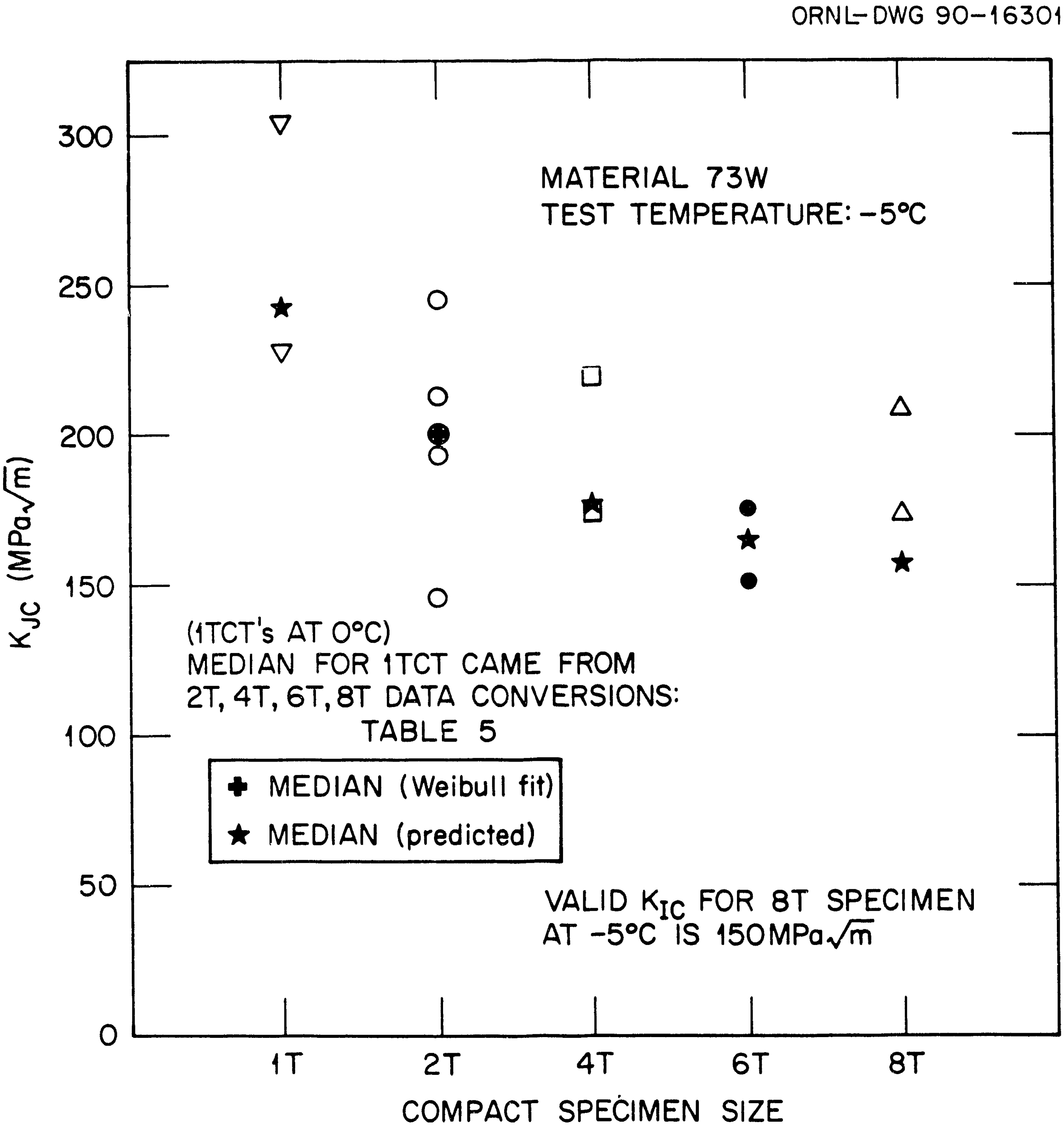

ORNL-DWG 90-16301
MATERIAL 73W
TEST TEMPERATURE: -5°C
(1TCT's AT 0°C)
MEDIAN FOR 1TCT CAME FROM
2T, 4T, 6T, 8T DATA CONVERSIONS:
TABLE 5
MEDIAN (Weibull fit)
MEDIAN (predicted)
VALID KIC FOR 8T SPECIMEN
AT -5°C IS 150MPa√m
KJC (MPa√m)
0
50
100
150
200
250
300
1T
2T
4T
6T
8T
COMPACT SPECIMEN SIZE

Fig. 8.

ORNL-DWG 90-16297R

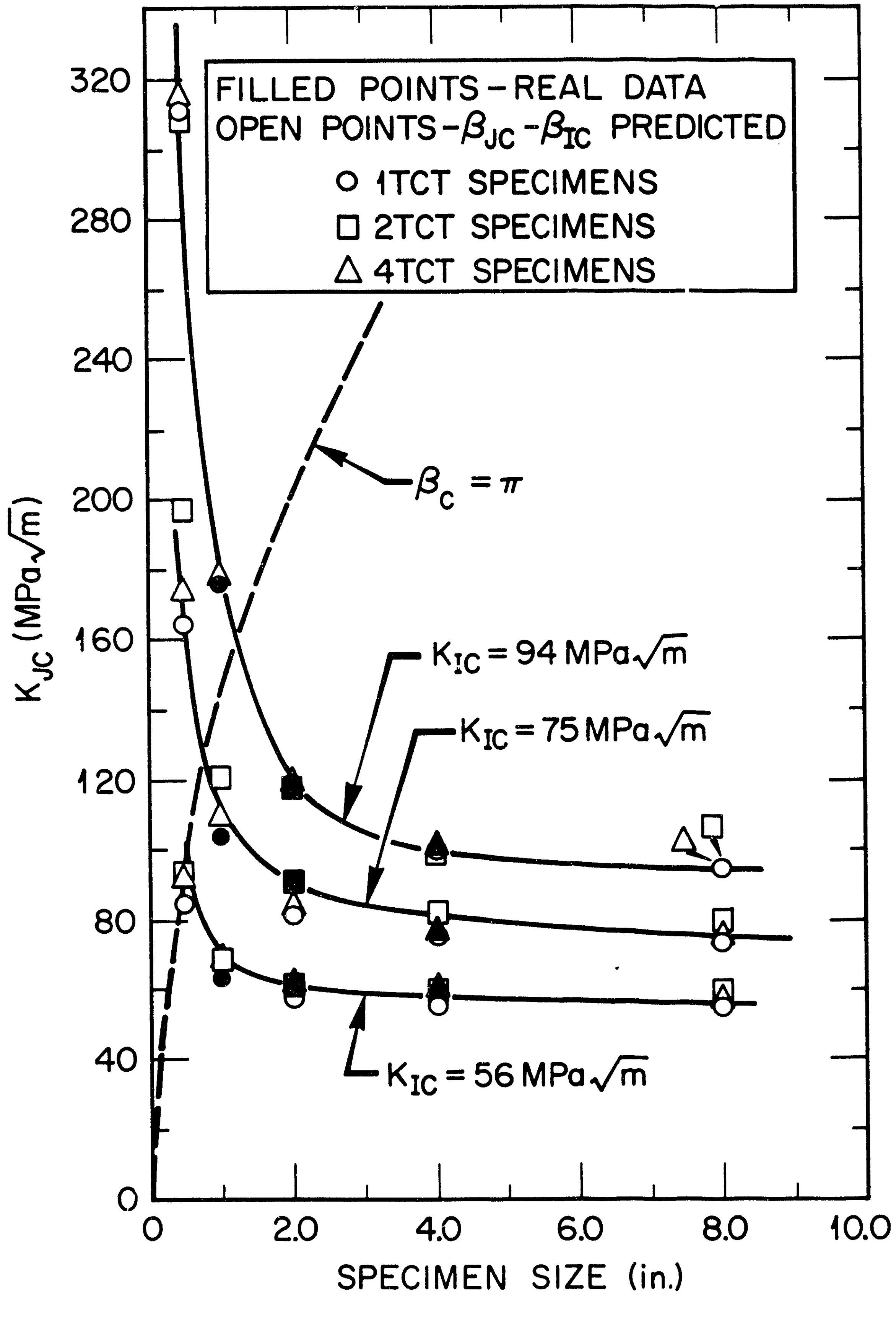


Fig. 9.

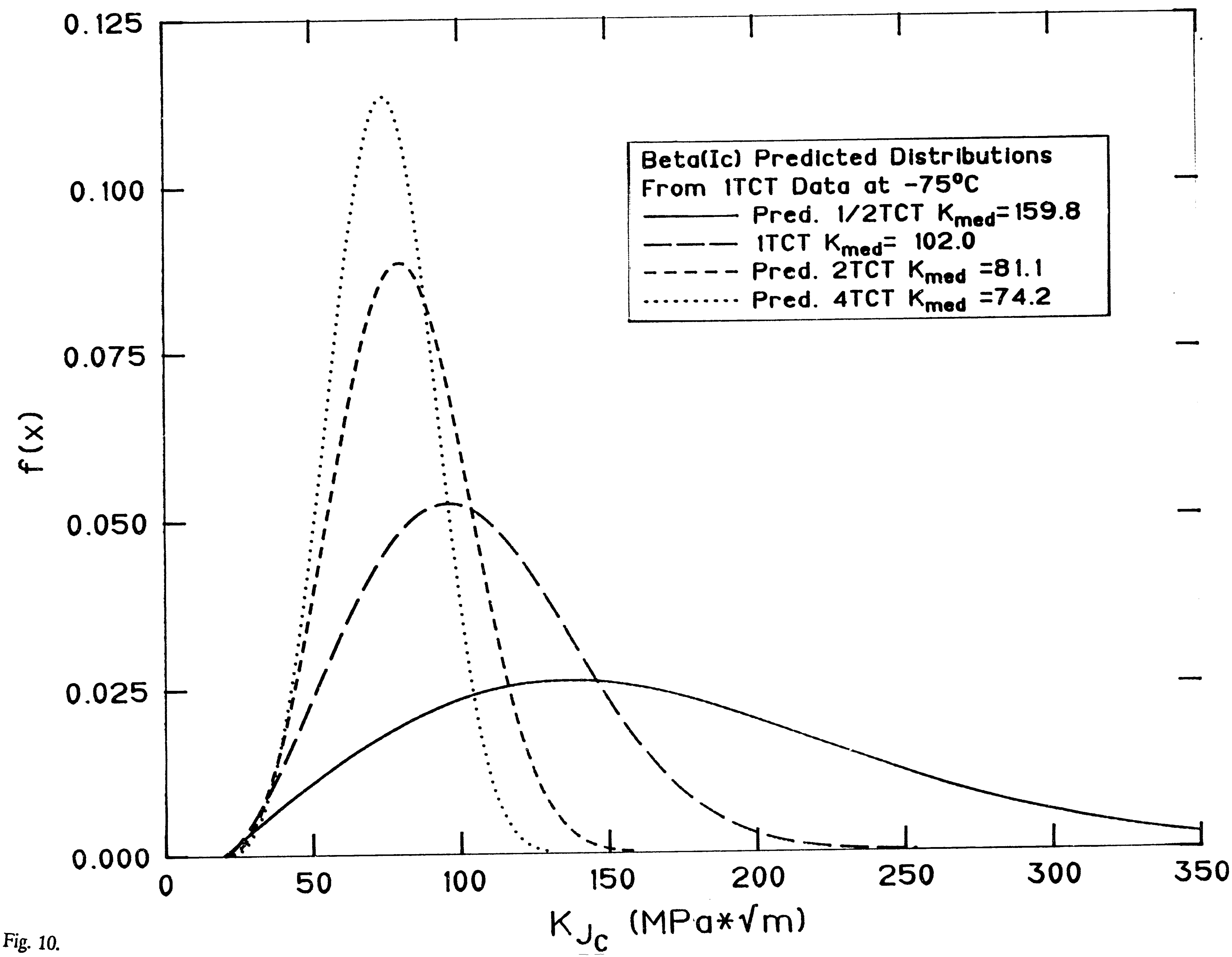
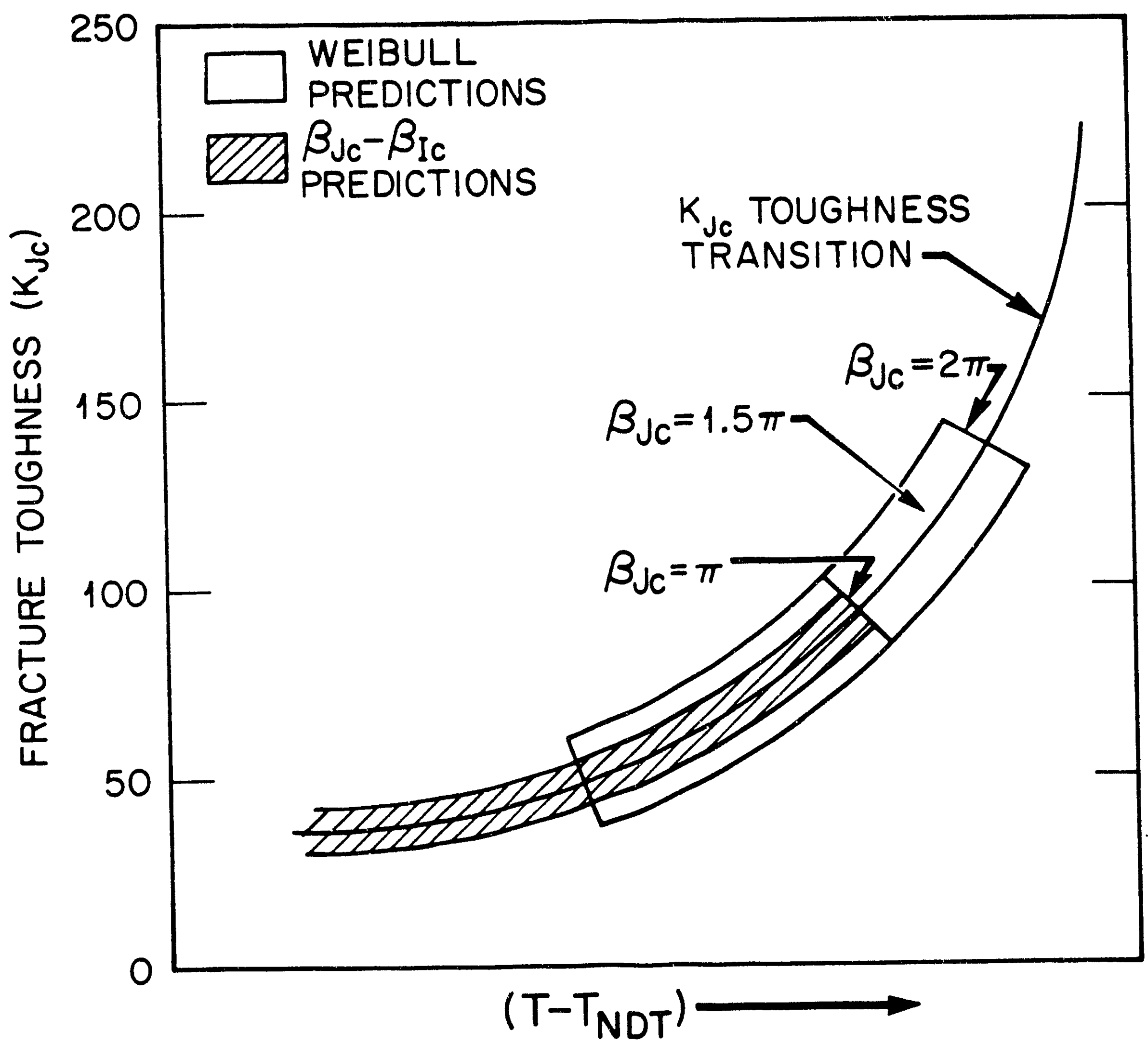

Beta(Ic) Predicted Distributions
From 1TCT Data at -75°C
Pred. 1/2TCT Kmed=159.8
1TCT Kmed= 102.0
Pred. 2TCT Kmed =81.1
Pred. 4TCT Kmed =74.2
f(x)
0.125
0.100
0.075
0.050
0.025
0.000
0
50
100
150
200
250
300
350
KJc (MPa*√m)

Fig. 10.

ORNL-DWG 90-16839

ZONES OF APPLICATION FOR PREDICTIVE MODELS FOR K_{Jc} DATA FROM 1/2 TCT SPECIMENS



ornl

Fig. 11.

Table. 1. Test conditions and number of replicate specimens used in statistical analysis

Material	Test temperature (°C)	Number of specimens					
		1/2TC(T)	1TC(T)	2TC(T)	4TC(T)	6TC(T)	8TC(T)
A 533 grade B Plate 13A	-150	18	17	12			
	-75	20	26	12	6		
	-18		6	2			
	24		5				
A 533 grade B welds							
72W	10			4	2	2	2
73W	-5			4	2	2	2

Table 2. Materials

Yield and tensile strengths of test materials

Material		Strength, MPa (ksi)	
		Yield	Ultimate
A 533 grade B		444 (64.4)	600 (87.0)
A 533 grade B SA weld	72W	499 (72.4)	608 (88.2)
	73W	490 (71.1)	600 (87.0)

Nominal chemical compositions

Material[a]	Composition (wt %)									
	C	Mn	P	S	Si	Cr	Ni	Mo	Cu	V
A 533 grade B Plate 13A	0.25	1.34	0.35[b]	0.040[b]	0.29		0.55	0.52		
A 533 grade B welds:										
72W	0.093	1.66	0.006	0.006	0.044	0.27	0.60	0.58	0.23	0.003
73W	0.098	1.56	0.005	0.005	0.045	0.25	0.60	0.58	0.31	0.003

[a]ASTM specifications for A 533 class 1.
[b]Maximum.

Table 3. Comparison of Weibull fitting parameters for best correlation coefficient

Test temperature (°C)	Size [C(T)]	Three fitting parameters			Fixed slope Two fitting parameters		
		Slope	K_{min}[a]	Correlation coefficient	Slope	K_{min}[b]	Correlation coefficient
-150	1/2T	1.7	25	0.991	4	10.5	0.975
-150	1T	1.6	34	0.993	4	24.5	0.961
-150	2T	3.0	24	0.990	4	19.0	0.998
-75	1/2T	1.1	89	0.991	4	42.5	0.933
-75	1T	3.3	0	0.993	4	0	0.993
-75	2T	4.7	0	0.983	4	13.5	0.983
-75	4T	1.8	44	0.986	4	6.0	0.982
-18	1T	0.9	109	0.988	4	0	0.914
24	1T	3.2	0	0.913	4	0	0.913

[a]K_{min} for best fit with b, K_o, and K_{min} variable.
[b]K_{min} for best fit with K_o and K_{min} variable.

Table 4. Size effect predictions using extremal statistics (Weibull slope of 4, comparing best K_{min} vs fixed K_{min})

Test temperature (°C)	Size [C(T)]	Median K_{Jc} (MPa√m)			
		Fit actual data		Extremal predictions from 1/2TC(T)	
		Best K_{min}	K_{min} = 20	Best K_{min}	K_{min} = 20
-75	1/2T	122.4	124.8		
-75	1T	102.4	98.6	109.9	108.2
-75	2T	102.6	102.1	99.3	94.1
-75	4T	86.4	85.3	90.4	82.3
-150	1/2T	40.6	39.8		
-150	1T	43.4	43.7	33.9	36.7
-150	2T	44.8	44.8	31.9	34.0
-150	4T			28.5	31.8

Table 5. Predicted specimen size effect, comparing extremal statistics and beta methods

Test temperature (°C)	Size [C(T)]	Median K_{Jc} (MPa√m)		
		Fit actual data	Predictions from 1TC(T)	
			Extremal	Beta
-75	1/2T	122.4	121.3	159.8
-75	1T	102.4		
-75	2T	102.6	85.8	81.1
-75	4T	86.4	72.1	74.2
-150	1/2T	40.6	48.2	47.2
-150	1T	43.7		
-150	2T	44.8	39.9	42.6
-150	4T		36.8	42.4

END

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