Productivity and Injectivity of Horizontal Wells

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Department of Petroleum Engineering
Stanford University
Stanford, CA 94305

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Principal Investigator:	Khalid Aziz
Co-Investigator:	Thomas A. Hewett
Research Associate:	Sepehr Arbabi
Administrative Assistant:	Marilyn Smith
Technical Project Manager (DOE):	Thomas B. Reid

Quarterly Report

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Summary of Technical Progress

A number of activities have been carried out in the last three months. A list outlining these efforts is presented below.

- The Ph.D. Dissertation on developing correlations for breakthrough time and post breakthrough behavior of horizontal wells was successfully defended in March. A paper on this work will be presented in the Fifth SPE Latin American and Caribbean Conference to be held in Rio de Janeiro, Brazil in September.

- Multi-lateral wells offer higher productivity than single horizontal wells. However, the increase in their productivity depends on the length and form of the laterals. We have initiated a simulation study to investigate the productivity of multi-lateral wells in terms of their arrangement and positioning.

- The experimental work on using horizontal wells as injectors and producers in a gas injection gravity drainage process is progressing. Experimental setup is near completion and trial runs will follow next.

- Work on generalized gridding methods continued by considering streamline grids. In this approach, gridblock boundaries are aligned along streamlines to better represent flow paths governed by heterogeneities.

- The correct well index for a typical partially penetrating horizontal well could be very different from the classical well index and can be calculated in a semi analytical fashion in which the well pressure is obtained analytically and the well block pressure is computed numerically. In the so called explicit well modeling approach, the need to calculate any well index is eliminated by fine gridding the well. We have studied this second approach and have shown that it can lead to incorrect results.

The last activity listed above is the subject of this quarterly report.

Effects of Grid Systems on Predicting Horizontal Well Productivity (Tasks 1 and 4)

Introduction

When solving the flow equations of a reservoir numerically, the block pressure cannot be assumed to be equal to the wellbore pressure. These two pressures can be related by a numerical, semi-analytical or analytical well model, which gives the appropriate well index. Peaceman [1] has derived expressions for well index under the condition of 2D flow from an isolated well under steady state or pseudo-steady state condition. Babu and Odeh [2, 3] have considered a more general case and have obtained an analytical solution for the steady-state or pseudo-steady state case with an arbitrarily located well in the block, provided the well is along one of the grid lines. Penmatcha *et al.* [4] have extended Babu and Odeh's model to the case of infinite well conductivity, which yields the expected non-uniform flux pattern along the wellbore.

Most current commercial simulators use the Peaceman model in spite of its limitations. Dietrich and Kuo [5] have compared horizontal well performance simulated by a uniform coarse grid with a non-uniform fine grid system using the Peaceman model. They concluded that the well productivity is overpredicted using a coarse grid and the conventional well model proposed by Peaceman. Dietrich and Kuo also applied an explicit modeling technique using a non-uniform grid system, in which the grid was refined around the well to represent the length and the diameter of the horizontal well. They also set the permeability of the wellbore gridblocks at a very high level to establish a negligible pressure gradient along the wellbore. At the same time, a very large well transmissibility connection factor was used for the wellbore at the heel to force the well and the well block pressures to be the same, thus deactivating the well model.

The numerical and analytical solutions presented here show that for the case considered by Dietrich and kuo, well productivity is underpredicted using a coarse grid system with a conventional well index, which is different from the results of Dietrich and Kuo. We also show that the reservoir performance simulated by a non-uniform fine grid can deviate significantly from that simulated by a uniform fine grid and that calculated by the appropriate analytical well model.

Well Model

The well index (WI), which accounts for the geometric characteristics of the well and the reservoir properties around the well, is used to relate the well bottomhole pressure and the well block pressure.

Peaceman Formula

Assuming single phase flow we can relate well and well block pressures through the radial form of Darcy's law:

$$q_p = WI \frac{k_{rp}}{\mu_p B_p}(P_{p,block} - P_{well})$$

$$WI = \frac{\theta k h}{ln(\frac{r_0}{r_w}) + S}$$

where θ is the angle open to flow and S is the skin factor.

Peaceman [1] derived the following expression for r_0 (for a well parallel to the x-axis):

$$r_0 = 0.28 \frac{[(k_z/k_y)^{1/2}\Delta y^2 + (k_y/k_z)^{1/2}\Delta z^2]^{1/2}}{(k_z/k_y)^{1/4} + (k_y/k_z)^{1/4}}$$

It is based on the assumptions of single phase flow, homogeneous reservoir, uniform grid, uniform permeability, an isolated well (the well is at least five gridblocks away from all boundaries and ten gridblocks away from other wells [6]), pseudo-steady state or steady state conditions, and block-centered well. Since a horizontal well cannot be far from the top or bottom boundary when the reservoir is thin, the accuracy of Peaceman's well model is restricted. Furthermore the flow around a horizontal well may not be two dimensional.

Babu and Odeh's Analytical Solution

Babu and Odeh [2] considered a more general case, where the well can be arbitrarily located at any position in a block, provided that it is parallel to one of the axes or gridlines. This analytical solution is based on steady-state (or pseudo-steady state)

However, Babu's analytical solution is based on the assumption of uniform flux along the wellbore. In reality, the flux pattern along the well is non-uniform, which is also the case for the problem considered by Dietrich and Kuo [5]. Recently, Penmatcha *at al.* [4] have developed a solution for a well with non-uniform influx and infinite conductivity. The wellbore is assumed to have a uniform pressure, resulting in non-uniform flux along the well length. Later we will use this analytical solution to evaluate the adequacy of numerically computed well productivity.

Grid Systems

Uniform Grids

In the previous section, we discussed the fact that the Peaceman well model is based on a uniform grid. For homogeneous, uniform permeability, single phase flow reservoir simulation, it is appropriate to use this model.

In practice, the limitation of computational resources requires that the grid be as coarse as possible with refinement in some regions; therefore we need to ensure that the well model is correct. All well models are based on some assumptions, so the model used needs to be appropriate for the problem being considered. We will show in the next section how this is accomplished.

Non-uniform Grids

The use of non-uniform grid spacing is often required in reservoir simulation. For example, it is necessary for 3-D simulations of stratified reservoirs or for well coning calculations [5]. Dietrich and Kuo [5] used the approach of grid refinement and deactivating the wellbore to realize explicit well modeling. In this approach, the wellbore is represented by a series of very small gridblocks scaled to represent the length and diameter of the horizontal well, the permeability of the wellbore is set at a high level and the transmissibility connection factor for the wellbore at the heel is also made very large, so that the well model itself can be removed from the problem and the effect of the well index is trivial. Then they assumed that this scheme will yield accurate solutions for horizontal well productivity.

However, we have found that this approach has two significant disadvantages:

1. Numerical Approximation:

   Aziz and Settari [6] showed some disadvantages of irregular grids considering the accuracy of finite-difference approximation. In most of current commercial simulators, the block-centered grids are used. Aziz and Settari have shown that this type of grid causes higher numerical errors than point-distributed grids. In the example below, we will show the divergence of the results by a non-uniform fine grid from both a uniform fine grid and an analytical solution.

2. Physical Restrictions:

   Palagi [7] discussed well models for Cartesian and Voronoi grids. The numerical results of block pressure obtained with large grid aspect ratios do not agree well with analytical solutions for locations close to the wells. In spite of this, the Peaceman's model provides the correct well pressure. The discrepancy of the pressure of gridblocks adjacent to the well is due to the assumption of linear flow used to derive geometric factors for grid connections in the Cartesian system, even in areas of predominantly radial flow like regions around the well. It is clear that this effect is more significant for large aspect ratios.

   Dietrich and Kuo's approach of using explicit well modeling actually uses the block pressure as the well pressure. This is possible because by explicit well modeling and the approach to remove the wellbore, the block pressure and the well pressure are the same. However, without the correct well index, and because the block pressure for large aspect ratio grid system will deviate a lot from the correct one, large errors are introduced.

Example Problem

We tried one example to test the effects of grid systems on predicting horizontal well productivity.

Model Description

We used a three-dimensional, homogeneous closed reservoir. A single horizontal producer is centered in the reservoir and partially penetrates the reservoir in the y-direction. We assume only oil phase exists in the reservoir, so it is a single phase problem. Both constant bottomhole pressure and constant flow rate constraints are used at the horizontal well. Horizontal well production rates are all evaluated under pseudo-steady state when the production rate is constrained. But when the bottomhole pressure is constrained, it is not possible to reach pseudo-steady state, we will elaborate further on this point later.

The properties and controlling parameters in simulation are shown in Table 1. The configuration of the non-uniform fine grid is exactly the same as that used in reference [5].

Table 1: Reservoir and Fluid Properties

K_x, K_y	= 100 mD (isotropic)
K_z	= 100 mD (isotropic)
K_x, K_y	= 150 mD (anisotropic)
K_z	= 5 mD (anisotropic)
Φ	= 0.2
C_t	= 3.0×10^{-5} psi^{-1} at P_{ini}
q_{max}	= 10000 STB/d
bhp_{min}	= 2001.521 psia
μ	= 1 cP
B	= 1 RB/STB
P_{ini}	= 3596.94 psi
Drainage area	= 2640 * 2640 ft^2
Thickness	= 100 ft
L_w	= 1000 ft
S	= 0.0
r_w	= 0.1875 ft

Analysis of Results

All the numerical results were obtained using a homogeneous single phase flow model in the Eclipse reservoir simulator. The analytical solution is used as the yardstick to measure the adequacy of simulation results.

Constant Bottomhole Pressure Constraint on the Well

(i) Results by the uniform fine grid system and the analytical solution

Figure 1 shows that simulations with the coarse grid underpredict well productivity over most of the simulated period. Figure 2 depicts an expanded region of Figure 1 from 20 to 25 days to better show differences among various results. Figure 3 is a typical plot used in well testing. We find that there is no unit slope line, which means pseudo-steady state was never reached in this case. Nevertheless, we will use this case to compare our results with those of Dietrich and Kuo's. We can see from Figures 1 and 2 that the curve of the uniform fine grid is closest to that of the analytical solution. In this case, the Grid Aspect Ratio (GAR) is equal to 6. It should be mentioned that simulations were done with no gravity in order to be consistent with the currently available analytical solution.

Figure 4 shows the flow rate ratio vs. well index ratio. WI_0 is the default well index value calculated by Eclipse for the coarse grid simulation, and increased WI is obtained by multiplying it by a factor. The flow rate ratio is the ratio of the coarse to uniform fine grid or coarse to analytical solution. We note that the Peaceman's well index, which is the default in Eclipse, has to be multiplied by a factor corresponding to the flow rate ratio of 1 to yield the correct coarse grid solution. In this case, the correct well index ratio is flow rate dependent and varies from 1.24 to 1.44. In Figure 4, we observe two different trends, one is when the well index ratio increases with increasing oil rate ratios, which corresponds to the period that the pressure derivative curve has a negative slope; the other is that when the well index ratio increases while the oil rate ratio decreases, it corresponds to the period when the derivative curve has a positive slope.

(ii) Different Schemes to Obtain Irregular Grids and Explicit Modeling Results

Figures 1 and 3 show the production rate of the horizontal well calculated by different grids and well models. There are three curves referring to the flow rates simulated by non-uniform fine grids. By setting high permeability to the wellbore and high WI to the heel, we can get slightly better results than those from the non-uniform fine grid with no adjustment of K or WI. While these results are close to those simulated by coarse grids at late times, they are still far from those of uniform fine grids and analytical solutions.

Dietrich and Kuo also evaluated coarse grid simulation by this method, the difference is that they used explicit modeling plus a non-uniform fine grid for their reference solution, which we have demonstrated to be wrong. One reason is that the Grid Aspect Ratio for their problem is about 66. It is large enough to cause a significant error in the pressure of grid blocks near the well. So their result that the coarse grid overpredicts well productivity is completely different from ours, since we see that coarse grid simulation underpredicts well productivity.

Constant Flow Rate Constraint on the Well

In this case, the slope of the pressure derivative curve is exactly unity (Figure 5), so we can identify the period after about 10 days as pseudo-steady state. Figures 6 and 7 show that the coarse grid simulation underpredicts productivity at the end of the pseudo-steady state period. Again to better show details in results, an expanded region of Figure 7 is displayed in Figure 8. Figure 9 shows that in coarse grid simulation, to obtain the correct horizontal well productivity, the well index should be multiplied by a factor of 1.25 for the isotropic case and by a factor of 1.07 for the anisotropic case. Figure 10 demonstrates similar results by the use of bottomhole pressure ratio.

Conclusions

An explicit modeling procedure employing high resolution grid systems near the well has been shown to be even worse than the coarse grid simulation results. Only when using the approach to deactivate the well model, are reasonable results are obtained, but they are still far from the correct solutions. Numerical approximations and physical restrictions are responsible for this deviation.

Horizontal well rates computed using a uniform coarse grid system and a conventional well model will under-predict well productivity for a well of the type considered.

The correct well index to be used in coarse grid systems can be computed and it is problem dependent.

Nomenclature

Q	Flow rate, STB/day
B	Formation volume factor, bbl/STB
k_r	Relative permeability
k_x	Permeability in the x-direction, mD
k_y	Permeability in the y-direction, mD
k_z	Permeability in the z-direction, mD
r_w	Wellbore radius, ft
r_0	Effective well radius, ft
S	Skin factor
h	Thickness, ft
C_t	Total compressibility, psi^{-1}
Δx	Gridblock size in the x-direction, ft
Δy	Gridblock size in the y-direction, ft
Δz	Gridblock size in the z-direction, ft
Φ	Porosity
bhp	Bottomhole pressure, psi
P	Pressure, psi
P_{ini}	Initial pressure, psi
L_{well}	Well length, ft
μ	Viscosity, cP

Subscripts

p	Phase
w	Well

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2. Babu, D.K. and Odeh, A.S.: "Productivity of a Horizontal Well," *SPE Reservoir Engineering*, pp. 417-421, November 1989.

3. Babu, D.K. and Odeh, A.S.: "Productivity of a Horizontal Well: Appendices A and B" paper SPE 18334 presented at the 63rd Annual Tech. Conf., 1988.

4. Penmatcha, R. V., Babu, D. K., and Aziz, K.: *Effects of Wellbore Friction Factor under Transient Conditions*, Chapter 5 of the Fourth Annual Report of the Project to US DOE, 1997.

5. Dietrich, J.K. and Kuo, S.S.: "Predicting Horizontal Well Productivity," *JCPT*, February 29, 1996.

6. Aziz, K. and Settari, A. *Petroleum Reservoir Simulation,* Elsivier Applied Science, New York, p. 75.

7. Palagi, C.L.: *Generation and Application of Voronoi Grid to Model Flow In Heterogeneous Reservoirs*, Ph.D. Dissertation, Chapter 6, Stanford University, 1992.

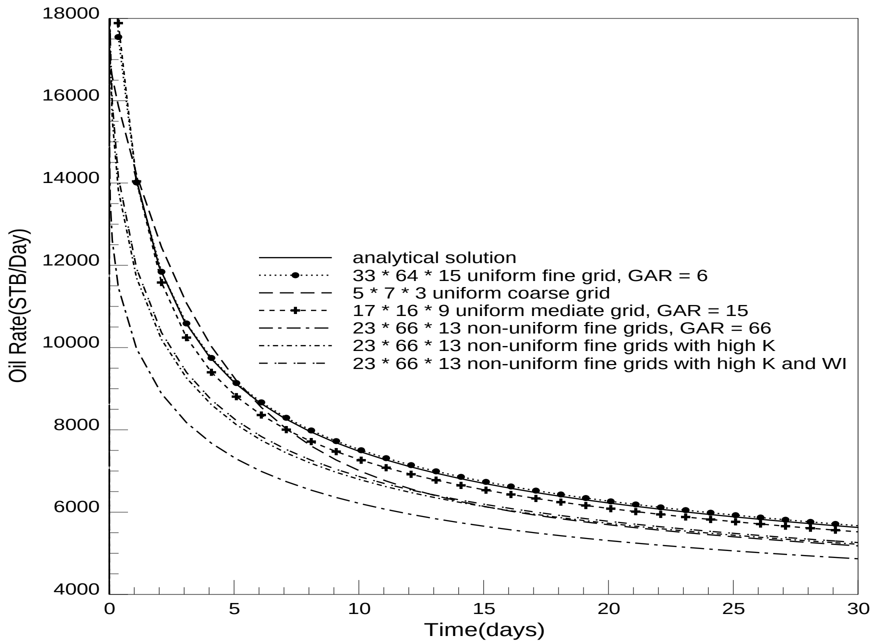

Figure 1: Influence of Grid System on Production Ratio for the Case of Constant Well Pressure Constraint

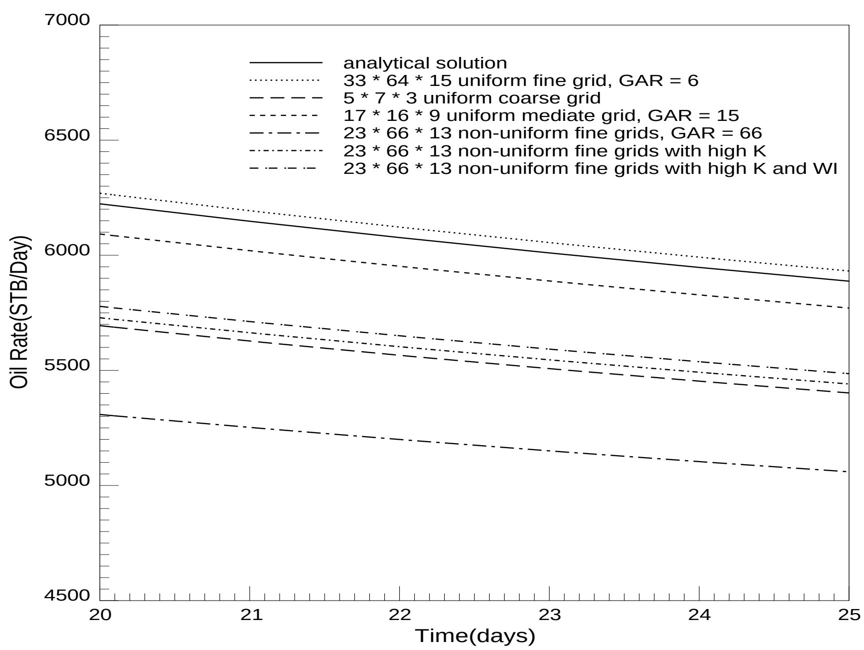

Figure 2: An Expanded Portion of Figure 1 from 20 to 25 Days

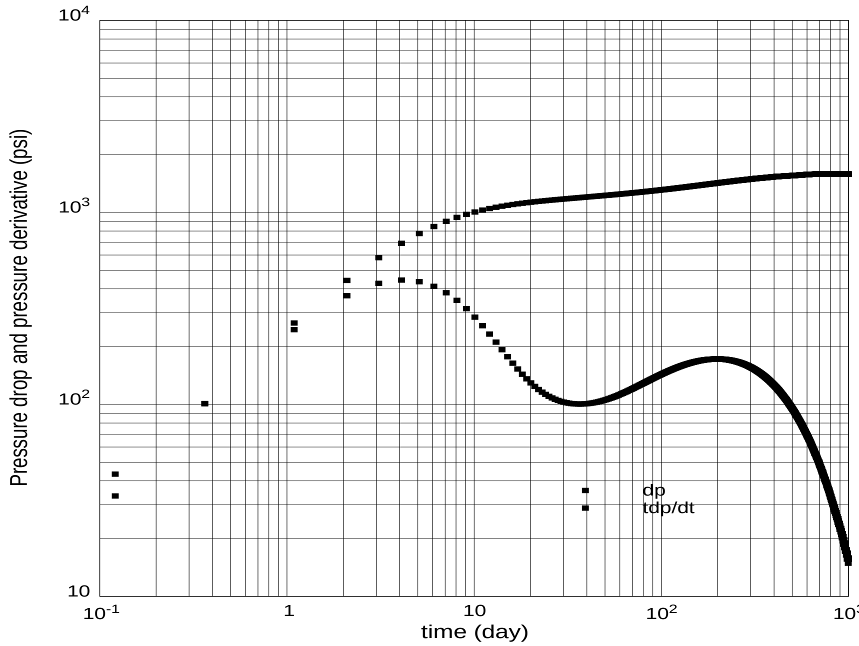

Figure 3: Approach to Pseudo Steady State for the Case of Constant Well Pressure Constraint

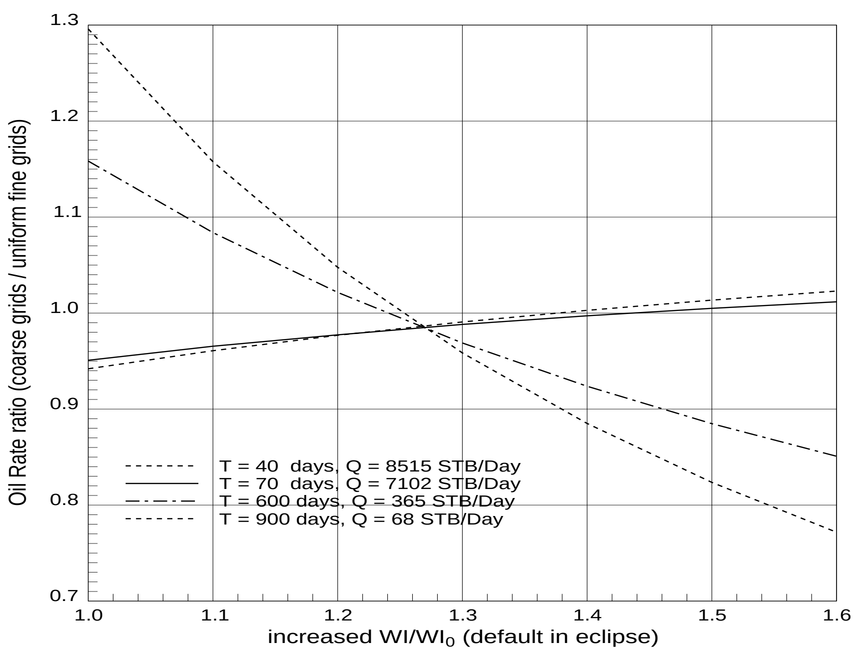

Figure 4: Adjustment Factor for WI to obtain Correct Flow Rate - Isotropic Case

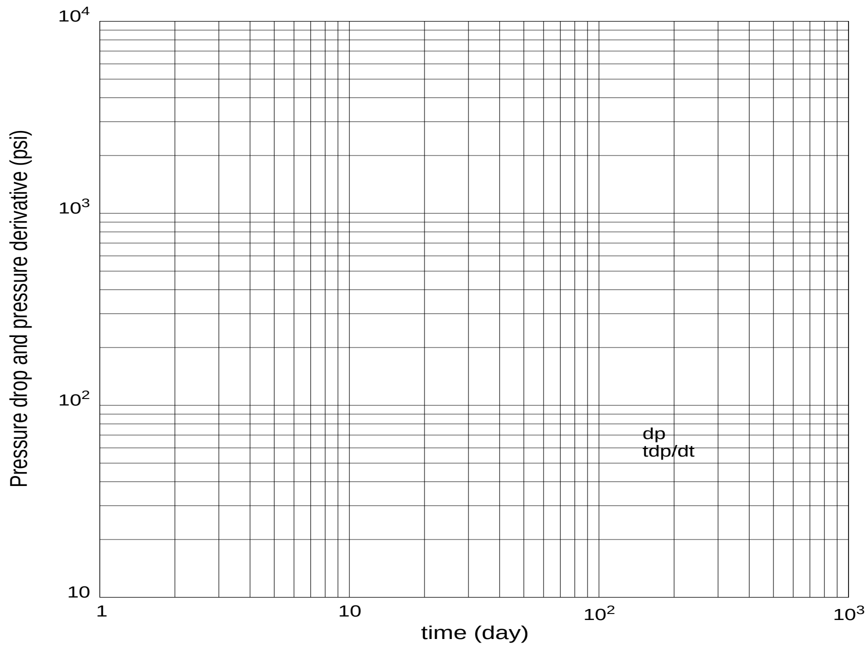

Figure 5: Approach to Pseudo Steady State for the Constant Flow Rate Case

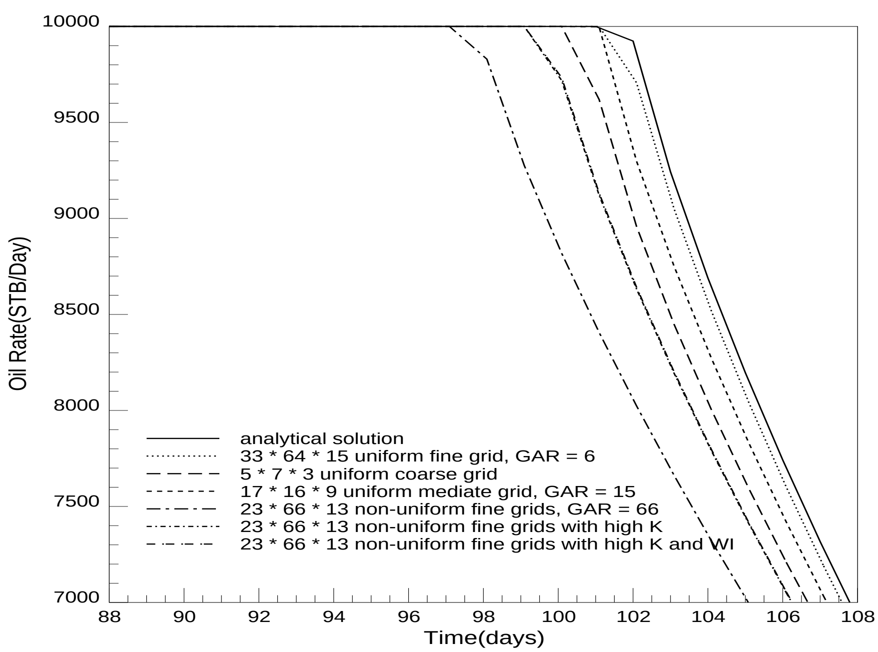

Figure 6: Results of Constant Flow Rate Set Initially Followed by a Period of Constant Wellbore Pressure Constraint

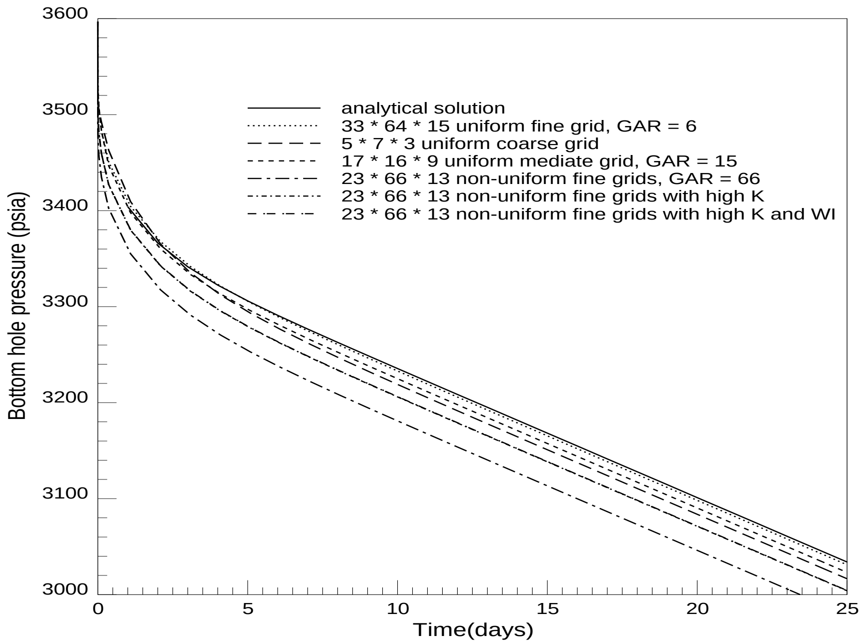

Figure 7: Bottomhole Pressure in the Case of Constant Flow Rate Constraint

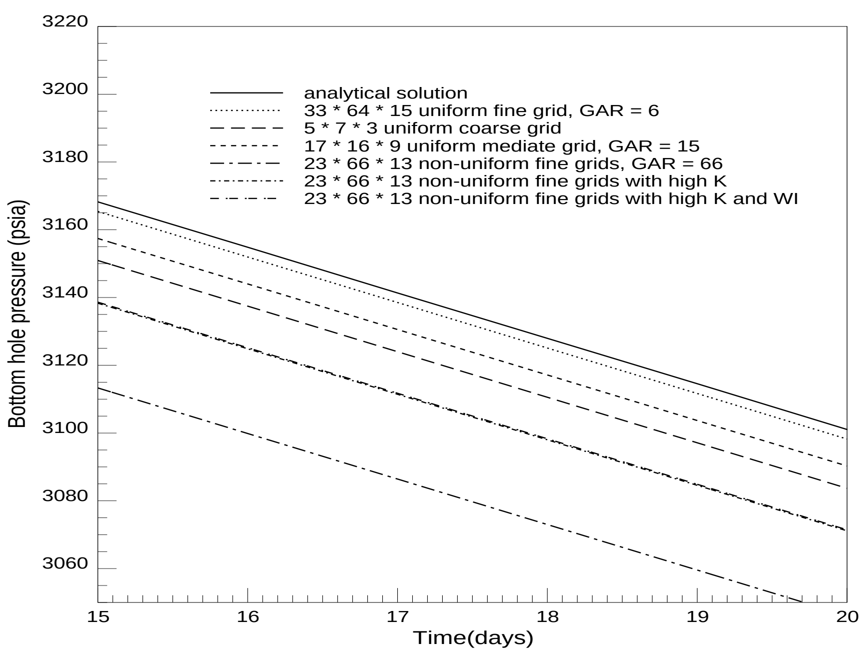

Figure 8: An Expanded Portion of Figure 7 from 15 to 20 Days

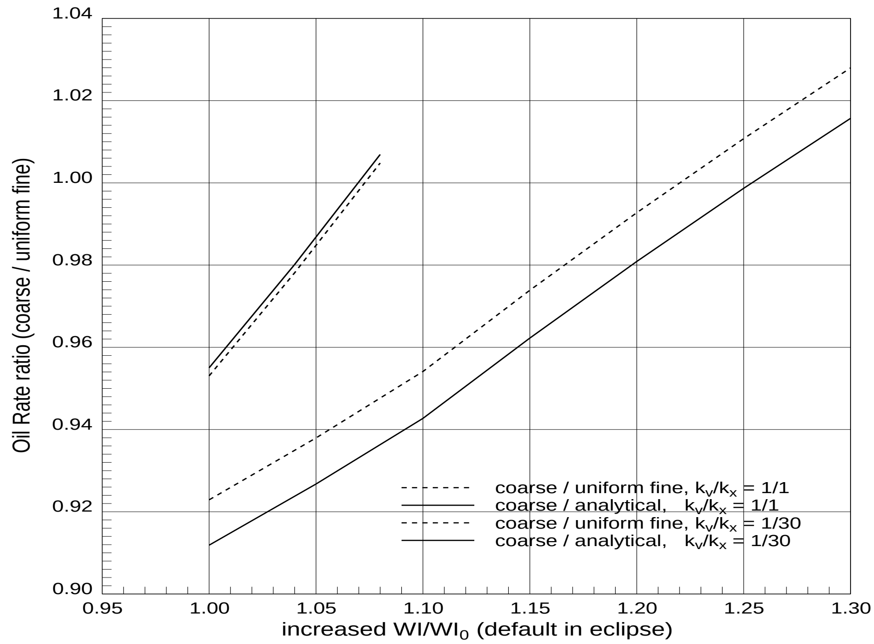

Figure 9: Variation of Oil Rate with Well Index for both Isotropic and Anisotropic Cases

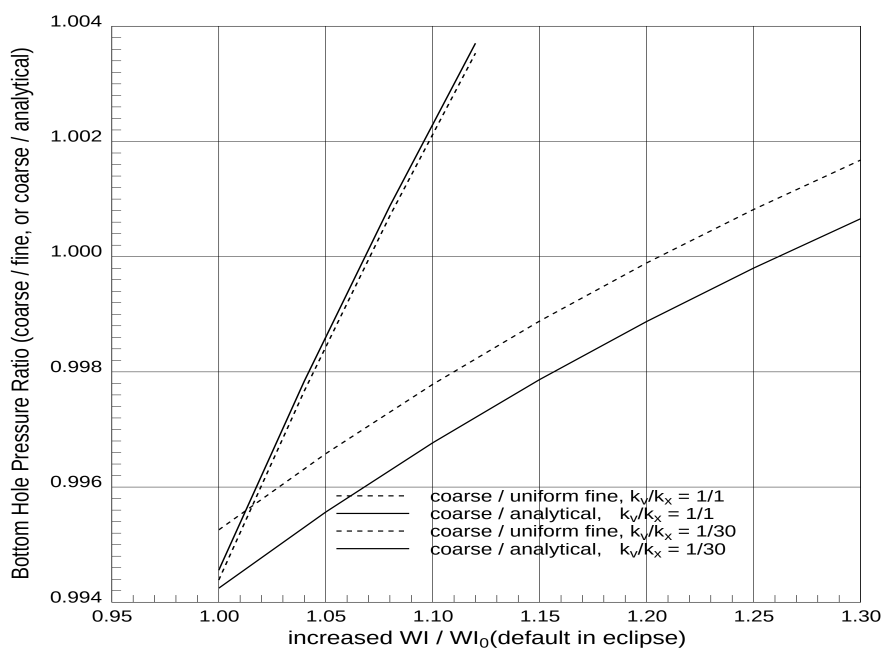

Figure 10: Variation of Bottomhole Pressure with Well Index for both Isotropic and Anisotropic Case