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A MIRCROWAVE FEL CODE USING WAVEGUIDE MODES *

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A free electron laser code, GFEL, is being developed for application to the LLNL tokamak current drive experiment, MTX. This single frequency code solves for the slowly varying complex field amplitude using the usual wiggler–averaged equations of existing codes, in particular FRED[1], except that it describes the fields by a 2D expansion in the rectangular waveguide modes, using coupling coefficients similar to those developed by Wurtele[2], which include effects of spatial variations in the fields seen by the wiggler motion of the particles. Our coefficients differ from those of Wurtele in two respects. First, we have found a missing $\sqrt{2}\gamma/a_w$ factor in his C_z; when corrected this increases the effect of the E_z field component and this in turn reduces the amplitude of the TM mode. Second, we have consistently retained all terms of second order in the wiggle amplitude. Both corrections are necessary for accurate computation. GFEL has the capability of following the TE_{0n} and $TE(M)_{m1}$ modes simultaneously. GFEL produces results nearly identical to those from FRED if the coupling coefficients are adjusted to equal those implied by the algorithm in FRED. Normally, the two codes produce results that are similar but different in detail due to the different treatment of modes higher than TE_{01}.

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I. Introduction

In this paper we describe a new free electron laser code, GFEL, that uses the vacuum waveguide modes as a basis set for describing the electromagnetic fields. Its basic structure follows that of the existing code FRED [1], but with an entirely new field algorithm, designed to operate only in the microwave regime. The development closely follows that of Wurtele[2]. We modify his mode coupling coefficients by calculating all terms to the consistent order. We also specifically correct his E_z coupling coefficient which we find to be in error at lowest order. We describe equivalent methods of applying these mode coupling coefficients. We demonstrate that there is a choice of coupling coefficients for GFEL that makes this code equivalent to FRED, even though heretofore the treatment in FRED of modes higher order than TE_{01} had been considered only approximate. We describe a simple modification that would allow FRED to accurately model the higher order modes for waveguide problems.

We show results for 34.6 ghz conditions to demonstrate the noticeable but not extreme differences that result when changing from the FRED-equivalent coefficients to the new (presumably more accurate) coefficients. The results of Fig 1 agree with those from FRED in all particulars, such as maximum field amplitudes, onset of phase jumps, etc., to within a few percent. Results from the new coefficients in Fig. 2 show a 7% higher growth rate and a larger fraction of 21 mode power, both in greater disagreement with the experiment [3]. Our calculation also predicts a very small TM_{21} power relative to the TE_{21} power, also in contradiction with the experiment [3]. It can be seen that there is *no* sudden turning over to $d\varphi_1/dz \approx 0$ after saturation, in sharp contradiction to the experiment, as already pointed out from the FRED results [4]. At the present time GFEL has a full set of betatron orbits and a space charge solution, both essentially identical to those of FRED. Improvements in the space charge solution are being developed. The taper algorithm in GFEL is still crude, but results close to those of FRED are obtained, both for the 34.6 ghz case and for the Microwave Tokamak Experiment (MTX) at 250 ghz at LLNL.

II. Laser equations with waveguide mode coupling coefficients

Here we quote directly from Wurtele's thesis for the free electron laser equations using the waveguide mode coupling coefficients, C_s. His table 5.1 gives the coefficients C_{xs} and C_{zs} for the first few TE_{mn} and TM_{mn} modes. We develop corrections to these coefficients further on in section IV. These coefficients are functions of the transverse dimensions x,y. The sum $C_s = C_{xs} + C_{zs}$ is to be used as a factor in the particle equations wherever the field amplitude a_s occurs, with a sum over modes $s \equiv m, n$, and using the x,y of the particular particle being followed. The same factor C_s appears on the right hand side of the field equations (now a pair for each mode rather than at each grid point for a gridded code) as a result of the Fourier mode transform of the current.

Wurtele defines the particle phase ψ_i relative to the design mode TE_{01}, denoted as mode $s = 1$. The particle equations are

$$\gamma_i' = -\sum_s \frac{\omega}{c} a_s a_w C_s \frac{\sin(\psi_i + \delta_s)}{\gamma_i}$$

$$\psi_i' = k_w - \delta k_1 - \frac{\omega}{2c\gamma_i^2}\left[1. + a_w^2 - \sum_s 2 a_w a_s C_s \cos(\psi_i + \delta_s)\right] + \frac{d\phi_1}{dz}$$

where $\delta k_1 \equiv \frac{\omega}{c} - k_1$ and $\delta_s \equiv (k_s - k_1)z + \phi_s(z) - \phi_1(z)$. The particle equations involve a sum over modes, s. The ψ_i' equation needs correction for the betatron orbits contribution. The field equations are

$$a_s' = \frac{\omega_{p,\mathrm{eff}}^2 a_w C_s}{2\omega c}\left\langle \frac{\sin(\psi_i + \delta_s)}{\gamma_i} \right\rangle$$

$$\phi_s' = \frac{1}{a_s}\frac{\omega_{p,\mathrm{eff}}^2 a_w C_s}{2\omega c}\left\langle \frac{\cos(\psi_i + \delta_s)}{\gamma_i} \right\rangle$$

In these field equations the C_s are really functions of the i'th particle's wiggler–averaged x, y and so are really to be brought inside the angle bracket average over the set of particles. In terms of these variables it is $\varphi_s = (k_s - k_1)z + \phi_s$ that is equivalent to the field phase in FRED. In the linear regime during exponential growth the field phases φ_s (not ϕ_s) of all modes are identical, once initial transients have died away (see Figs 1,2).

III. Lowest order coupling coefficients; equivalent methods

The standard description for a mode code (as in Wurtele) is to separate the TE and TM modes, each with their own mode coupling coefficients, but it is possible to define an equivalent combined field amplitude, e, where $e^2 = e_{TE}^2 + e_{TM}^2$ and an equivalent combined mode coupling factor is given by $C^2 = C_{TE}^2 + C_{TM}^2$. The fact that the two pieces are combined by adding them in quadrature will be seen below to lessen the net effect of E_z.

We understand that FRED reduces the source amplitude in the field equation by an 0.5 factor (and doubles the e_{ztot}^2 power) for all modes except TE_{01}. In GFEL there is an equivalence to using either a) the same C_s on the γ' equation and on the e' equation or b) a source reduction factor in the field equation as in FRED but which varies in value mode by mode and is equal to C_s^2. The inverse of this source reduction factor is to be used as a field power correction factor. In particular, GFEL can nearly match FRED results by either a) choosing $C_{21} = 0.707$, so that $C_{21}^2 = 0.5$ in GFEL, matching the source reduction factor 0.5 in FRED, or b) setting $C_{21} = 1$ and instead using a source reduction factor equal to 0.5 in the field equation for the 21 mode and doubling the 21 mode power. This equivalence demonstrates that FRED could be easily modified to include the equivalent of the mode coupling coefficients, simply by replacing its 0.50 source modification factor by C_s^2.

To lowest order and exclusive of the spatial sinusoidal factors, the coupling coefficients are $C_{xTE} = J_0(\rho)k_y/k_\perp$, $C_{xTM} = J_0(\rho)k_x/k_\perp$, and $C_{zTM} = -\frac{\sqrt{2}\gamma}{a_w}\frac{k_\perp}{k}J_1(\rho)\frac{K_2(R)}{K_1(R)}$. where $\rho = k_x x_w = k_x\sqrt{2}a_w/\gamma k_w$, $R = a_w^2/2(1+a_w^2)$, $K_1(R) = J_0(R) - J_1(R)$, $K_2(R) = J_0(R) + J_1(R)$. We here divide through by $K_1(R)$, so as to normalize $C_{01} = 1$.

During the development of this work, Scharlemann pointed out that we had missed the C_z piece, which comes from the E_z of the TM mode. In fact we were using C_z from Wurtele's table 5.1 which is missing the $\sqrt{2}\gamma/a_w$ factor shown above. The E_z effect thus incorrectly calculated is relatively minor. Apparently, Wurtele's incorrect form was also used for the calculations in a published paper[5]. The corrected value of C_z will clearly increase (by about a factor of 5 for 34.6 ghz), and since C_z is negative increase the reduction in the net 21 mode coupling factor. With this change we now calculate $C_{zTM} = -0.27$ and $C_{xTM} = 0.47$. The net TM coefficient is then given by $C_{TM} = C_{xTM} + C_{zTM} = 0.47 - 0.27 = 0.20$. We see that the net C_{TM} is reduced by about 60%, a sizeable reduction of the TM part of the 21 mode. However the net effect on the combined C_{21} is much less.

The net TE coefficient is $C_{TE} = C_{xTE} = J_0(\rho)\frac{k_y}{k_\perp} \approx 0.94 \times 0.861 = 0.81$. The combined 21 mode coefficient (34.6 ghz) is $C_{21}^2 = C_{TE}^2 + C_{TM}^2 = 0.65 + 0.04 = 0.69$, to be compared to the 0.50 source reduction factor used in FRED. We see that the effect of E_z is comparable to the effect of $J_0(\rho)$ that reduces both C_{xTE} and C_{xTM}. Since $C_{TM}^2 << C_{TE}^2$ the TM_{21} mode should be greatly suppressed relative to the TE_{21} —this result still stands with the full corrections of sec. IV and is in disagreement with the experiment [3]. Our other corrections given in section IV will further modify the coefficients by decreasing C_{01} and increasing C_{21} even more. For the three methods [1) full corrections, 2)Wurtele's lowest order (but C_z factor corrected), 3)FRED equivalent] the coefficients are $C_{01}^2 = 0.83, 1.0, 1.0$, and $C_{21}^2 = 0.88, 0.69, 0.50$. The detailed results from using methods 1 and 3 for 34.6 ghz (see Figs 1,2) show significant but not extreme differences.

IV. Corrections to Wurtele's coefficients

Wurtele[2] calculated modifications to the FEL equations due to radial variations of waveguide modes. His results, as presented in his Table 5.1, include the Bessel functions $J_0(\rho)$ and $J_1(\rho)$, where $\rho = k_x x_w$, the implication being that ρ is not so small that the Bessel functions can be replaced by their leading-order approximations. However, Wurtele dropped terms of order $a_w^2/\gamma^2 \sim \rho^2$ compared to one at some points in the analysis, so the results of his Table 5.1 have significance only at leading order. One goal of this present paper is to present results obtained by consistently retaining next-order terms in the a_w/γ expansion.

There are, in particular, four places in the calculation where higher-order terms should be kept:

(1) in evaluating t' in the energy equation

$$\gamma' = (t' e/mc^2)\mathbf{v}\cdot\mathbf{E} \quad , \tag{1}$$

$1/v_\parallel$ should be expanded to second order rather than replaced by c^{-1}; hence we take

$$t' = \frac{c}{v_0}\left(1 + \frac{a_w^2}{2\gamma^2} + \frac{a_w^2}{2\gamma^2}\cos\chi\right) + \mathcal{O}\left(\frac{a_w^4}{\gamma^4}\right)$$

where $\chi = k_w z$ and $v_0 = c(1-\gamma^{-2})^{1/2}$.

(2) in evaluating the transverse coordinate $x = \int dz\, v_w/v_\parallel$, we must again retain second-order corrections to $1/v_\parallel$, yielding

$$x = x_0 + \frac{2^{1/2} c a_w}{v_0 k_w \gamma} \left(\left(1 + \frac{3}{4}\frac{a_w^2}{\gamma^2}\right) \sin\chi + \frac{1}{12}\frac{a_w^2}{\gamma^2} \sin 3\chi \right) + \mathcal{O}\left(\frac{a_w^4}{\gamma^4}\right) \quad .$$

This correction generates additional Bessel functions (associated with the $\sin 3\chi$ term) and corrections to the argument ρ of the existing ones.

(3) Even without this correction to x, the expansion of the transverse field variation $\cos(k_x x - m\pi/2)$ should include a J_2 term in order to be second-order accurate in a_w/γ; we have

$$\cos(k_x x - m\pi/2) = c_1 \left(J_0(\rho) + 2J_2(\rho)\cos 2\chi\right) + \mathcal{O}\left(\frac{a_w^4}{\gamma^4}\right) + \text{nonresonant terms} \quad , \qquad (2a)$$

where $c_1 = \cos(k_x x_0 - m\pi/2)$. This expression is also valid with the corrections to x. Similarly, the expansion of $\sin(k_x x - m\pi/2)$ should include a J_3 term; incorporating also the corrections to x, we have:

$$\begin{aligned}\sin(k_x x - m\pi/2) =& 2c_1 \left(J_1(\rho_1)\sin\chi + (J_3(\rho) + J_1(\rho_2))\sin 3\chi\right) \\ &+ \mathcal{O}\left(\frac{a_w^5}{\gamma^5}\right) + \text{nonresonant terms} \quad , \qquad (2b)\end{aligned}$$

where

$$\begin{aligned}\rho &= k_x 2^{1/2} \frac{c}{v_0 k_w} \frac{a_w}{\gamma} \\ \rho_1 &= \rho\left(1 + \frac{a_w^2}{\gamma^2}\right) \\ \rho_2 &= \rho \frac{a_w^2}{12\gamma^2}\end{aligned}$$

In Eqs. (2a) and (2b), "nonresonant terms" denotes terms proportional to $\sin 2m\chi$ and $\cos[(2m+1)\chi]$, respectively; these terms do not give rise to "slowly varying" contributions to γ'. Note, to the order of our calculation, one can replace $J_2(\rho)$, $J_1(\rho_2)$, and $J_3(\rho)$ by their leading-order approximations.

(4) in evaluating the wiggler average of $\psi = \int dz\, k_w + k_s - \omega t + \phi$ [which leads to the "axial Bessel functions" $J_m(R)$ with $R = a_w^2/2(1 + a_w^2)$], if R is to be considered of order

unity, (that is, if we are worried about small corrections to R), then we should retain terms through order a_w^4/γ^4 in evaluating $t(z)$ from $v_\parallel^{-1}$; thus with $\epsilon \equiv a_w^2/2(\gamma^2-1)$,

$$v_\parallel^{-1} = v_0^{-1}\left(1+\epsilon+\frac{9}{4}\epsilon^2+(\epsilon+3\epsilon^2)\cos 2\chi+\frac{3}{4}\epsilon^2\cos 4\chi+\ldots\right)$$

$$\omega t = \omega\int\frac{dz}{v_\parallel} = \omega\bar{t}+R_1\sin 2\chi+R_2\sin 4\chi+\mathcal{O}\left(\frac{a_w^4 R}{\gamma^4}\right)$$

$$\bar{t} = \int\frac{dz}{v_0}\left(1+\frac{a_w^2}{2(\gamma^2-1)}+\frac{9}{4}\frac{a_w^4}{(\gamma^2-1)^2}+\ldots\right)$$

$$R_1 = R\frac{c}{v_0}\frac{1+(3/2)a_w^2/\gamma^2}{1-\gamma^{-2}}$$

$$R_2 = \frac{3}{16}R\frac{a_w^2}{\gamma^2}$$

Using these expressions, Wurtele's expressions for the waveguide fields [his Eqs. (A.27)-(A.30)], and the relation $\mathbf{v}\cong\mathbf{v}_w = 2^{1/2}(a_w/\gamma)c\mathbf{e}_x\cos\chi$ in Eq. (1) and performing the usual wiggler average, we obtain $\gamma' = \gamma'_\perp+\gamma'_\parallel$, where $\gamma'_\perp$ and $\gamma'_\parallel$ are respectively, the energy gain rates due to the transverse and longitudinal components of $\mathbf{E}$, and are given by

$$\gamma'_\perp \cong \sum_s \frac{a_s a_w}{\gamma}\frac{\omega}{c}c_1 s_2\sin(\psi+\delta_s)\,\kappa\frac{c}{v_0}\,G_\perp$$

$$\gamma'_\parallel \cong -\sum_s \frac{a_s a_w}{\gamma}\frac{\omega}{c}c_1 s_2\sin(\psi+\delta_s)\frac{2^{1/2}k_\perp\sigma}{k_s}\frac{\gamma}{a_w}\,G_\parallel$$

$$G_\perp = J_0(\rho)K_1+J_2(\rho)K_3+\frac{a_w^2}{\gamma^2}\left(J_1(R)+\frac{1}{4}J_2(R)+\frac{3}{4}J_0(R)\right)$$

$$G_\parallel = J_1(\rho_1)K_2+\left(J_1(\rho_2)+J_3(\rho)\right)K_4 \quad .$$

Here,

$$K_1 = J_0(R_1)-J_1(R_1)(1+R_2)$$

$$K_2 = J_0(R_1)+J_1(R_1)(1+R_2)$$

$$K_3 = J_0(R)+J_2(R)$$

$$K_4 = J_1(R)-J_2(R)$$

$$s_2 \equiv \sin(k_y y-n\pi/2) \quad ,$$

and $\kappa = k_y/k_\perp$ for TE modes and $k_x/k_\perp$ for TM modes, while $\sigma = 0$ for TE modes and 1 for TM modes. We can display these results in an updated version of Wurtele's table, our Table I. where now $S_1 = \sin(m\pi x/a)$, $C_1 = \cos(m\pi x/a)$, $S_2 = \sin(n\pi y/b)$, $C_2 = \cos(n\pi y/b)$. Besides the presence of more second-order terms and the differences resulting from sign conventions, this table also differs from Wurtele's through the factor $2^{1/2}\gamma/a_w$ present in the C_{zs} terms. It can be straightforwardly verified that the particle phase and the mode amplitude and phase equations have the same coupling coefficients as the γ' equation, as asserted by Wurtele.

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Figure Captions

Fig. 1. Results from GFEL modeling of laser growth and saturation for 34.6 ghz, for FRED–equivalent coefficients. Four modes are shown, 01–solid line, 21–short dash, 41 medium dash, 61–long dash.

Fig 2. The same as Fig 1 for the corrected Wurtele coefficients.

Table I. Mode coupling coefficients

	C_{xs}	C_{zs}
TE_{01}	K_1C_2	0
TE_{11}	$\frac{k_y}{k_\perp}\frac{c}{v_0}G_\perp S_1C_2$	0
TM_{11}	$\frac{k_x}{k_\perp}\frac{c}{v_0}G_\perp S_1C_2$	$-\frac{2^{1/2}\gamma}{a_w}\frac{k_\perp}{k}G_\parallel S_1C_2$
TE_{21}	$-\frac{k_y}{k_\perp}\frac{c}{v_0}G_\perp C_1C_2$	0
TM_{21}	$-\frac{k_x}{k_\perp}\frac{c}{v_0}G_\perp C_1C_2$	$\frac{2^{1/2}\gamma}{a_w}\frac{k_\perp}{k}G_\parallel C_1C_2$

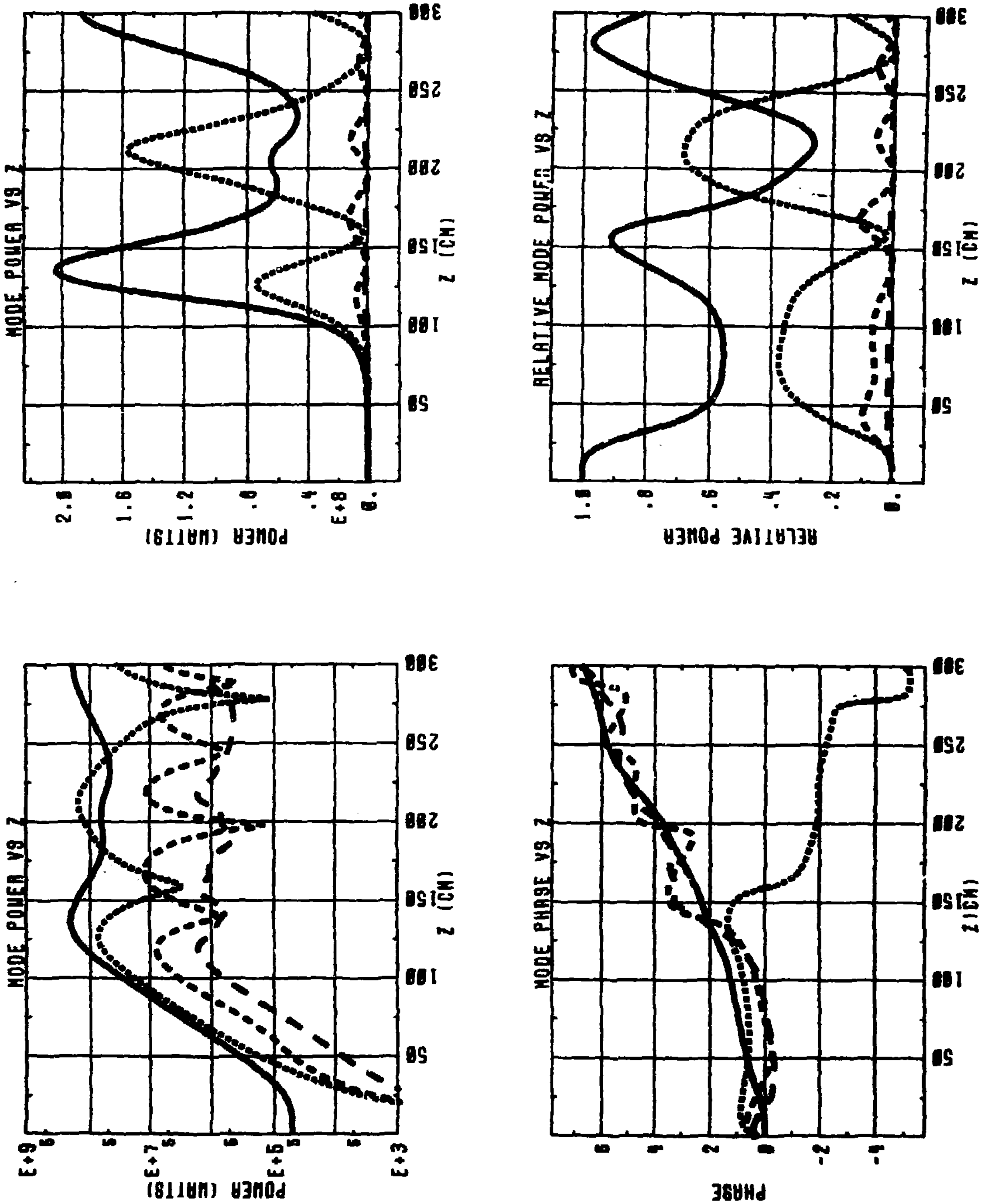
MODE POWER VS Z
POWER (WATTS)
Z (CM)
RELATIVE MODE POWER VS Z
RELATIVE POWER
MODE PHASE VS Z
PHASE

FIGURE 1

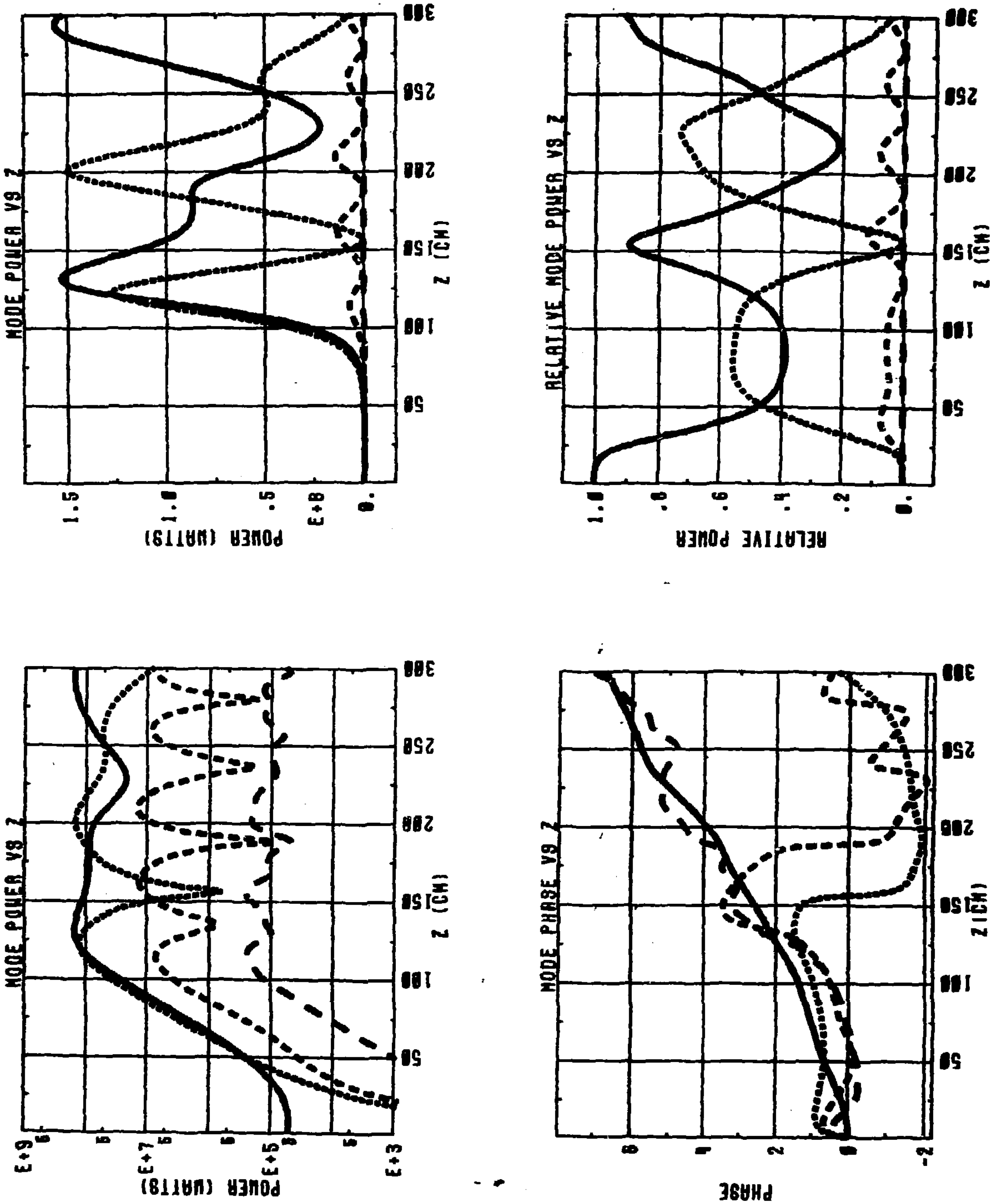
MODE POWER VS Z
POWER (WATTS)
Z (CM)
RELATIVE MODE POWER VS Z
RELATIVE POWER
Z (CM)
MODE POWER VS Z
POWER (WATTS)
Z (CM)
MODE PHASE VS Z
PHASE
Z (CM)

FIGURE 2