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EXTENSION OF THE CONSISTENT Q FORMALISM TO ODD-A NUCLEI IN THE W-Pt REGION

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ABSTRACT

It is shown that the approach of the Consistent Q Formalism, which has proved successful in the Interacting Boson Model description of even-even nuclei, can be extended to odd A nuclei within the framework of the Interacting Boson-Fermion Model. The proposed method describes the transition between the SU(3) and O(6) symmetry limits of the U(6/12) boson fermion group, and can be applied to the odd neutron W, Os, Pt nuclei. As in the even-even case, a number of parameter-free predictions emerge for the transitional region concerning energies, B(E2) values and also single particle structure factors, and some of these are compared to existing data for the odd Os nuclei.

1. INTRODUCTION

The Consistent Q Formalism[1] (CQF) has proved to be a particularly attractive starting point in the description of the collective structure of a broad range of even-even nuclei, within the framework of the Interacting Boson Model[2] (IBM). Indeed, a number of impressive examples of the successful application of this approach are discussed in a separate contribution[3] to these proceedings. The method is predicated on maintaining a consistent form for the IBM boson quadrupole operator in both the Hamiltonian and in the description of E2 transitions, and leads to a number of essentially parameter-free predictions concerning the behavior of relative energies and B(E2) values across a wide region of nuclei. It is the purpose of this contribution to show that this approach can be extended to the odd-A case also, within the framework of the Interacting

Boson-Fermion Model[4] (IBFM), where the greatly enhanced complexity of the general Hamiltonian makes the simplification offered by the CQF parameterization even more valuable.

To begin, it is worthwhile reviewing the basic characteristics of the CQF in the even-even case. The Hamiltonian is defined by

$$H = -\kappa Q_B \cdot Q_B - \kappa' L \cdot L \tag{1}$$

where the boson quadrupole operator, Q_B, is

$$Q_B = (s^+\tilde{d} + d^+s)^{(2)} + (\chi/\sqrt{5})\ (d^+\tilde{d})^{(2)} \tag{2}$$

The E2 operator is then constrained to take the same form as Q_B, i.e.

$$T(E2) = \alpha Q_B \tag{3}$$

For a given boson number N, the parameter χ alone determines the structure of the wave functions and can be varied in the range defined by the values which generate the SU(3) and O(6) limits, namely, $-\sqrt{35}/2$ to 0. The advantages of this approach lie chiefly in its simplicity. It involves at least one less parameter than the conventional approach, since the symmetry breaking mechanism is now represented by χ, rather than an extra term in H, but the freedom to vary the structure of the E2 operator is removed. In addition, since the wave functions and E2 operator are uniquely specified by χ (and N), relative B(E2) values and energies depend only on these two parameters. There is also a degree of simplicity gained at the intuitive, or interpretive, level since the changes in structure which result from a change in the equilibrium nuclear shape are now ascribed simply to changes in the form of the quadrupole operator and can, in fact, be simply related[5] to the geometrical β and γ deformation variables.

2. EXTENSION TO ODD-A NUCLEI

In order to formulate an equivalent approach to the CQF for the boson-fermion Hamiltonian it is clearly necessary to develop a compatible parameterization for the fermion degrees of freedom. As in the even-even case, the starting point is to consider the SU(3) and O(6) symmetries as "benchmarks" for the general IBFM Hamiltonian. In the odd-A case, the symmetry structure stems from a group of the form U(6/m) where m specifies the dimension of the assumed single particle space, and what follows will concentrate on the U(6/12) case. This corresponds to single particle orbits with j=1/2, 3/2 or 5/2, and is therefore appropriate to the negative parity states in the odd-neutron nuclei at the end of the N=82-126 shell.

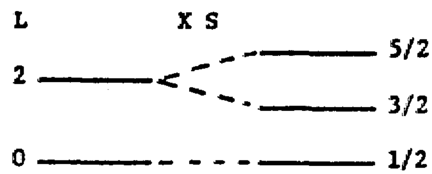
The SU(3) limit of U(6/12) is defined by the group chain decomposition[6]

$$U^B(6) \times U^F(12) \supset \boxed{U^B(6) \times U^F(6) \times SU^F(2)}$$

$$\supset U^{BF}(6) \times SU^F(2) \supset SU^{BF}(3) \times SU^F(2) \qquad (4)$$

$$\supset O^{BF}(3) \times SU^F(2) \supset \text{Spin}\ (3)$$

The boxed portion of chain (4) represents a pseudo-orbital angular momentum decomposition[7] of the fermion space, and constitutes the crucial feature in obtaining the required parameterization of the fermion operators. This technique corresponds to treating the fermion angular momenta as arising from the coupling of a pseudo L quantum number with L=0 or 2 to a pseudo spin of 1/2.



The analogy with the s,d boson space is immediately obvious, and allows a fermion quadrupole operator Q_F to be defined in an equivalent way to Q_B (eq. 2).

$$Q_F = G_F^{(2)}(0,2) + G_F^{(2)}(2,0) + (\chi/\sqrt{5})\ \ G_F^{(2)}(2,2) \qquad (5)$$

The fermion generators $G_F^{(2)}(\ell,\ell')$ are given in detail elsewhere[8,9]. They simply consist of appropriate combinations of the fermion annihilation and creation operators, such that $G_F^{(2)}(0,2)$ or $G_F^{(2)}(2,0)$ involve couplings of the type (j,j') = (1/2,3/2), (1/2,5/2), while $G_F^{(2)}(2,2)$ involves (j,j') = (3/2,3/2), (5/2,5/2) and (3/2,5/2). The CQF for odd-A nuclei can now be defined by demanding that χ take identical values in Q_B and Q_F.

In the IBFM Hamiltonian which corresponds to the SU(3) group chain (4) the quadratic Casimir operator of the group $SU^{BF}(3)$ generates a quadrupole interaction of the form

$$Q \cdot Q = (Q_B + Q_F) \cdot (Q_B + Q_F) \qquad (6)$$

where Q_B and Q_F are defined by eqs. 2 and 5 with $\chi = -\sqrt{35}/2$. It is then easy to show that when $\chi=0$, the Q·Q interaction reduces to the form

$$C_{20^{BF}(6)} - C_{20^{BF}(5)} \tag{7}$$

which are the Casimir operators required in place of that of $SU^{BF}(3)$ to produce the O(6) limit of U(6/12). However, as in the even-even case, the O(6) limit produced in this way is not the most general one, in that the O(6) and O(5) contributions are constrained to be equal. The first success of this approach, therefore, can be found by considering the magnitudes of these terms found in an earlier fit[9] to the nucleus ^{195}Pt which, by virtue of its core nucleus ^{196}Pt, is the obvious odd-A candidate to exhibit this symmetry. In this previous calculation, no restriction was placed on relative sizes of the two terms but, in fact, the fit yielded 33.5 and 35.0 keV for the coefficients of the O(6) and O(5) terms, respectively.

In a transitional situation, where χ takes a value intermediate between $-\sqrt{35}/2$ and 0, the contributions to the Hamiltonian from the Casimir operators of $U^{BF}(6)$, $O^{BF}(3)$ and Spin (3) remain diagonal, so that the symmetry breaking mechanism is contained only within the Q·Q term, and hence is uniquely specified by χ. Thus, just as in the even-even case, the wave functions depend only on χ, and the boson number N. Also, if the E2 operator is defined as

$$T(E2) = \alpha\ (Q_B+Q_F)$$

then all relative B(E2) values are likewise uniquely determined. Thus, a situation totally analogous to that in the case of the even-even CQF is obtained, in that the behavior of relative energies, B(E2)'s and in this instance, single particle structure factors, can be predicted across the transition from deformed to γ-unstable structure.

3. CHARACTERISTICS OF THE SU(3)-O(6) TRANSITION

In order to understand the behavior of various observables across the transitional region, it is instructive to begin by recalling the properties of the SU(3) limit of U(6/12). The features of this limit have been explored in detail in ref. 10, where it was also demonstrated that the low lying levels in ^{185}W represent a good empirical example of this symmetry. The theoretical level scheme is illustrated in fig. 1.

The states group into the various (λ,μ) representations of $SU^{BF}(3)$, and within each representation, the equivalent of one or

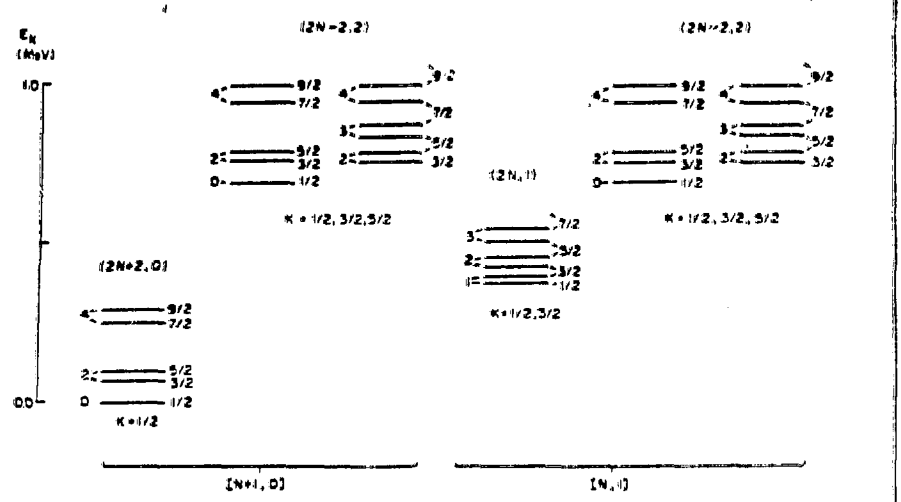


Fig. 1. The SU(3) scheme of U(6/12). Representations are labelled by the (λ,μ) quantum numbers of $SU^{BF}(3)$ and by the $[N_1,N_2]$ labels of $U^{BF}(6)$. The pseudo-L quantum number is given on the left of the levels.

more odd-A rotational bands can be assigned, and are labelled by their appropriate K values. The $SU^{BF}(3)$ representations themselves fall into one of two possible representations of the group $U^{BF}(6)$, which are distinguished by the quantum numbers [N+1,0] and [N,1]. In fact, it was demonstrated in ref. 10 that it is the K=1/2 and 3/2 bands of the (2N,1) representation which form the ground state structure in ^{185}W, and this situation can be realized within the symmetry scheme by a suitable adjustment of the relative sizes of the constants multiplying the Casimir operators of $U^{BF}(6)$ and $SU^{BF}(3)$ in the Hamiltonian.

The correct empirical ordering of representations for ^{185}W is illustrated in fig. 2a. An additional label has also been introduced in this figure, which proves useful in tracking the behavior of the SU(3) states through the transition to O(6) structure. It is evident in fig. 2a that the pseudo-L values given on the left of the levels themselves group into rotational band structures, which can be distinguished by means of a pseudo-projection quantum number K_p, as shown. The behavior of the states within each pseudo-K band as $\chi\to 0$ is then displayed in fig. 2b, the boson number and all other coefficients being kept constant. This figure is necessarily schematic, since the rotational band structure is eventually lost as the O(6)

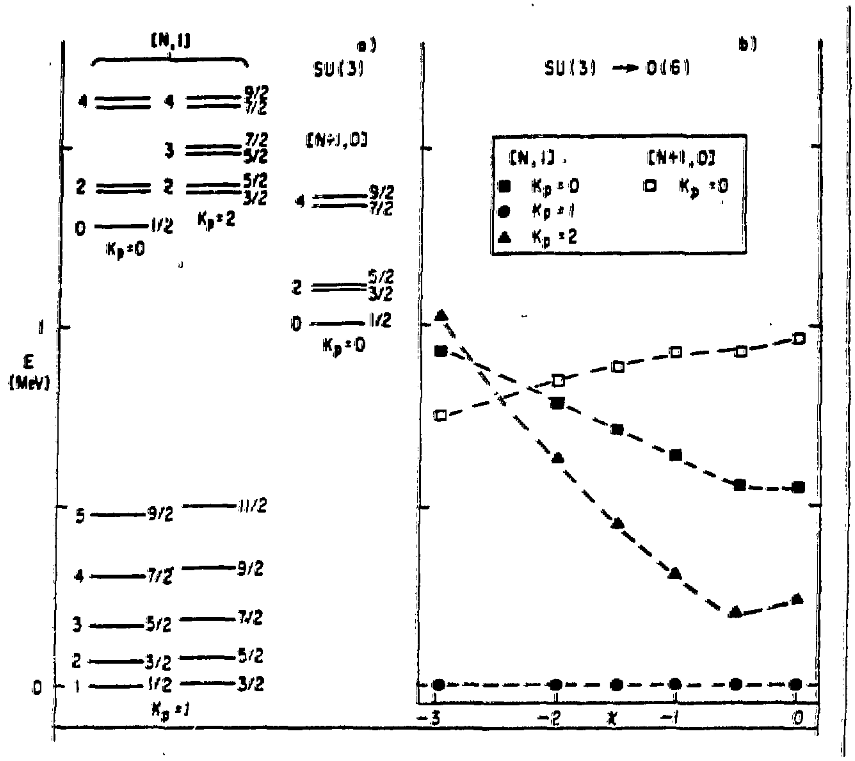


Fig. 2. (a) The SU(3) limit. Bands have been labelled by the pseudo-projection quantum number K_p. (b) Schematic indication of the evolution of the bands as χ changes from its SU(3) value (−2.958) to O(6) (0).

limit is approached. Nevertheless, the most important features are evident, namely, that the K_p=1 states remain lowest in energy while the most significant change is the rapid descent of the K_p=2 states, which eventually mix strongly with the ground state structure around χ = −0.5.

The empirical situation is depicted in fig. 3. At the top of this figure, the "benchmark" SU(3) and O(6) structures of ^{185}W and ^{195}Pt are shown. The transitional region of interest spans the odd Os nuclei, and throughout these isotopes, the low lying structure mimics that of ^{185}W, as evident in the bottom half of fig. 3. The first four low spin states are those originating from the near-degenerate K=1/2 and K=3/2 bands, and the single particle strength resides in the first 5/2 and second 3/2 states in all cases. Moreover, the ratio of the 5/2 and 3/2 strength in each case remains rather

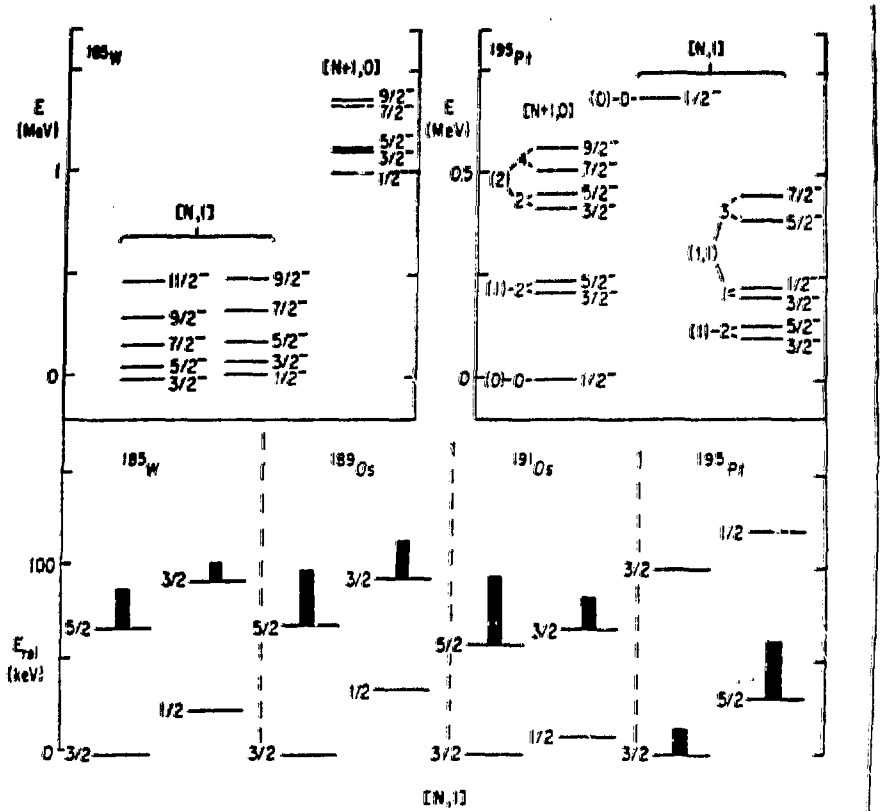


Fig. 3. (Top) The characteristic SU(3) and O(6) structure in ^{185}W, ^{195}Pt and (Bottom) the (d,t) single particle structure factors across the region in the low lying group of states shown (see text for details). Data are from refs. 11-15.

constant. However, while these strengths can be reasonably well described in ^{185}W in terms of the appropriate Coriolis-mixed Nilsson orbits, their magnitudes grow in the odd Os nuclei, to the extent that they can no longer be accounted for in a simple Nilsson framework[11,12]. Of course, such problems are hardly surprising in this region given that the neighboring even-even nuclei no longer exhibit the characteristics of axially symmetric rotors.

In ^{195}Pt, the situation changes in two important respects. Firstly, the analogues of the four lowest states in the W and Os nuclei, which are characterized by $U^{BF}(6)$ quantum numbers [N,1] in fig. 2, no longer form the ground state structure, but appear slightly higher in energy, as is evident in the upper portion of

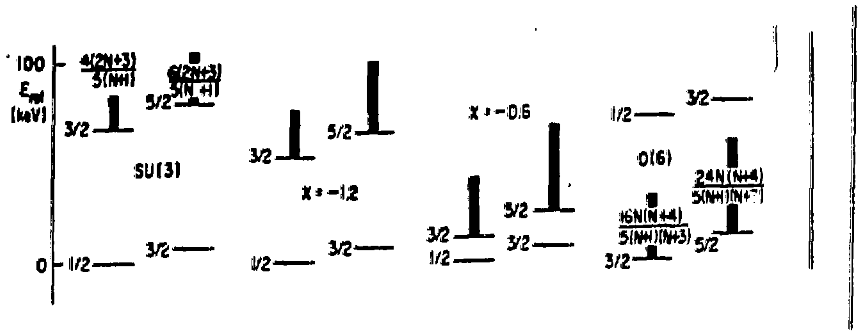


Fig. 4. Predicted changes in the (d,t) structure factors for the lowest lying states. From the schematic calculation of fig. 2b.

fig. 3. In addition, the order of the 3/2, 5/2 and 1/2, 3/2 couplets is reversed. The single particle structure factors, however, still follow the pattern established in the W, Os nuclei. The calculated structure of the low lying states across the transition region is shown in fig. 4. It is clear that the most important empirical features of the transitions emerge naturally from this description. The single particle structure factors are in fact predicted to maintain a constant ratio, S(5/2):S(3/2) = 3:2, independent of χ, while the absolute magnitudes grow as $\chi \to 0$. Moreover, the reversal in the ordering of the two pseudo-spin couplets is also reproduced. Referring back to fig. 2, the mechanism which produces this re-ordering can be easily understood. The mixing which takes place between the $K_p=1$ and $K_p=2$ structures involves only states with the same pseudo-L quantum numbers, since the Casimir of $O^{BF}(3)$ remains diagonal in the transitional Hamiltonian. Hence the L=2 states of the two structures mix, those of the $K_p=1$ band being pushed below those with L=1, since the latter L value is absent from the $K_p=2$ configuration.

Finally, it should be noted that only χ has been varied to produce fig. 4. Details concerning the ordering and correct energies of states can be improved by variation of the remaining diagonal terms in the Hamiltonian. In particular, the correct positioning of the states of fig. 4 in the case of ^{195}Pt requires an adjustment of the $U^{BF}(6)$ contribution.

In the preceding discussion of the structure factors of the low lying states, the lowest order transfer operator of the form $\zeta_j a^+_j$ has been assumed to describe the (d,t) reaction in this region, with $\zeta_{3/2} = \zeta_{5/2}$. More generally, with this operator, the ratio of structure factors for any two states with the same spin will depend only on χ and N. An example is shown in fig. 5, where the ratio for the $3/2_2$ state in the $K_p=1$ band, and the $3/2_1$ state in the Kp=2 band is plotted. However, another feature of this

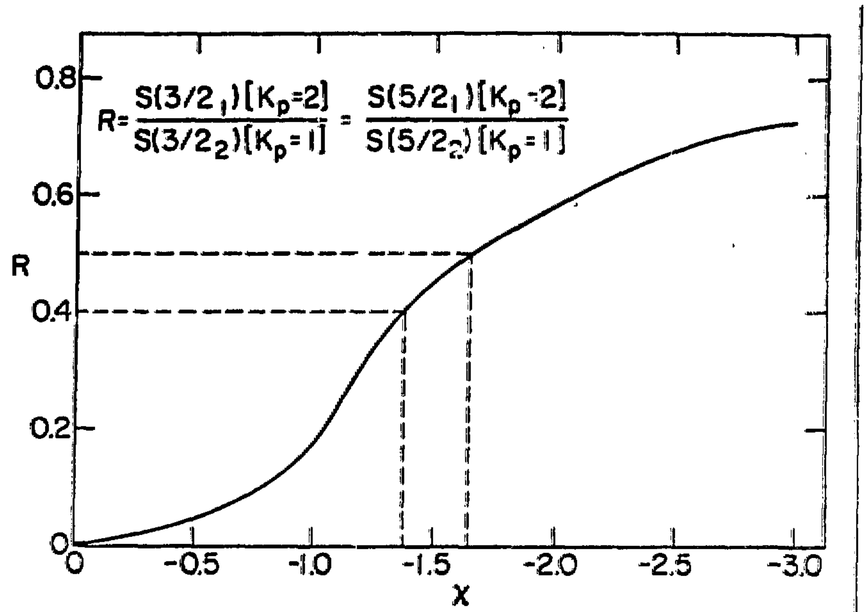


Fig. 5. Predicted ratio of indicated (d,t) structure factors as a function of χ, for N=9. The dashed lines correspond to the mean of the two empirical ratios for ^{189}Os.

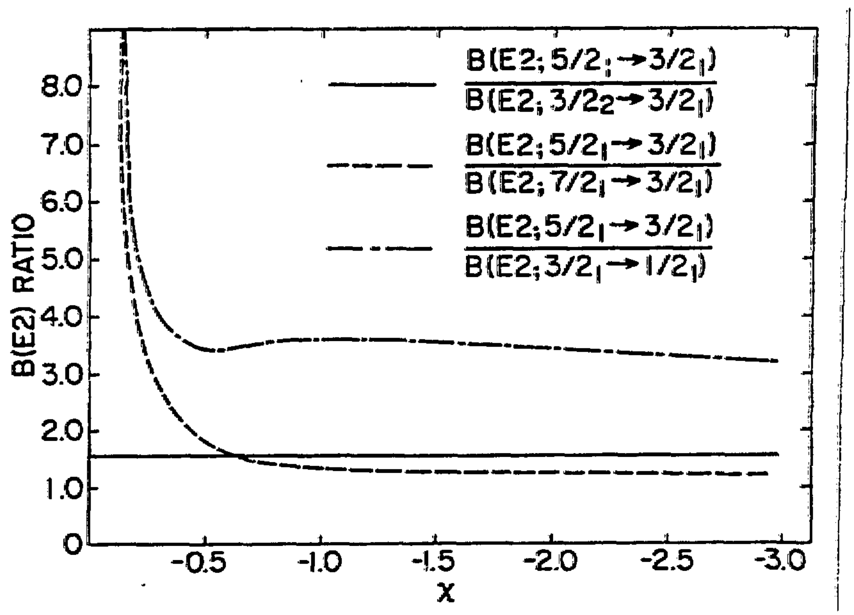


Fig. 6. Predicted B(E2) ratios vs. χ for N=9.

Hamiltonian is that the squares of the matrix elements of $a^{+}_{5/2}$ and $a^{+}_{3/2}$ are always in the ratio 3:2 for two members of a pseudo spin couplet with L=2. As pointed out already in the context of fig. 4, this ratio is constant for all χ values. Thus the curve of fig. 5 applies equally to the ratio of structure factors from the accompanying 5/2 states in each case. The curve has been drawn for N=9, which is appropriate to ^{189}Os, and in that case, the experimental values are R(3/2) = 0.43(6) and R(5/2) = 0.48(7), and hence consistent with equality. The range corresponding to their mean value and error is drawn on the figure, and defines a range of χ values for ^{189}Os, centered on $\chi = -1.5$.

Similar types of predictions can be made for B(E2) values, again couched in terms of ratios to avoid the necessity to specify the effective charge. Some examples are shown in fig. 6, again for N=9, for the lowest lying states. Here, the transition to O(6)-like structure around $\chi = -0.5$ is particularly evident. Also, one ratio maintains a constant value throughout, and this feature again arises from the fact that the pseudo-L symmetry is conserved.

To conclude, the results of figs. 5 and 6 can be combined and compared with the low lying structure in ^{189}Os, as shown in fig. 7. Note that in the experimental part of this figure, the low lying $9/2^-$ and $7/2^-$ states which are known[11] to originate from the $h_{9/2}$ shell model orbit are omitted, since they lie outside the U(6/12) basis being considered here. However, there is very little mixing between these states and the j=1/2, 3/2, 5/2 orbits, so that their neglect should not significantly affect the comparison with the remaining levels.

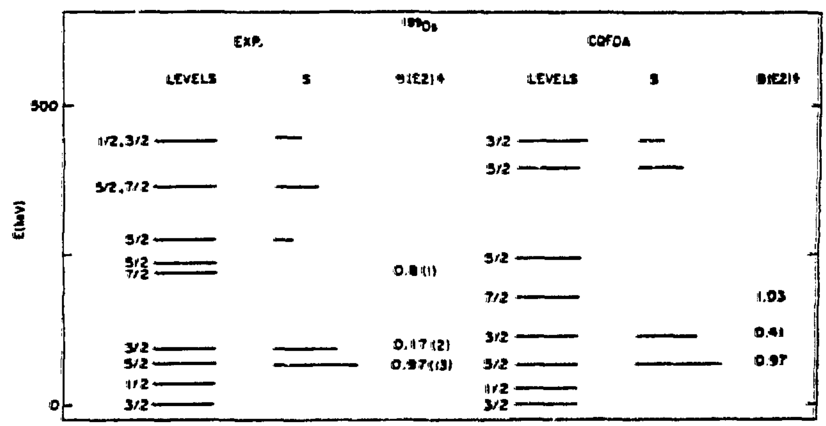


Fig. 7. Calculated and empirical negative-parity structure in ^{189}Os. Lines under S give the relative magnitudes of the (d,t) structure factors. B(E2)↑ implies B(E2) value from the ground state in e^2b^2.

The theoretical description has been obtained with $\chi = -1.5$, as determined from fig. 5. In addition, a negative value of the Spin(3) coefficient has been employed in order to reproduce the empirical inversion of the J, J+1 couplets. At first glance, this may seem unphysical, but the total rotational energy in the odd-A Hamiltonian consists of the sum of the $O^{BF}(3)$ and Spin (3) terms and this, in fact, remains positive.

The agreement for the (d,t) structure factors is excellent, the data reflecting both the 3:2 ratio for each 5/2-3/2 couplet, as well as the predicted absolute magnitudes. The three strongest predicted B(E2) strengths from the ground state are also shown, and these coincide with the strongest measured values[13]. However, in this case, there is a discrepancy of a factor of two for the $3/2_2$ state. The other apparent discrepancy is the existence of an additional $5/2^-$ state in the empirical level scheme, which cannot be accounted for by the theory. The $5/2^-$ spin assignment for this state, suggested in ref. 11, has been questioned in ref. 13, where $3/2^-$ is tentatively proposed, and (n,γ) studies are currently in progress at Brookhaven National Laboratory to clarify this and other uncertainties in the ^{189}Os level scheme.

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