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Alan Minkoff
Steve Swartz
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May 1982

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Center for Computational Research in Economics and Management Science
Massachusetts Institute of Technology
Cambridge, Massachusetts

June 25, 1982

Executive Summary

Analysis of The Validity of The Coefficient Estimates and Forecasting Properties of the RDFOR Models - A Summary Report

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Alan Minkoff
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The Regional Demand FORcasting model (RDFOR) is a simple econometric model currently used by the DOE within the Midterm Energy Forecasting System. Econometric models are often used to provide baseline forecasts of near- to mid-term economic behavior. From the point of view of the policymaker, it is desirable to ascertain as objectively as possible the degree to which these econometric forecasts can be trusted. This paper illustrates, within the context of the industrial and residential sectors of RDFOR, a number of diagnostic tools of general interest which are useful in assessing model reliability.

We present three separate lines of analysis. First, the regression diagnostics of Belsley, Kuh and Welsch (1980) are used to analyze the quality of the model-data interaction. We present information related to the collinearity of the data (the Variance Decomposition) and the sensitivity of the coefficient estimates to the individual data elements (the various regression diagnostics, such as DFFITS, DFBETAS, HAT, and RSTUDENT). Our conclusions are twofold. There is extreme collinearity in the data, which frustrates statistical accuracy and hampers efforts to improve the specification by using more complicated lag structures or by including additional explanatory variables. We also observe evidence that the model assumption of homogeneous behavior across the data set is quite possibly false. We have found evidence of a significant change in behavior during the early and middle 1970's, as well as of differences in price and income response among the DOE regions. These results cast doubt on coefficient estimates and forecasts based on the model.

Second, to more directly understand the quality of RDFOR forecasts, we have conducted several ex-post forecasts over the years 1975-1978. Such forecasts use actual, as opposed to forecast, values of the exogenous series. They are commonly used to assess the quality of forecasting models, as they allow us to separate forecast errors into components based on model and data errors. Our results indicate that RDFOR provides little useful detail in its forecasts over fuels and DOE regions. The median absolute percent error of forecast across regions and fuel types in the industrial sector is 28% after four years (the mean absolute percent error is 40%); the median absolute percent error of forecast in the residential sector is 89% after four years (the mean absolute percent error is over 1500%). We conclude from these results that OLS-based RDFOR forecasts of regional fuel demands are of little value.

Third, we consider an alternative set of robust estimates known as the Bounded-Influence (BIF) estimators. The gains that can be obtained through the use of BIF estimators have been illustrated by forecasts developed with the RDFOR specification. We find that the BIF-based forecasts are relatively superior to the OLS based forecasts. The range of percent error improves considerably by using the BIF coefficients. Thus the problems encountered in producing forecasts of energy demand using the RDFOR specification can be lessened significantly through the use of the BIF estimators.

Several other econometric issues arose during our analysis of RDFOR. We are skeptical about the usefulness of simple difference equation structures in intermediate-term forecasting - they seem unrobust to various common modeling problems. We are dubious about the correctness of the "share equation" specification: shares are not between 0 and 1 in the specification, and do not sum to one; also, the gas service customer series does not perform well. The above issues reinforce the diagnostic conclusions which already cast a pall over the reliability of RDFOR.

The current report complies with all of the tasks in the area of RDFOR mentioned in the "Research Issues Described for EIA Research Contract EX-76-A-01-2295, dated 29 April 1981."

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Table of Contents

	Page
1. Introduction	1
2. The RDFOR Model - An Overview	4
2.1 Model Documentation	4
2.2 RDFOR Modeling Strategy and Estimation Procedures: Some Issues	5
2.3 Model Specification Used in Current Study	11
2.4 Data	14
3. Estimation Results	16
3.1 Residential Sector	16
3.2 Industrial Sector	32
4. Diagnostic Forecasts	47
5. Alternative Estimation Procedures	71
5.1 Bounded Influence Estimators	74
5.2 Empirical Results	82
Appendix A: Results of More Complicated Estimation Procedures	A-1
Appendix B: Definition of the Estimates	B-1

1. Introduction

The Regional Demand FORcasting (RDFOR) Model comes to us at the end of an evolutionary chain spanning most of the last decade. Loosely based on work done by David Knapp and David Nissen (1976), the specification underwent numerous changes both before and after it was integrated into the Mid-term Energy Forecasting System (MEFS) maintained by the U.S. Department of Energy. RDFOR was used extensively by the DOE to provide forecasts of fuel demand dissagregated over DOE regions, relevant fuels, and economic sectors for several years. Except for the industrial sector, it has since been replaced by more complicated structural models.

RDFOR is a simple statistical model. It is a collection of reduced form equations which can be used to project trends in energy demand into the near future. The RDFOR user should keep in mind that because of its simplicity, its scope is inherently limited. The relevance of statistical parameter estimates is conditional on the stability of the modeled structure over the range of the model's application. In the case of reduced form equations such as RDFOR, issues of stability resolve themselves into questions concerning changes in the entire economic sector being modeled. The RDFOR user implicitly assumes that the energy market did not change 'very much' during the period 1961-1978, where 'very much' is very loosely defined. The RDFOR user also assumes that the energy market will not change 'very much' during the forecast period.

Such limitations are not unique to RDFOR; all simulation models operate under the assumption that all major changes in the economy that might be correlated with the series being modeled enter into the model as exogenous data. Everything else is presumably captured by the specification of the model and its estimated parameters. Simple reduced form models sacrifice complexity of detail and wide-ranging applicability in order to achieve ease of use, accessability, and adequate degrees of freedom for estimation. The economy usually exhibits much inertia, which can be used by a reduced form specification to form specific inferences about the future, provided the future does not differ greatly from the past.

The analysis presented in this report has been structured as follows: Section 2 explains the model and the data construction methodology used in our analysis. Section 3 presents coefficient estimates and diagnostics using the ordinary least squares (OLS) estimator for the parameters of the equations of the industrial and residential sectors of RDFOR. Section 4 contains diagnostic forecasts (based on OLS coefficient estimates) over the last quarte of the historical data set which are subsequently used to illustrat issues relating to behavioral homogeneity over time and across DOE regions. Section 5 contains an explanation of an alternative set of robust estimators known as the Bounded-Influence (BIF) estimato which are avilable within the TROLL software environment and an examination of OLS and BIF estimates of the model, diagnostic information relevant to the results, and forecasts comparing the values of the various sets of coefficient estimates within the context of the forecasting model.*

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*Appendix A narrates the results of using more complicated estimation procedures. Appendix B describes the theoretical details of the BIF estimators described in Section 5.

2. The RDFOR Model: An Overview

The demand for fuels within each sector of the United States economy is broken down by RDFOR into two distinct analytical areas First, the total demand for energy in each sector is specified as a function of a price index, a measure of economic activity, demographic factors, and the amount of energy demanded in the previous year. Total demand is then shared out among individual fuels by a series of equations relating the share of demand for each fuel as a function of the relative price of that fuel, demographic factors and the share of demand taken up by the fuel in the previo year. The different sector submodels differ only in the definitic of economic activity used within each submodel and in the number c fuels considered.

2.1 Model Documentation

Because the RDFOR specification has changed each time anyone has used the model, a brief history of its documentation is relevant. The basic model was developed by Knapp and Nissen in 19 The first adaptation of the model by the DOE was documented in the NEO-77 Draft Report (1977). More recently, Synergy, Inc. has beer charged with documenting the entire MEFS system; their papers contain an extensive description of the model. None of these documer are entirely clear as to the functional form, data construction techniques, or parameter constraints used to develop their estima Moreover, revisions of the data make it difficult to reconstruct the series used by these authors. As a result of these problems, we have not tried to replicate any former econometric work done or the model. Instead, we have tried to test the viability of the RDFOR approach to estimating fuel demand, which has been to apply simple econometric methods to a largely synthetic database (SEDS).

Recently the EIA has commissioned several studies of their operating models. While these cannot, by their nature, be taken as a definitive reflection of DOE practice, they nonetheless provide a good picture of the strengths and weaknesses of the specification. Chief among the studies relevant to RDFOR are the statistical analyses of David Freedman (1981) and the diagnostic analysis of Kuh, Lahiri, and Swartz (1981). The Freedman study focuses on the statistical concepts behind the specification and the data base, and delimits the degree to which such a system can be a useful analytic tool. The Kuh, Lahiri, and Swartz study focuses on the data-specification interface and the quality of the information available from the data base as organized by the RDFOR system.

2.2 RDFOR Modeling Strategy and Estimation Procedures: Some Issues

The RDFOR Modeling strategy is potentially faulty. Total energy demand in any sector, for instance, is arrived at by agents operating within the sector as the solution to a maximizing problem into which all prices of factors used in that sector enter. Changes in the relative prices of labor, capital, and raw materials (for example) would change the production plans of firms and change the amount of energy demanded in the industrial sector. Since prices of complements and substitutes of energy are not included in the specification, their effects are relegated to the error term. It is likely that wages and interest rates are correlated with the price of fuel or total value added in the industrial sector; this correlation would bias the estimates. Any significant change in the relative prices of factors in an economic sector would make the coefficient estimates obtained from data gathered before the changes less meaningful, and forecasts using those coefficient estimates would be systematically wrong. The RDFOR user assumes that factor input ratios in the residential and industrial sectors will remain constant during both the estimation and forecast periods

Another potential problem with the RDFOR modeling strategy is that it treats all fuels symmetrically. RDFOR implicitly assumes that fuels are perfect substitutes for one another, ignoring the fact that certain fuels cannot be easily used for certain things. Were the price of coal or liquid gas relative to electricity or natural gas to fall dramatically, we would not expect to see people begin to cook, refrigerate, or heat their homes with liquid gas or coal. RDFOR does not incorporate any such technological or habitual constraints except in the lagged dependent variables of the various equations. During an extended forecast, when the lagged dependent variable is last period's prediction, RDFOR predictions tend to stray significantly from the levels we might expect given these constraints.

Various theoretical considerations of this sort are generally ignored by RDFOR documentation. In fact, the original RDFOR documentation (NEO 77 Draft, p. D-19) states that:

> "...the estimating form for this model is quite far removed from any underlying theory of consumer or producer behavior. The level of aggregation of the data and the inability to measure actual consumption... and stocks of capital, make it impossible to impose a functional structure on the model which directly reflects neoclassical economic theory. What is presented is a highly reduced form of a nondeducible structure."

The error in this logic lies in the assumption that a nondeducible structure does not imply something about the reduced form. Economic theory tells us a number of things which should be correlated with energy demand -- all of those things should be included in a reduced form equation. It is clear that the functional form of the structural equations of the energy sector is not yet well understood in the economics profession. Better reduced form equations, however, which at least do not contradict simple theory, surely do exist.

In our development of the RDFOR specification, we stuck quite close to the structure presented in the NEO 77 Draft.* The theoretical basis of the specification is therein detailed as:

$$QDIV = f(PDIV, Y, HDAY, CDAY, QDIV(-1), POP) \qquad (1)$$

$$Q_i/QDIV = g_i(P_i/PDIV, GSC, Q_i(-1)/QDIV(-1)) \qquad (2)$$

where QDIV is a Divisia index of the quantity of energy demanded; PDIV is a Divisia index of the price of energy demanded: Y is a measure of economic activity (Y is YPER, permanent income, in the residential sector and VADD, value added, in the industrial sector); HDAY and CDAY are heating and cooling degree days; POP is population; Q_i is the quantity demanded of the i^{th} fuel; P_i is the price of the

*The discussion of RDFOR modeling strategy begins in the NEO 77 Draft on p. D-18. The general form of the model is on p. D-21. Compare these with the Synergy documentation p. 78.

i^{th} fuel; and GSC is a measure of the availability of gas service. The construction of the series used in the model will be explained later in this section.

Practical considerations have led to the imposition of a log linear functional form onto equations (1) and (2). We have already discussed why the RDFOR model might be unreliable in the presence of data far different from that encountered in the estimation period. The log linear form imposes constant elasticity onto the estimation output, which is necessary within the MEFS framework. Constant elasticity is a good approximation as long as the data remain close to those encountered during the estimation period. Since we should not use the model during forecast periods with divergent data anyway, the constant elasticity assumption is practically appealing, although the restriction strongly limits the range of application of the model.

After imposing the log linear functional form, the NEO 77 Draft goes on to suggest a number of changes in the specification. First* the report suggests a number of changes in the lag structure on the exogenous series in (1) and (2). On the one hand, the NEO 77 Draft correctly argues that the lagged endogenous variable is necessary to pick up the effects of stickiness in demand due to capital investments, habits, and uncertainty over future conditions. Immediately thereafter, however, the NEO 77 Draft suggests replacing the lagged endogenous variable by a geometric

*The NEO 77 Draft discusses the lag structures on pp. D-23 to D-26; the lag structures are discussed in the Synergy documentation on pp. 80 and 81.

lag on price and a fixed lag structure on income.* Considering income in terms of a fixed lag structure appears reasonable and we have adopted it. Replacing the lagged endogenous variable by the price lag structure does not appear advisable, though, and we have not done so in our work. We have found that the data involved in (1) and (2) already suffer from extreme collinearity. Simultaneously estimating the AR1 correction and the geometric price lag results in intolerable collinearity.† We have found that even state-of-the-art code cannot estimate parameters and related covariances using the available data. The argument for a lag structure of some sort is potentially compelling, yet the lagged dependent variable should not be dropped and the data are not informative enough to determine the parameters associated with a lag structure on price. Since this element of the lag structure is not computationally tractable, we will not consider it further.

For similar reasons, we have not estimated auto-regressive error structures on the RDFOR equations. Although RDFOR documentation historically argues for the existance of autocorrelation within the specification,° we find no strong evidence of it in our work.

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*Because the algebraic transformations involved in estimating an AR1 error and a Kyock geometric lag are similar (both involving the difference between the equation and the equation lagged one period times a coefficient), the various sources are prone to confusing algebraic errors which inhibit understanding exactly what they mean.

†We are assuming here that the equation D-8 on p. D-25 in the NEO 77 Draft (compre with (III-31) on p. 81 of Synergy) is the result of simultaneously dropping the lag term from the specification and using a Koyck transformation to estimate the geometric lag structure coefficients.

°The autoregressive transformation is discussed on pp. D-26 to D-27 in the NEO 77 Draft and on pp. 87 and 98 of Synergy. The NEO 77 Draft discusses the cross-sectional issues associated with autoregression on p. D-30.

It is doubtful whether our data could handle the AR transformation in any case, so it is fortunate that it is not necessary to do so.

Another line of argument in the NEO 77 Draft proposes extensive disaggregation of the parameters of (1) and (2) across the cross-sectional dimension of the specification. In the various versions of RDFOR documentation the depth of disaggregation changes a great deal, but all indicate that estimation should be run on some subsample of the total data set. Theory cannot help us decide an appropriate level of geographical disaggregation (should demand behavior in Boston be constrained to the same as demand behavior in Washington, D.C.? Portland, Maine? Stockbridge, Mass.?) The NEO 77 Draft recognizes* that "the estimation problem (is) one of delineating an appropriate cross section for each of the estimates." The methodology they proposed, however, was "... to start at the coarsest level of aggregation and work down to finer levels as inappropriate results were encountered." In other words, one should search for a disaggregation plan that most nearly duplicates one's prior beliefs. It seems no less efficient to choose appropriate parameter estimates and ignore the estimation results altogether. As the RDFOR data base has grown and parameter estimates have shifted,** the level of cross-sectional disaggregation has apparently changed as well to keep parameter estimates near to 'believable' levels (negative price response, positive income response, and so on).

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*This quote and the next one are from p. D-31 of the NEO 77 Draft.

**The Synergy report lists, for example, coefficients evidently constrained over quite a different regional aggregation plan (pp. 108-109, and pp. 118-119) than those presented in the NEO 77 Draft report (pp. D-72 and D-74). No explanation of the "dummy variable" or of the inclusion of the economic activity variable in the share equations in the Synergy specification is offered.

In our initial estimation we have used a more modest disaggregation scheme. We have estimated the RDFOR equations over a cross-sectional time-series database spanning the ten DOE regions and the eighteen years from 1961-1978. In order to differentiate between inter- and intra-regional effects, we have included a dummy for each region. This brings the number of parameters to be estimated to 15 in (1) and to 13 in (2). The evidence strongly indicates that there is correlation among the residuals from different regions; to estimate the equations using the Zellner method, though, would add over fifty coefficients to an already heavily burdened data set.

Should one be interested in exploring disaggregation, it is not clear that geographic splits are necessarily desirable. Dissaggregation would generally improve the performance of the model only if the smaller units exhibited significantly greater behavioral homogeneity in some well-defined sense. Perhaps the residential model could be disaggregated over income level and urban/rural distinctions, or the industrial model over industry type. The present data set could not support these lines of inquiry; present efforts towards disaggregation are constrained more by the dataset than by any other consideration.

2.3 Model Specification Used in the Current Study

Given the strategy we have outlined, we developed the following estimating equations for the residential sector:

$$QDIVR_{it} = a_i + b \cdot PDIVR_{it} + c \cdot YPER_{it} + d \cdot HDAY_{it} + e \cdot CDAY_{it} + h \cdot QDIVR_{it-1} \quad (3.1)$$

$$QDFR_{it}/QDIVR_{it} = a_i + b \cdot PDFR_{it}/PDIVR_{it} + c \cdot GSCR_{it} + h \cdot QDFR_{it-1}/QDIVR_{it-1} \quad (3.2)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$QNGR_{it}/QDIVR_{it} = a_i + b \cdot PNGR_{it}/PDIVR_{it} + c \cdot GSCR_{it} + h \cdot QNGR_{it-1}/QDIVR_{it-1} \quad (3.5)$$

where all demand relationships specified in the above equations are taken to be linear in the logarithms of the variables.

i - cross-sectional index over the DOE regions (1-10)

t - time index over the years 1961-1978

a_i, b, c, d, e, h - parameters to be estimated (note that the parameters for each equation are not constrained to be equal. We have given them the same names for convenience, and to conform to the documentation)

$QDIVR_{it}$ - the Divisia index of quantity demanded

$PDIVR_{it}$ - the Divisia index of price

$YPER_{it}$ - per capita permanent income

$HDAY_{it}$ - heating degree days

$CDAY_{it}$ - cooling degree days

$Q\begin{Bmatrix}DF\\EL\\LG\\NG\end{Bmatrix}R_{it}$ - quantity of distillate fuels, electricity, liquid gas, and natural gas demanded, respectively

$P\begin{Bmatrix}DF\\EF\\LG\\NG\end{Bmatrix}R_{it}$ - price of distillate fuels, electricity, liquid gas, and natural gas, respectively.

Note that we omitted the equations for electricity (3.3) and liquid gas (3.4) for brevity -- they are similar to (3.2) and (3.5) in an obvious way. The industrial sector is specified similarly, as:

$$QDIVI_{it} = a_i + b \cdot PDIVI_{it} + c \cdot VADD_{it} + d \cdot HDAY_{it} + e \cdot CDAY_{it} + h \cdot QDIVI_{it}-1 \quad (4.1)$$

$$QCOI_{it}/QDIVI_{it} = a_i + b \cdot PCOI_{it}/PDIVI_{it} + c \cdot GSCI_{it} + h \cdot QCOI_{it-1}/QDIVI_{it-1} \quad (4.2)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$QROI_{it}/QDIVI_{it} = a_i + b \cdot PROI_{it}/PDIVI_{it} + c \cdot GSCI_{it} + h \cdot QROI_{it-1}/QDIVI_{it-1} \quad (4.7)$$

where all demand relationships specified in the equations are taken to be linear in the logarithm of the variables.

i - cross-sectional index over the DOE regions (1-10)

t - time index over the years 1961-1978

a_i,b,c,d,e,h - parameters to be estimated (note that the parameters for each equation are not constrained to be equal. We have given the same names for convenience, and to conform to the documentation)

$QDIVI_{it}$ - the Divisia index of quantity demanded

$PDIVI_{it}$ - the Divisia index of price

$VADD_{it}$ - value added

$HDAY_{it}$ - heating degrees days

$CDAY_{it}$ - cooling degree days

$Q\begin{Bmatrix}CO\\DF\\EL\\LG\\NG\\RO\end{Bmatrix}I_{it}$ - quantity of coal, distillate fuels, electricity, liquid gas, natural gas, and residual oils demanded in the industrial sector, respectively

$\begin{Bmatrix}CO\\DF\\EL\\LG\\NG\\RO\end{Bmatrix}I_{it}$ - price of coal, distillate fuels, electricity, liquid gas, natural gas, and residual oils in the industrial sector, respectively.

Note that we omitted the equations for distillate fuels (4.3), electricity (4.4), liquid gas (4.5), and natural gas (4.6) -- they are similar to (4.2) and (4.7) in the obvious way.

2.4 Data

Independent of the quality of the RDFOR specification, demand forecasts using the model are affected by the quality of the associated SEDS database. It should be noted in passing that David Wood (1981) has found large discrepancies between regional data maintained by the EIA in the RDIM database and regional series constructed from the SEDS database. Since most regional series containing the same prices or quantities across different regions are perfectly correlated, it is likely that both databases have been constructed from national series rather than primary sources. There is no need to detail the impact such data has had on the nature of the results presented in this paper. We leave issues concerning the quality of the database to others, however, and focus on questions concerning the possibility of making useful forecasts of future energy demand using RDFOR and SEDS.

SEDS is a state-level database. As such, we have had to construct all of the series necessary for the estimation of RDFOR at the regional level. A description of our data-construction methods follows. The only way in which our construction differs from that described in earlier RDFOR documents is in the Divisia Indices. Apparently, the decision was made for pragmatic reasons early in the development of RDFOR to use an 'index' which is only vaguely similar to a true geometric Divisia Index. We have constructed a true Divisia index, and use it throughout this paper.

Quantity data

QCOI, QDFI, QELI, QLGI, QNGI, QROI, QDFR, QELR, QLGR, and QNGR were all constructed as sums of the data for each state within the respective DOE region.

Price data

PCOI, PDFI, PELI, PLGI, PNGI, PROI, PDFR, PELR, PLGR, and PNGR were all constructed as weighted averages of the data for each state with the respective region, where the weights are the quantity of that fuel consumed in each state divided by the total quantity of that fuel consumed in the region.

Economic Activity data

VADD is the sum of the state data for states within each region. YPER is the weighted sum of the state data for states within each region, where the weights are the population of the state divided by the population of the region.

Geographic data

HDAY and CDAY are the weighted sums of the state data for states within each region, where the weights are the population of the state divided by the population of the region. GSCI and GSCR are sums of state data for states within each region.

Divisia indexes

Within the RDFOR documentation an error has developed and propagated itself.* In our earlier draft** we duplicated the Divisia index as defined in the Synergy documentation and as it is currently used in RDFOR. In this paper, we have constructed a true geometric Divisia index of price and quantity.

QDIVI, QDIVR, PDIVI, and PDIVR are therefore weighted sums of the quantities and prices of fuels k where the weights ω_k are:

$$\omega_k = (((P_{kt} \cdot Q_{kt})/\sum_k (P_{kt} \cdot Q_{kt})) \cdot ((P_{kt-1} \cdot Q_{kt-1})/\sum_k (P_{kt-1} \cdot Q_{kt-1})))^{\frac{1}{2}} \quad (5)$$

$$W_k = \frac{\omega_k}{\sum_k \omega_k}$$

where the change in the logarithm of a Divisia price index is a weighted average of the changes in the logarithms of the individual prices and where the change in the logarithm of a Divisia quantity index is also a weighted average of the changes in the logarithm of the individual quantities.

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*The Divisia is incorrectly presented in the NEO 77 Draft (p. D-22) and repeated in the Synergy documentation (pp. 78-79).

**We copied the Synergy formula in hopes of replicating their presumably authorative documentation.

3. Estimation Results

We will present the results of OLS estimation of the parameters of (3.1) - (3.5) and (4.1) - (4.7) in this section. We will begin with the residential sector, and present the total quantity equation followed by the four share equations. These are followed by results for the industrial sector total energy demand equation, and finally the industrial sector share equations.

All results are presented as they were taken from TROLL output detailing results of the OLS estimation procedures. Coefficient names in some charts differ from those described in Section 2 to obviate the need to remember the meanings of abstract symbols. Output will often omit insignificant or near-zero values from charts for clarity's sake.

3.1 Residential Sector

Exhibit 3.1 is the TROLL output from OLS estimation of equation 3.1.* The significant price coefficient of -.156 (corresponding to a long-run price elasticity of -.85) and income coefficient of .15 (corresponding to a long-run income elasticity of .82) are broadly in conformity with our theoretical intuition, although the price coefficient seems slightly higher than is usually estimated.** None of the coefficients are insignificant. Looking at only these coefficients, the results seem acceptable.

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*In the TROLL output, NOB is the number of observations, NOVAR is the number of regressors, CRSQ is the corrected R^2, SER is standard error of the regression, SSR is the sum of squared residuals, DW is the Durbin Watson statistic, and the other statistics are explained in the text.

**A good summary of empirical work concerning these coefficients may be found in Hartman, R.S., "Frontiers in Energy Demand Modeling," Annual Review of Energy, Vol. 4, 1979.

NOB = 180 NOVAR = 15
CRSQ = 0.989 SER = 0.028 SSR = 0.132 DW(0) = 1.870
COND(X) = 528.204 MAX:HAT = 0.189 RSTUDENT = -3.146 DFFITS = -1.021

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A1	-3.183	0.423	-7.523	0.
A10	-3.092	0.416	-7.441	0.
A2	-3.197	0.424	-7.544	0.
A3	-3.135	0.421	-7.447	0.
A4	-2.922	0.403	-7.246	0.
A5	-3.21	0.428	-7.494	0.
A6	-2.913	0.402	-7.252	0.
A7	-3.174	0.427	-7.427	0.
A8	-3.205	0.429	-7.474	0.
A9	-2.93	0.393	-7.457	0.
B	-0.156	0.034	-4.559	0.
C	0.15	0.055	2.703	0.008
D	0.312	0.043	7.227	0.
E	0.056	0.02	2.851	0.005
H	0.817	0.037	22.049	0.

Exhibit 3.1. OLS Estimates of Total Residential Demand

The statistic labeled COND(X) in Exhibit 3.1, however, indicates a potentially serious problem with the OLS analysis. COND(X) is the scaled condition number of the explanatory variable data; it is the ratio of the largest to the smallest singular value of the familiar data matrix X and is uniquely related to the eigenvalues of (X'X) and $(X'X)^{-1}$.* Singular values are the square roots of the eigenvalues of X; except in totally degenerate cases, there is one singular value for each column of the matrix. COND(X) reflects the degree of collinearity among the exogenous data series. As values of COND(X) greater than 30 indicate potential collinearity problems, our 528 is certainly cause for concern.

Collinearity diagnostics, as developed in Belsley, Kuh, and Welsch (1980), enable us to decompose the X-matrix and apportion the estimated variance of each coefficient among the rows of the X-matrix. The 'variance decomposition' associated with the equation

*The condition number and collinearity analysis in general are explained more fully in Belsley, Kuh and Welsch, Regression Diagnostics, pp. 98-117.

(3.1) is presented as Exhibit 3.2. The rows are ordered by the magnitude of the associated singular value, which is printed next to the row label in the first column of the chart. A colline relationship shows up among the singular values as a large ratio among pairs of them, large being defined (as mentioned above) as a ratio greater than 30. The 'condition index' of comumn 2 is the ratio of the largest singular value among those in column 1 and the singular value associated with each row. Note that the values of the condition indices range from that reported in OLS output as COND(X) (the ratio of the largest to the smallest singular value) down to 1 (the ratio of the largest singular value to itself). The remainder of the chart contains a column for each coefficient, and indicates for each coefficient how much of its estimated variance is due to each row of the chart. The values in each column of this part of the decomposition sum to 1; hence all of the estimated variance of each coefficient is apportioned among the rows. If any row is responsible for a significant (say, >.5) share of the variance of two or more series, then a collinear relationship among those series is indicated. If a series is involved in two or more such relationships, its presence in relationships associated with smaller condition numbers might be masked by the large value(s) in the row(s) above. Practically, one identifies a potential collinear relationship by a large condition index, and then identifies series potentially involved in that relationship by noting which columns of the decomposition either have larg entries in the row itself or have large entries in rows above the one in question. Regressions among the indicated rows can identif actual near-dependencies among the data.

	SING.VAL	COND.IND	CONST	PRICE	INCOME	HEATING	COOLING	QUANT(-1)
ROW1	0.004	528.204	0.974	0.01	0.001	0.953	0.195	0.001
ROW2	0.014	145.648	0.022	0.01	0.066	0.046	0.783	0.075
ROW3	0.037	55.069	0.003	0.085	0.933	0.001	0.022	0.852
ROW4	0.577	3.509		0.739				0.048
ROW5	0.955	2.122		0.009				0.012
ROW6	1.	2.025						
ROW7	1.	2.025						
ROW8	1.	2.025						
ROW9	1.	2.025						
ROW10	1.	2.025						
ROW11	1.002	2.022						
ROW12	1.002	2.021						
ROW13	1.019	1.988		0.003				0.003
ROW14	1.268	1.598		0.142				0.008
ROW15	2.025	1.		0.002				

Exhibit 3.2. Variance Decomposition of the Total Demand Equation for the Residual Sector

The variance decomposition in Exhibit 3.2 indicates three serious collinear relationships. The first row indicates a relationship among the constant terms and HDAY. The second row indicates an independent relationship between CDAY and, presumably, the constant terms again. The third row indicates a near-relationship between YPER and $QDIVR_{-1}$ (and possibly the constants, HDAY and CDAY). Subsequent regression analysis, accomplished by regressing TPPR, YPER, and HDAY in turn on the remaining explanatory variables, $QDIVR_{-1}$, and the regional dummies indicate three relationships with an R^2 greater than .9. Since the relationships overlap, describing any one of them as being "among" a certain subset of the variates would be misleading; effectively, there are three independent and strong linear near-relationships among the fifteen right-hand-side series.

A number of major problems could be associated with this extreme collinearity. Certainly the standard errors of our estimates and of test statistics we might wish to develop with our data are larger than they would be if the collinearity were less extreme. Extensive collinearity also increases the sensitivity of our estimates and sub sequent forecasts to very small changes in the sample or to errors in the data. Given the state of the data base and the complexity of the Divisia structure in the dependent variable, such sensitivity is especially serious. Finally, in small samples such as this one, collinearity makes it difficult to distinguish among linear combinations of the parameters. No solution can be found for this collinea problem within the sample and specification; prior information from outside the model must be used to sharpen the inferences we can make within the model.*

The statistics labeled MAX:HAT, RSTUDENT, and DFFITS in Exhibit 3.1 are the largest members of sets of diagnostic measures designed to measure potential areas of extreme sensitivity of the data-specification fit. One member of each of these sets is associated with each of the data elements. The information in each HAT and RSTUDENT underlies that contained in the respective DFFITS and in th closely related DFBETAS,** which measure differences between regress results using the entire sample and results using the sample with on row deleted. DFFITS is the difference in the fitted or predicted va of the endogenous variable resulting from such a deletion, scaled by measure of the standard error of the regression adjusted so as to be

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* Work on using prior information to cope with collinearity problems in progress at the Center for Computational Research in Economics Management Science by David Belsley. See CCREMS Technical Report Collinearity and Forecasting, by David Belsley.

** The diagnostic measures discussed in this paper are summarized and developed beyond the scope of this paper in Belsley, Kuh, and Wels Regression Diagnostics, pp. 12-21 and 28.

statistically independent of the error. Similarly, DFBETAS is the difference in the parameter estimates caused by the deletion of a row of data scaled by a similarly adjusted measure of the coefficients' standard errors. "Large" DFFITS and/or DFBETAS suggest that one row of data has a relatively strong influence on the regression estimates, and probably merits closer scrutiny. Useful rules of thumb have been developed to flag noteworthy values of the statistics. If n is the number of observations and p the number of estimated parameters then DFFBETAS greater than $2/\sqrt{n}$ (.149 in our sample) or DFFITS greater than $2\sqrt{p/n}$ (.577 in our sample) are considered noteworthy.

HAT and RSTUDENT are fundamental diagnostics more centrally placed within the regression theory. HATs are the diagonal elements of the least-squares projection matrix, also called the hat matrix. HAT is most useful as a diagnostic indicating an outlying observation among the exogenous data. As with the other diagnostics, rules of thumb have been developed to indicate whether or not an individual HAT is noteworthy; an element of HAT is called a <u>leverage point</u> and is considered signficiant if it exceeds 2p/n. RSTUDENT, the studentized residuals, are scaled residuals from the OLS regression. Values of RSTUDENT above 2 indicate significantly large residuals associated with a data point.

A list of data points associated with at least one significant diagnostic is presented as Exhibit 3.3. These diagnostics reveal several potential problems within the analysis. First, over two-thirds of the significant DFFITS are associated with post-embargo data, indicating a potential breakdown or shift in the modeled relationship

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .57735
HAT -- .166667
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	INCOME	HEATING	COOLING	QUAN(-1)
REG 1, 1964								0.182	
REG 1, 1966									0.158
REG 1, 1976	2.601		0.749		0.327				
REG 2, 1974					-0.161				
REG 2, 1976					0.258				
REG 2, 1978	-2.532		-0.88		-0.466		-0.168	0.17	
REG 3, 1974				-0.186			0.154	0.156	
REG 4, 1961						0.166			
REG 4, 1963	-2.08		-0.678	0.288			-0.333		
REG 4, 1968				-0.182			0.178		
REG 4, 1973				0.169			-0.223		
REG 4, 1974				-0.26			0.265		
REG 4, 1977					0.187				
REG 6, 1961	-3.146		-1.021			-0.165		0.161	0.266
REG 6, 1962			0.591			0.32			-0.372
REG 6, 1963				0.19				-0.21	
REG 6, 1969	2.194				-0.173				
REG 6, 1971				0.294		-0.176	-0.274		0.183
REG 6, 1972	2.457		0.7			-0.259			0.267
REG 6, 1974				-0.166					
REG 6, 1975	-2.346		-0.695	-0.183				0.186	
REG 6, 1976								0.185	
REG 6, 1977	2.036		0.646		0.204	0.154			
REG 6, 1978		0.189		0.151			-0.171		
REG 7, 1961						-0.161			
REG 7, 1963								-0.156	
REG 7, 1978				0.29		-0.196	-0.246		
REG 8, 1963								-0.231	
REG 8, 1965								0.252	
REG 8, 1968								-0.188	
REG 8, 1972								-0.155	
REG 8, 1973					0.171				
REG 8, 1974					0.231	-0.182			0.172
REG 8, 1977	-2.362		-0.699		-0.174		0.167		
REG 8, 1978	2.032		0.672			0.369			-0.296
REG 9, 1971				-0.186			0.211		
REG 9, 1974	-2.004		-0.626			0.306			-0.318
REG 10, 1964	2.112		0.584		0.164				
REG 10, 1967							-0.155		
REG 10, 1972									0.159
REG 10, 1973					0.153				
REG 10, 1974					0.243				
REG 10, 1976								0.168	
REG 10, 1977			-0.593	0.18		-0.26		-0.189	0.212

Exhibit 3.3 Significant Diagnostics from the Residential Sector Total

after 1974. Second, the significant DFBETAS are often bunched around specific regions, so that region 2, for instance, has a large and disproportionate impact on the price coefficient and region 6 has a large impact on the coefficient of the lagged term. These cluster effects indicate that the assumption of behavioral homogeneity (constant slope coefficients) across regions is suspect. In fact, over three-quarters of all influential observations are from southern and western regions; it is not absurd to propose that people in those regions might react differently to changes in, say, fuel price or temperature. Finally, we note that a disproportionate number of diagnostics are located at the ends of the time series. In addition to casting doubt on the homogeneity of the relationship with respect to exogenous shocks happening in time, this also indicates a potential breakdown in the linearity of the specification. Since there are so few influential HAT diagnostics, we conclude that aberrant exogenous data is not nearly so much of a problem as is a systematic misfitting of endogenous data throughout the data set.

Let us step back for a moment and review our findings. Basic coefficient estimates of the specification seem intuitively plausible, although the price coefficient seems a little large. Our dataset seems well balanced with few multivariate outliers. Diagnostic evidence indicates that the equation might be tested for time and cross-sectional homogeneity, as misfitting of endogenous data seems systematic. An alarming collinearity problem, discussed in the context of Exhibit 3.2, causes us to worry about the stability of our coefficient estimates in the face of data errors and alternate data construction methods. We will have to appraise trial forecasts of residential energy demand in the light of these findings.

We now turn our attention to the share equations (3.2) - (3.5). OLS estimates of the parameters of these equations are presented as Exhibits 3.4 - 3.7. In all four equations we find price coefficients with the 'correct' sign; three of these are statistically significant We note that the gas service customer variable does not seem to be performing well. It is negatively correlated with the distillate fuel share; since one might imagine distillate fuel to be the closest substitute for gas in home heating, it is surprising to find that the model suggests that the less gas we make available to customers, the lower is distillate fuel's share in total energy demand. Electricity should also be a substitute for gas (in cooking, for example), and in Exhibit 3.5 we find that GSCR indeed does have a significant positive coefficient. The least significant coefficient on the gas serv variable comes in the gas equation, oddly enough; the supply of gas to customers does not significantly affect natural gas demand. We are not satisfied with the performance of this series within the specification.

Although COND(X)s for these regressions are not as large as that which we observed in Exhibit 3.1, serious collinearity is potentially indicated in all four share equations. Since the larges condition indexes in Exhibit 3.2 were associated with the relationship betwen HDAY, CDAY, and the constants, which are not included in the share equations, the collinearity we observe in the share equations is akin in magnitude to the similar collinear relationshi

NOB = 180 NOVAR = 13
CRSQ = 0.999 SER = 0.122 SSR = 2.505 DW(0) = 2.026
COND(X) = 54.559 MAX:HAT = 0.187 RSTUDENT = -6.981 DFFITS = -2.199

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A1	0.411	0.238	1.725	0.086
A10	0.057	0.219	0.258	0.797
A2	0.[illegible]	0.247	2.681	0.008
A3	0.459	0.224	2.05	0.042
A4	0.138	0.243	0.57	0.569
A5	0.653	0.242	2.696	0.008
A6	0.284	0.123	2.317	0.022
A7	0.344	0.165	2.079	0.039
A8	0.23	0.15	1.53	0.128
A9	0.24	0.104	2.301	0.023
B	-0.069	0.064	-1.08	0.282
C	-0.42	0.178	-2.354	0.02
H	0.866	0.04	21.554	0.

Exhibit 3.4. OLS Estimates of the Distillate Fuels Share Equation in the Residential Sector

NOB = 180 NOVAR = 13
CRSQ = 1.000 SER = 0.022 SSR = 0.079 DW(0) = 2.298
COND(X) = 295.379 MAX:HAT = 0.149 RSTUDENT = -3.226 DFFITS = 1.131

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A1	0.641	0.185	3.462	0.001
A10	0.591	0.165	3.578	0.
A2	0.642	0.206	3.111	0.002
A3	0.669	0.203	3.301	0.001
A4	0.736	0.211	3.484	0.001
A5	0.686	0.225	3.047	0.003
A6	0.633	0.205	3.082	0.002
A7	0.591	0.187	3.159	0.002
A8	0.531	0.164	3.24	0.001
A9	0.605	0.202	3.	0.003
B	-0.10[illegible]	0.038	-2.711	0.007
C	0.096	0.03	3.21	0.002
H	0.922	0.029	31.989	0.

Exhibit 3.5. OLS Estimates of the Electricity Share Equation in the Residential Sector

NOB = 180 NOVAR = 13
CRSQ = 0.999 SER = 0.105 SSR = 1.831 DW(0) = 2.341
COND(X) = 80.968 MAX:HAT = 0.154 RSTUDENT = -4.688 DFFITS = -1.248

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A1	1.032	0.25	4.134	0.
A10	0.941	0.263	3.575	0.
A2	0.997	0.211	4.719	0.
A3	1.082	0.244	4.436	0.
A4	1.544	0.374	4.131	0.
A5	1.523	0.309	4.929	0.
A6	1.397	0.294	4.749	0.
A7	1.412	0.291	4.854	0.
A8	1.192	0.251	4.755	0.
A9	1.037	0.213	4.868	0.
B	-0.185	0.041	-4.515	0.
C	0.166	0.15	1.102	0.272
H	0.735	0.052	14.21	0.

Exhibit 3.6. OLS Estimates of the Liquid Gas Share Equation in the Residential Sector

NOB = 180 NOVAR = 13
CRSQ = 1.000 SER = 0.043 SSR = 0.309 DW(0) = 1.868
COND(X) = 110.858 MAX:HAT = 0.192 RSTUDENT = -4.355 DFFITS = -1.330

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A1	0.623	0.153	4.067	0.
A10	0.5	0.141	3.556	0.
A2	0.737	0.164	4.501	0.
A3	0.661	0.166	3.984	0.
A4	0.565	0.174	3.241	0.001
A5	0.779	0.18	4.322	0.
A6	0.589	0.148	3.985	0.
A7	0.594	0.147	4.042	0.
A8	0.487	0.13	3.754	0.
A9	0.653	0.153	4.271	0.
B	-0.202	0.051	-3.925	0.
C	-0.028	0.062	-0.461	0.646
H	0.888	0.024	37.743	0.

Exhibit 3.7. OLS Estimates of the Natural Gas Share Equation in the Residential Sector

we observed in Exhibit 3.2 (between YPER and $QDIVR_{-1}$). Exhibit 3.8 is the variance decomposition for the electricity share equation, which is typical of the four decompositions. We notice that the strongest relationship is one among the relative price, the lagged quantity share, and the constant terms. This indicates a relation between the relative price and lagged quantity share with the means removed, an artifact of the slow adjustment within the energy sector of the economy.

VARDCOM - VARIANCE DECOMPOSITION PROPORTIONS

	SING.VAL	CONO.INO	CONST	PRICE	GAS SER	LAG SHARE
ROW1	0.007	295.379	0.998	0.896	0.032	0.982
ROW2	0.057	34.129	0.001	0.104	0.1	0.012
ROW3	0.064	30.51			0.868	0.006
ROW4	1.	1.951				
ROW5	1.	1.951				
ROW6	1.	1.951				
ROW7	1.	1.351				
ROW8	1.	1.951				
ROW9	1.	1.951				
ROW10	1.	1.951				
ROW11	1.008	1.936				
ROW12	1.083	1.802				
ROW13	1.951	1.				

Exhibit 3.8. Variance Decomposition Matrix for the Electricity Share Equation Within the Residential Submodel

Exhibits 3.9 to 3.12 contain the significant diagnostics from the share equations. Perhaps the first thing to notice here is the concentration of significant DFFITS and RSTUDENTS in a small subset of regions. Three quarters of the significant studentized residuals (including a few very large elements and most of the largest ones) in the distillate fuels equation are in Region 6. Half of the significant RSTUDENTs in the liquid and natural gas share equations are

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT -- .1444444
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SER	LAG SHARE
REG 3, 1978						0.174	
REG 4, 1978							0.153
REG 6, 1961	-6.381		-2.199	-1.039	1.34	0.252	0.949
REG 6, 1962	-2.59	0.178	-1.205	-0.88	0.785	0.348	0.949
REG 6, 1963	2.4	0.187	1.152	0.868	-0.711	-0.313	-0.937
REG 6, 1965	2.267		0.726	0.425	-0.394		-0.403
REG 6, 1966	4.228		1.142	0.364	-0.451		-0.239
REG 6, 1967	-4.29		-1.25	0.281		-0.453	-0.521
REG 6, 1968	3.51		0.888	0.188	-0.191		
REG 8, 1971	-2.183		-0.597	0.19		-0.168	-0.259
REG 6, 1973	3.928		0.976	-0.172	0.198		0.162
REG 6, 1974		0.173		0.374	-0.399		-0.364
REG 6, 1976	-2.475		-0.773	0.413	-0.425		-0.323
REG 6, 1977					0.18	-0.157	
REG 8, 1961						-0.183	
REG 9, 1962	-2.545		-0.693				-0.186
REG 9, 1963				-0.18			
REG 9, 1966				-0.256			
REG 9, 1967				-0.173			
REG 9, 1970	2.23		0.569	0.308			-0.161
REG 9, 1976					0.152		
REG 9, 1977			0.56		0.257		0.151
REG 10, 1961		0.152					
REG 10, 1962						0.151	

Exhibit 3.9. Significant Diagnostics for the Distillate Fuels Share Equation in the Residential Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT -- .1444444
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SER	LAG SHARE
REG 1, 1961				-0.187			0.215
REG 1, 1966	2.301		0.61				
REG 1, 1973				-0.183			0.194
REG 1, 1976	-2.016		-0.627			0.275	
REG 2, 1973	2.481		0.671	-0.272	0.245		0.28
REG 2, 1976			-0.568	0.234	-0.169		-0.254
REG 2, 1978	-3.226		-0.919			0.268	
REG 3, 1961		0.148					
REG 3, 1962				-0.179		0.297	0.202
REG 3, 1966	-2.097		-0.605			-0.25	
REG 3, 1976				0.152			-0.164
REG 3, 1978				-0.199			0.223
REG 4, 1976				0.163			-0.177
REG 5, 1977				0.15	-0.183		
REG 6, 1964					-0.16		
REG 7, 1963	2.213		0.614			-0.177	
REG 7, 1976	-2.012		-0.56				
REG 7, 1977					-0.164		
REG 8, 1963	2.207		0.655	-0.192	0.227	-0.188	0.157
REG 8, 1964	-2.512		-0.75	0.344	-0.371		-0.31
REG 8, 1966					-0.151		
REG 8, 1974				0.249	-0.276		-0.223
REG 8, 1975					0.185		
REG 8, 1977	2.904		1.131	0.659	-0.771		-0.572
REG 8, 1978	-2.108		-0.69		0.244		
REG 9, 1964				-0.208	0.172		0.202
REG 9, 1977				-0.154			0.17
REG 10, 1961		0.149				-0.387	
REG 10, 1962						-0.26	
REG 10, 1964	-2.078		-0.58			0.21	
REG 10, 1973						0.167	

Exhibit 3.10. Significant Diagnostics for the Electricity Share Equation in the Residential Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT -- .1444444
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SER	LAG SHARE
REG 1, 1964	2.378		0.618	0.192		0.175	-0.174
REG 1, 1977				-0.172	0.159	-0.234	
REG 1, 1978				-0.191		-0.236	0.16
REG 2, 1964	3.207		0.794	0.15			
REG 2, 1965	-2.775		-0.769	0.326			-0.349
REG 3, 1962						-0.167	
REG 3, 1964				0.166		0.183	
REG 5, 1963					0.164		
REG 6, 1963			-0.55		0.25		
REG 6, 1964					-0.297		
REG 8, 1963	-2.112		-0.587			0.219	
REG 9, 1962					-0.15		
REG 9, 1964							0.15
REG 9, 1966			-0.547	0.211			-0.282
REG 9, 1975	-4.689		-1.248	-0.158	-0.41	0.179	
REG 9, 1976			-0.674	-0.522			0.503
REG 9, 1977		0.149		-0.392			0.385
REG 9, 1978	2.582	0.154	1.102	0.828			-0.82
REG 10, 1961	-2.603	0.15	-1.096	0.189		0.801	
REG 10, 1962	2.371		0.936	0.222	-0.293	-0.424	-0.308
REG 10, 1963						0.252	
REG 10, 1964	3.189		0.893		-0.163	-0.281	-0.159
REG 10, 1967	2.281		0.573				
REG 10, 1975	-4.097		-1.176		-0.343	-0.403	
REG 10, 1976				-0.178			0.176

Exhibit 3.11. Significant Diagnostics for the Liquid Gas Share Equations in the Residential Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT -- .1444444
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SER	LAG SHARE
REG 1, 1976						0.183	
REG 1, 1978						0.19	
REG 2, 1961					0.217	-0.15	
REG 2, 1966					0.154		
REG 2, 1975					0.217		
REG 3, 1961		0.149					
REG 3, 1966						0.171	
REG 3, 1974					0.165		
REG 4, 1963			0.553	-0.271		-0.247	0.258
REG 4, 1964				-0.159			0.155
REG 4, 1973	-2.466		-0.645	-0.197		-0.179	0.186
REG 4, 1976				0.156			-0.159
REG 6, 1975	2.732		0.827	0.345	0.15		-0.358
REG 6, 1976			0.555		0.308	-0.153	
REG 6, 1977		0.146			0.162		
REG 6, 1978		0.18					
REG 8, 1975	2.082						
REG 8, 1977	-2.323		-0.697		-0.346		
REG 9, 1978				-0.163			0.149
REG 10, 1961		0.192	-0.849	-0.347		0.416	0.401
REG 10, 1962		0.156					
REG 10, 1963				0.166			-0.184
REG 10, 1964	2.037		0.645	0.277	-0.193		-0.301
REG 10, 1969	2.754		0.712			0.218	
REG 10, 1973	-2.128		-0.641	0.219		-0.176	-0.243
REG 10, 1975	4.227		1.189	-0.268		0.356	0.313
REG 10, 1976	-2.966		-1.008	0.589	-0.348		-0.617
REG 10, 1977	-4.355		-1.33	0.591	-0.55		-0.625

Exhibit 3.12. Significant Diagnostics for the Natural Gas Share Equations in the Residential Sector

from region 10. These concentrations indicate either data problems within individual regions or that behavior in those regions is very different than behavior in the rest of the model. These concentratio flag areas in need of further study.

The concentration of significant diagnostics in the post-embargo period that we noticed for the total demand equation is not so obviou here. This indicates that the share equations have a more homogenous response in the pre- and post-embargo periods. Although residential demand seems to have changed a great deal after the 1974 crisis, the share of energy needs met by each fuel seems to have been stable.

The results from the share equations are not initially discouraging. Price coefficients all have correct signs and most are significa Although collinearity might confuse the importance of inertia and pric response, it is not so bad as in the total demand equation. The diagnostics tell us that some of the smaller regions (6, 8, and 10) seem to be totally out of place in some of the share equations, and that the Southwest does not seem to fit well at all. What remains to be seen when we look at the diagnostic forecasts is whether or not the data and specification are sufficiently strong to identify the chief forces at work in apportioning total fuel demand among the different fuels.

3.2 Industrial Sector

Turning now to the industrial sector, we present the result of OLS estimation of equation (4.1), the total demand equation for th industrial sector in Exhibit 3.13. We notice immediately an insignificant price coefficient (corresponding to a long-run price elasticit

NOB = 180 NOVAR = 15
CRSQ = 0.965 SER = 0.038 SSR = 0.242 DW(0) = 1.851
COND(X) = 572.082 MAX:HAT = 0.188 RSTUDENT = 3.066 DFFITS = -0.946

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A01	-3.895	0.644	-6.053	0.
A02	-4.144	0.658	-6.3	0.
A03	-4.047	0.651	-6.218	0.
A04	-4.004	0.634	-6.316	0.
A05	-4.339	0.675	-6.427	0.
A06	-3.782	0.621	-6.095	0.
A07	-3.762	0.643	-5.851	0.
A08	-3.362	0.624	-5.39	0.
A09	-3.904	0.616	-6.335	0.
A10	-3.573	0.621	-5.759	0.
B	-0.009	0.019	-0.488	0.626
C	0.295	0.035	8.482	0.
D	0.105	0.058	1.805	0.073
E	0.02	0.027	0.749	0.455
H	0.659	0.038	17.16	0.

Exhibit 3.13. Coefficient Estimates for the Total Demand Equation for the Industrial Sector

of only .02) along with a strong positive value added coefficient of .295 (corresponding to a reasonable long-run value added elasticity of .86). This does not please our economic intuition at all.

Several stories could be told explaining the absence of a significant price effect. If one believes that the industrial sector is well approximated by a fixed-coefficient model in the short-run, or in periods of great uncertainty, then one would not be surprised to find a lack of price responsiveness over a time period characterized first by extreme stability and then by extreme uncertainty. Another problem lies in the inclusion of value added in the list of regressors. Presumably, changes in the real price of non-labor factors and resources will systematically change the total value of productive activity. This second relationship between price and value added would tend to confuse the estimation of price and value added coefficients.

The extreme collinearity indicated by a COND(X) of 572 will only add to our problems concerning the identification of the price coefficient. The three strong collinear relationships indicated in the variance decomposition, shown as Exhibit 3.14, include two which seem to be between the weather variables and the constant term, and one between the value added and lagged dependent variable series. The collinearity would lead us to suspect that the price coefficient could be lost through sensitivity to data problems rather than an indication of no 'real' price effect.

The significant diagnostics, presented as Exhibit 3.15, indicate a rather worse performance across time, but a better performance across regions, than did the diagnostics for the residential sector. Most of the significant diagnostics are post-embargo. The large RSTUDENTs are mostly associated with late year data from large and influential manufacturing sectors like the Northeast, New York/New Jersey, and the West. We might expect the industrial sector to perform better over regional cross-sections than the residential sector.

VARDCOM - VARIANCE DECOMPOSITION PROPORTIONS

	SING.VAL	COND.IND	CONST	PRICE	VALUE ADDED	HEATING	COOLING	QUANT(-1)
ROW1	0.004	572.082	0.986	0.002	0.126	0.765	0.197	0.104
ROW2	0.007	296.425	0.01	0.046	0.842	0.218	0.002	0.703
ROW3	0.015	138.554	0.004		0.032	0.017	0.801	0.013
ROW4	0.617	3.412		0.77				0.085
ROW5	0.903	2.332		0.103				0.042
ROW6	1.	2.106						
ROW7	1.	2.106						
ROW8	1.	2.106						
ROW9	1.	2.106						
ROW10	1.	2.106						
ROW11	1.001	2.103						
ROW12	1.003	2.099						
ROW13	1.034	2.037		0.054				
ROW14	1.137	1.852		0.012				0.053
ROW15	2.106	1.		0.012				

Exhibit 3.14. The Variance Decomposition of the Total Demand Equation in the Industrial Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .143071
DFFITS -- .57735
HAT -- .166667
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	VALUE ADDED	HEATING	COOLING	QUANT(-1)
REG 1, 1964								-0.152	
REG 1, 1973								0.215	
REG 1, 1975				-0.157	-0.151	0.25			-0.214
REG 1, 1975	2.895		0.806		0.375				
REG 1, 1977					0.205				
REG 2, 1962	3.066		0.886					-0.395	
REG 2, 1971	-2.489		-0.654						
REG 2, 1973							-0.161		
REG 2, 1975	-2.356		-0.774	-0.341	-0.23	0.299	0.251		-0.224
REG 2, 1976	2.336		0.696		0.343				-0.206
REG 2, 1978					-0.21				
REG 3, 1967								0.214	
REG 3, 1973							-0.156		
REG 3, 1974				-0.15					
REG 3, 1975				-0.204		0.204			-0.189
REG 3, 1976					0.165				
REG 4, 1962						-0.216			
REG 4, 1968				0.227			-0.225		
REG 4, 1969				0.166			-0.155		
REG 4, 1974				-0.173			0.2		
REG 4, 1976					0.184				
REG 4, 1978				0.151					
REG 5, 1967								0.206	
REG 6, 1962						-0.24			
REG 6, 1973					-0.186				0.172
REG 6, 1975	-2.947		-0.946	-0.264	-0.356		0.232	0.244	
REG 6, 1978		0.188	-0.74	0.488	-0.316	-0.24	-0.398	-0.191	0.168
REG 7, 1961			-0.637					0.163	
REG 7, 1967	-2.341		-0.812	-0.342			0.262	0.542	
REG 7, 1976	2.768		0.779			-0.208			0.287
REG 7, 1978							-0.183		
REG 8, 1965								0.209	
REG 8, 1967	-2.009							0.164	
REG 8, 1970	-2.037				0.155			-0.215	
REG 9, 1976				0.196			-0.195		
REG 9, 1978	-2.166		-0.811		-0.464	-0.246	0.22	-0.159	0.211
REG 10, 1972	2.219		0.582		-0.174				

Exhibit 3.15. Significant Diagnostics for the Total Demand Equation of the Industrial Sector

OLS estimates of equations (4.2) - (4.7) are presented as Exhibits 3.16 - 3.21. Half of the point estimates on the price share series are positive, although none of the positive coefficients are significant. The gas service customers variable does not perform well again; only one of the six fuels (liquid gas) shows a reasonable coefficient estimate on that series. The coefficients on the lagged dependent variable are generally larger than we have seen so far, indicating that the majority of the explanatory power of the share equations seems to arise from simple auto-regression of the fuel share.

Collinearity problems arise in all of the share equations, although none of them are extremely severe. Exhibit 3.22, the variance decomposition of the electricity share equation, is typical of the decompositions of the share equations. Price and lagged quantity share are strongly related to the constant terms. In some of the other share equations price does not enter into this relationship, but lagged quantity share almost always does. This indicates that quantity shares have remained virtually constant over the estimation period, and that coefficients on the lagged term might tend to be picked up in part by the constant series coefficients.

Significant diagnostics for the industrial share equation are presented as Exhibits 3.23 - 3.28. The regional bunching of large diagnostics that we noticed in the residential share equations is also evident in several of the industrial equations. Regions 1 and 2, for instance, are fit very poorly in the coal and

NOB = 180 NOVAR = 13
CRSQ = 0.999 SER = 0.164 SSR = 4.501 DW(0) = 2.025
COND(X) = 46.383 MAX:HAT = 0.224 RSTUDENT = 8.613 DFFITS = 2.304

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A01	0.156	0.185	0.845	0.4
A02	0.507	0.257	1.899	0.059
A03	0.577	0.283	2.043	0.043
A04	0.53	0.257	2.058	0.041
A05	0.695	0.343	2.024	0.045
A06	0.53	0.262	2.022	0.045
A07	0.438	0.235	1.868	0.063
A08	0.321	0.167	1.922	0.056
A09	0.473	0.223	2.119	0.036
A10	0.26	0.148	1.759	0.08
B	0.008	0.042	0.19	0.85
C	-0.097	0.063	-1.541	0.125
H	0.952	0.026	36.796	0.

Exhibit 3.16. Coefficient Estimates for the Coal Share Equation in the Industrial Sector

NOB = 180 NOVAR = 13
CRSQ = 0.999 SER = 0.118 SSR = 2.321 DW(0) = 1.950
COND(X) = 70.128 MAX:HAT = 0.211 RSTUDENT = 4.076 DFFITS = 1.132

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A01	0.76	0.215	3.542	0.001
A02	0.972	0.27	3.597	0.
A03	1.096	0.283	3.876	0.
A04	1.187	0.299	3.973	0.
A05	1.183	0.338	3.5	0.001
A06	1.096	0.318	3.448	0.001
A07	0.881	0.246	3.582	0.
A08	0.964	0.221	4.369	0.
A09	1.087	0.287	3.785	0.
A10	0.993	0.231	4.304	0.
B	-0.156	0.096	-1.625	0.106
C	0.044	0.046	0.975	0.331
H	0.724	0.06	12.118	0.

Exhibit 3.17. Coefficient Estimates for the Distillate Fuels Share Equation in the Industrial Sector

NOB = 180 NOVAR = 13
CRSQ = 1.000 SER = 0.038 SSR = 0.241 DW(0) = 1.915
COND(X) = 153.481 MAX:HAT = 0.152 RSTUDENT = -4.653 DFFITS = -1.475

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A01	0.605	0.145	4.17	0.
A02	0.696	0.165	4.207	0.
A03	0.75	0.176	4.264	0.
A04	0.782	0.19	4.125	0.
A05	0.798	0.193	4.128	0.
A06	0.674	0.161	4.191	0.
A07	0.585	0.141	4.147	0.
A08	0.532	0.121	4.409	0.
A09	0.679	0.162	4.191	0.
A10	0.576	0.14	4.107	0.
B	-0.102	0.031	-3.272	0.001
C	0.027	0.016	1.707	0.09
H	0.872	0.031	28.012	0.

Exhibit 3.18. Coefficient Estimates for the Electricity Share Equation in the Industrial Sector

NOB = 180 NOVAR = 13
CRSQ = 0.998 SER = 0.139 SSR = 3.242 DW(0) = 1.828
COND(X) = 35.737 MAX:HAT = 0.185 RSTUDENT = 3.454 DFFITS = 1.016

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A01	-0.096	0.138	-0.69	0.491
A02	-0.162	0.168	-0.965	0.336
A03	-0.11	0.157	-0.698	0.486
A04	-0.078	0.169	-0.463	0.644
A05	-0.208	0.225	-0.926	0.356
A06	-0.203	0.228	-0.892	0.374
A07	-0.104	0.165	-0.627	0.531
A08	-0.007	0.089	-0.082	0.935
A09	-0.135	0.165	-0.814	0.417
A10	-0.072	0.081	-0.885	0.378
B	0.105	0.089	1.188	0.237
C	0.121	0.054	2.218	0.028
H	0.956	0.028	34.693	0.

Exhibit 3.19. Coefficient Estimates for the Liquid Gas Share Equation in the Industrial Sector

NOB = 180 NOVAR = 13
CRSQ = 1.000 SER = 0.074 SSR = 0.909 DW(0) = 1.858
COND(X) = 144.139 MAX:HAT = 0.183 RSTUDENT = -7.351 DFFITS = -2.115

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A01	0.984	0.178	5.518	0.
A02	1.257	0.242	5.199	0.
A03	1.447	0.282	5.124	0.
A04	1.438	0.296	4.866	0.
A05	1.734	0.331	5.235	0.
A06	1.694	0.361	4.687	0.
A07	1.27	0.265	4.789	0.
A08	1.022	0.234	4.369	0.
A09	1.437	0.294	4.89	0.
A10	1.122	0.228	4.912	0.
B	-0.307	0.051	-6.003	0.
C	-0.009	0.029	-0.325	0.745
H	0.733	0.05	14.769	0.

Exhibit 3.20. Coefficient Estimates for the Natural Gas Share Equations in the Industrial Sector

NOB = 180 NOVAR = 13
CRSQ = 0.999 SER = 0.149 SSR = 3.704 DW(0) = 1.685
COND(X) = 67.497 MAX:HAT = 0.177 RSTUDENT = -4.929 DFFITS = -1.233

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
A01	0.59	0.319	1.846	0.067
A02	0.617	0.362	1.707	0.09
A03	0.624	0.348	1.794	0.075
A04	0.618	0.331	1.867	0.064
A05	0.598	0.41	1.458	0.147
A06	0.499	0.337	1.479	0.141
A07	0.299	0.255	1.174	0.242
A08	0.421	0.202	2.084	0.039
A09	0.501	0.32	1.565	0.12
A10	0.429	0.217	1.973	0.05
B	0.034	0.054	0.634	0.527
C	0.003	0.064	0.049	0.961
H	0.881	0.044	19.99	0.

Exhibit 3.21. Coefficient Estimates for the Residential Oil Share Equation in the Industrial Sector

	SING.VAL	COND.IND	CONST	PRICE	GAS SERV	LAG SHARE
ROW1	0.013	153.481	0.988	0.478	0.095	0.977
ROW2	0.053	37.056		0.056	0.85	0.021
ROW3	0.11	17.905	0.008	0.465	0.055	0.002
ROW4	1.	1.967				
ROW5	1.	1.967				
ROW6	1.	1.967				
ROW7	1.	1.967				
ROW8	1.	1.967				
ROW9	1.	1.967				
ROW10	1.	1.967	0.001			
ROW11	1.012	1.944	0.001			
ROW12	1.044	1.885	0.002			
ROW13	1.967	1.		0.001		

Exhibit 3.22. The Variance Decomposition of the Electricity Share Equation in the Industrial Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT -- .1444444
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SERV	LAG SHARE
REG 1, 1961				-0.227			0.385
REG 1, 1962				-0.152			0.258
REG 1, 1963							0.181
REG 1, 1965							0.163
REG 1, 1967	2.091		0.575	-0.183			0.257
REG 1, 1970	-3.36		-0.857		-0.199	-0.152	
REG 1, 1973							0.165
REG 1, 1974	3.219		1.144	0.485			-0.79
REG 1, 1976	-2.237		-0.769	-0.255	0.159		0.473
REG 1, 1977		0.158	0.578	0.324		-0.175	-0.426
REG 1, 1978		0.152	-0.591	-0.345		0.2	0.43
REG 6, 1961	-2.212		-0.592		-0.215		
REG 6, 1971					-0.158		
REG 6, 1973	8.613		2.304	0.455	0.369	0.28	-0.807
REG 6, 1977		0.165					
REG 6, 1978		0.154		0.201	0.4		-0.163
REG 7, 1972						0.162	
REG 8, 1962						0.158	
REG 9, 1961	2.492		0.722	0.293	0.215	-0.321	
REG 9, 1962	-2.514		-0.639				-0.15
REG 9, 1973					0.181		
REG 9, 1976		0.151	0.589	-0.157	-0.466		
REG 9, 1977					-0.378		
REG 9, 1978		0.224		0.186		-0.18	
REG 10, 1961				0.202			
REG 10, 1972				-0.172			
REG 10, 1974				0.169			
REG 10, 1975				0.197			
REG 10, 1976			0.583		-0.4		
REG 10, 1977					0.193		

Exhibit 3.23. Significant Diagnostics for the Coal Share Equation in the Industrial Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT -- .1444444
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SERV	LAG SHARE
REG 1, 1961	-2.269		-0.652		0.274		
REG 1, 1965	2.13		0.605	0.295			-0.282
REG 1, 1977	2.379		0.642		0.212	-0.21	
REG 1, 1978					-0.161	0.151	-0.155
REG 2, 1961					0.172		
REG 2, 1964	-2.347		-0.579				
REG 2, 1965	4.076		1.132	0.443		0.17	-0.494
REG 2, 1978	2.317		0.622		0.161	-0.156	0.154
REG 3, 1973						0.163	-0.149
REG 3, 1977	3.032		0.963	-0.413	0.399		0.422
REG 4, 1961						0.151	
REG 4, 1965				0.194			-0.183
REG 4, 1977				-0.169			0.184
REG 5, 1976					0.176		
REG 6, 1968	-2.007		-0.586	-0.264	-0.153		0.284
REG 6, 1977	2.664		0.848	-0.205	-0.465		0.246
REG 6, 1978				-0.254			0.28
REG 7, 1961	-2.5		-0.669		0.231		
REG 7, 1976						-0.153	
REG 7, 1978					0.158	-0.305	
REG 8, 1962						-0.165	
REG 8, 1971	2.066		0.57	0.253			-0.262
REG 8, 1972						-0.203	
REG 9, 1968				0.185			-0.198
REG 9, 1969					-0.151		
REG 9, 1970				-0.178			0.168
REG 9, 1974					-0.158		
REG 9, 1978		0.211	0.588		-0.311	0.452	
REG 10, 1974					-0.305		
REG 10, 1975				0.155			

Exhibit 3.24. Significant Diagnostics for the Distillate Fuel Share Equation in the Industrial Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT -- .1444444
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SERV	LAG SHARE
REG 2, 1961	4.155		1.472	1.036	-0.571	0.332	-1.03
REG 2, 1963	-2.931		-0.733	-0.17			0.169
REG 2, 1975				-0.161			0.168
REG 2, 1976				0.291	-0.155	0.161	-0.299
REG 2, 1977				-0.178			0.187
REG 2, 1978				0.186			-0.196
REG 3, 1964				-0.151			
REG 3, 1978				-0.281			0.284
REG 4, 1961						-0.191	
REG 6, 1961					-0.155		
REG 6, 1975			0.589		-0.421		
REG 7, 1973						-0.199	
REG 7, 1975			0.579			-0.359	
REG 7, 1976	-3.737		-1.261	0.456		0.771	-0.537
REG 7, 1978			0.645			-0.409	
REG 8, 1966	2.083						
REG 8, 1972						-0.17	
REG 8, 1973						-0.28	
REG 9, 1961						0.185	
REG 9, 1963				-0.194			0.203
REG 9, 1979		0.152				0.341	
REG 10, 1974	2.312		0.947	0.585	-0.726	0.23	-0.556
REG 10, 1976					-0.173		
REG 10, 1977	-4.653		-1.475	-0.305	0.843		0.235

Exhibit 3.25. Significant Diagnostics for the Electricity Share Equation in the Industrial Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT -- .1444444
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SERV	LAG SHARE
REG 2, 1961							0.188
REG 2, 1978							0.157
REG 4, 1962				0.151	-0.169		
REG 4, 1964					-0.176		
REG 4, 1965	-2.245		-0.566				
REG 4, 1969					-0.149		
REG 5, 1974					0.159		
REG 6, 1964	3.258		0.905		-0.392		
REG 7, 1970	-2.803		-0.78	0.321		-0.356	
REG 7, 1971						-0.151	
REG 7, 1974					0.314	-0.163	
REG 7, 1975						-0.169	
REG 7, 1976	3.001		1.016	0.284	0.317	-0.563	0.342
REG 7, 1978						0.162	
REG 8, 1967					0.158		
REG 8, 1972				-0.181		0.224	
REG 8, 1973						-0.153	
REG 8, 1978				0.204		-0.205	
REG 9, 1964	-2.466		-0.643	-0.204		0.194	
REG 9, 1966					-0.204		0.209
REG 9, 1969	3.454		0.909				-0.3
REG 9, 1977					0.179		-0.266
REG 9, 1978		0.185					
REG 10, 1961	-3.395		-0.98	-0.431	0.204	0.348	-0.393
REG 10, 1962				0.201	-0.218		
REG 10, 1964	-2.901		-0.712	-0.269			
REG 10, 1965				-0.167			
REG 10, 1970				0.266			
REG 10, 1973				-0.159			
REG 10, 1974	2.154		0.578	0.221	0.231		
REG 10, 1978							0.149

Exhibit 3.26. Significant Diagnostics for the Liquid Gas Share Equations in the Industrial Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT -- .1444444
RSTUDENT -- 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SERV	LAG SHARE
REG 1, 1961				-0.282			0.269
REG 1, 1967					0.223		
REG 1, 1976	-2.34		-0.68	0.353	-0.218		-0.36
REG 2, 1961	-7.351		-2.115	0.936	-0.797	0.469	-1.048
REG 2, 1963				0.229			-0.212
REG 2, 1967	-3.884		-1.008	0.223	-0.321		-0.252
REG 2, 1968	2.238		0.804	0.503	-0.431	0.182	-0.553
REG 2, 1974			-0.554		0.274		
REG 2, 1977				0.167			-0.188
REG 3, 1975					0.168		
REG 4, 1961						0.241	
REG 4, 1976	-2.041						
REG 5, 1962				-0.151			
REG 7, 1975						0.303	
REG 7, 1978						0.151	
REG 8, 1974						-0.207	
REG 8, 1975	-2.105		-0.598	-0.15		-0.27	0.19
REG 8, 1978		0.154					
REG 9, 1978		0.183	-0.947	-0.193		-0.703	0.254
REG 10, 1976					-0.229		-0.167
REG 10, 1977					0.29		
REG 10, 1978			0.603		0.407		0.167

Exhibit 3.27. Significant Diagnostics for the Natural Gas Share Equation in the Industrial Sector

SIGNIFICANT DIAGNOSTICS
CUTOFFS:
DFBETAS -- .149071
DFFITS -- .537484
HAT — .1444444
RSTUDENT — 2

	RSTUDENT	HAT	DFFITS	CONST	PRICE	GAS SERV	LAG SHARE
REG 2, 1965				0.15			
REG 3, 1975					-0.157		
REG 4, 1976					0.188		
REG 4, 1977					0.16		
REG 5, 1972	2.026		0.563		-0.151		-0.209
REG 6, 1965	-4.929		-1.233	0.246			-0.287
REG 6, 1966	-2.389		-0.683	-0.3	0.164		0.34
REG 6, 1968				-0.169			0.194
REG 6, 1969				-0.227			0.271
REG 6, 1971	-2.56		-0.809	-0.411			0.467
REG 6, 1972	2.945		1.141	0.673	-0.377	-0.167	-0.822
REG 6, 1973	4.511		1.199	0.294			-0.368
REG 6, 1974				-0.187	0.265		0.218
REG 6, 1975				-0.251	0.235		0.3
REG 6, 1976				-0.243			0.279
REG 6, 1977				-0.302			0.357
REG 6, 1978				-0.277			0.324
REG 7, 1969						0.155	
REG 7, 1970	-2.141		-0.655	0.308		-0.37	-0.211
REG 7, 1971						-0.226	
REG 7, 1972	2.269		0.709		-0.185	0.293	-0.151
REG 7, 1974					-0.276		
REG 7, 1975				0.18		-0.215	
REG 7, 1977				0.151		-0.176	
REG 7, 1978				0.168		-0.231	
REG 8, 1961						-0.192	
REG 9, 1978		0.177				0.149	
REG 10, 1975			-0.555		-0.281		

Exhibit 3.28. Significant Diagnostics for the Residual Oil Share Equation in the Industrial Sector

electricity share equations, respectively. Both of these poor fits show up as high DFBETAS on the constant terms and on the lagged dependent variable, indicating that the pattern of inertia which characterizes most of the other regions is not in force in the nonconforming regions. Similarly, Region 6 is highly influential in the residual oils share equations. Most of the Exhibits indicate that a small number of regions don't fit well in each share equation.

These share equation diagnostics also seem to be skewed towards the post-embargo data. Almost every large HAT, indicating a multivariate outlier, points to data from the last three years. Most of the large DFFITS point either to post-1974 data or to data from 1961 to 1962, indicating the possibility both of the need for a nonlinear specification and of time heterogeneity. None of these findings are new; they merely reinforce our need to test the RDFOR model for the different problems they suggest.

4. Diagnostic Forecasts

We have been alerted by the diagnostic information presented in Section 3 that RDFOR is potentially weak, in the sense of heterogenous behavior, over both its time and its cross-sectional dimension. Two approaches suggest themselves as a means of exploring the implications of this potential weakness. We might decide that statistical testing of the homogeneity hypotheses underlying RDFOR is called for. On the other hand, we could conduct trial forecasts over the last few years of the data and get a less rigorous but perhaps an intuitively more informative picture of RDFOR's abilities and deficiencies.

We will pursue the second course of action. Due to the extreme collinearity within the data set, our ability to conduct powerful statistical tests is weakened. Tests of cross-sectional homogeneity involve estimation of models with well over 50 parameters, a heavy burden indeed for 180 data elements to carry. Tests of homogeneity over time indicate that a sharp behavioral change occurs during the early seventies, but cannot provide more information concerning the exact period in which behavior changed. We feel that the use of ex-post forecasts provides more insight into the model under these circumstances.

Referring to our earlier draft,* one should note that the diagnostic forecasts presented there are not comparable to those presented in Exhibit 4.1 at all. Because we were forecasting only the Synergy-defined 'Divisia' index, we did not produce a forecast of energy

*Kuh, Lahari, and Swartz, "On Certain Properties of Estimates of the RDFOR Total Energy Demand Equations for the Residential Sector - A Preliminary Analysis," pp. 38-41

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975.........	637.0	1230.1	1026.6	1184.5	2857.8	979.0	765.2	371.1	929.3	277.7	10258.2
1976.........	696.8	1327.6	1086.6	1278.4	2964.9	985.9	773.6	386.8	904.8	274.6	10680.1
1977.........	679.0	1283.0	1064.1	1331.8	2884.7	1052.6	754.3	340.2	852.3	268.0	10509.9
1978.........	660.7	1248.3	1085.8	1320.9	2972.5	1066.2	763.3	371.3	866.3	276.6	10632.9
FORECAST DEMAND											
1975.........	497.1	861.8	887.9	1542.5	5692.0	2270.5	2369.2	1003.9	981.3	558.0	16664.2
1976.........	775.5	1196.7	1028.5	1981.7	13670.0	4350.1	5909.4	2410.0	1460.9	1354.6	34137.5
1977.........	1026.9	1582.6	1140.7	2228.8	23219.7	5704.8	9898.7	3606.6	1838.3	2345.2	52592.1
1978.........	1183.8	1874.3	1215.9	2362.2	32356.9	6169.5	13710.0	4430.7	2052.9	3370.5	68726.8
FORECAST ERROR											
1975.........	-139.9	-368.3	-138.7	358.0	2834.2	1291.4	1604.1	632.8	52.0	280.3	6406.0
1976.........	78.7	-130.9	-58.1	703.3	10705.1	3364.1	5135.8	2023.2	556.1	1080.0	23457.4
1977.........	347.9	299.6	76.6	897.0	20335.0	4652.2	9144.4	3266.4	986.0	2077.1	42082.2
1978.........	523.0	626.1	130.0	1041.3	29384.4	5103.4	12946.7	4059.4	1186.6	3093.9	58094.8
PERCENT ERROR											
1975.........	-22.0	-29.9	-13.5	30.2	99.2	131.9	209.6	170.5	5.6	101.0	62.4
1976.........	11.3	-9.9	-5.3	55.0	361.1	341.2	663.9	523.1	61.5	393.3	219.6
1977.........	51.2	23.4	7.2	67.4	704.9	442.0	1212.3	960.2	115.7	775.0	400.4
1978.........	79.2	50.2	12.0	78.8	988.5	478.7	1696.1	1093.3	137.0	1118.4	546.4

Exhibit 4.1. Total Energy Demand in the Residential Sector

consumption at all. Rather, our first report presented the anti-log of the Synergy-defined 'Divisia', a unitless forecast useful only in assessing the quality of the Divisia equation. The forecasts presented in this paper are based on the summation over fuel type of the individual fuel forecasts from the fuel share equations, which are actually estimated in trillions of btus.

Ex-ante forecasts of fuel demand derive their accuracy (or lack thereof) both from the specification itself and from the artful construction of necessary exogenous data. Ex-post forecasts are a useful diagnostic tool because they abstract from the qualities of predicted exogenous data and indicate the power of the specification alone to forecast future values of the endogenous series. That we have obtained such low quality forecasts for RDFOR indicates better than any other evidence how much caution the analyst must bring to mid-term ex-ante forecasts using the model.

Forecasts over the period 1975-1978 for the residential sector using actual exogenous data and coefficients estimated using data from 1961-1974 are presented as Exhibit 4.1.* They are horrible. The residential model significantly overpredicts several of the quantity Divisias in the first forecast period; in subsequent periods the model compounds its initial errors and adds still more misprediction to the accumulated error. The far right column of Exhibit 4.1 has the forecasts for total United States energy demand -- forecast error builds up to a 545% total percent error in four years.

*All of the forecasts which we present will label information concerning actual and forecast levels, and actual and percent errors on the vertical axis, with the ten DOE regions and the US total on the horizontal axis above the graph.

This substantial error might arise from improper separation of effects over the period 1961-1974, erroneous cross-sectional homogeneity assumptions, and/or the inability of the model specification to cope with the energy market during the mid-seventies. Collinearit among the regressors makes it difficult for straightforward OLS analysis to determine which quantity changes were due to inertia, which to price effects, which to income changes, and so on. Compared to the forecasts, energy demand was relatively stable in the 1975-197 time period; the difference-equation structure of RDFOR is extremely unrobust in the face of shocks such as those which occurred in 1974 (and presumably could reoccur).

Abstracting for a moment from the gross over-estimation of the total residential energy demand, RDFOR also exhibits a wide variation in its ability to accurately capture regional demand. Looking across the bottom of Exhibit 4.1, which are the percent errors of forecast for the total demand equations, we find that the Eastern states fit the specification comparatively well, while the Midwest and the Western regions fit comparatively poorly. Clearly, the present specification is not adequate to the task of predicting regional detail, even beyond its shortcomings in the total demand forecasts.

The vast overprediction of total energy demand indicates heterogeneity of behavior over time; the large differences in the accuracy of demand forecasts among regions indicate heterogeneity of behavior over the cross-section. Evidence of improper model behavior among fuels can be found in Exhibits 4.2 to 4.5, the share equation forecasts of individual fuel demand in the residential sector. Because RDFOR views each fuel as a perfect substitute for every other fuel,

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975..........	375.3	584.9	303.7	160.2	505.3	38.6	55.9	31.6	14.1	53.1	2122.7
1976..........	422.8	645.6	323.3	186.5	568.0	29.0	62.8	28.2	16.5	55.6	2338.2
1977..........	405.6	625.5	314.7	178.4	556.4	40.1	60.1	23.4	20.3	59.1	2283.6
1978..........	390.7	601.4	293.3	163.9	569.6	45.0	66.8	25.3	22.7	59.0	2237.7
FORECAST DEMAND											
1975..........	63.7	69.7	69.9	35.4	83.7	12.4	15.8	7.4	3.3	13.0	374.2
1976..........	14.0	11.4	20.0	9.5	18.2	4.5	5.2	2.1	1.0	3.9	89.8
1977..........	3.9	2.5	7.0	3.1	5.0	1.9	2.0	.8	.3	1.4	27.9
1978..........	1.4	.7	2.9	1.2	1.7	.9	.9	.3	.1	.6	10.8
FORECAST ERROR											
1975..........	-311.6	-515.3	-233.8	-124.8	-421.6	-26.2	-40.1	-24.2	-10.8	-40.1	-1748.5
1976..........	-408.7	-634.2	-303.3	-177.0	-549.8	-24.5	-57.6	-26.1	-15.5	-51.7	-2248.4
1977..........	-401.6	-623.0	-307.7	-175.3	-551.4	-38.2	-58.0	-22.7	-20.0	-57.7	-2255.7
1978..........	-389.3	-600.7	-290.4	-162.7	-567.9	-44.0	-65.9	-25.0	-22.5	-58.4	-2226.9
PERCENT ERROR											
1975..........	-83.0	-88.1	-77.0	-77.9	-83.4	-67.8	-71.8	-76.7	-76.4	-75.6	-82.4
1976..........	-96.7	-98.2	-93.8	-94.9	-96.8	-84.4	-91.7	-92.4	-94.2	-93.0	-96.2
1977..........	-99.0	-99.6	-97.8	-98.2	-99.1	-95.2	-96.6	-96.8	-98.3	-97.6	-98.8
1978..........	-99.6	-99.9	-99.0	-99.2	-99.7	-97.9	-98.6	-98.8	-99.4	-99.0	-99.5

Exhibit 4.2. Demand for Distillate Fuels in the Residential Sector

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975..........	88.2	147.4	207.2	446.1	387.3	244.8	110.5	49.9	190.6	123.1	1995.1
1976..........	93.7	151.7	218.6	462.9	394.5	248.2	109.9	52.6	196.5	128.3	2056.9
1977..........	93.8	154.1	231.2	507.7	414.9	281.0	116.7	54.9	201.6	131.5	2187.6
1978..........	95.8	138.3	255.6	528.4	424.8	303.8	125.3	59.8	213.5	137.8	2283.2
FORECAST DEMAND											
1975..........	103.2	157.1	220.4	488.2	594.3	280.9	176.9	60.9	180.7	211.8	2474.5
1976..........	117.6	167.9	246.1	539.9	858.8	312.8	274.8	74.8	179.4	346.7	3118.8
1977..........	134.6	177.2	275.4	597.8	1182.3	350.2	404.1	91.4	179.1	531.1	3923.2
1978..........	155.2	187.6	310.0	657.6	1563.6	390.3	569.9	111.1	181.6	780.1	4907.0
FORECAST ERROR											
1975..........	15.0	9.7	13.2	42.1	207.1	36.1	66.4	11.0	-9.9	88.7	479.4
1976..........	23.9	16.2	27.5	77.0	464.3	64.6	165.0	22.1	-17.0	218.3	1061.9
1977..........	40.8	23.0	44.2	90.0	767.5	69.1	287.4	36.5	-22.5	399.6	1735.6
1978..........	59.4	49.2	54.4	129.2	1138.8	86.5	444.6	51.3	-31.9	642.3	2623.8
PERCENT ERROR											
1975..........	17.1	6.6	6.4	9.4	53.5	14.8	60.1	21.9	-5.2	72.1	24.0
1976..........	25.5	10.7	12.6	16.6	117.7	26.0	150.1	42.0	-8.7	170.1	51.6
1977..........	43.5	14.9	19.1	17.7	185.0	24.6	246.2	66.4	-11.2	303.9	79.3
1978..........	61.9	35.6	21.3	24.5	268.1	28.5	354.7	85.8	-14.9	466.1	114.9

Exhibit 4.3. Demand for Electricity in the Residential Sector

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975.........	29.7	31.9	44.4	218.0	293.9	201.1	189.1	67.1	30.6	12.2	1117.8
1976.........	31.7	33.6	46.2	227.9	301.6	200.3	187.0	67.8	29.0	12.4	1137.5
1977.........	33.4	34.9	46.7	236.7	290.5	195.7	181.6	65.6	28.0	12.5	1125.5
1978.........	33.9	33.7	44.6	229.4	281.8	194.5	177.4	78.9	41.5	13.8	1129.7
FORECAST DEMAND											
1975.........	228.8	279.0	235.6	749.9	3605.5	1624.7	1778.9	776.1	399.0	255.3	9932.7
1976.........	569.4	758.5	479.9	1222.8	11646.0	3762.8	5250.7	2205.0	1022.7	924.2	27842.0
1977.........	832.4	1212.8	635.7	1463.7	21118.7	5143.3	9146.5	3412.0	1484.6	1733.6	46183.4
1978.........	983.5	1544.1	721.8	1573.3	30063.3	5614.0	12824.1	4236.3	1748.1	2513.2	61821.7
FORECAST ERROR											
1975.........	199.1	247.2	191.2	531.9	3311.7	1423.6	1589.8	709.0	368.4	243.1	8814.8
1976.........	537.7	724.9	433.7	994.8	11344.4	3562.5	5063.6	2137.2	993.7	911.9	26704.5
1977.........	799.1	1177.9	589.0	1227.1	20828.2	4947.5	8964.9	3346.4	1456.7	1721.1	45057.9
1978.........	949.6	1510.3	677.3	1343.8	29781.6	5419.5	12646.7	4157.3	1706.6	2499.4	60692.0
PERCENT ERROR											
1975.........	670.7	777.3	431.1	244.0	1126.9	707.8	840.7	1055.8	1204.3	1996.7	788.6
1976.........	1695.7	2154.6	938.0	436.5	3761.4	1778.8	2707.7	3153.8	3430.8	7361.8	2347.6
1977.........	2395.1	3377.5	1260.9	518.5	7170.3	2527.7	4936.3	5098.2	5210.8	13758.5	4003.3
1978.........	2799.4	4477.1	1518.8	585.7	10570.2	2786.0	7127.7	5267.2	4111.9	18074.9	5372.6

Exhibit 4.4. Demand for Liquid Gas in the Residential Sector

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975.........	143.8	465.9	471.3	360.2	1671.4	494.6	409.7	222.4	694.0	89.3	5022.6
1976.........	148.6	496.7	498.5	401.2	1700.8	508.5	413.9	238.1	662.9	78.3	5147.5
1977.........	146.3	468.5	471.5	409.0	1622.9	535.7	395.9	196.2	602.4	64.9	4913.2
1978.........	140.3	474.8	492.4	399.1	1696.4	522.9	393.7	207.2	588.6	66.0	4981.4
FORECAST DEMAND											
1975.........	101.4	356.0	362.0	269.0	1408.5	352.4	397.7	159.6	398.3	78.0	3882.9
1976.........	74.5	258.9	282.5	209.6	1147.1	270.0	378.7	128.1	257.8	79.8	3086.9
1977.........	56.0	190.1	222.5	164.2	913.7	209.4	346.0	102.4	174.2	79.0	2457.5
1978.........	43.7	142.0	181.1	130.1	728.2	164.3	315.1	83.1	123.1	76.7	1987.4
FORECAST ERROR											
1975.........	-42.4	-109.9	-109.3	-91.1	-262.9	-142.1	-12.1	-62.8	-295.7	-11.4	-1139.8
1976.........	-74.1	-237.8	-216.0	-191.6	-553.7	-238.5	-35.2	-110.1	-405.1	1.5	-2060.5
1977.........	-90.3	-278.4	-248.9	-244.8	-709.2	-326.3	-49.9	-93.8	-428.2	14.1	-2455.6
1978.........	-96.6	-332.8	-311.3	-269.0	-968.2	-358.6	-78.7	-124.1	-465.5	10.6	-2994.1
PERCENT ERROR											
1975.........	-29.5	-23.6	-23.2	-25.3	-15.7	-28.7	-3.0	-28.3	-42.6	-12.7	-22.7
1976.........	-49.9	-47.9	-43.3	-47.7	-32.6	-46.9	-8.5	-46.2	-61.1	2.0	-40.0
1977.........	-61.7	-59.4	-52.8	-59.9	-43.7	-60.9	-12.6	-47.8	-71.1	21.8	-50.0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

its forecasts predict a complete shift from heating oil, and a near-complete shift from natural gas, towards electricity and liquid gas (which have become relatively cheaper over time). Note that RDFOR shifts its overprediction of total residential demand almost totally into the liquid gas market. That RDFOR would suggest an almost total shift toward the use of propane lights, refrigerators, and central heaters in such a short time is indicative of the inadequacy of its fuel-sharing mechanism.

This improper representation of substitutes is not the only problem with the share equation specification. In fact, calling these equations "share" equations at all is a misnomer. Even under the best of circumstances, the sum of four "shares" would not equal 1. Each "share" is actually the quotient of a quantity of btu's and a unitless index.

A summary of the residential ex-post forecasting ability of RDFOR is included as Exhibit 4.6, which contains the percent error portion of Exhibits 4.1 through 4.5. It includes, therefore, percent forecast errors for each fuel and for total fuel consumption, in each region and across the entire country. Obvious patterns in these percent errors document again the evidence for heterogeneity which we have discussed earlier.

Before turning to a closer look at the forecasts themselves, we need to deal with the argument that the poor forecasting ability of RDFOR in our hands is due to our use of a true Divisia index as opposed to the index referred to in earlier RDFOR documentation. Especially in light of the collinearity within the data, the choice

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
DISTILLATE FUELS											
1975.........	-83.03	-88.09	-76.99	-77.90	-83.43	-67.79	-71.76	-76.73	-76.43	-75.57	-82.37
1976.........	-96.68	-98.24	-93.81	-94.90	-96.80	-84.39	-91.74	-92.45	-94.21	-93.02	-96.16
1977.........	-99.03	-99.61	-97.78	-98.24	-99.11	-95.19	-96.64	-96.80	-98.35	-97.63	-98.78
1978.........	-99.65	-99.89	-99.00	-99.25	-99.70	-97.93	-98.62	-98.75	-99.37	-98.98	-99.52
ELECTRICITY											
1975.........	17.07	6.59	6.35	9.43	53.47	14.76	60.13	21.93	-5.20	72.08	24.03
1976.........	25.51	10.70	12.59	16.63	117.68	26.01	150.14	42.03	-8.67	170.13	51.62
1977.........	43.47	14.94	19.12	17.73	184.99	24.60	246.17	66.39	-11.17	303.91	79.34
1978.........	61.94	35.60	21.31	24.45	268.09	28.47	354.67	85.78	-14.93	466.14	114.92
LIQUID GAS											
1975.........	670.70	777.33	431.13	244.02	1126.87	707.79	840.73	1055.79	1204.30	1996.69	788.57
1976.........	1695.74	2154.61	938.03	436.48	3761.42	1778.85	2707.69	3153.56	3430.61	7361.77	2347.63
1977.........	2395.14	3377.53	1260.87	518.52	7170.27	2527.73	4936.33	5098.17	5210.82	13758.46	4003.28
1978.........	2799.42	4477.07	1518.84	585.67	10570.21	2786.01	7127.69	5267.16	4111.93	18074.89	5372.59
NATURAL GAS											
1975.........	-29.48	-23.60	-23.19	-25.30	-15.73	-28.74	-2.95	-28.25	-42.60	-12.72	-22.69
1976.........	-49.87	-47.87	-43.33	-47.75	-32.56	-46.91	-8.50	-46.23	-61.10	1.96	-40.03
1977.........	-61.73	-59.42	-52.80	-59.86	-43.70	-60.91	-12.60	-47.80	-71.09	21.79	-49.98
1978.........	-68.87	-70.09	-63.22	-67.40	-57.07	-68.58	-19.98	-59.90	-79.09	16.11	-60.10
TOTAL DEMAND											
1975.........	-21.96	-29.94	-13.51	30.23	99.17	131.91	209.63	170.54	5.60	100.95	62.45
1976.........	11.30	-9.88	-5.35	55.01	361.06	341.22	663.87	523.06	61.46	393.31	219.64
1977.........	51.24	23.35	7.20	67.36	704.94	441.97	1212.27	960.24	115.69	775.00	400.41
1978.........	79.16	50.16	11.97	78.84	988.54	478.66	1698.10	1093.29	136.98	1118.41	546.42

Exhibit 4.6. Percent Errors of Forecast Within the Residential Sector Submodel

of index clearly influences subsequent coefficient estimates. The choice among indices is similar in many respects to the choice of disaggregation strategies and functional forms. As we argued earlier, although it is true that none of these questions can be directly answered by appeal to theory, it is nonetheless inappropriate to base ones choice solely on pragmatic grounds. Statistical properties of parameter estimates and forecasts are based on the prior hypothesis of a fixed specification, which includes the methods used to construct the series used in the specification. To the extent that one's specification is grounded to some extent in theory, subsequent statistical analysis is at least partially defensible against charges of arbitrary manipulation. Choosing an index because it 'works' over the sample period provides no basis to believe that it will continue to 'work' in the forecast period. One goal of econometric forecasting analysis is the separation of the world-view of the modeler from the world-view imposed on data by the specification; this aim is frustrated when the choice of specification is made result-dependent. The index defined in the earlier RDFOR documentation is certainly a function of quantities and prices. Its 'weights' do not sum to one, it imposes no clear structure on the disaggregate data, and there seems to be no clear justification of its structure anywhere. For these reasons, we feel justified in choosing the index which RDFOR was purporting to use all along, and which is the most widely accepted among the many possible indexes which could be chosen.

To understand the mediocre performance of the RDFOR specification, we have broken down the forecasts of the model into their individual components. Exhibit 4.7 is a detailed analysis of the

	ACTUAL DIVISIA	FORECAST DIVISIA	FORECAST LN(DIVISIA)	CONSTANT EFFECT	TOTAL PRICE EFFECT	TOTAL INCOME EFFECT	LA E
1975....	1.282	1.917	.651	.219	.009	.216	
1976....	1.328	2.627	.966	.219	.000	.220	
1977....	1.328	3.383	1.219	.219	-.010	.227	
1978....	1.363	4.185	1.432	.219	-.010	.235	

Exhibit 4.7. Decomposition of Region 5 Forecasts of QDIVR5

Region 5 forecast of the Divisia Index; we focus on Region 5 merely because it is typical of the rest of the model. The first column i the actual Divisia computed from the data available to us. Column is the RDFOR forecast of this Divisia. Column 3 is the natural log of column 2, and represents the actual values produced by the RDFOR equation for Region 5. Columns 4 through 7 are the actual exogenou (or lagged endogenous) series multiplied by their estimated coeffi- cient. RDFOR predicts the regional Divisias by summing a constant term (Column 4), the product of the price coefficient and the price Divisia (Column 5), the product of the income inefficient and the lo of permanent income (Column 6), and the product of the lag term coe cient and the lag of the past period's quantity Divisia. Hence, Columns 4 through 7 sum to Column 3, the log of the forecast quantit Divisia, and are the sources of the misprediction of Region 5's resi dential fuel demand (which rose to 1000% by the 1978 forecast).

We are immediately struck, upon looking at Exhibit 4.7, by the the fact that the exogenous data hardly varies at all. The specific tion of RDFOR is behaving like a perturbed difference equation. Remember that the lagged term has a coefficient of .8 - we observe that the errors in the forecasts of the Divisia quantity index are large, and that the first difference of the index is decreasing over time (.31 in 1976, .25 in 1977, and .22 in 1978). This is exactly how we would expect a difference equation to behave.

We believe that Exhibit 4.7 illustrates an essential problem with RDFOR. Because the exogenous data are relatively stable during the forecast period, we would need unbelievably large coefficient estimates before the exogenous data would be able to force any significant change in the predicted series. In the face of significant change in energy demands, the difference equation would converge to a new level solely on the basis of the size of the lagged term's coefficient. The data are not adequate for estimating how the model should react to such a shock, since no completed shock-response cycle is contained within the estimation period. The lag coefficient is over-burdened with the job of controlling the behavior of ten difference equations simultaneously. It is no wonder that some of the forecasts rapidly wander off towards a relatively distant stationary point, nor is it surprising that the series does not converge by the end of four years (a shock of one unit in our system would decay as 1 + .8 + .64 + .512 + .41 + .33 + .26 + .20 ...). It does not seem realistic to model in the short or mid-term with a specification that takes this long to stabilize.

Exhibits 4.8 to 4.11 present breakdowns of the share equation forecasts for Region 5 similar to Exhibit 4.7. Columns 1 and 2 are actual and forecast levels of fuel demand, respectively, for each year of the forecast. Since the model actually predicts the log of the quotient of fuel demand and the Divisia quantity index, Columns

3 and 4 are the actual and forecast values of the "shares" of each fuel (note the loose use of the word "share"; due to the relative units of the fuels and the index, the "shares" have arbitrary sizes), while Column 5 is the log of Column 4, the forecast value. These quotients are forecast as the sum of columns 6-8, respectively, the constant term, the price coefficient times the log of the quotient of price and the Divisia price index, and the lag term coefficient times the value of last period's "share". Note that the exogenous data are once again stationary, and that once again the different forecasts wander off from the actual values at a rate that decreases with time. The direction of the movement is determined by the relative magnitudes of the sum of the exogenous effects and the size of the quantity "share"; the speed and duration of the movement is determined by the nearness of the lag coefficient to 1. Notice how swiftly the difference equation structure substitutes away from distillate fuels (home heating oil) and towards liquid gas (propane). The use of RDFO is partly justified by its regional and cross-fuel detail; that the individual fuel forecasts are based on two of these unrobust difference equation structures perhaps explains why the finer detail of the RDFO forecast is of so little use.

The sensitivity of the specification arises from two separate causes. We have indicated that the functional form of the specification causes it to behave like a difference equation. Beyond that, we noted that the difference equations perform badly, at least in part, because the adjustment and forcing function coefficients are constrained to be equal across all ten regions. Exhibit 4.12 present

	ACTUAL FUEL DEMAND	FUEL DEMAND FORECAST	ACTUAL FUEL SHARE	FUEL SHARE FORECAST	FORECAST LOG OF FUEL SHARE	CONSTANT EFFECT	PRICE EFFECT	LAG SHAR EFFECT
1975	505.277	51.532	394.131	26.887	3.292	-1.691	-.042	5.02
1976	567.964	7.407	427.773	2.819	1.036	-1.691	-.039	2.76
1977	556.412	1.431	419.031	.423	-.860	-1.691	-.041	.87
1978	569.567	.361	418.004	.086	-2.452	-1.691	-.038	-.72

Exhibit 4.8. Decomposition of Share Equation Forecasts - Region 5, QDFR5

	ACTUAL FUEL DEMAND	FUEL DEMAND FORECAST	ACTUAL FUEL SHARE	FUEL SHARE FORECAST	FORECAST LOG OF FUEL SHARE	CONSTANT EFFECT	PRICE EFFECT	LAG SHAR EFFECT
1975	387.261	712.643	302.075	371.818	5.918	1.044	-.340	5.21
1976	394.529	1200.219	297.148	456.817	6.124	1.044	-.333	5.41
1977	414.865	1881.237	312.433	556.101	6.321	1.044	-.325	5.60
1978	424.793	2777.784	311.755	663.707	6.498	1.044	-.328	5.78

Exhibit 4.9. Decomposition of Share Equation Forecasts - Region 5, QELR5

	ACTUAL FUEL DEMAND	FUEL DEMAND FORECAST	ACTUAL FUEL SHARE	FUEL SHARE FORECAST	FORECAST LOG OF FUEL SHARE	CONSTANT EFFECT	PRICE EFFECT	LAG SHARE EFFECT
1975	293.880	9026.230	229.235	4709.386	8.457	6.053	-.071	2.47
1976	301.600	46032.200	227.156	17520.360	9.771	6.053	-.080	3.79
1977	290.480	106248.400	218.759	31407.470	10.355	6.053	-.087	4.38
1978	281.750	173567.100	206.775	41471.070	10.633	6.053	-.071	4.65

Exhibit 4.10. Decomposition of Share Equation Forecasts - Region 5, QLGR5

	ACTUAL FUEL DEMAND	FUEL DEMAND FORECAST	ACTUAL FUEL SHARE	FUEL SHARE FORECAST	FORECAST LOG OF FUEL SHARE	CONSTANT EFFECT	PRICE EFFECT
1975	1671.400	1438.876	1303.741	750.726	6.621	-.060	.013
1976	1700.800	1177.780	1280.991	448.277	6.105	-.060	.004
1977	1622.899	931.340	1222.197	275.308	5.618	-.060	-.004
1978	1696.399	730.173	1244.982	174.463	5.162	-.060	-.007

Exhibit 4.11. Decomposition of Share Equation Forecasts - Region 5, QNGR5

	NEW ENG.	NY/NJ	MID ATL.	S. EAST	MID WEST	S. CENT.	CORNBELT	MOUNTAIN	S. WEST
Actual	1.26007	1.16428	1.28565	1.54035	1.28200	1.56146	1.35695	1.36572	1.41928
With	1.29507	1.38331	1.21498	1.22951	1.91665	1.16864	1.94462	1.48476	1.00472
Without	1.23611	1.15067	1.17032	1.53404	1.30337	1.79115	1.35244	1.38166	1.38478
Actual	1.36601	1.23699	1.35909	1.62807	1.32772	1.57149	1.36555	1.41398	1.42458
With	1.30479	1.57084	1.17811	1.01938	2.62735	.89394	2.61930	1.60822	.77898
Without	1.19500	1.11867	1.07341	1.47920	1.27544	1.90792	1.32016	1.42441	1.37114
Actual	1.34195	1.21564	1.37556	1.73977	1.32785	1.72421	1.37349	1.33622	1.40754
With	1.31236	1.73353	1.14562	.87367	3.38291	.71499	3.31764	1.70440	.63336
Without	1.14017	1.07663	.93414	1.38749	1.21737	2.00215	1.24211	1.48490	1.33685
Actual	1.32403	1.15256	1.44623	1.76626	1.36259	1.80181	1.42167	1.47030	1.47135
With	1.33291	1.89291	1.13022	.76902	4.18526	.59034	4.07214	1.80400	.54120
Without	1.10612	1.05826	.84223	1.33667	1.18274	1.96323	1.20173	1.55439	1.33587

Exhibit 4.12. Comparison of RDFOR Forecasts of Divisia Quantity Indices Using Coefficients Estimated Both With and Without Cross-Equation Constraint

forecasts of the Divisia indexes with these cross-sectional constraints removed. For each year of the forecast, the actual Divisias, the Divisias forecast with coefficients estimated using the cross-equation constraints, and the Divisias forecast without using the constraints are presented in turn. The latter model, in which coefficients are estimated separately for each region, performs far better than the constrained model. Many of the individual equations have 'undesirable' coefficient estimates (price coefficients, for example, range from -.97 to .432), yet the unconstrained model forecasts are closer to the true Divisias after four periods in eight of the ten regions. These forecasts indicate that the cross-equation constraints are indeed a significant cause of the poor performance of the RDFOR model.

Exhibit 4.13 presents total demand forecasts in the industrial sector. Four year total forecast error for the industrial model was only 9.1%. Note, though, that the cross-sectional dispersion of that error is hardly smooth. Midwestern energy demand is overstated by 32% by the fourth period, while Southeastern demand is barely overstated at all. Significant movements of economic activity occurring after the estimation period cannot be captured by the specification, suggesting that Sun-belt migration of industry will not be adequately reflected by the forecasts. That the industrial sector performs so much better than the residential sector, though, indicates that industrial behavior is perhaps more homogeneous.

Exhibits 4.14 to 4.19 are forecasts of the industrial share equations of individual fuel demand. Note that United States total percent forecast error ranges from 33% underprediction of distillate fuel and

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975.........	282.5	639.4	2160.5	2144.3	4131.5	4733.1	766.1	584.2	1234.8	518.0	17194.4
1976.........	331.9	759.4	2255.5	2234.0	4215.8	5116.1	848.9	618.6	1235.7	523.9	18139.8
1977.........	355.1	704.8	2121.4	2281.9	4182.2	5388.5	854.7	637.8	1259.9	537.1	18323.3
1978.........	343.0	673.0	2061.2	2319.9	4106.6	6455.3	794.3	630.7	1116.8	588.4	18089.1
FORECAST DEMAND											
1975.........	308.6	732.4	2301.2	2284.2	4327.7	5242.9	776.8	583.6	1279.6	538.2	18375.3
1976.........	309.2	725.4	2263.0	2271.9	4530.6	5041.1	742.8	602.0	1273.6	543.4	18303.0
1977.........	316.4	743.5	2280.7	2297.0	4917.7	5072.0	720.7	623.2	1296.2	559.8	18827.3
1978.........	325.3	755.9	2280.9	2328.9	5434.5	5317.7	719.6	654.6	1339.4	586.5	19743.3
FORECAST ERROR											
1975.........	26.0	93.0	140.7	139.9	196.1	509.9	10.7	-.6	44.8	20.2	1180.9
1976.........	-22.7	-34.0	7.6	37.8	314.8	-75.0	-106.1	-16.6	37.9	19.5	163.2
1977.........	-38.7	38.6	159.3	15.1	735.5	-316.4	-134.0	-14.6	36.4	22.7	503.9
1978.........	-17.7	82.8	219.7	9.0	1328.0	-137.5	-74.7	23.9	222.6	-1.9	1654.3
PERCENT ERROR											
1975.........	9.2	14.6	6.5	6.5	4.7	10.8	1.4	-.1	3.6	3.9	6.9
1976.........	-6.8	-4.5	.3	1.7	7.5	-1.5	-12.5	-2.7	3.1	3.7	.9
1977.........	-10.9	5.5	7.5	.7	17.6	-5.9	-15.7	-2.3	2.9	4.2	2.8
1978.........	-5.2	12.3	10.7	.4	32.3	-2.5	-9.4	3.8	19.9	-.3	9.1

Exhibit 4.13. Total Energy Demand in the Industrial Sector

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975..........	4.3	159.2	1170.6	435.3	1601.5	91.6	88.6	145.1	63.2	37.6	3797.2
1976..........	3.0	212.6	1179.0	413.6	1495.5	94.1	104.8	155.2	80.9	47.9	3786.4
1977	3.5	171.5	1012.8	417.1	1484.7	95.7	111.1	175.8	102.3	38.9	3613.4
1978..........	2.5	138.9	976.0	500.1	1349.7	80.3	99.7	167.4	82.7	30.2	3427.8
FORECAST DEMAND											
1975..........	5.0	215.4	1197.4	472.2	1888.2	106.5	91.1	103.5	60.9	24.3	4164.4
1976..........	4.7	224.8	1181.8	480.8	2282.0	128.6	96.2	86.2	70.1	21.6	4576.8
1977..........	4.5	245.9	1239.5	518.0	2785.6	162.6	101.9	76.3	80.2	19.3	5233.7
1978..........	4.3	261.5	1247.7	536.4	3354.1	204.1	110.5	66.7	91.2	17.1	5893.8
FORECAST ERROR											
1975..........	.7	56.1	26.8	36.9	286.7	14.9	2.4	-41.7	-2.3	-13.3	367.3
1976..........	1.7	12.2	2.8	67.3	786.6	34.5	-8.6	-69.0	-10.8	-26.3	790.4
1977..........	1.0	74.4	226.7	100.9	1300.9	66.9	-9.2	-99.5	-22.1	-19.6	1620.3
1978..........	1.8	122.6	271.8	36.3	2004.4	123.8	10.8	-100.7	8.5	-13.1	2466.2
PERCENT ERROR											
1975..........	15.5	35.3	2.3	8.5	17.9	16.2	2.8	-28.7	-3.7	-35.3	9.7
1976..........	56.3	5.8	.2	16.3	52.6	36.7	-8.2	-44.5	-13.4	-54.8	20.9
1977..........	29.3	43.4	22.4	24.2	87.6	69.9	-8.3	-56.6	-21.6	-50.4	44.8
1978..........	73.2	88.3	27.8	7.3	148.5	154.1	10.9	-60.1	10.2	-43.3	72.0

Exhibit 4.14. Demand for Coal in the Industrial Sector

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975.........	21.8	38.9	88.7	129.2	138.4	145.3	47.5	58.0	78.0	50.6	796.3
1976.........	25.1	44.1	104.4	164.7	172.3	164.4	56.8	62.6	79.0	60.0	933.5
1977.........	34.5	54.4	137.0	204.2	179.3	238.8	57.8	69.0	101.3	72.6	1148.8
1978.........	27.1	65.5	116.0	194.9	175.4	250.2	64.1	67.7	117.2	75.8	1154.0
FORECAST DEMAND											
1975.........	25.4	40.5	72.5	125.2	116.9	129.7	43.1	56.5	83.5	51.3	744.6
1976.........	24.6	39.1	69.7	124.5	104.6	127.6	40.3	62.1	87.5	57.2	737.1
1977.........	24.5	38.4	66.8	124.6	98.9	129.0	39.6	68.0	97.0	62.9	749.8
1978.........	24.4	38.5	67.3	128.7	98.7	126.7	39.9	75.0	108.1	68.7	775.9
FORECAST ERROR											
1975.........	3.6	1.6	-16.2	-4.0	-21.6	-15.6	-4.3	-1.5	5.5	.7	-51.7
1976.........	-.5	-5.0	-34.7	-40.2	-67.7	-36.8	-16.5	-.6	8.5	-2.8	-196.4
1977.........	-10.0	-16.0	-70.2	-79.6	-80.4	-109.7	-18.2	-1.0	-4.3	-9.7	-399.0
1978.........	-2.8	-27.0	-48.7	-66.2	-76.7	-123.5	-24.2	7.2	-9.2	-7.1	-378.1
PERCENT ERROR											
1975.........	16.8	4.2	-18.3	-3.1	-15.6	-10.7	-9.1	-2.6	7.0	1.4	-6.5
1976.........	-2.0	-11.4	-33.3	-24.4	-39.3	-22.4	-29.1	-.9	10.7	-4.6	-21.0
1977.........	-29.1	-29.3	-51.2	-39.0	-44.8	-46.0	-31.4	-1.4	-4.2	-13.4	-34.7
1978.........	-10.1	-41.2	-42.0	-34.0	-43.7	-49.4	-37.7	10.7	-7.8	-9.4	-32.8

Exhibit 4.15. Demand for Distillate Fuels in the Industrial Sector

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975.........	64.0	142.7	251.0	493.4	558.1	289.6	94.9	59.6	195.9	155.0	2304.0
1976.........	69.5	150.0	272.4	546.3	620.8	328.0	92.3	66.6	210.0	169.0	2524.7
1977.........	71.1	158.8	282.6	554.9	656.4	368.4	98.2	72.1	217.6	155.0	2635.0
1978.........	74.8	147.4	307.1	553.7	677.1	391.5	103.5	78.6	221.1	176.1	2730.9
FORECAST DEMAND											
1975.........	66.5	147.1	255.8	485.6	509.3	291.8	85.8	55.9	181.6	170.1	2249.5
1976.........	67.3	143.8	253.0	490.4	490.0	301.0	85.0	60.9	184.2	180.7	2256.2
1977.........	69.7	143.9	252.6	503.0	483.4	316.2	86.2	66.5	191.1	194.4	2307.1
1978.........	72.5	145.3	255.1	516.2	480.4	330.6	88.0	72.6	199.1	210.2	2370.0
FORECAST ERROR											
1975.........	2.5	4.5	4.7	-7.7	-48.8	2.2	-9.1	-3.6	-14.3	15.1	-54.5
1976.........	-2.1	-6.2	-19.4	-55.9	-130.8	-27.0	-7.3	-5.7	-25.8	11.6	-268.5
1977.........	-1.4	-14.8	-29.9	-51.9	-173.0	-52.3	-12.0	-5.5	-26.5	39.4	-327.9
1978.........	-2.3	-2.1	-52.0	-37.5	-196.7	-60.8	-15.5	-6.0	-22.0	34.1	-360.9
PERCENT ERROR											
1975.........	4.0	3.1	1.9	-1.6	-8.7	.8	-9.6	-6.1	-7.3	9.7	-2.4
1976.........	-3.1	-4.1	-7.1	-10.2	-21.1	-8.2	-7.9	-8.6	-12.3	6.9	-10.6
1977.........	-1.9	-9.3	-10.6	-9.4	-26.4	-14.2	-12.2	-7.7	-12.2	25.4	-12.4
1978.........	-3.1	-1.5	-16.9	-6.8	-29.0	-15.5	-15.0	-7.6	-10.0	19.4	-13.2

Exhibit 4.16. Demand for Electricity in the Industrial Sector

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975.........	26.8	23.1	41.1	86.8	140.8	174.9	70.8	22.2	81.9	7.7	676.2
1976.........	30.1	24.0	42.1	94.4	183.5	162.6	117.5	24.1	77.9	8.9	765.3
1977.........	35.1	26.6	47.1	98.1	217.1	152.5	118.6	28.4	64.9	10.2	798.6
1978.........	33.3	31.0	52.1	108.2	211.9	136.9	104.1	24.0	68.7	14.3	784.5
FORECAST DEMAND											
1975.........	24.4	20.3	37.9	78.7	87.4	147.7	56.7	30.6	70.7	10.6	565.1
1976.........	25.7	18.6	37.2	79.6	67.4	120.2	52.6	43.7	67.6	13.4	526.1
1977.........	27.5	17.7	37.3	81.9	54.2	102.7	50.0	61.2	66.4	16.8	515.7
1978.........	29.5	16.9	37.3	84.1	44.9	90.8	48.4	83.4	66.0	21.0	522.3
FORECAST ERROR											
1975.........	-2.4	-2.8	-3.2	-8.2	-53.4	-27.1	-14.2	8.4	-11.1	2.9	-111.1
1976.........	-4.4	-5.4	-4.9	-14.8	-116.1	-42.4	-64.9	19.6	-10.4	4.5	-239.2
1977.........	-7.5	-9.0	-9.8	-16.2	-162.8	-49.8	-68.6	32.8	1.4	6.6	-282.9
1978.........	-3.8	-14.1	-14.8	-24.1	-167.1	-46.1	-55.6	59.4	-2.7	6.7	-262.2
PERCENT ERROR											
1975.........	-9.0	-12.2	-7.7	-9.4	-37.9	-15.5	-20.0	37.8	-13.6	38.3	-16.4
1976.........	-14.6	-22.6	-11.6	-15.6	-63.3	-26.1	-55.2	81.1	-13.3	51.2	-31.3
1977.........	-21.5	-33.7	-20.8	-16.5	-75.0	-32.6	-57.8	115.6	2.2	64.2	-35.4
1978.........	-11.4	-45.5	-28.4	-22.3	-78.8	-33.7	-53.5	247.9	-3.9	46.9	-33.4

Exhibit 4.17. Demand for Liquid Gas in the Industrial Sector

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975..........	49.7	160.2	426.5	823.7	1503.1	3911.3	445.4	245.9	741.8	224.0	8531.8
1976..........	45.9	180.6	432.6	756.1	1517.3	4217.4	457.3	253.2	706.5	194.9	8761.9
1977..........	52.8	152.6	394.5	691.5	1401.8	4347.1	445.8	238.3	701.8	209.5	8635.7
1978..........	51.5	136.5	396.9	670.9	1464.9	4392.0	397.4	236.2	553.0	239.1	8538.5
FORECAST DEMAND											
1975..........	47.3	163.8	520.4	939.8	1527.7	4476.7	484.7	286.0	793.4	222.4	9462.1
1976..........	46.1	154.1	512.0	916.1	1395.9	4283.7	453.7	295.6	771.6	210.8	9039.6
1977..........	46.2	149.7	478.5	888.2	1307.9	4287.0	428.1	294.6	763.3	205.3	8848.8
1978..........	46.2	142.5	469.0	879.8	1269.6	4493.9	417.8	296.8	770.3	205.5	8991.5
FORECAST ERROR											
1975..........	-2.3	3.6	93.9	116.1	24.6	565.4	39.2	40.0	51.5	-1.7	930.4
1976..........	.2	-26.5	79.4	160.1	-121.4	66.3	-3.7	42.5	65.0	15.8	277.8
1977..........	-6.6	-2.9	84.0	196.7	-93.9	-60.1	-17.7	56.3	61.5	-4.2	213.0
1978..........	-5.4	6.0	72.1	208.9	-195.3	101.9	20.4	60.6	217.3	-33.6	453.0
PERCENT ERROR											
1975..........	-4.7	2.3	22.0	14.1	1.6	14.5	8.8	16.3	6.9	-.8	10.9
1976..........	.5	-14.7	18.3	21.2	-8.0	1.6	-.8	16.8	9.2	8.1	3.2
1977..........	-12.5	-1.9	21.3	28.4	-6.7	-1.4	-4.0	23.6	8.8	-2.0	2.5
1978..........	-10.4	4.4	18.2	31.1	-13.3	2.3	5.1	25.7	39.3	-14.1	5.3

Exhibit 4.18. Demand for Natural Gas in the Industrial Sector

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
ACTUAL DEMAND											
1975.........	116.0	115.3	182.6	175.9	189.6	120.4	18.8	53.4	74.0	43.1	1089.0
1976.........	158.3	148.1	225.0	259.0	226.5	149.6	20.1	56.9	81.4	43.3	1368.0
1977.........	158.1	141.0	247.4	316.2	242.9	185.9	23.2	54.2	71.8	50.9	1491.7
1978.........	153.7	153.7	213.1	292.1	227.6	204.3	25.5	56.8	74.0	52.9	1453.6
FORECAST DEMAND											
1975.........	139.9	145.3	217.2	182.7	198.2	90.5	15.5	51.2	89.5	59.5	1189.5
1976.........	140.7	145.0	209.4	180.4	190.6	80.0	15.0	53.5	92.8	59.8	1167.1
1977.........	144.0	147.9	206.0	181.4	187.5	74.5	14.8	56.5	98.2	61.2	1172.1
1978.........	148.4	151.1	204.5	183.7	186.9	71.6	14.9	60.1	104.7	64.0	1189.9
FORECAST ERROR											
1975.........	23.9	30.0	34.7	6.8	8.7	-30.0	-3.3	-2.2	15.5	16.4	100.5
1976.........	-17.6	-3.0	-15.6	-78.7	-35.8	-69.6	-5.1	-3.4	11.4	16.5	-200.9
1977.........	-14.1	6.9	-41.4	-134.7	-55.3	-111.4	-8.4	2.3	26.4	10.3	-319.6
1978.........	-5.3	-2.5	-8.6	-108.4	-40.7	-132.8	-10.6	3.4	30.7	11.1	-263.7
PERCENT ERROR											
1975.........	20.6	26.0	19.0	3.9	4.6	-24.9	-17.7	-4.1	21.0	38.0	9.2
1976.........	-11.1	-2.1	-6.9	-30.4	-15.8	-46.5	-25.5	-5.9	14.0	38.1	-14.7
1977.........	-8.9	4.9	-16.7	-42.6	-22.8	-59.9	-36.1	4.2	36.7	20.1	-21.4
1978.........	-3.4	-1.7	-4.1	-37.1	-17.9	-65.0	-41.6	5.9	41.5	21.0	-18.1

Exhibit 4.19. Demand for Residual Oils in the Industrial Sector

liquid gas demand to a 70% overprediction of coal demand. Again we notice that within each fuel type individual regional predictions vary greatly in their share of forecast error. Note that the Region 8 (Mountain States) forecasts usually behave very differently than do the other regions' forecasts. The large eastern regions (1 and 2) generally have better forecasts than do the western regions. Note the strong correlation between poor fits indicated by the diagnostics and poor forecasts here. For example, Region 1 had a substantial number of influential diagnostics for the coal share equations, and the forecast errors of New England coal demand in the industrial sector are also large. These forecasts certainly fit closely with the evidence presented to us by the diagnostics.

Exhibit 4.20 is the percent forecast errors for the entire industrial model. The results clearly indicate that while the total demand forecast is only moderately bad, information on individual regions and fuels varies quite a bit over the entire forecast. The policy-maker is probably not interested in errors relative to standard error of forecast as much as in absolute error. This evidence indicates that the RDFOR detail over a five-year forecast is likely to be subject to significant absolute error.

It would not be wise to assume that the relatively better performance of the RDFOR model when cross-sectional constraints are removed indicates a direction in which a quick fix of the model could be found. We believe that these constraints are only one of several major problems within the RDFOR specification. RDFOR exhibits numerous significant shortcomings; time and resources spent on repairing it could surely be better spent on more sophisticated energy models with a greater probability of potential gain.

	NORTH-EAST	NY/NJ	MID-ATLANTIC	SOUTH-EAST	MIDWEST	SOUTH-CENTRAL	CORN BELT	MOUNTAIN	SOUTH-WEST	NORTH-WEST	US TOTALS
COAL											
1975.........	15.49	35.26	2.29	8.48	17.90	16.23	2.76	-28.70	-3.69	-35.33	9.67
1976.........	56.33	5.76	.24	16.26	52.60	36.73	-8.20	-44.47	-13.39	-54.82	20.88
1977.........	29.25	43.39	22.38	24.18	87.62	69.91	-8.27	-56.80	-21.64	-50.41	44.84
1978.........	73.21	88.27	27.85	7.26	148.50	154.13	10.88	-60.14	10.22	-43.34	71.95
DISTILLATE FUELS											
1975.........	16.75	4.17	-18.26	-3.07	-15.58	-10.71	-9.13	-2.64	7.04	1.39	-6.49
1976.........	-1.99	-11.42	-33.27	-24.42	-39.30	-22.39	-29.07	-.90	10.72	-4.64	-21.04
1977.........	-29.07	-29.34	-51.22	-38.99	-44.82	-45.96	-31.42	-1.38	-4.25	-13.40	-34.73
1978.........	-10.15	-41.24	-41.98	-33.97	-43.75	-49.36	-37.73	10.69	-7.81	-9.38	-32.77
ELECTRICITY											
1975.........	3.98	3.14	1.88	-1.56	-8.75	.76	-9.56	-6.07	-7.29	9.74	-2.36
1976.........	-3.06	-4.11	-7.13	-10.23	-21.07	-8.24	-7.88	-8.57	-12.28	6.89	-10.64
1977.........	-1.94	-9.34	-10.60	-9.35	-26.35	-14.19	-12.17	-7.68	-12.17	25.42	-12.44
1978.........	-3.11	-1.46	-16.94	-6.77	-29.05	-15.54	-14.97	-7.58	-9.96	19.36	-13.22
LIQUID GAS											
1975.........	-9.02	-12.17	-7.71	-9.39	-37.93	-15.51	-20.04	37.82	-13.58	38.28	-16.43
1976.........	-14.56	-22.56	-11.63	-15.65	-63.27	-26.09	-55.21	81.09	-13.33	51.23	-31.26
1977.........	-21.46	-33.68	-20.84	-16.54	-75.01	-32.65	-57.82	115.62	2.18	64.21	-35.43
1978.........	-11.43	-45.51	-28.41	-22.27	-78.84	-33.66	-53.46	247.86	-3.94	46.86	-33.42
NATURAL GAS											
1975.........	-4.67	2.26	22.00	14.09	1.64	14.46	8.80	16.29	6.95	-.76	10.90
1976.........	.46	-14.70	18.35	21.17	-8.00	1.57	-.80	16.77	9.21	8.13	3.17
1977.........	-12.55	-1.93	21.29	28.45	-6.70	-1.38	-3.97	23.62	8.76	-2.00	2.47
1978.........	-10.40	4.43	18.17	31.13	-13.33	2.32	5.14	25.65	39.29	-14.05	5.31
RESIDUAL OILS											
1975.........	20.63	25.99	18.99	3.86	4.57	-24.87	-17.69	-4.09	21.03	38.03	9.23
1976.........	-11.11	-2.06	-6.93	-30.37	-15.82	-46.53	-25.49	-5.90	14.02	38.14	-14.68
1977.........	-8.91	4.90	-16.74	-42.62	-22.78	-59.94	-36.13	4.22	36.68	20.12	-21.42
1978.........	-3.42	-1.65	-4.05	-37.11	-17.86	-64.98	-41.58	5.90	41.51	20.96	-18.14
TOTAL DEMAND											
1975.........	9.22	14.55	6.51	6.53	4.75	10.77	1.40	-.09	3.63	3.89	6.87
1976.........	-6.84	-4.47	.34	1.69	7.47	-1.47	-12.49	-2.69	3.07	3.72	.90
1977.........	-10.88	5.48	7.51	.68	17.59	-5.87	-15.67	-2.29	2.89	4.23	2.75
1978.........	-5.15	12.31	10.66	.39	32.34	-2.52	-9.40	3.79	19.93	-.32	9.15

Exhibit 4.20. Percent Errors of Forecast Within the Industrial Sector Submodel

5. Alternative Estimation Procedures

The ordinary least-squares (OLS) estimator of the coefficients of the classical linear regression model has a number of desirable qualities. Under the appropriate assumptions (see e.g., Theil (1971)) it is unbiased, the maximum likelihood estimator, consistent, and asymptotically normal. The salient argument favoring the use of OLS, though, seems to be the Gauss-Markov Theorem: among all unbiased linear estimators of the regression model, the OLS estimator is BLUE (Best Linear Unbiased Estimator) in that its covariance matrix differs from the covariance matrix of any other linear estimator by a positive definite matrix.

The Gauss-Markov Theorem establishes that, for at least one choice criterion, OLS is the optimal estimator of the coefficient of the linear model. What is often overlooked in the econometrics literature is that other criteria exist by which to judge estimators. Many of these are at least as important as efficiency. For example, OLS and many other commonly used maximum likelihood techniques have an unbounded influence function; any small subset of the data can have an arbitrarily large influence on their coefficient estimates. In a world of fat-tailed or asymmetric error distributions, data errors, and imperfectly specified models, it is just that data in which we have the least faith that often exerts the most influence upon the OLS estimates.

Estimators whose influence functions are not unbounded are known collectively as <u>bounded-influence</u> (BIF) estimators. They are members of a larger family of statistical techniques (robust techniques) which are designed to safeguard against model failure by sacrificing efficiency in the absence of deviations from the statistical model in favor of decreased sensitivity to the presence of such problems. In the linear regression model, BIF estimators have a number of advantages. They safeguard against increases in estimated parameter variance caused by fat-tailed error distributions unrelated to the explanatory variable data. When there is reason to believe that the error contamination is related to the data (keypunch errors which are associated with large data values, or breakdown in the specification at extreme values of the data), then bias as well as variability is a concern. BIF estimators control the bias inherent in such situations by limiting the influence that aberrant data can have on the final estimates Subject to a constraint on the influence any data point can have, the BIF estimator recently developed by Krasker and Welsch (1982) can be shown to meet the necessary conditions for an efficient estimator.

We will illustrate the gains that can be had through the use of BIF estimators by considering the forecasts developed with the RDFOR specification. Using coefficient estimates based on the first three quarters of the time series data associated with the models, we have found that *ex post* forecasts based on BIF estimates are significantly better than those based on OLS estimates. BIF estimates seem to lead to a decrease in the largest errors of the OLS based forecasts while providing virtually the same accuracy in other instances. Since we are chiefly concerned with avoiding very large errors in applied work, this result argues that BIF estimators deserve consideration when developing forecasts with the standard linear regression model.

Because RDFOR is a reduced form model, and because of the structural problems mentioned earlier, it cannot be expected to fit the demand patterns manifested by the real economy in any exact way. The Kuh, Lahiri, and Swartz analysis found RDFOR particularly bad at fitting the response to the energy price shocks of the early 1970's. Moreover, the specification does not perform well for the very early data values in the time range. Large and systematic trends in various diagnostic measures indicate that potential biases mar the accuracy of the OLS coefficient estimates. The collinearity of the data both magnify the effect of these biasing forces and increase the sensitivity of forecasts to whatever biases occur. For these reasons, we are skeptical of the ability of the OLS estimator to produce good forecasts. Bounded influence estimators should perform better in an environment such as this.

We have organized our exposition in this section as follows. In Section 5.1, we attempt to present a coherent intuitive explanation of the various robust estimators available in the BIF package on TROLL; a more technical exposition is included as Appendix B for those so inclined. Section 5.2 contains OLS and BIF estimates of the equations necessary to produce estimates of coal demand in the industrial section, a comparison of forecasts based on these results, and a discussion of additional or complementary insights such ex-post BIF forecasts provide within the context of our analysis. Our emphasis is on the ability of BIF to provide diagnostic insights, rather than on the potential of BIF estimates to 'rescue' the model.

5.1 Bounded-Influence Estimators*

The OLS estimator can be intuitively justified from a number of perspectives. Geometrically, it can be viewed as a projection operator into the space spanned by the exogenous data series considered as n-vectors (where there are n observations). Statistically, OLS is, of course, the maximum likelihood estimator of the linear regression model for i.i.d. normally distributed errors. Due to the special nature of the above operations, they are both computationally equivalent to minimizing the sum of the squared residuals with respect to the coefficients of the regression model - hence the estimator's name.

That the influence function of the OLS estimator is unbounded can also be understood from a variety of perspectives. The following thought experiment is typical. Suppose that we knew the 'real' coefficients. We could identify some subset of observations which were fit least well by those coefficients. Allow that subset of the data to become more and more aberrant. As it does so, the associated 'real' residuals would grow without bound. The least-squares estimator will shift its guesses of the true values of the coefficients in such a way that these large residuals are minimized at the expense of the residua OLS will adjust its coefficient estimates based on information from a small, aberrant set of data points; OLS is therefore unbounded.

*This section is intended to be intuitive rather than rigorous. For a more complete technical development of this material, and for a description of software which implements these estimators, see Computational Procedures for Bounded-Influence and Robust Regression, by Peters, Samarov, and Welsch (CCREMS TR-30), a portion of which has been extracted to become Appendix B.

The systematic downweighting of aberrant, or influential, observations will not cause the coefficient estimates of BIF estimators to be inconsistent. Should the information embodied in influential observations be due to some unmodeled mechanism, we would like to isolate and ignore that information when developing coefficient estimates. If, on the other hand, the influential observations are generated by the vicissitudes of chance as embodied in the error process, their symmetric downweighting will not create bias in the estimates. In general, there is no direct cost of bias in the coefficient estimates resulting from the methods used by BIF estimators to downweight influential data.

Accurate identification of influential observations relies on prior knowledge of the true coefficient values of the model. Our thought experiment was simplified by the fact that we assumed perfect information regarding the 'real' coefficient values, which enabled us to pick out influential observations. Finding such observations in a real sample is problematic because the very aberrations we are looking for cloud our conception of the true coefficient values and make aberrations less obvious; influential data hides itself. A feasible strategy in the absence of perfect foreknowledge of the coefficient values (a common situation, after all) is to develop some concept of the central tendency of the model that is robust to potentially influential observations and use that concept to identify candidate influential observations. Downweighting these 'bad' data, we would develop a new idea of the model's central tendency, identify 'bad' data within this new view, revise our outlook once again, and so on. Huber (1973, 1975, and 1981) worked out the procedures necessary to implement this iterative process and proved its convergence in some specific cases.

We have been discussing influential observations as though they were a phenomena having to do only with the residuals of the model. Actually, influential observations enter into the computation of the OLS coefficient estimates both as a function of the residual and as a function of the magnitude of the X-data. The solution to the problem of minimizing the sum of the squares of the residuals is equivalent to the problem of solving a set of equations, called normal equations, which are the derivatives of the sum of the squared residuals with respect to the coefficient estimates (because at the minimum of a function, that function's derivatives must be zero). The normal equations of the OLS estimator contain terms in both the residuals and the X-data; most bounded-influence estimators downweight both outlying X-data and observations corresponding to large residuals in order to achieve boundedness.

Corresponding to these two sources of unboundedness are two distinct mathematical techniques which simultaneously downweight influential observations in the calculation of most BIF coefficient estimates. The first involves changing the function through which residuals enter the sum which is minimized to find the estimates. By convention, this function is called the robustifying criterion function; we refer to it as the ρ-function. OLS corresponds to an estimator for which the ρ-function is always $(\cdot)^2$. Minimum absolute error estimators are occasionally used, for which the ρ-function is always $|\cdot|$. For robust estimation of the linear model, a cutoff value related to the number of data values and the desired efficiency of the estimator in well-behaved circumstances is calculated and used to identify residuals as large. As implemented first by Huber (1974, 1977) and later in all of the BIF estimators we shall consider, the ρ-function of choice is to allow a residual to enter the sum as a function of its square if it is less than the cutoff, and as a function of its absolute value if it is greater than the cutoff. Here greater than and less than obviously refer to the absolute value of the residual. In the robust estimator due to Huber, this is the only kind of downweighting that occurs. Since even the most abberrant of data still enter the sum as a function of the absolute value of the error, the Huber estimator is clearly unbounded (although it is more robust than OLS when influential data are present).

When we say that the cutoffs are determined as a function of the desired efficiency of the BIF estimator, we are referring to the relative efficiency with respect to the OLS estimator. Since the comparison of efficiencies involves placing an ordering on square matrices, and since no clear and widely accepted ordering is known, a number of proxies are implemented by various researchers. One suggestion is to compare the determinants of the OLS and BIF covariance matrices. Another is to use the ratio of the greatest eigenvalues of the two matrices. These two measures bound a range of plausible relative efficiencies relating to sums and averages of the eigenvalues of the different matrices. Since economists seem to think more in terms of the determinants of the covariance matrices, we have used that measure in our analysis.

The second mathematical technique used to downweight influential observations in the calculation of BIF coefficient estimates involves the identification of outlying values in the space of the exogenous series. BIF estimators are differentiated by the sophistication of the tools they use to determine whether or not a particular observation is 'different' from the other, 'normal' observations. We will discuss the methods used by the various estimators in turn.

The BIF estimation procedure due to Mallows (1973, 1975) uses two separate iterative processes, governed by two separate cutoffs, to downweight influential observations. The first iterative process calculates X-weights based on a robust distance measure in a manner similar to the iterative process we described earlier. Initial X-weights are calculated to downweight outliers; the revised X-matrix is analyzed and new X-weights are calculated; and so on. This weighted X-matrix is used in the second iterative procedure, where further downweighting occurs corresponding to the size of the residuals arising from coefficient estimates developed from the most recent version of the X-matrix. As a data-analysis tool, the Mallows estimator is intuitively clear. The choice of separate cutoffs for the separate sources of contamination (X-outliers and large residuals) allows the user a greater degree of flexibility than can be had from the other BIF estimators. Remember, though, that outlying data that fits well increases the efficiency of the estimates. Any downweighting of X-data which does not take account of the quality of the fit of that X-data cannot produce efficient estimates because the outlying points, if they contain good information, would contribute much to the precision of the estimates. Since the Mallows estimator develops its X-weights before it even considers the fit of the data, the other BIF estimators are to be preferred on purely statistical grounds.

The Schweppe BIF estimator (see Handschin et al. (1975)) uses a relatively simple algorithm to develop initial X-weights which both provides bounded influence and does not downweight 'good' outliers. It accomplishes this feat through the use of the diagonal elements of the OLS projection matrix $X(X'X)^{-1}X'$, called the HAT values.[18] The HAT values are used to compute initial X-weights through a simple, noniterative process. Although the quality of the outliers influences the initial X-weights, this simple algorithm does not optimally combine information about outliers in X-space and poorly fit observations. For example, the Schweppe algorithm is not sophisticated enough to identify nearly replicated outliers well. Imagine one outlier in X-space duplicated over and over in the sample. As the outlier appears more often, it seems less different in some naive way (there are lots of other points like it in the sample!) and the Schweppe algorithm downweights the outliers less severely. This and other problems indicate that the Schweppe algorithm is too simple to be consistently useful as a BIF estimator.

Given the bound on efficiency arising from the cutoff value of the ρ-function, but otherwise completely buying the statistical model, the BIF estimator developed by Krasker and Welsch (1979) can be shown to meet the necessary conditions for an efficient BIF estimator. Its algorithm is difficult to explain intuitively; the interested reader is referred to Appendix B. If one wants to use a bounded-influence estimator but otherwise stick closely to the worldview underlying the OLS estimator, then the Krasker-Welsch estimator with its Gauss-Markov-like efficiency properties and its minimal violation of the data is clearly the estimator of choice. If, on the other hand, one is more interested in data analysis and skeptical about efficiency results of all kinds then the choice of the estimator is less clear. Since the different estimators use different methods to guard against model failure, they can provide different kinds of information about the data and the fit; one cannot easily choose arbitrarily among the estimators.

The boundedness criterion is intuitively attractive, and should enter into the decision regarding an estimator for the linear model. Even the most non-interventionist analyst among us should have some limit, albeit large, on the amount of influence any one data point can have on the coefficient estimates; one cannot trust the data or oneself too much, after all. As a practical matter, then, some very lenient cutoff associated with a very moderate loss in efficiency vis á vis the OLS estimator is probably always called for. Should there be no problem at all within the data set, BIF estimates will turn out to be quite close to OLS estimates. The small reduction in the power of inference is not often likely to be important; the reduction of uncertainty concerning some of the model assumptions which impacts on the usefulness of the inference is often likely to be important. This gain, plus the diagnostic information available as a result of BIF estimation, argues strongly for using BIF estimators for the linear model.

5.2 The Empirical Results

When choosing a reduced-form, econometric model over a structural, 'engineering' model for forecasting purposes, one sacrifices detail and scope for simplicity and inexpensiveness. Unfortunately, in complicated situations the basic assumptions prerequisite to empirical analysis using relatively unsophisticated estimators are often flagrantly violated. Using more advanced estimators can be infeasible because of a lack of good data, further breakdown of statistical assumptions, and so on. Either the simplicity and moderate cost originally desired will not be achieved through the means of the econometric model, or the results obtained will be questionable on methodological and practical grounds.

Guarding against potential statistical problems by including them in the specification or in the estimator can be like tilting with windmills; without a reasonable framework, one's imagination can find hideous problems in even the most innocent of error processes. Robust estimators provide a desirable alternative in that they usually require little more from the data than do the simpler estimators they replace, and they generally guard against a wider range of potential problems than do parametric specification tests - not by 'correcting' for them but by minimizing the damage which they can create. The type of robust procedure chosen should correspond to the class of problems one anticipates in the data and model being used. In subsequent analysis of results based on both simple and robust estimators, differences in estimates or forecasts can flag instances in which one's fears were well-founded. The robust procedure is then justified both practically and diagnostically, and one can pinpoint the areas of the analysis which should be taken skeptically.

For the RDFOR model, Kuh, Lahiri, and Swartz (1981) found little, if anything, right among the assumptions behind the coefficient estimates used to develop forecasts of energy demand with the model. A logical extension of their work is the use of alternative estimators which might sail the stormy seas with RDFOR better than OLS. BIF estimators were chosen because the quality of the data and specification seemed to be the greatest single problem with the OLS RDFOR results. It was hoped that BIF estimates would eliminate or mitigate strange patterns of time and regional sensitivity manifested in the OLS results. In a limited sense, that hope proved well-founded. Ex post forecasts of industrial fuel demand (using data from 1960-1974 to create coefficients and then forecast demand for 1975-1978) have been significantly improved in terms of percent error using BIF estimates. Generally, BIF performed better in the eastern and mid-western DOE regions but worse in the western regions. Odd behavior in the western regions seemed to reduce our ability to accurately forecast behavior in the eastern regions - improvement in the latter forecasts led to a significant increase in the accuracy of total U.S. forecasts since the eastern regions consume the largest share of fuel. We present a detailed comparison of the ex post forecasts following a look at the coefficient estimates and the diagnostic information which best explains the difference among them.

Adjusted coefficient estimates for the RDFOR total demand equation using OLS and the various BIF estimators tuned to the 95% efficiency level are presented on the next page as Exhibit (5.1).* Remember, when using the RDFOR specification for forecasting the region-specific constant terms (A1-A10 in the chart) must be adjusted for the fact that the series proxying heating and cooling demand are dropped. For that reason, reported values for A1-A10 are actually estimated values for those coefficients plus the in-region mean of the heating and cooling degree days series times their coefficients (D and E, respectively).**

Except for the constant terms (and their associates the weather coefficients) these coefficients show little variation over the estimators we used. The constant terms are always at least a little bit less negative in the columns corresponding to the BIF estimators. The price coefficients B are slightly more negative, the value-added coefficients C slightly less positive, and the lagged endogenous coefficients H slightly closer to 1 in the BIF columns. Comparing these coefficients, we conclude that aberrant data accounts for a small overprediction of aggregate industrial demand.

The adjusted coefficient estimates for the RDFOR coal share equation, presented as Exhibit 2, embody quite a bit more variation among the different estimators. While OLS finds the share of energy demand allocated to coal to be a difference process with a positive constant and an autoregressive coefficient of .9, BIF

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*Results here in Section 5 are not comparable with the results in Section 3, since here the time bounds omit data for 1975-1978.

**Full statistical information on these coefficients is provided elsewhere - see Kuh, Lahiri, and Swartz (1981).

Exhibit 5.1

Adjusted coefficient estimates for the RDFOR total demand equation

	Estimates from OLS	BIF Estimates: Huber	BIF Estimates: Schweppe	BIF Estimates: Kr.-Welsch	BIF Estimates: Mallows
A1	-2.689	-2.594	-2.594	-2.605	-2.629
A2	-2.915	-2.813	-2.814	-2.827	-2.852
A3	-2.832	-2.731	-2.732	-2.744	-2.769
A4	-2.845	-2.745	-2.745	-2.757	-2.781
A5	-3.097	-2.988	-2.989	-3.001	-3.028
A6	-2.632	-2.541	-2.541	-2.552	-2.574
A7	-2.564	-2.470	-2.471	-2.481	-2.504
A8	-2.174	-2.094	-2.095	-2.104	-2.123
A9	-2.767	-2.669	-2.669	-2.681	-2.705
A10	-2.965	-2.690	-2.690	-2.724	-2.809
B	-.050	-.063	-.063	-.062	-.057
C	.278	.268	.268	.269	.272
D	.040	.017	.017	.019	.025
E	.033	.033	.033	.034	.037
H	.662	.669	.669	.669	.668

Exhibit 5.2

Adjusted coefficient estimates for the RDFOR coal share equation

	Estimates from OLS	BIF Estimates: Huber	BIF Estimates: Schweppe	BIF Estimates: Kr.-Welsch	BIF Estimates: Mallows
AC1	.007	-.195	-.204	-.222	-.162
AC2	.350	-.000	-.012	-.036	.050
AC3	.468	.035	.020	-.005	.103
AC4	.395	.028	.015	-.007	.087
AC5	.498	.028	.012	-.012	.093
AC6	.273	-.018	-.026	-.034	.009
AC7	.255	-.029	-.039	-.055	.020
AC8	.263	.019	.010	-.013	.072
AC9	.279	.027	.018	.001	.061
AC10	.339	.096	.088	.073	.154
GC	-.148	-.088	-.086	-.082	-.087
SC	-.081	.007	.009	-.002	.039
TC	.921	.992	.994	.996	.986

ignores virtually all information and sets the constant terms AC1-AC10 to zero, the relative price effect SC to zero, and the autoregressive parameter TC to 1. BIF seems to say, almost across the board, that the best available guess as to the share of total energy demand which will be met by coal is last period's share.

New England (region 1) provides an interesting example of the kind of insight one can develop from an analysis of these coefficients. In the BIF estimates, AC1 is the only significantly non-zero constant term. Apparently, New England alone amongst its regional peers has experienced a significant reduction in the share of its energy needs met by coal. We would be surprised if New England consumed much coal at all; the data bears us out. Nonetheless, since the magnitudes of the various shares are similar regardless of the magnitude of aggregate demand in the regional data, a region like New England can have an impact on the share equation disproportionate to its share of total coal consumption. Compared to the BIF estimates, it seems as though the OLS coefficients interpret at least some of the drop in New England consumption as evidence that the coefficient of autoregression of shares (TC) should be further from 1 (that is, that shares are influenced by included series other than the lagged endogenous share) and/or that the relative price effect should matter. Since the evidence from region 1 seems relatively different than that from other regions, BIF ignores it and finds that neither price nor (except in a few regions) the constant term seem to matter.

Differences in the coefficient estimates among the OLS and BIF estimators extend diagnostic insight available from other sources. Since BIF estimators downweight observations which have a large influence upon the final coefficient estimates, the information they contain is most similar to that contained in the diagnostic DFFITS.* DFFITS is a measure of the impact an observation has on its own fitted value; it can be broken into two components, one of which (HAT) measures the degree to which the observation is an outlier in X-space and the other of which is the scaled residual (RSTUDENT) associated with the observation. Should data associated with a high DFFITS have a large impact on any specific coefficient estimate, it could alternatively be detected through a measure (DFBETAS) of the impact an observation has on a parameter estimate. Diagnostic analysis using these measures is complementary to that using the various BIF estimators; one can either develop BIF estimates and then use the diagnostics to pinpoint sources and effects of whatever differences in coefficients occur, or one can first check for trouble in the diagnostics and then study the results when the damages are limited in a controlled way.

Within the context of the BIF coefficient estimates, the diagnostics corresponding to the coal share equation presented as Exhibit 3 serve as a useful means of confirming the channels through which BIF and OLS interpret the data differently. The fact that the diagnostics are distributed so unevenly across the

*The reader is again referred to Belsley, Kuh, and Welsch (1980) for details.

Exhibit 5.3

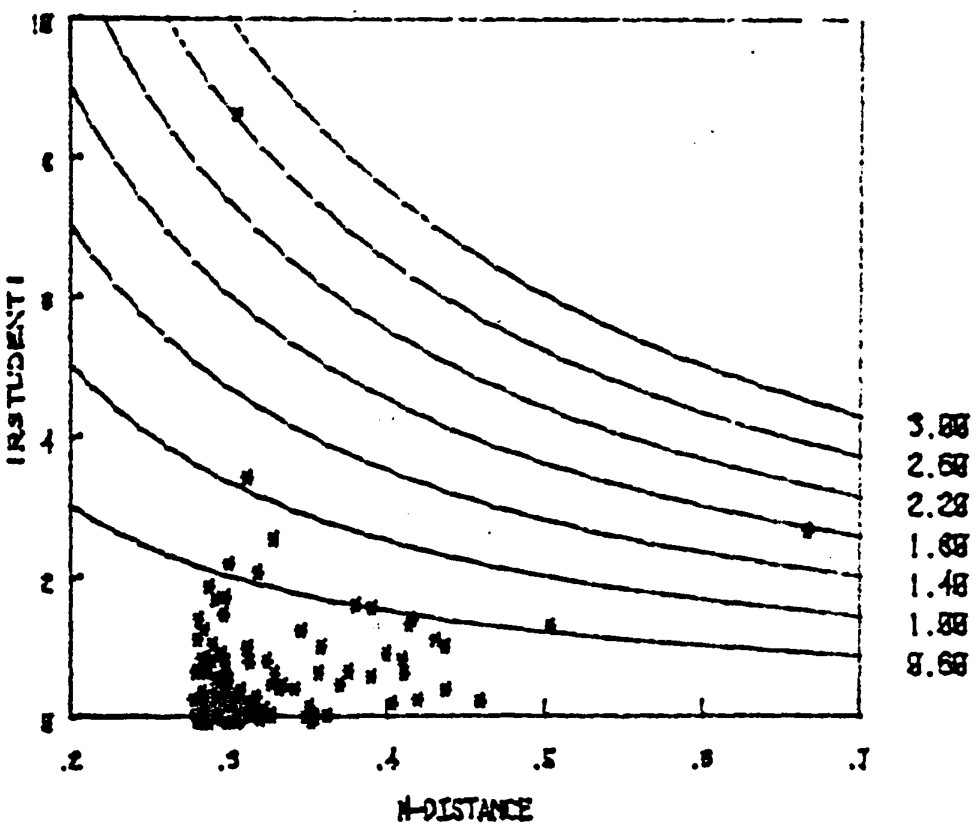
Diagnostics from OLS estimation of the RDFOR coal share equation

		RSTUDENT	HAT	DFFITS	DFBETAS Constant	Rel. Price	Gas Avail.	Lag Share
Region 1,	1961	1.3957	.1466	.5785	-.1439	.0472	-.0150	.3926
Region 1,	1962	.9867	.1383	.3952	-.0917	.0323	-.0013	.2602
Region 1,	1970	-3.5097	.0891	-1.0979	-.2884	-.0761	-.4102	.2391
Region 1,	1972	-1.1784	.1569	-.5083	-.2330	.0225	-.0769	.3464
Region 1,	1973	-1.3777	.2027	-.6947	-.3875	.1340	.0139	.5462
Region 1,	1974	2.7484	.3082	1.8346	1.0232	-.5923	-.6423	-1.4371
Region 6,	1961	-2.1552	.0924	-.6875	-.0586	.0669	-.2600	-.1105
Region 6,	1973	8.6989	.0856	2.6609	.0924	.0352	-.7166	-.6510
Region 8,	1972	.7050	.1449	.2902	-.1387	.1922	.0860	.0508
Region 8,	1973	.8995	.1443	.3694	-.2204	.2590	-.0068	.0822
Region 9,	1961	2.6315	.0977	.8661	.3834	-.3610	.2384	-.1725
Region 9,	1962	-2.2730	.0832	-.6846	-.1904	-.0906	.1047	-.2399
Region 9,	1966	-1.7975	.0820	-.5374	.0540	.0283	.1700	-.0840
Region 9,	1973	1.6866	.1268	.6426	.0602	.0974	.4055	-.0892
Region 10,	1961	1.6533	.1328	.6469	.1850	-.1275	.4005	-.0107
Region 10,	1968	-1.0820	.1606	-.4733	.0881	.0127	.3518	-.0422
Region 10,	1969	-1.4764	.1479	-.6150	.0641	.0427	.4409	.0110
Region 10,	1974	1.3256	.1073	.4594	.0952	.0194	.2262	-.1361

ten DOE regions confirms our suspicions about cross-regional problems. The preponderence of large positive DFBETAS associated with region 1 on the lag share coefficient show that region 1 data generally have the effect of making that coefficient lower than it otherwise would be (the deletion diagnostics are a scaled difference between coefficients estimated without and with the specific observation). The observation corresponding to region 1, 1974 is noteworthy as an observation which OLS interprets in the constant term instead of as caused change, an interpretation which BIF finds congenial among the rest of the (downweighted) data. Also noteworthy is the observation from Region 6, 1973. The residual from that observation is almost eight times as large as the standard error of the error distribution. We have since determined that the data error is in the endogenous 'share' series for that observation.

An alternative way of presenting diagnostic information like the BIF weights or DFFITS, which can be broken into two separate components, as mentioned before, is in the form of contour plots. Exhibit 4 contains a contour plot of DFFITS for the coal share equation. The contours are arc equi-DFFITS lines, broken into the effect due to RSTUDENT (the scaled residual) and to M-distance (outliers in X-space). Notice that except for the one observation on the far right of the plot (which data is the observation associated with Region 1, 1974, and which is the point in X-space most deviant from the other data), most of the problems with the results

Exhibit 5.4
DFFITS CONTOUR PLOT



are due to poorly fitting endogenous observations as opposed to aberrant exogenous data. This might suggest that various important series have been omitted from the regression, that coefficient constraints restrict the specification from representing important interregional variation, or that the aggregate demand/share modeling strategy is inadequate to the task of predicting energy demand.

Another important repository of diagnostic information arising out of the BIF estimation procedure are the final weights with which BIF scales the exogenous data. Exhibit 5 lists the thirty worst points as measured by the OLS DFFITS and the weights each of the BIF estimators assigned to those points. Immediately obvious in this chart is the preponderence of downweighted data from Regions 1, 6,9, and 10. Note that not all of the 'bad' points from the point

Exhibit 5.5

Final weights from BIF estimates of the RDFOR coal share equation, ordered by the OLS DFFITS

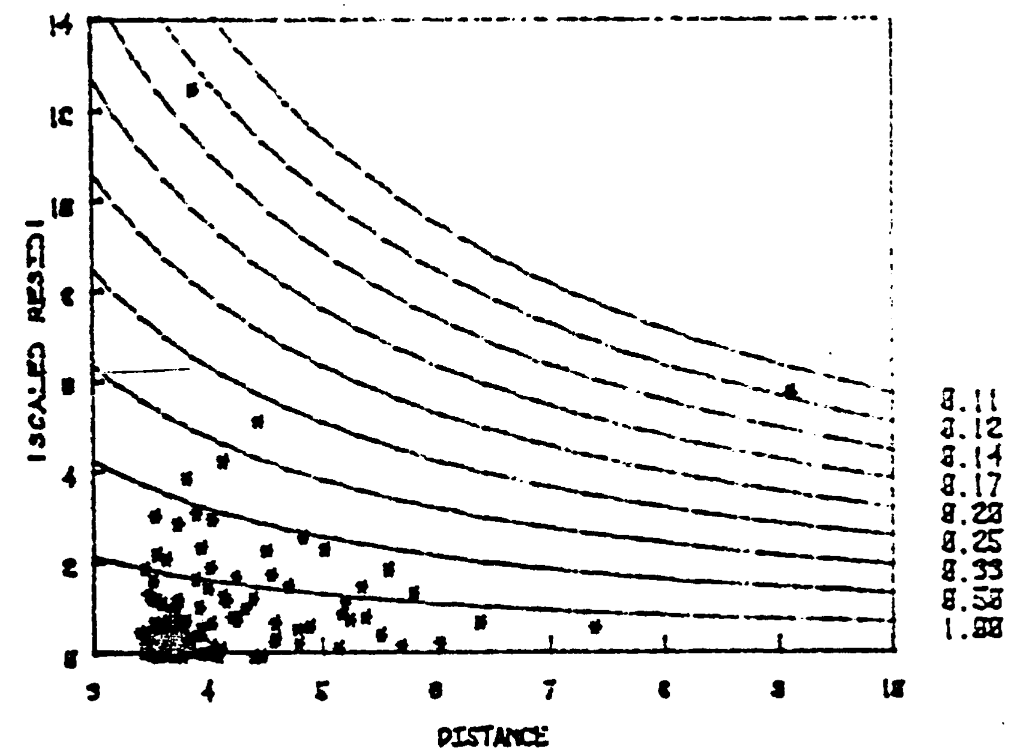
	OLS DFFITS	BIF final weights			
		Huber	Schweppe	Kr.-Welsch	Mallows
Region 6, 1973	2.6609	.1079	.1083	.1290	.1415
Region 1, 1974	1.8346	.2388	.2059	.1190	.2167
Region 1, 1970	-1.0979	.2539	.2561	.2712	.3294
Region 9, 1961	.8661	.3180	.3170	.3507	.4212
Region 1, 1973	-.6947	1.0000	1.0000	1.0000	.8580
Region 6, 1961	-.6875	.4447	.4441	.5100	.5738
Region 9, 1962	-.6846	.3403	.3420	.4172	.4501
Region 10, 1961	.6469	.5528	.5433	.5228	.7819
Region 9, 1973	.6426	.5266	.5170	.4908	.7220
Region 10, 1969	-.6150	.7408	.7218	.5825	.9801
Region 1, 1961	.5785	.8961	.8722	.7582	1.0000
Region 1, 1967	.5631	.4347	.4366	.5047	.5668
Region 9, 1966	-.5374	.4525	.4559	.5692	.6020
Region 6, 1971	-.5170	.6234	.6290	.7930	.7974
Region 1, 1972	-.5083	1.0000	1.0000	1.0000	.9923
Region 10, 1968	-.4733	1.0000	1.0000	.7629	1.0000
Region 10, 1972	-.4609	.5719	.5763	.6564	.7079
Region 10, 1974	.4594	.5662	.5616	.5944	.8041
Region 6, 1967	-.4182	.8041	.8136	1.0000	1.0000
Region 7, 1972	.3926	.7458	.7367	.7537	1.0000
Region 6, 1964	.3822	.4320	.4359	.5672	.5720
Region 8, 1973	.3694	1.0000	1.0000	.9896	1.0000
Region 10, 1966	.3384	.7162	.7202	.8308	.9661
Region 2, 1971	-.3377	.6707	.6792	.9228	.8839
Region 8, 1962	-.3273	.7639	.7669	.9291	1.0000
Region 10, 1971	.2893	.6610	.6636	.7877	.9176
Region 2, 1963	.2609	.9903	1.0000	1.0000	1.0000
Region 1, 1968	-.2574	.8477	.8615	1.0000	1.0000
Region 10, 1965	-.2561	.9027	.8844	.8499	1.0000
Region 6, 1970	.2258	.5955	.6003	.7812	.7929

of view of OLS and DFFITS remain 'bad' points from the point of view of BIF. For example, it seems that downweighting data from Region 1 for 1970 and 1974 was sufficient to alleviate whatever was wrong with data from Region 1, 1972 and 1973; the latter two points were not downweighted at all by any of the algorithms except the Mallows procedure. This suggests, perhaps, that the data from 1972 and 1973 was still fairly outlying in X-space, but that it fit reasonably well after the other data were properly downweighted.

Just as the OLS DFFITS can be broken down into components due to aberrant X data and large residuals, so can these BIF weights be broken into components due to X-outliers and large residuals as well. A contour plot of the BIF weights for the Krasker-Welsch estimator is presented in the next page as Exhibit 6. The Krasker-Welsch algorithm gives the smallest weight to the Region 1, 1974 observation, which again (compare with Exhibit 4) is on the right-hand side of the diagram. By and large, the BIF algorithm expands the scaled residuals of the more aberrant data points, which is what we would expect of BIF - we sacrifice some fit of the worst points in order to better fit the majority of points. Again we find that the majority of downweighting is due to poorly fit left-hand side data.

In considering the various diagnostic possibilities attendant to the BIF estimators, it should be clear that we have done more than would the average analyst. Practically, one would seldom use all of these different diagnostic techniques simultaneously. Depending on intent and the arena in which results are to

Exhibit 5.6
BIF WEIGHT CONTOUR PLOT



be presented, one might choose the coefficients generated by the BIF algorithm, the weights used to create those coefficients, or the diagnostics arising directly from the OLS algorithm to analyze potential weaknesses in the statistical model. Charts of the diagnostics have the advantage of thouroughness, yet they are often overwhelming to the casual user; the contour plots are both easy to use and informative of the nature of the breakdown encountered in the original model.

Our present purpose in amassing diagnostic information is to assess the reliability of energy demand forecasts for coal based on the RDFOR equation system. We might conclude that although

RDFOR's OLS forecast of aggregate energy demand seems fairly robust to data problems of the kind that BIF estimators can correct for (remember, the total demand equation's coefficients changed only slightly, in a direction which would produce lower forecasts of total energy demand, when using the BIF algorithms), the share equations seem more sensitive to data errors. Regions with lower absolute energy demand will have just as much impact on the OLS forecasts of share equation coefficients as will regions using a great deal of energy. We should remember that a large data error in Region 6 might hurt the OLS forecasts in that region. Regions 1, 6, 9 and 10 fit very poorly in the OLS estimation process; since none of those regions are especially heavy coal consumers, relaxing their influence on the rest of the data might have a significant impact on the overall accuracy of energy demand forecasts. Finally, we remarked earlier that it was possible that important forces were omitted from the RDFOR specification. If those forces are dependent on time, as for instance an increasing impact of conservation or supply side forces, or a change over time in the reaction to relative price, then our forecasts are irrecoverably damaged. The best we can hope for from BIF coefficients in this situation is that the time dependent forces were prevalent over the majority of our data points, and that our estimatin procedure can therefore discard information from those points which do not seem to fit as well.

In order to test the forecasting ability of the RDFOR specification, we have constructed ex post forecasts using actual exogenous series and coefficient estimates corrected for the omission of the weather and gas availability series. Percent errors of forecasts of energy demand for the years 1975-1978 are presented on the next page as Exhibit 5.7.

Given our expectations as to the quality of these forecasts (based on the diagnostic evidence which we have already presented) the patterns of improvement in our forecasts which can be gained through use of BIF coefficient estimates are hardly surprising. Using BIF coefficients improves the ability of the specification to forecasts demand in Region 1-5, the regions in which the larger part of U.S. coal is used. It also improves forecasts in Region 6, in which we identified one large data error. These gains were achieved at the expense of forecasts for Regions 7-9, for which the BIF forecasts are somewhat worse. Apparently Region 10's data is difficult to make sense of within either algorithm's interpretation of the data. Note that the overall effect of the changes in predictive accuracy is to produce remarkable improvement in aggregate coal demand forecasts; BIF forecasts were off by from one and a quarter to five percent, as opposed to a 24% forecast error in the OLS forecast after four years.

We have been comparing percent forecast errors, as opposed to the standard error of forecast, for a number of reasons. Since BIF can recognize whether or not a new observation is like those upon which it was based, it predicts a tighter standard error for a good observation and a looser standard error for an outlier. In fact, should BIF recognize a point as an outlier, we probably should not

Exhibit 5.7

Percent errors of forecast using OLS and BIF coefficient estimates of the RDFOR industrial coal demand forecasting model

	OLS	BIF			
		Huber	Schweppe	Kr.-Welsch	Mallows
Region 1					
1975......	17.681%	-.172%	-.869%	-1.444%	-.282%
1976......	62.502%	15.088%	13.403%	12.239%	14.405%
1977......	37.106%	-19.281%	-21.116%	-22.273%	-20.122%
1978......	87.407%	-7.730%	-10.596%	-12.400%	-8.898%
Region 2					
1975......	27.409%	20.459%	20.231%	19.851%	19.418%
1976......	-5.776%	-15.776%	-16.100%	-16.583%	-17.169%
1977......	21.316%	-.439%	-1.086%	-1.517%	-3.777%
1978......	51.841%	17.854%	16.835%	16.119%	13.012%
Region 3					
1975......	1.566%	-2.381%	-2.484%	-2.356%	-2.741%
1976......	-1.199%	-7.200%	-7.365%	-7.313%	-7.243%
1977......	19.750%	4.928%	4.546%	5.131%	3.497%
1978......	24.214%	6.151%	5.661%	6.223%	4.933%
Region 4					
1975......	8.674%	4.329%	4.217%	4.355%	4.250%
1976......	16.532%	7.232%	6.991%	7.293%	7.054%
1977......	24.434%	4.963%	4.491%	5.533%	3.017%
1978......	7.397%	-13.197%	-13.712%	-12.677%	-15.042%
Region 5					
1975......	-.694%	-4.244%	-4.348%	-4.315%	-4.260%
1976......	9.561%	-2.152%	-2.458%	-1.911%	-3.469%
1977......	16.154%	-4.690%	-5.210%	-4.062%	-7.463%
1978......	34.130%	2.074%	1.284%	3.086%	-2.231%
Region 6					
1975......	.613%	-4.896%	-4.775%	-4.053%	-5.492%
1976......	3.451%	-13.065%	-12.975%	-10.995%	-15.978%
1977......	13.432%	-17.962%	-17.980%	-14.475%	-23.893%
1978......	51.073%	-4.573%	-4.720%	1.180%	-14.935%
Region 7					
1975......	.501%	-3.239%	-3.354%	-3.428%	-2.508%
1976......	-12.409%	-24.898%	-25.215%	-24.651%	-25.642%
1977......	-14.683%	-34.182%	-34.659%	-33.701%	-35.691%
1978......	.549%	-30.913%	-31.654%	-30.066%	-33.696%
Region 8					
1975......	-10.418%	-11.757%	-11.812%	-11.997%	-12.071%
1976......	-13.805%	-17.815%	-17.949%	-18.117%	-18.793%
1977......	-18.096%	-26.980%	-27.231%	-27.055%	-29.270%
1978......	-9.910%	-22.165%	-22.520%	-22.306%	-25.185%
Region 9					
1975......	-7.213%	-6.221%	-6.236%	-6.967%	-5.132%
1976......	-18.871%	-28.790%	-29.050%	-28.827%	-30.857%
1977......	-28.176%	-43.754%	-44.131%	-43.468%	-47.150%
1978......	-.721%	-29.635%	-30.314%	-28.936%	-35.885%
Region 10					
1975......	-45.936%	-40.304%	-40.406%	-41.816%	-44.345%
1976......	-62.968%	-61.592%	-61.820%	-62.812%	-66.883%
1977......	-56.047%	-56.965%	-57.375%	-58.509%	-64.631%
1978......	-42.499%	-45.588%	-46.258%	-47.767%	-56.286%
US Totals					
1975......	1.408%	-2.316%	-2.417%	-2.382%	-2.517%
1976......	2.910%	-6.314%	-6.552%	-6.265%	-7.190%
1977......	13.664%	-4.703%	-5.153%	-4.282%	-7.121%
1978......	23.916%	-1.845%	-2.479%	-1.274%	-5.070%

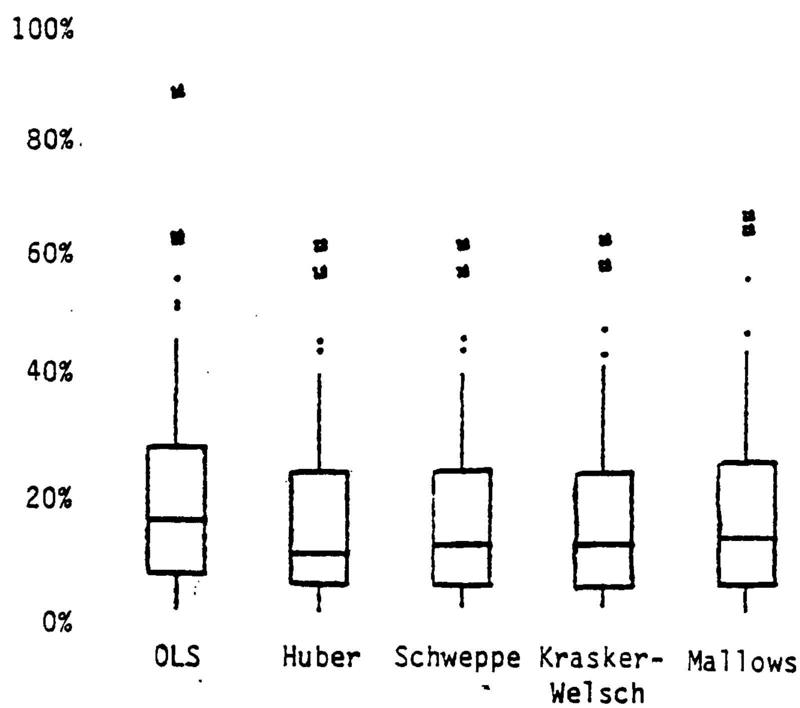
attempt to forecast it with the linear model at all. Also, it is not clear that the proper criterion upon which we should judge forecasts is the standard error of forecast in any case; we are concerned with mistakes in forecasts for their own sake, regardless of whether they come from uncertainty in the error term, from drift in the exogenous data, or from uncertainty as to the reality of the specification or the statistical model.

Just as large charts of diagnostic information are somewhat imposing, so are large charts of diagnostic percent error deviations imposing. In a situation where distributions of numbers need to be compared, a useful didactic tool is the box plot, an example of which is presented as Exhibit 5.8. The box plot is best considered as an aerial view of a distribution - in this case, the distribution of the absolute values of the percent errors of forecast arising out of each of the estimators we have examined. The line in the center of the plot for each series is the median of that series. The lower and upper line of the box containing the median line are the first and third quartiles, respectively; their difference is called the midspread. The lines attached to the top and bottom of the box represent the extent of the third quartile plus the midspread and the first quartile minus the midspread. The dots and squares, finally, are the points that are respectively between one and one and a half midspreads from the region spanned by the box and more than one and a half midspreads from that region.

The box plot of Exhibit 5.8 graphically represents the superiority of the BIF-based forecasts over the OLS-based forecasts.

Exhibit 5.8

OLS and BIF percent errors of forecast of industrial coal demand in the RDFOR model, presented as a group of Box Plots



The median OLS absolute percent error is approximately 17%; the median Huber absolute percent error of forecast is 11%. The range of percent errors was improved by the BIF coefficients. We can conclude that the problems encountered in producing forecasts of energy demand using the RDFOR specification can be lessened significantly through use of more resistant estimators such as the BIF estimators considered in this paper.

In the case of the RDFOR model, it is hard to distinguish between the various BIF algorithms. Since by-and-large there are few influential X-observations, we might expect the Huber algorithm (which ignores the exogenous data completely) to do slightly better than the other three estimators. Under these circumstances it is not possible to distinguish among the three BIF estimators. Another confusing factor is the experimental nature of the efficiency calibration algorithms for the various estimators; the same efficiency level can lead to markedly different average weights and aggregate downweighting. It is not likely, in any case, that any one of these estimators will dominate the others; each should be a useful tool for the analyst.

While it is hard to defend a model such as RDFOR as an accurate device for creating forecasts which are to be used for policy analysis, such models have their place among the portfolio of tools which should be available to the analyst. Such reduced-form specifications are easier to develop and validate than more complicated structural tools, and certainly much cheaper to run and alter during the construction of specific policy prescriptions. Exactly because of their simplicity, though, they are likely to be subject to a variety of ills related to their interaction with the real world through the data with which we provide them. We have demonstrated a range of tools which are useful to catalogue the various failures occurring in the specific model being used. If the list of problems includes potential data errors; inconsistency of small parts of the data with other, larger segments; occasional, unpredictable, yet significant departures of the behavior being modeled from the specification, and so on, then the potential for improved performance at a small additional cost through the use of bounded-influence estimators should be seriously considered.

Appendix A

Results[1] of More Complicated Estimation Procedures

This Appendix amplifies the argument against using more complicated estimation procedures as cited in the RDFOR documentation of NEO(77) draft. While we have stuck quite close to Ordinary Least Squares analysis, we have also mentioned that the RDFOR documentation suggests several more complicated estimation strategies. We have attempted to duplicate these procedures as closely as possible and estimate (A.1) in strict accordance with the documentation. The specification we have used in TROLL is

$$TQQR_{it} = a_i + b*TPPR_{it} + c*YPER_{it} + d*HDAY_{it} + e*CDAY_{it} + h*TQQR(-1)_{it} + \varepsilon_{it} \quad (A.1)$$

where all series are in logs and where:

$TQQR_{it}$	-	a Divisia index of total energy use
$TPPR_{it}$	-	a Divisia index of energy price
$YPER_{it}$	-	"permanent" per capita disposable income
$HDAY_{it}$	-	heating degree days
$CDAY_{it}$	-	cooling degree days
i	-	a cross-sectional index over DOE regions
t	-	a time index over the years 1962-1978
a,b,c, d,e,h	-	parameters to be estimated
ε_{it}	-	a normally distributed error term

The sources of the data and the specification are discussed in earlier sections of the paper.

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[1]These results have been taken from an earlier draft of ours entitled "Draft Report on Certain Properties of the Estimates of the RDFOR, Total Energy Demand Equation for the Residential Sector - A Preliminary Analysis" where we duplicated the Divisia index as defined in the Synergy documentation and as it is currently used in RDFOR.

Attempts to use non-linear least squares to estimate the coefficients of the transformed equation were frustrated by the computational singularity of the Hessian at the NLLS solution; even using state-of-art code, the NLLS estimates have computationally infinite variance. Estimates of the reduced form of the equation cannot reject the restrictions imposed by the transformations yet provides widely varying estimates of the parameters of transformation. Both of these estimation problems result from the severe data collinearity we have encountered. Because we cannot adequately estimate these transformed equations, we have ignored them in the analysis of the model.

The first transformation we studied was to add a geometric lag to the price term in (A.1), yielding

$$TQQR_{it} = a_i + b^* \sum_{k=0}^{\infty} g^k TPPR_{i,t-k} + c^* YPERM_{it} + \\ + dHDAY_{it} + eCDAY_{it} + hTQQR_{i,t-1} \cdot \qquad (A.2)$$

The most common way to estimate such an equation is to multiply the entire equation by g, lag it one period, and subtract the result from (A.2) leaving

$$TQQR_{it} = a_i(1-g) + bTPPR_{it} + cYPER_{it} - cgYPER_{it-1} + dHDAY_{it} \\ - dgHDAY_{it-1} + eCDAY_{it} - egCDAY_{it-1} + (h+g)TQQR_{it-1} \qquad (A.3) \\ - hgTQQR_{i,t-2} \cdot$$

This equation can be estimated in two ways. First, one can estimate it directly using nonlinear methods. TROLL results of an attempt to do so are presented as Exhibit A.1. Unbiased point estimates are obtained which correspond well with our earlier results. A long-run price elasticity of .47 and a long-run income elasticity of 1.02 are obtained. No standard errors or t-statistics are presented because it was infeasible to calculate them; the Hessian matrix at the solution was indefinite. We cannot trust these estimates, for their standard error are essentially infinite.

Exhibit A.1

NLLS Estimation of (A.1) with Geometric Price Lag

CRSQ = 1.000 SER = 0.031 SSR = 0.138 DW(0) = 1.904

COEF	ESTIMATE
A1	-1.976
A10	-2.028
A2	-1.918
A3	-1.739
A4	-1.472
A5	-1.676
A6	-1.541
A7	-1.957
A8	-2.207
A9	-1.597
B	-0.099
C	0.252
D	0.328
E	0.035
G	0.151
H	0.753

A second way of estimating the parameters of (A.3) is to estimate it using OLS without the constraints among the parameters. That is, we estimate a separate coefficient for TPPR, YPER, YPER(-1), etc. Results from this analysis are listed as Exhibit A.2. Note that relaxing the nonlinear constraints does not change the SSR at all; we cannot reject the constraints. Nonetheless, parameter estimates of g vary from .027 to .474. This variability at the solution is caused by collinearity. Relatively large movements in parameter estimates need not be accompanied by much movement in the SSR.

Exhibit A.2

OLS Reduced Form Estimation of (A.1) with Geometric Price Lag

CRSQ = 1.000 SER = 0.031 SSR = 0.138 DW(0) = 1.871
COND(X) = 1246.580 MAX:HAT = 0.273 RSTUDENT = 2.731 DFFITS = -0.941

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
TPPR2L0	-0.092	0.037	-2.488	0.014
YPER2L0	0.367	0.167	2.198	0.03
YPER2L1	-0.174	0.17	-1.022	0.309
HDAY2L0	0.336	0.051	6.591	0.
HDAY2L1	-0.044	0.066	-0.667	0.506
CDAY2L0	0.037	0.025	1.479	0.141
CDAY2L1	0.001	0.024	0.03	0.976
TQQR2L1	0.899	0.089	10.058	0.
TQQR2L2	-0.092	0.086	-1.075	0.284
CONS2.1	-1.912	0.794	-2.408	0.017
CONS2.2	-1.789	0.801	-2.233	0.027
CONS2.3	-1.765	0.797	-2.215	0.028
CONS2.4	-1.502	0.769	-1.953	0.053
CONS2.5	-1.683	0.824	-2.042	0.043
CONS2.6	-1.554	0.757	-2.054	0.042
CONS2.7	-1.904	0.796	-2.393	0.018
CONS2.8	-2.098	0.79	-2.655	0.009
CONS2.9	-1.589	0.741	-2.146	0.034
CONS2.10	-1.948	0.767	-2.541	0.012

Through manipulations similar to those used in constructing (A.3) we can construct transformations useful in estimating the autocorrelation lag coefficient ρ and the case of both autocorrelation and a geometric price lag. These are, respectively

$$\begin{aligned} TQQR_{it} &= a_i(1-\rho) + bTPPR_{it} - b\rho TPPR_{i,t-1} + cYPER_{it} - c\rho YPER_{i,t-1} \\ &+ dHDAY_{it} - d\rho HDAY_{i,t-1} + e\,CDAY_{it} - e\rho CDAY_{i,t-1} \\ &+ (h+\rho)TQQR_{i,t-1} - \rho hTQQR_{i,t-2} \end{aligned} \tag{A.4}$$

and

$$\begin{aligned} TQQR_{it} &= a_i(1-\rho)(1-g) + bTPPR_{it} - b\rho TPPR_{i,t-1} + cYPER_{it} - c(g+\rho)YPER_{i,t-1} \\ &+ c\rho gYPER_{i,t-2} + dHDAY_{i,t} - d(\rho+g)HDAY_{i,t-1} + dg\rho\, HDAY_{i,t-2} \\ &+ eCDAY_{it} - e(\rho+g)CDAY_{i,t-1} + e\rho g\, CDAY_{i,t-2} + (h+g+\rho)TQQR_{i,t-1} \\ &- (hg+g\rho+\rho h)TQQR_{i,t-2} + hg\rho TQQR_{i,t-3} \end{aligned} \tag{A.5}$$

Estimates for (A.4) are presented as Exhibit A.3 [NLLS] and Exhibit A.4 [OLS]. Estimates for (A.5) are presented as Exhibits A.5 [NLLS] and Exhibit A.6 [OLS].

The problems plaguing estimation of (A.3) continue to hinder estimation of (A.4) and (A.5). Although in neither case is the SSR sufficiently small to reject the transformations, it proves impossible to get reliable estimates of either ρ or g. Estimates of ρ from (A.4) vary from .672 to .027 in the OLS estimation as compared to the NLLS estimate of .184. Estimates of ρ and g from A.5 are apparently imaginary, a result definitely outside the spirit of the specification. The unconstrained estimates are nonsensical precisely because of the numerical instability of the estimation problem.

Were we to adopt these transformations, it seems clear that at best we would obtain very poor estimates of the parameters associated with them. The Durbin-Watson statistic from Exhibit A.1 does not directly justify the autocorrelation correction, and no good theoretical justification for the geometric price lag has been advanced. For these reasins, we have not proceeded with the analysis of (A.5).

Exhibit A.3

NLLS Estimation of (A.1) with Autocorrelation Adjustment

CRSQ = 1.000 SER = 0.031 SSR = 0.138 DW(0) = 1.935

COEF	ESTIMATE
A1	-1.9
A10	-1.961
A2	-1.735
A3	-1.704
A4	-1.384
A5	-1.583
A6	-1.459
A7	-1.862
A8	-2.142
A9	-1.519
B	-0.121
C	0.272
D	0.326
E	0.034
H	0.739
RHO	0.184

Exhibit A.4

OLS Reduced Form Estimation of (A.1) with Autocorrelation Adjustment

CRSQ = 1.000 SER = 0.031 SSR = 0.135 DW(0) = 1.860
COND(X) = 1314.550 MAX:HAT = 0.288 RSTUDENT = 2.843 DFFITS = 1.382

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
TPPR2L0	-0.22	0.079	-2.77	0.006
TPPR2L1	0.149	0.082	1.817	0.071
YPER2L0	0.25	0.178	1.409	0.161
YPER2L1	-0.068	0.179	-0.383	0.702
HDAY2L0	0.326	0.051	6.427	0.
HDAY2L1	-0.035	0.065	-0.532	0.596
CDAY2L0	0.04	0.025	1.625	0.106
CDAY2L1	0.004	0.024	0.148	0.883
TQOR2L1	0.883	0.089	9.907	0.
TQOR2L2	-0.062	0.087	-0.72	0.473
CONS2.1	-2.023	0.79	-2.561	0.011
CONS2.2	-1.911	0.798	-2.395	0.018
CONS2.3	-1.885	0.793	-2.377	0.019
CONS2.4	-1.634	0.766	-2.132	0.035
CONS2.5	-1.812	0.82	-2.209	0.029
CONS2.6	-1.684	0.754	-2.233	0.027
CONS2.7	-2.018	0.792	-2.548	0.012
CONS2.8	-2.194	0.785	-2.794	0.006
CONS2.9	-1.709	0.738	-2.317	0.022
CONS2.10	-2.052	0.763	-2.69	0.008

Exhibit A.5

NLLS Estimation of (A.1) with Both Autocorrelation Adjustment and Geometric Price Lag

CRSQ = 1.000 SER = 0.030 SSR = 0.122 DW(0) = 1.951

COEF	ESTIMATE
A1	-2.491
A10	-2.535
A2	-2.348
A3	-2.323
A4	-2.018
A5	-2.22
A6	-2.06
A7	-2.488
A8	-2.709
A9	-2.337
B	-0.074
C	0.163
D	0.364
E	0.059
G	0.038
H	0.771
RHO	0.12

Exhibit A.6

OLS Reduced Form Estimation of (A.1) with Both Geometric Price Lag and Autocorrelation Adjustment

CRSQ = 1.000 SER = 0.029 SSR = 0.105 DW(0) = 2.043
COND(X) = 1816.410 MAX:HAT = 0.365 RSTUDENT = 3.261 DFFITS = 1.689

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
TPPR3L0	-0.22	0.085	-2.576	0.011
TPPR3L1	0.103	0.085	1.219	0.225
YPER3L0	0.249	0.178	1.402	0.163
YPER3L1	-0.775	0.278	-2.791	0.006
YPER3L2	0.725	0.202	3.59	0.
HDAY3L0	0.344	0.052	6.567	0.
HDAY3L1	-0.086	0.063	-1.372	0.172
HDAY3L2	-0.049	0.064	-0.766	0.445
CDAY3L0	0.047	0.026	1.794	0.075
CDAY3L1	0.009	0.025	0.378	0.706
CDAY3L2	0.027	0.024	1.128	0.261
TGDR3L1	0.931	0.086	10.878	0.
TGDP3L2	-0.161	0.116	-1.387	0.168
TGDP3L3	-0.002	0.087	-0.018	0.986
CONS3.1	-1.23	0.991	-1.241	0.217
CONS3.2	-1.12	1.	-1.12	0.265
CONS3.3	-1.108	0.995	-1.115	0.267
CONS3.4	-0.921	0.961	-0.958	0.34
CONS3.5	-0.979	1.025	-0.955	0.341
CONS3.6	-1.01	0.949	-1.064	0.289
CONS3.7	-1.282	0.996	-1.287	0.201
CONS3.8	-1.44	0.988	-1.458	0.147
CONS3.9	-1.	0.926	-1.08	0.282
CONS3.10	-1.352	0.961	-1.407	0.162

Appendix B*

Some notes on robust estimation algorithms

S. Peters, A. Samarov and R. Welsch

Definitions of the Estimators

The standard linear regression model is generally written as

$$Y = X\beta + \varepsilon,$$

where Y is an n×1 response variable, X is an n×p matrix containing p explanatory variables $X_1,\ldots,X_p$ as columns, β is a p vector of unknown coefficients, and ε is a normal random n-vector with $E(\varepsilon) = 0$ and $E(\varepsilon\varepsilon^T) = \sigma^2 I$.

These assumptions, particularly the normality assumption, should be regarded as defining the "central model", and since we fully expect small violations of these assumptions, and our aim is to present an estimator which is not too sensitive to these small violations.

Fitting the linear model is accomplished by attempting to find an estimator $\hat{\theta} = (\hat{\beta},\hat{\sigma})$ of the coefficients β and scale σ which minimizes

$$Q(\beta,\sigma) = d\cdot\sigma + \sum_{i=1}^{n} \rho\left(\frac{y_i - x_i^T\beta}{\sigma v(x_i)}\right) \sigma\, u(x_i)v^2(x_i) \qquad (1)$$

where x_i^T denotes the i-th row of the X matrix, $v(x_i)$ and $u(x_i)$ are positive functions, and $\rho(\cdot)$ is often called a robustifying criterion function. The estimator $\hat{\theta} = (\hat{\beta},\hat{\sigma})$ is sometimes called a general M-estimator [see Maronna, Bustos, Yohai (1979)].

The functions $u(x_i)$ and $v(x_i)$ are included in (1) to downweight the influence of outlying points in the X-space while preserving reasonably high efficiency of the estimates [see Krasker, Welsch (1982), Maronna, Bustos, Yohai (1979)]. These functions may depend on the entire X matrix and not just on x_i.

The minimization of the function $Q(\beta,\sigma)$ may be viewed as a generalization of the least squares method for which $\rho(t) = t^2/2$, $d = \frac{n-p}{2}$, $u(\cdot) = v(\cdot) = 1$. It is usually assumed [see Huber (1977)] that $\rho \geq 0$ is a convex function, and

$$0 < \lim_{|t|\to\infty} \frac{\rho(t)}{|t|} < \infty. \qquad (2)$$

With these assumptions Q is a convex function of $\theta = (\beta,\sigma)$ [see Huber (1977)], and unless the minimum $\hat{\theta} = (\hat{\beta},\hat{\sigma})$ occurs on the boundary $\sigma = 0$, it can equivalently be characterized by the p+1 equations:

$$\text{(a)} \quad \sum_{i=1}^{n} u(x_i)v(x_i)\psi\left(\frac{r_i}{\hat{\sigma} v(x_i)}\right) x_i = 0$$

$$\text{(b)} \quad \sum_{i=1}^{n} u(x_i)v^2(x_i)\,\chi\left(\frac{r_i}{\hat{\sigma} v(x_i)}\right) = d \qquad (3)$$

where $r_i = y_i - x_i^T\hat{\beta}$ is the residual, $\psi(\cdot) = \rho'(\cdot)$ and $\chi(t) = t\psi(t) - \rho(t)$.

*Source: TR No. 30, "Computational Procedures for Bounded-Influence and Robust Regression (TROLL: BIF and BIFMOD)," 1981.

We consider here only one ρ-function, that of Huber [Huber (1964) and (1977)] which is defined as follows:

$$\rho_c(t) = \begin{cases} \frac{1}{2} t^2 & \text{for } |t| < c \\ c|t| - \frac{1}{2} c^2 & \text{for } |t| \geq c \end{cases} \qquad (4)$$

with corresponding psi-function

$$\psi_c(t) = \rho_c'(t) = \begin{cases} t & \text{for } |t| < c \\ -c & \text{for } t \leq -c \\ c & \text{for } t \geq c \end{cases} \qquad (5)$$

Then the chi-function in the scale equation (3b) has the form

$$\chi_c(t) = \frac{1}{2} \psi_c^2(t) \qquad (6)$$

which corresponds to Huber's "proposal-2" estimates [Huber (1964) and (1977)]. Parameter d in (3) is chosen to make the scale estimate $\hat{\sigma}$ asymptotically unbiased for normal errors, e.g., for $u(x) = v(x) = 1$:

$$d_H = (n-p) \cdot E\, \chi_c(\eta) \qquad (7)$$

where the expectation is taken for a standard normal random variable η. Notice that for the OLS estimator, which corresponds to the cut-off parameter $c=\infty$ in (7), parameter $d = (n-p)/2$.

The psi-function (5) satisfies the conditions (2) and is proved to be optimal in different contexts - see Huber (1964), Hampel (1968), Krasker and Welsch (1982) Maronna, Bustos, and Yohai (1979).

When the psi-function (5) is used, equations (3a) and (3b) can be rewritten as the equations for a weighted least squares (WLS) estimator of the form:

$$\text{(a)} \quad \sum_{i=1}^{n} u(x_i)\, w_c\left(\frac{r_i}{\hat{\sigma} v(x_i)}\right) r_i x_i = 0$$

$$\text{(b)} \quad \sum_{i=1}^{n} u(x_i)\, w_c^2\left(\frac{r_i}{\hat{\sigma} v(x_i)}\right) r_i^2 = 2d\hat{\sigma}^2 \qquad (8)$$

where $w_c(z) = \dfrac{\psi_c(z)}{z}$.

Several different robust estimators can be obtained from (8) by the different choices of functions $u(\cdot)$ and $v(\cdot)$. We consider here the following four robust estimators.

I. Huber M-estimators (Huber, 1964 and 1977).

In (8) set

$$u(x_i) = v(x_i) = 1, \text{ and the constant } d = d_H \text{ as in (7)}.$$

These estimates, even though they control for large residuals, are not qualitatively robust (Hampel, 1971 and 1974) in the sense that they may have an arbitrarily large asymptotic bias when the assumptions defining the "central model" are satisfied only approximately. This is due to the fact that for these estimates the influence of one single observation (y,x) can be arbitrarily large. This property violates one principle of qualitative robustness proposed by Hampel (1974) which, in essence, states that the influence of small portions of the data on the parameter estimates should be bounded. Several estimators have been proposed to correct this drawback of classical M-estimates for regression.

II. Mallows Estimates (Mallows, 1973, 1975)

In (8) set

$$v(x_i) \equiv 1, \; u(x) = \frac{\psi_b[(x^T\hat{A}^{-1}x)^{\frac{1}{2}}]}{(x^T\hat{A}^{-1}x)^{\frac{1}{2}}}, \tag{9}$$

where $\psi_b(\cdot)$ is defined by (5) and the p×p matrix $\hat{A}$ satisfies the equation

$$\hat{A} = \frac{g_1(c)}{n} \sum_{i=1}^{n} u^2(x_i)x_i x_i^T, \tag{10}$$

and

$$g_1(z) = E\psi_z^2(\eta) = E \min(\eta^2, z^2) = z^2 + (1-z^2)(2\Phi(z)-1) - 2z\phi(z), \tag{11}$$

where η is a standard normal variable, and $\phi(z)$ and $\Phi(z)$ denote its density and distribution function, respectively.

The constant d is given by

$$d = d_H \frac{1}{n} \sum_{i=1}^{n} u(x_i). \tag{12}$$

Mallows' estimator guarantees the qualitative robustness in the sense of Hampel (1974) by downweighting the outlying points in the X-space. However, any downweighting of X-data that does not include some consideration of how the response fits the remainder of the data cannot produce efficient estimates because the outlying points, if they are "good", contribute much to the precision of $\hat{\beta}$ and therefore increase the efficiency of the estimator. The following estimator proposed by Schweppe [see Handschin et al. (1975)] has the potential to help overcome some of the efficiency problems of the Mallows' estimator.

III. Schweppe Bounded-Influence Estimator

The functions $u(\cdot)$ and $v(\cdot)$ are defined on the observed data as follows

$$u(x_i) \equiv 1, \; v^2(x_i) = 1-h_i, \; i=1,\ldots,n, \tag{13}$$

where $h_i = x_i^T(X^TX)^{-1}x_i$ are the diagonal elements of the hat matrix, and the constant

$$d = \frac{(n-p)}{2n} \sum_{i=1}^{n} g_1\left(c\sqrt{1-h_i}\right). \tag{13'}$$

These estimators can provide bounded influence with the additional property that if the outlier is "good", i.e., $(y-x^T\beta)/(\sigma\sqrt{1-h})$ is small, there is no do weighting. See Hill (1977) and Denby and Larsen (1977) for Monte Carlo comparison of the Mallows and Schweppe estimators along with several others. Huber (1981) also discusses this estimation procedure.

IV. Krasker-Welsch Estimator [Krasker and Welsch (1979)]

In (3) or (4) set

$$u(x) \equiv 1, \; v^2(x) = \frac{1}{x^T\hat{A}^{-1}x}, \tag{14}$$

where the p×p matrix $\hat{A}$ satisfies the equation

$$\hat{A} = \frac{1}{n} \sum_{i=1}^{n} g_1\left(\frac{c}{(x_i^T\hat{A}^{-1}x_i)^{\frac{1}{2}}}\right) x_i x_i^T . \tag{15}$$

The constant d is given by

$$d = \frac{n-p}{2n} \sum_{i=1}^{n} g_1\left(\frac{c}{(x_i^T\hat{A}^{-1}x_i)^{\frac{1}{2}}}\right). \tag{16}$$

Krasker and Welsch (1982) show that so defined the estimator $\hat{\beta}$ satisfies a necessary condition for achieving the highest efficiency among all weighted least squares estimators with sensitivity $\leq c$ (in the strong sense that its asymptotic covariance matrix differs from all others by a non-negative definite matrix).

Research Background

Hampel (1974) for p=1 and Krasker (1980) and Krasker, Welsch (1982) for $p \geq 1$ considered the following optimality problem: find the estimators which are asymptotically efficient subject to a given bound on the sensitivity. Krasker and Welsch (1982) proposed the following definition of sensitivity which essentially measures the maximum possible influence of a single observation on the set of linear combinations of coefficient estimates $\hat{\beta}$:

$$\gamma = \sup_{y,x} \{\Omega^T(y,x)\ V^{-1}\Omega(y,x)\}^{\frac{1}{2}}$$

where $\Omega(y,x)$ is the influence function of an estimator (for the definition of $\Omega(y,x)$ see Hampel (1974) and $V = E\Omega\Omega^T$ is the asymptotic covariance matrix of the estimator. Notice that this measure of sensitivity is always greater than or equal to $\sqrt{p}$:

$$\gamma \geq \sqrt{p}\ . \tag{17}$$

For the coefficient estimates $\hat{\beta}$ the influence function is

$$\Omega(y,x) = u(x)\ \ w\left(\frac{y-x^T\beta}{\sigma v(x)}\right)\ (y-x^T\beta)B^{-1}x$$

where B is a p×p matrix

$$B = -\frac{\partial}{\partial\beta}E\left[u(x)\,w\left(\frac{y-x^T\beta}{\sigma v(x)}\right)\ (y-x^T\beta)\right]\ x\ . \tag{18}$$

The asymptotic covariance is

$$V = \sigma^2 B^{-1}\ AB^{-1}\ , \tag{19}$$

where

$$A = E\ u^2(x)\ w^2\ \left(\frac{y-x^T\beta}{\sigma v(x)}\right)\ \left(\frac{y-x^T\beta}{\sigma}\right)^2\ xx^T \tag{20}$$

and the expectation in (18) and (20) is over the joint distribution of (y,x). It follows that the sensitivity of an arbitrary WLS estimator is

$$\gamma = \sup_{y,x}\left[u(x)\ w\ \left(\frac{y-x^T\beta}{\sigma v(x)}\right)\ \left|\frac{y-x^T\beta}{\sigma}\right|\ \{x^TA^{-1}x\}^{\frac{1}{2}}\right]. \tag{21}$$

This expression has a nice interpretation. If we ignore the weights for the moment, we see that the influence of (y,x) depends on two components. The first is the scaled residual $(y - x^T\beta)/\sigma$. The second is the expression $(x^TA^{-1}x)^{\frac{1}{2}}$, which should be thought of as a robust measure of "distance" in X-space.

Suppose that we desire an estimator whose sensitivity γ is $\leq$ a, where a is some positive number greater than $\sqrt{p}$ in view of (17). There might be many such estimators. One reasonable way to choose among them would be to find that estimator which is "as close as possible" to least squares, subject to the constraint $\gamma \leq a$. By this we mean that we will downweight an observation only if its influence would otherwise exceed the maximum allowable influence. An observation whose influence is below the maximum will be given a weight of one, as would all the observations under least squares. Formally, suppose we require $\gamma \leq a$ for the number $a > 0$. If, for a given observation (y_i, x_i), we have

$$\left|\frac{y_i - x_i^T\hat{\beta}}{\hat{\sigma}}\right| \{x_i^T A^{-1} x_i\}^{\frac{1}{2}} \leq a, \tag{22}$$

we then want $u(x_i)\, w\left(\frac{y_i - x_i^T\beta}{\sigma v(x_t)}\right) = 1$. Otherwise, we will downweight this observation just enough so that its influence equals the maximum allowable influence, i.e., we choose $u(x_i)\, w\left(\frac{y_i - x_i^T\beta}{\sigma v(x_i)}\right)$ so that

$$u(x_i)\, w\left(\frac{y_i - x_i^T\hat{\beta}}{\sigma v(x_i)}\right) \left|\frac{y_i - x_i^T\hat{\beta}}{\hat{\sigma}}\right| \{x_i^T A^{-1} x_i\}^{\frac{1}{2}} = a. \tag{23}$$

Krasker and Welsch (1982) have shown that weights of the following form

$$u\,(x_i) \equiv 1$$

$$v\,(x_i) = (x_i^T A^{-1} x_i)^{-\frac{1}{2}}$$

$$w\,(t) = \min\,(1,\ a/|t|)$$

with

$$t = \left(\frac{y_i - x_i^T\beta}{\sigma}\right) (x_i^T\, A^{-1}\, x_i)^{\frac{1}{2}}$$

satisfy the first-order necessary condition for efficiency subject to $\gamma \leq a$. Notice that in this case the weights are independent of σ because of the way A is defined by (20).

Recall that under our "central model", the conditional distribution of $(y - x^T\beta)/\sigma$, given x, is N(0,1). Let η denote a random variable whose distribution, given x, is N(0,1). Plugging the optimal weights into the expression (20) for A, we find

$$A = E \min \left\{1, \frac{a}{\left|\frac{y-x^T\beta}{\sigma}\right| \{x^T A^{-1} x\}^{\frac{1}{2}}}\right\}^2 \left(\frac{y-x^T\beta}{\sigma}\right)^2 xx^T$$

$$= E_x[E_{\eta|x} \min \left\{\eta^2, \frac{a^2}{x^T A^{-1} x}\right\}] xx^T$$

$$= E_x g_1\left(\frac{a}{(x^T A^{-1} x)^{\frac{1}{2}}}\right) xx^T , \qquad (24)$$

where $g_1(t)$ is defined by (11) and $E_x(\cdot)$ is the expectation over the distribution of x. To estimate A we solve

$$\hat{A} = \frac{1}{n} \sum_{i=1}^{n} g_1\left(\frac{a}{(x_i^T \hat{A}^{-1} x_i)^{\frac{1}{2}}}\right) x_i x_i^T. \qquad (25)$$

Notice that the matrix A enters through the "robust distances" $d(x_i) = (x_i^T A^{-1} x_i)^{\frac{1}{2}}$ only. This suggests that it may be more convenient to rewrite (25) in terms of robust distances $d(x_i)$. From (25) we can easily obtain

$$d^2(x_i) = x_i^T \left[\frac{1}{n} \sum_{j=1}^{n} g_1 \left(\frac{a}{d(x_j)}\right) x_j x_j^T\right]^{-1} x_i, \; i=1,\ldots,n . \qquad (26)$$

Multiplying both sides of (26) by $g_1\left(\frac{a}{d(x_i)}\right)$ we have

$$d^2(x_i) \cdot g_1 \left(\frac{a}{d(x_i)}\right) = nh_i(w) , \; i=1,\ldots,n, \qquad (27)$$

where $h_i(w)$, $i=1,\ldots,n$ are the diagonal elements of the hat matrix corresponding to the weighted X-matrix $\langle g_1^{\frac{1}{2}}\left(\frac{a}{d(x_i)}\right)\rangle \cdot X$, where the expression in angle brackets is an nxn diagonal matrix.

We observe that, since $g_1(t) \leq t^2$ for all $t \geq 0$, equation (27), and hence (25), will have a solution only if $a > \sqrt{p}$ (cf. (17)). Then, if the sensitivity bound is very large compared to the x-distances $d(x_i)$, say $d(x_i) < \frac{a}{3}$, $i=1,\ldots,n$, the x-weight function $g_1(t)$ will be practically equal to 1 ($g_1(3) \approx .995$), and we will have from (26) that $d(x_i) \approx \sqrt{nh_i}$, $i=1,\ldots,n$, where h_i are the diagonal elements of the original hat matrix $H = X(X^TX)^{-1}X^T$. On the other hand, if there exist outlying points in the X-space, i.e., $d(x_i) \gg a$, the corresponding diagonal element $h_i(w)$ of the "weighted" hat matrix will be reduced to the "balanced design" $h_i(w) \approx \frac{a^2}{n}$, which follows from (27) and the fact that $g_i(t) \approx t^2$ for small t.

More generally, robust x-distances d(x) are defined in the form $d(x) = (x^TC^{-1}x)^{\frac{1}{2}}$ where C is some robust measure of multivariate dispersion (e.g., robust second moment matrices) [see Gnanadesikan and Kettenring (1972), Devlin, et al. (1975), Martin (1980)] . The robust second moment matrix C is usually defined [cf. Maronna (1976)] by the equation.

$$C = \frac{1}{n} \sum_{i=1}^{n} g\,(x_i^TC^{-1}x_i)\; x_ix_i^T\;, \qquad (28)$$

(25) being a special case of (28) with $g\,(x_i^TA^{-1}x_i) = g_1\left(\frac{a}{(x_i^TA^{-1}x_i)^{\frac{1}{2}}}\right)$.

If we constrain v(x) to be identically one, then we obtain the Mallows class of estimators. For a given bound on γ they are not as efficient as the Krasker-Welsch form at the central model, but possess other interesting properties [(Maronna, Bustos, Yohai (1979)].

With $v(x) \equiv 1$, we obtain from (21)

$$\gamma = \sup_{y,x}\left[u(x)\; w_c\;\left(\frac{y-x^T\beta}{\sigma}\right)\;\left|\frac{y-x^T\beta}{\sigma}\right|\;(x^TA^{-1}x)^{\frac{1}{2}}\right]$$

where A satisfies

$$A = E\;[u^2(x)\; w_c^2\;\left(\frac{y-x^T\beta}{\sigma}\right)\;\left(\frac{y-x^T\beta}{\sigma}\right)^2\; xx^T]$$

$$= E_x\;[u^2(x)\; xx^t\; E_{\eta|x}\;\left(w_c^2\;\left(\frac{y-x^T\beta}{\sigma}\right)\left(\frac{y-x^T\beta}{\sigma}\right)^2\right)] \qquad (29)$$

$$= E_x\;[u^2(x)\; xx^T\; g_1(c)]\;.$$

To keep $\gamma \leq a$, there are now many choices since $u(x)$ and $w_c(t)$ are both free. Maronna, Bustos, and Yohai (1980) have shown that the highest efficiency within the Mallows class is achieved by

$$u(x) = \min\left(1, \frac{b}{(x^T A^{-1} x)^{1/2}}\right) = w_b\left((x^T A^{-1} x)^{1/2}\right)$$

$$w_c(t) = \min\left(1, \frac{c}{|t|}\right)$$

with $\gamma = b \cdot c \leq a$.

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