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Investigation of Ultrasonic Methods for Residual Stress Measurement

Bendix Kansas City Division

G. D. Swanson

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INVESTIGATION OF ULTRASONIC METHODS
FOR RESIDUAL STRESS MEASUREMENT

By G. D. Swanson

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INVESTIGATION OF ULTRASONIC METHODS FOR RESIDUAL STRESS
MEASUREMENT

BDX-613-3490, Topical Report, Published April 1986

Prepared by G. D. Swanson

Three ultrasonic methods were investigated for nondestructively measuring residual stress in metals. Shear wave birefringence using a multiarticulated sample positioner and motorized transducer rotator was found to be very sensitive, but the results strongly depend on the material texture. Rayleigh wave measurements using a corner reflector and motorized goniometer have marginal sensitivity, but alternate methods using wedge, knife edge, or needle point contacts have sensitivity comparable to shear wave birefringence. The use of wave-propagation coefficients in conjunction with combinations of measurement methods appears to be at least as promising a technique as use of shear horizontal waves and promises to require much smaller transducers.

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SUMMARY

Three ultrasonic methods were investigated for nondestructive measurement of residual stress in metal parts. Experiments were designed such that all measurements could be made using only single surface contact to a flat area 25 mm square. This requirement limited consideration to those versatile methods that require only a small contact area regardless of part size. Even smaller transducers and curved surface capability can then be developed as needed in future requirements.

A multiarticulated transducer and sample positioner with a motorized transducer rotator was fabricated for shear wave birefringence measurements. It was found to be sensitive to one part in 8000 when using bee's honey as a uniform thickness couplant between the specimen and a piezoelectric shear contact transducer. Attempts were made to find an ultrasonic frequency that maintained the stress birefringence effect and was independent of material texture. The results were inconclusive, and some appeared inconsistent with those of previous researchers. Some rolled materials show extremely strong birefringence with both propagation modes existing simultaneously and having well defined time separation for a single transducer oriented at 45 degrees to the principal texture directions.

A corner reflector with motorized goniometer was fabricated for Rayleigh surface wave velocity measurements. It showed a capability for differentiating between materials of the same alloy but having different heat treatments. However, other surface wave velocity measurement methods using wedges, knife edges, or needle points were found to have greater sensitivity. The corner reflector can also measure shear and longitudinal wave velocity for some metals.

Wave-propagation coefficients appear promising for determining the plane stress components when using a combination of birefringence and surface wave measurements. A complete solution can only be made by assuming the thickness stress to be zero or knowing its value. Shear horizontal waves can produce the same result independently, without the aid of birefringence, but require development of smaller electromagnetic acoustic transducers.

DISCUSSION

SCOPE AND PURPOSE

The purpose of this work is to evaluate ultrasonic methods for nondestructively measuring residual stresses in metals. Residual stresses are common in many forged, heat treated, and machined metal parts. If these residual stresses are large, they can degrade the performance of parts by causing warpage, breakage, or reduced margin of safety and can cause the ultimate failure of the part to perform its intended function. Nondestructive measurement methods can directly aid in production troubleshooting and have potential as quality assurance tools.

The scope of this work includes evaluation of the following ultrasonic techniques. (1) Shear wave birefringence using a rotatable shear wave contact transducer is investigated for measuring the average bulk stress through a part section. This includes an attempt to separate stress effects from texture effects by changing the frequency of the ultrasonic wave. (2) A corner reflector with motorized rotatable surface wave goniometer is evaluated for measuring stresses in a surface layer. Knife edges are also evaluated for transmitting and receiving Rayleigh surface waves. A range of frequencies is used to probe successively deeper into the material and identify any stress gradients. (3) The theory of wave propagation coefficients provides guidance on how to combine measurements in an attempt to separate stress effects from texture effects using only measurements from a single surface.

PRIOR WORK

Three previous projects have evaluated destructive and semi-destructive residual stress measurement techniques. One project¹ compared blind hole drilling and X-ray techniques on welded HP 9-4-20 steel plate. Another project² investigated a hole-in-hole technique, while another determined sensitivity for the instrumented indentation technique.

One investigation of residual stress measurement by ultrasonic techniques has been made at Sandia National Laboratories.³ Both attenuation and velocity measurement techniques were included in that study.

One old⁴ and three recent⁵⁻⁷ reviews of ultrasonic techniques for residual stress measurement served as keys to the literature and as aids in choosing the techniques investigated here. They also identified the critical problems to be surmounted in separating the residual stress effect from sample texture and other competing effects.

ACTIVITY

A key consideration in all work reported here was that each technique should be adaptable to measurement on small parts using only contact to a single surface. Work reported thus includes couplant evaluations, transducer adaptations, and fixture design in addition to preliminary evaluation of the chosen measurement techniques. Some of the techniques that were marginal, not sufficiently sensitive, or otherwise caused problems are included in the appropriate sections for reporting completeness. Most of the discussion focuses on the successful evaluations and adaptations and describes the supporting theory.

All ultrasonic stress measurement methods assume finite linear elastic deformation⁸⁻¹¹ and experimentally determine either directly or indirectly the velocity of ultrasonic wave propagation or a difference between two velocities.¹² The stress is then calculated from the theoretical dependence on the known second and third order elastic constants or on the previously measured stress-acoustic constant for that particular material.¹³⁻¹⁹

Shear Wave Birefringence

Shear wave birefringence²⁰⁻²⁹ is one of the earliest ultrasonic methods to show good correlation with applied stress and promise for residual stress measurement. The stress level in the material is directly related to the difference in propagation velocity between shear waves polarized parallel versus perpendicular to the stress direction and propagating perpendicular to the stress direction. For initially isotropic materials, the proportionality constant between the velocity difference and the stress includes only second and third order elastic constants.⁸⁻¹⁹ More complex techniques are required to separate the effect of texture from the effect of residual stress for materials of any other initial symmetry.³⁰⁻⁵²

A means of stressing samples is needed in order to verify any technique for separating the texture effect from the stress effect. In order to provide known stresses to samples cut from thick materials and also to samples cut from thin materials, two stressing fixtures were fabricated. One fixture was designed to uniaxially compress a 25-mm cube cut from thick material along any one of its three axes. The other fixture was designed for uniaxial tension of a 25-mm wide sample cut from bar or sheet material. Each fixture was designed for ultrasonic transducer coupling to a 25-mm square area on a single surface.

Figure 1 shows the force transducer and a 25-mm cube sample mounted in the compression fixture. A second cube sample is displayed in the foreground. The fixture allows a compression load to be applied using a hydraulic press, a universal testing machine, or other suitable device. The load applied is read using the electronic readout for the strain gage force transducer, which is mounted in line with the cube sample. The clamping bolts are then tightened, and the external load is released. Some relaxation occurs, and the new reduced load is read. The fixture and stressed sample can now be freely transported or water immersed as needed for ultrasonic testing. Measurements can be made independently for stress applied along each cube axis, a necessary feature for evaluating anisotropic or textured materials.

Figure 2 shows a 25-mm wide bar sample mounted in the tension fixture and a second bar sample in the foreground. The stress applied to the sample is determined using clamp-on friction strain gages. The stress is applied through the lever arm by tightening the loading bolt. The fixture and stressed sample can then be freely transported as needed for ultrasonic testing. A separate sample must be cut for each desired test direction of interest in the part. Samples 13 mm thick are shown, but thinner samples are easily accommodated. The high mechanical advantage and small travel distance available for the lever arm require close control on the diameter and distance between the sample attachment holes.

Several viscous materials were evaluated for coupling the shear wave energy from the ultrasonic transducers into the desired samples. Evaluation criteria included ease of application, removal, and sample clean up, as well as low signal attenuation, reproducible bonding, and ability for use while the transducer is being rotated. Liquids will not transmit shear waves, and solids

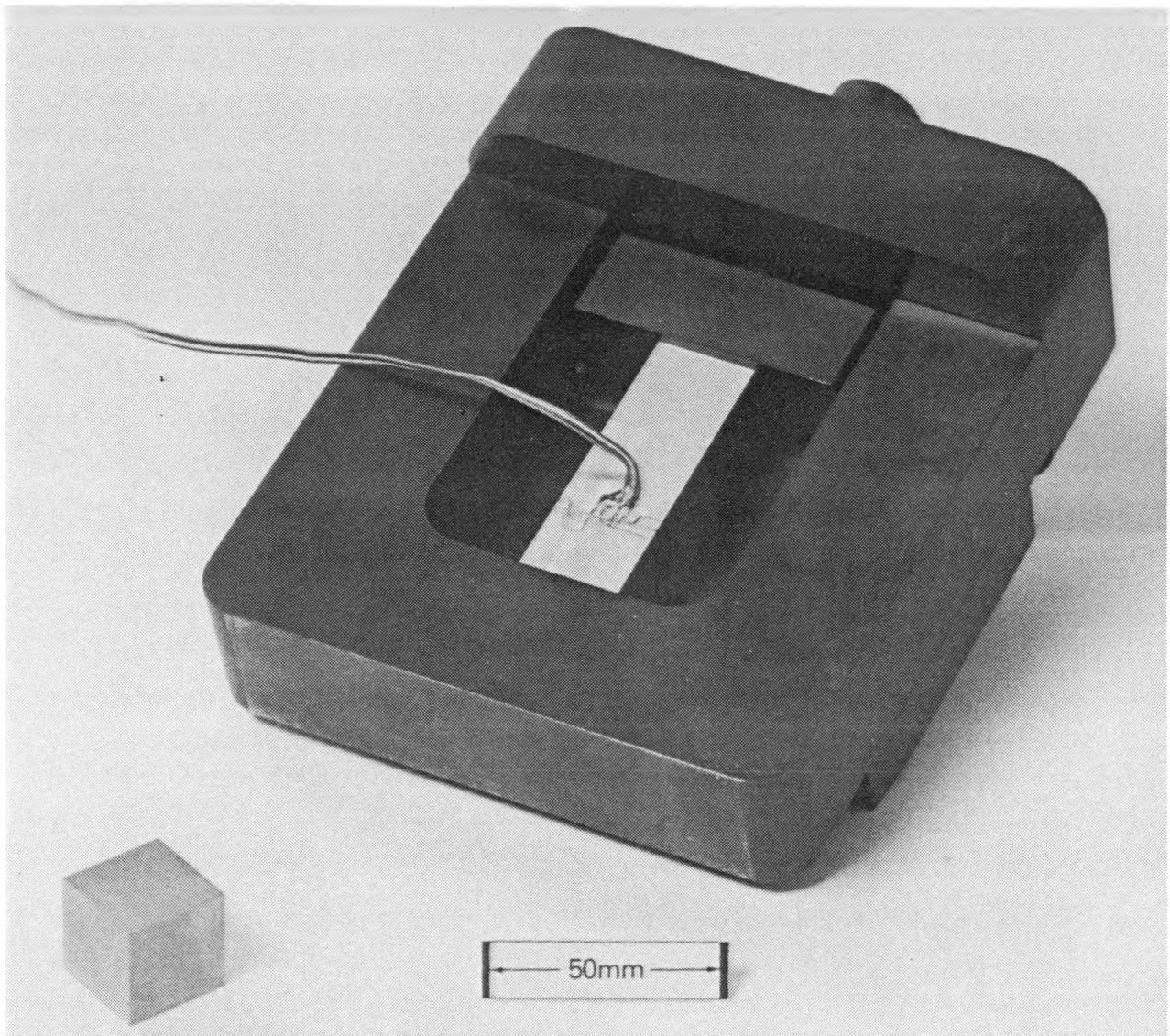


Figure 1. Compression Fixture With Cube Sample and Force Transducer Installed (Second Cube in Foreground)

cannot be used during rotation. A very convenient and usable couplant turned out to be bee's honey. It meets all criteria well and can be cleaned up easily with warm water. Viscous resins have slightly lower attenuation but are more difficult to remove from the sample and transducer.

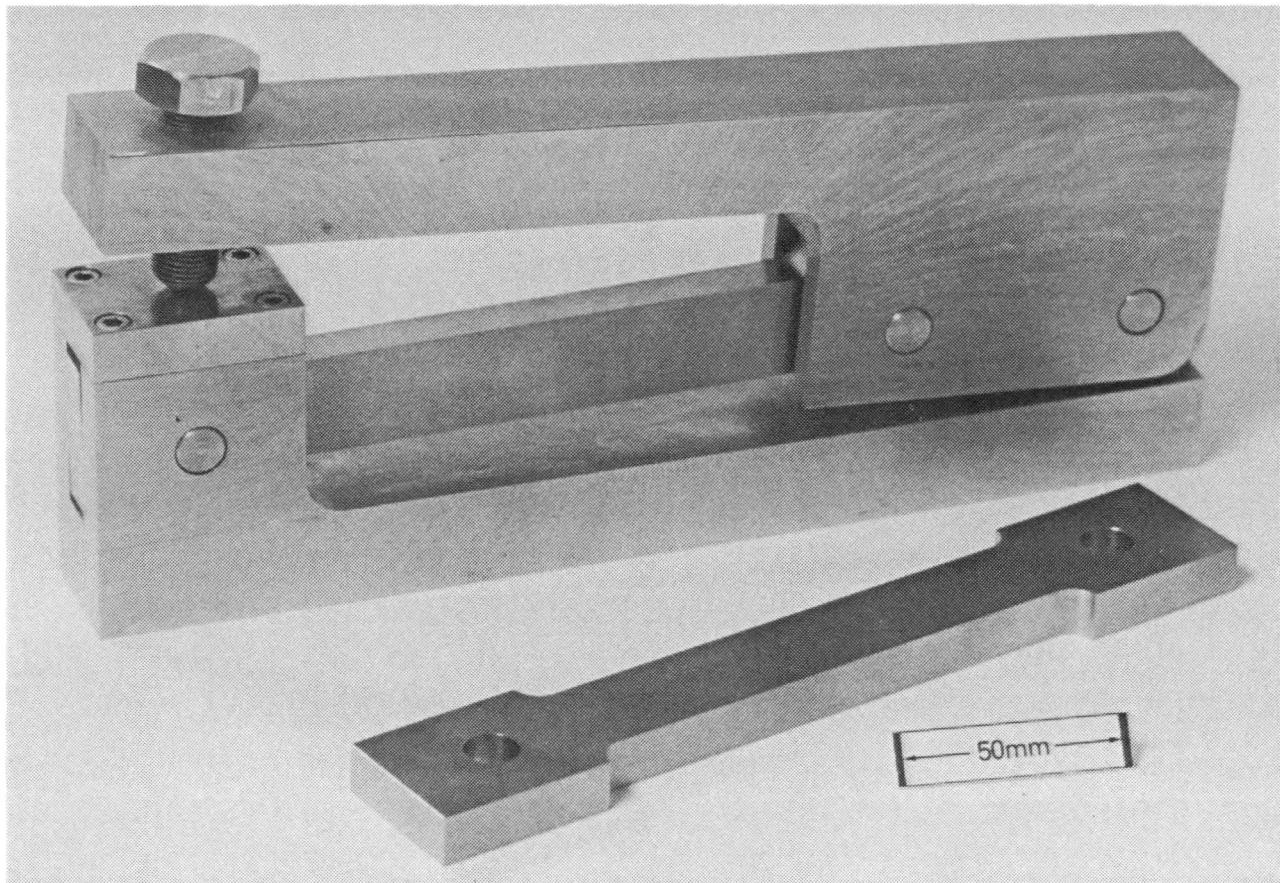


Figure 2. Tension Fixture With Bar Sample Installed and Second Bar Sample in Foreground

A controlled application procedure is essential for reliable and reproducible bonding of the transducer to the sample. Not only must the transducer and sample surfaces be flat and smooth, but also a consistent amount of couplant must be applied and compressed to a uniform thin layer. Very careful hand application can produce variations in pulse-echo transit times of ± 1 to 2 ns out of a total of 8,000 ns, but slightly less care can drastically change the pulse shape and increase the transit time variation to ± 10 ns or more. Bonding reproducibility can be enhanced and operator independence can be achieved by replacing hand application with use of a transducer and sample positioning device.

The transducer holder shown in Figure 3 was fabricated to provide three-dimensional sample positioning with two-angle transducer orientation and motorized variable speed axial transducer rotation. A compression sample is shown positioned for measurement. The shear wave transducer has a collar ring attached to maintain a constant couplant thickness during rotation.²⁵ A force transducer on axis with the ultrasonic transducer monitors the normal force. Two hundred grams is ample normal force for a 12.7-mm diameter transducer with ring, as shown in the lower left corner of Figure 4. Thin honey couplant layers of about 0.0127 mm work well but require care in ring attachment to achieve. Rotation speeds to 20 degrees per second can be applied before the echo shapes begin to deteriorate. Most measurements were made at a more conservative rate of 12 degrees per second. Repeated 0°-90°-0° rotations reproduce to ± 3 ns out of 8,000 ns. The transducer polarization plane is easily checked by rotating the transducer on the edge surface of a set of glass microscope slides clamped together.²⁵ The maximum echo amplitude occurs when the polarization direction is parallel to the length of the slides.

Evaluation of the birefringence technique thus far has included aluminum alloy 6061-T6, stainless steel alloy 304, titanium alloy 6Al-4V, a tool steel, brass, and copper alloy 172. Measured birefringence on metal pieces 3 mm to 25 mm thick ranged from a travel time difference of a few nanoseconds to two microseconds in the absence of external stress. Bar stock of titanium alloy 6Al-4V exhibits by far the largest birefringence. The wave packet separates directly into two distinct components on a 13-mm thick sample when the transducer is oriented at 45 degrees to the rolling direction⁴¹ so that birefringence can be measured directly without any transducer rotation. Moderate stresses then provide additional birefringence only a fraction as large as the initial birefringence because of texture. This is an extreme

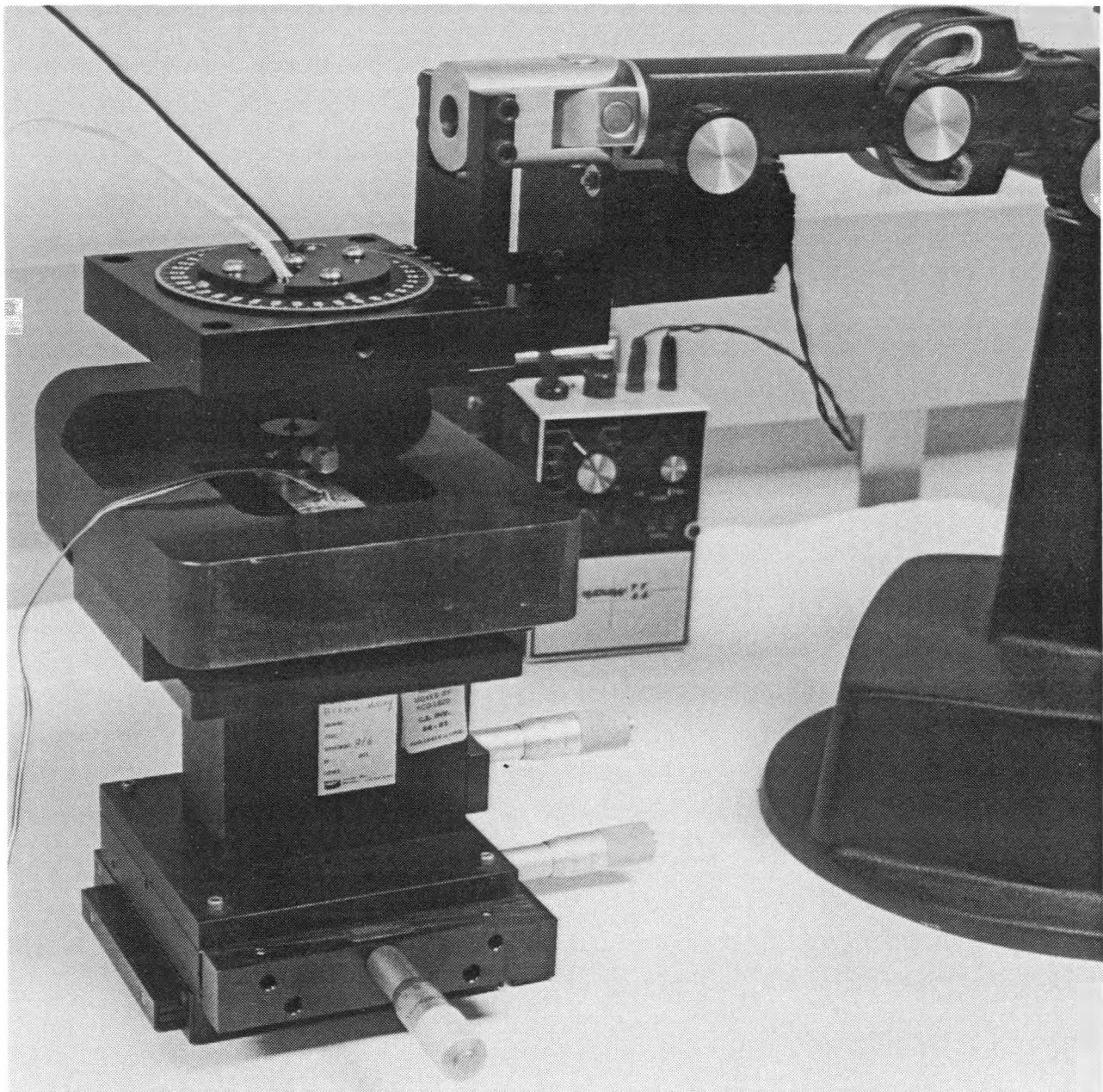


Figure 3. Rotatable Transducer Holder Positioned for Measurements on a Compression Sample Held by the Three-Axis Positioning Table

example showing the necessity of separating the texture contribution so that the stress effect can be found independently.

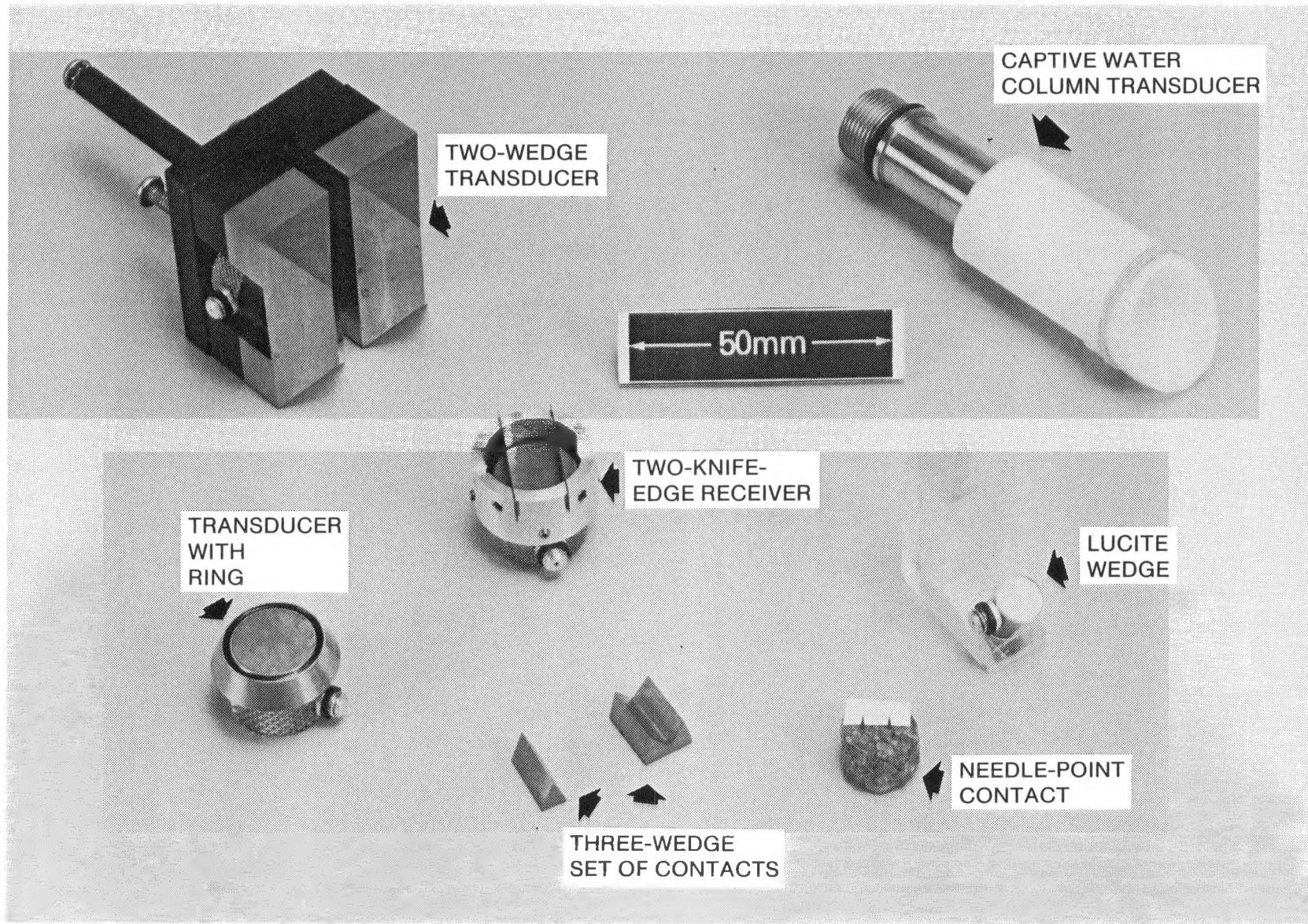


Figure 4. Ultrasonic Transducers With Assorted Coupling Adaptations

Quite different results are exhibited by a less severely anisotropic rolled bar of aluminum alloy 6061-T6, for which the wave components do not separate directly but produce apparent changes in time delay as the transducer is rotated. The results are shown in Figure 5 for a transducer having 1-MHz resonant frequency and operated at a 3-MHz measurement frequency with a rotation rate of 12 degrees per minute between data points. Note the sinusoidal shape of the curve as the transducer is rotated. Tests on successive days produced essentially identical curves with the angular location within 1 degree and the absolute pulse echo time delay within 2 ns. The data are independent of the angular rotation direction of the transducer. The total birefringence effect is 124 ns, with the texture axes being at 14° relative to the nominal 0° direction of fixture setup. Later work showed separation for later echoes at 5 MHz when very short pulses are used.

Previous research^{31,41} has indicated that a frequency may exist that produces texture-independent measurement results. This work thus included measurements on the rolled bar of Figure 5 over an angular range at several frequencies from 1 to 25 MHz. The sine wave shape flattens out as frequency increases and is essentially straight at 25 MHz. However, the flat frequency is about 15 MHz for a 25-mm cube also of aluminum alloy 6061-T6. Arora and James⁴¹ indicate texture independence for a 5-MHz frequency on their titanium alloy 6Al-4V samples. However, the Bendix Kansas City Division (BKC) titanium alloy 6Al-4V tensile bar produces about 2 microseconds time delay difference between 0° and 90°, with direct pulse component separation at 5 MHz. This implies that, if a texture-independent frequency truly exists, it may be different for each material texture of the same alloy and thus not independently capable of compensation even for a single alloy. Further study will be done. Signal attenuation makes it difficult to use high frequencies for thick sections of some metals.

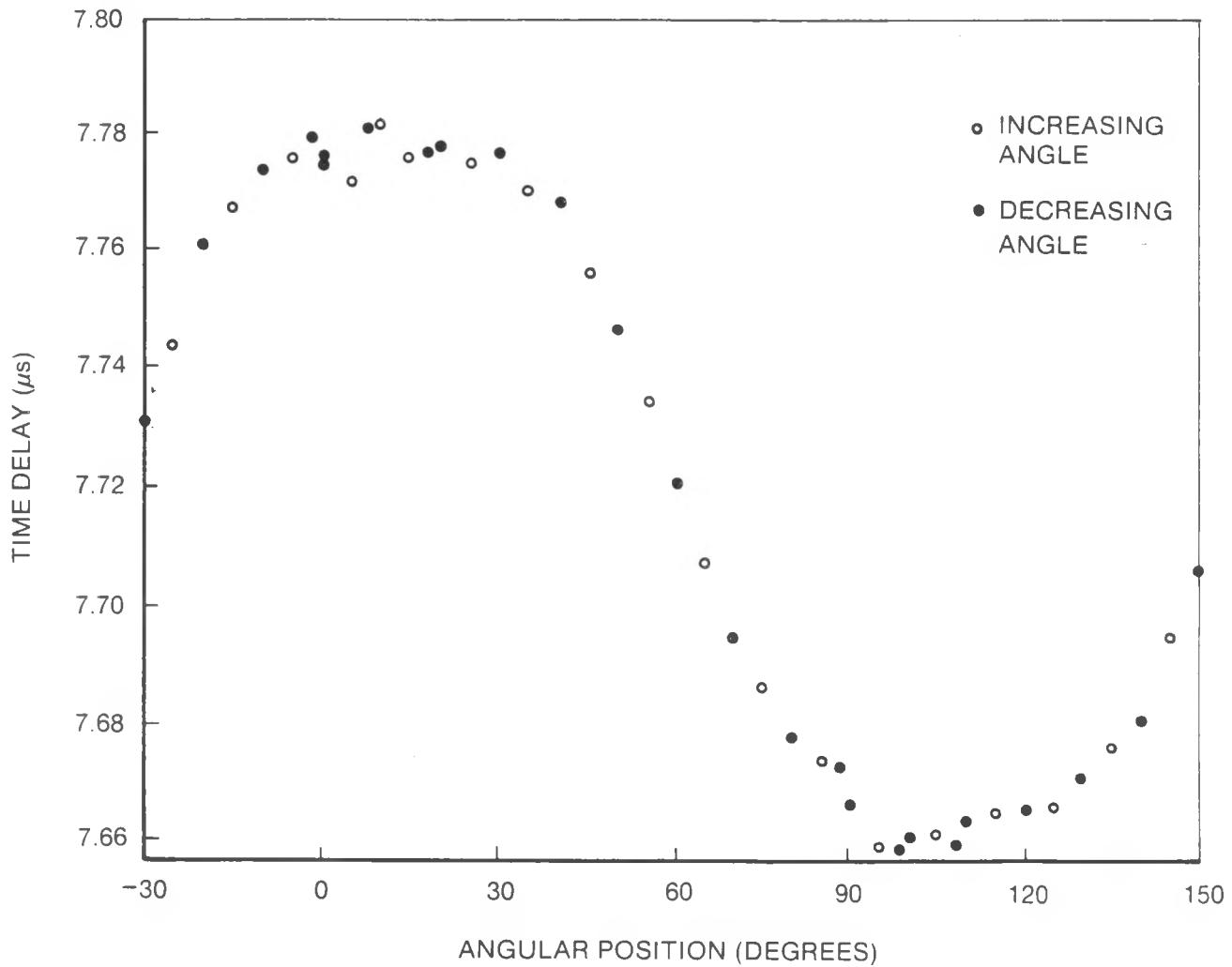


Figure 5. Pulse-echo Time Delay Versus 3-MHz Shear Wave Polarization Angle Relative to the Nominal Rolling Direction for Rolled Bar of Aluminum Alloy 6061-T6

Surface Wave Velocity

Rayleigh, or surface wave velocity,⁵³⁻⁷⁰ is sensitive to material microstructure, alloy, hardness, creep, and temperature as well as residual stress state. Surface waves are confined to travel in a surface layer nominally one wavelength deep. This leads to the possibility of investigating residual stress gradients by

using waves of different frequencies. In combination with shear wave birefringence, a versatile measurement capability is feasible for investigating surface stresses, stress gradients, and through-the-thickness average stresses. Several methods for transmitting and receiving surface waves are evaluated.

The corner reflector shown in Figure 6 has a copper alloy 172 sample attached. Figure 7 shows the corner reflector in contact with a tensile sample, such as that in Figure 2, all being immersed in a tank of water. Motorized transducer rotation occurs at the constant speed chosen such that a chart of amplitude versus angle can be obtained. The entire set of equipment except the strip chart recorder is shown in Figure 8. Two plug-in units for the transmitter/receiver cover the frequency range from 1 to 90 MHz.

A typical amplitude versus angle test result for a stainless steel alloy 304 sample using a 5-MHz transducer operated at 35 MHz is shown in Figure 9. The sample reflection peak at 0° , the reference surface reflection peak at 90° , and the sample and reference surface wave dips are very sharp for high frequencies, and the resultant noise also is present. Much less noise exists at lower frequencies, and the peaks and dips are still very usable (Figure 10) for half of a C-shape copper alloy 172 fracture toughness sample. The small peaks for critical reflection of longitudinal and shear waves are clearly seen.⁵⁵ The effect of varying the heat treatment has been studied⁷¹ using this equipment.

The same sample whose birefringence is reported in Figure 5 was also studied for difference in surface wave velocities. The velocity in the rolling direction (0°) was 3017 m/s, and that transverse to the rolling direction was 2946 m/s. The sensitivity is 1 part in 600. An aluminum single crystal cube

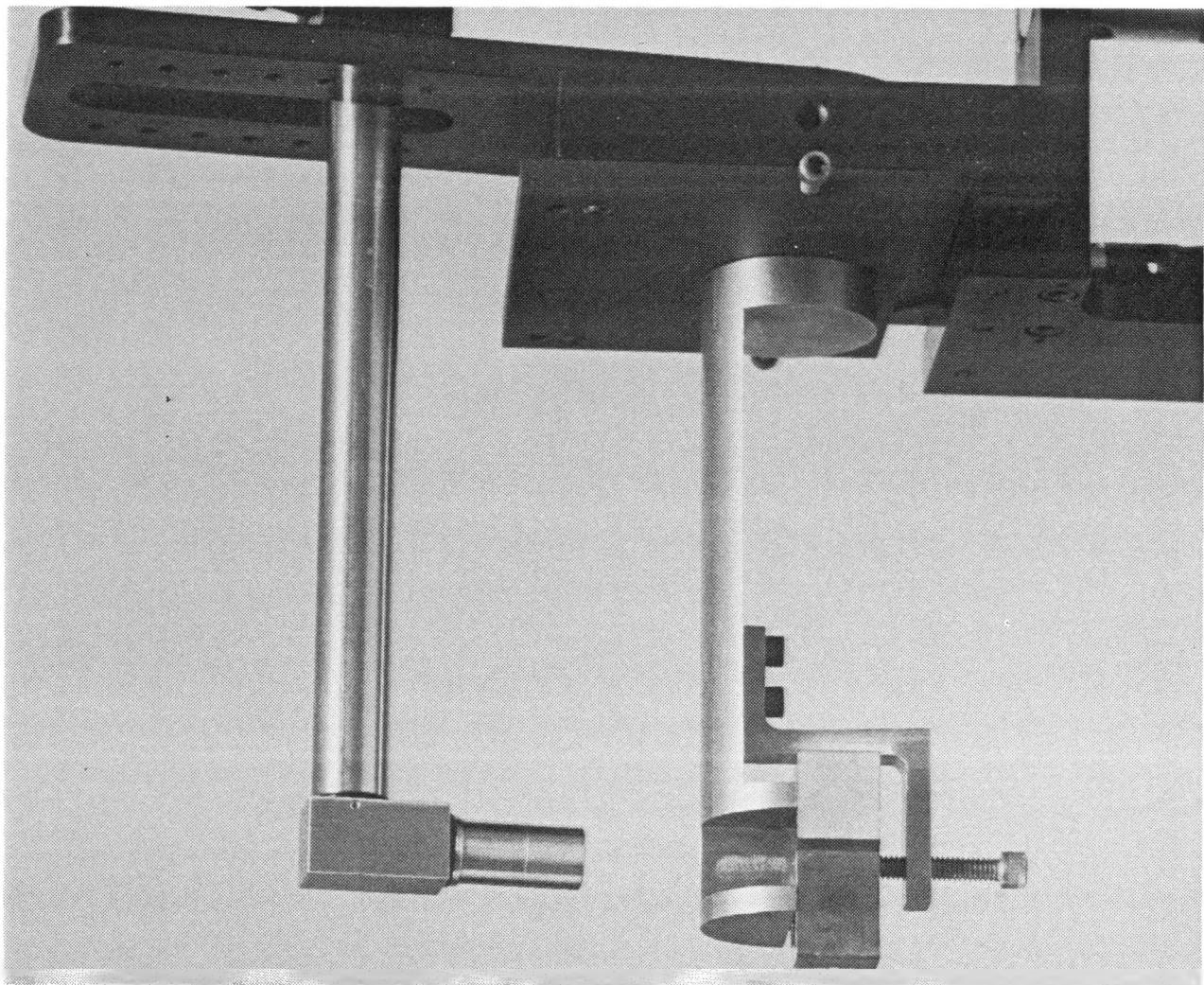


Figure 6. Ultrasonic Goniometer With Rotatable Immersion Transducer, Corner Reflector, and Mounted Half of C-Shape Fracture Toughness Sample of Copper Alloy 172

25 mm on a side yielded differences of about 1 percent for perpendicular directions on any face. This is a higher difference than expected for a single crystal.

The corner reflector has two limitations: (1) Water immersion with at least a 50-mm distance to the reflector corner is required, and (2) the sensitivity limit of about 1 part in 600 is

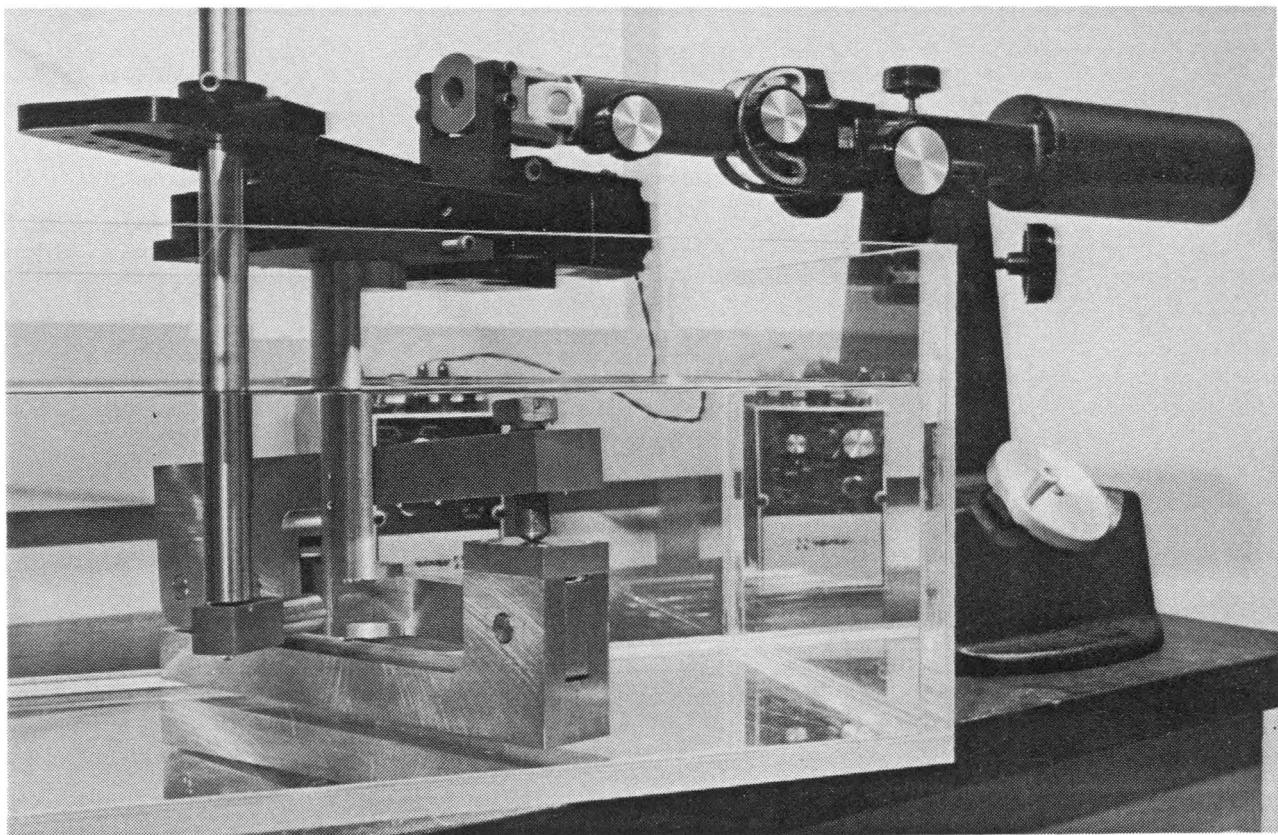


Figure 7. Ultrasonic Goniometer With Corner Reflector in Contact With a Tensile Sample Immersed in a Tank of Water

low compared to birefringence, which has a sensitivity of easily 1 part in 10,000 for samples 12 mm thick. A higher sensitivity for surface wave measurements and also elimination of need for any couplant is claimed for knife edge transducers.⁶⁹

The two-wedge transducer shown in the upper left of Figure 4 was prepared and evaluated. It works well on both flat and gently curved surfaces, with moderate normal coupling force required. It was found that for samples thinner than the 25 mm between wedge contacts, a series of mode-converted shear and longitudinal back-wall reflections could be confounded with the surface wave signal. To eliminate this problem and provide for a

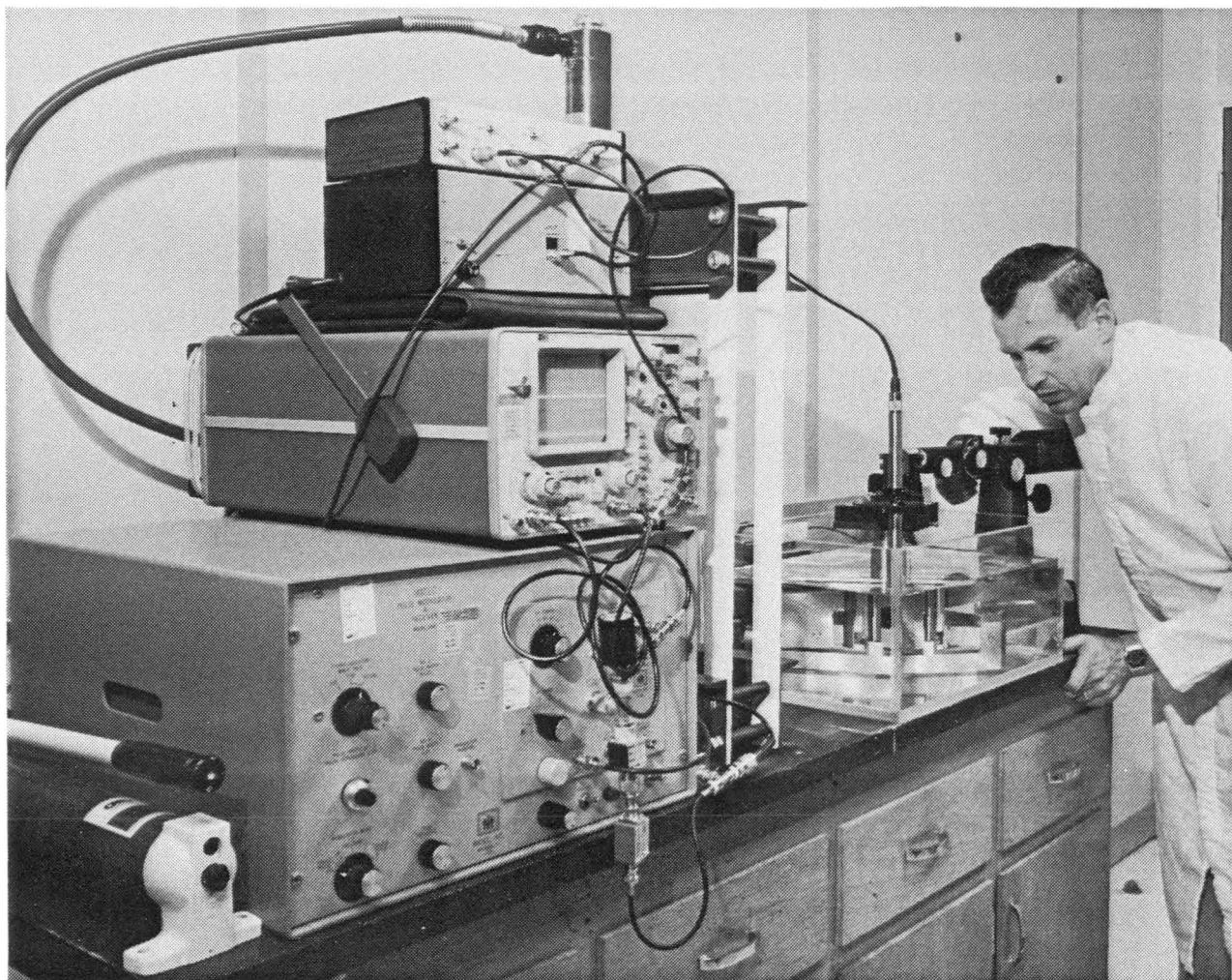


Figure 8. Ultrasonic Transmitter/Receiver and Associated Electronic Equipment With the Critical Angle Reflection Equipment

direct-overlap measurement of travel time, the three-wedge set of contacts shown at the lower center of Figure 4 was prepared. The two-wedge unit is the receiver to be bonded to one transducer, while the single-wedge is bonded to the transmitting transducer. However, the very short two-wedge unit confounds the reverberations from the nearest receiving wedge with the first received signal from the farthest receiving wedge.

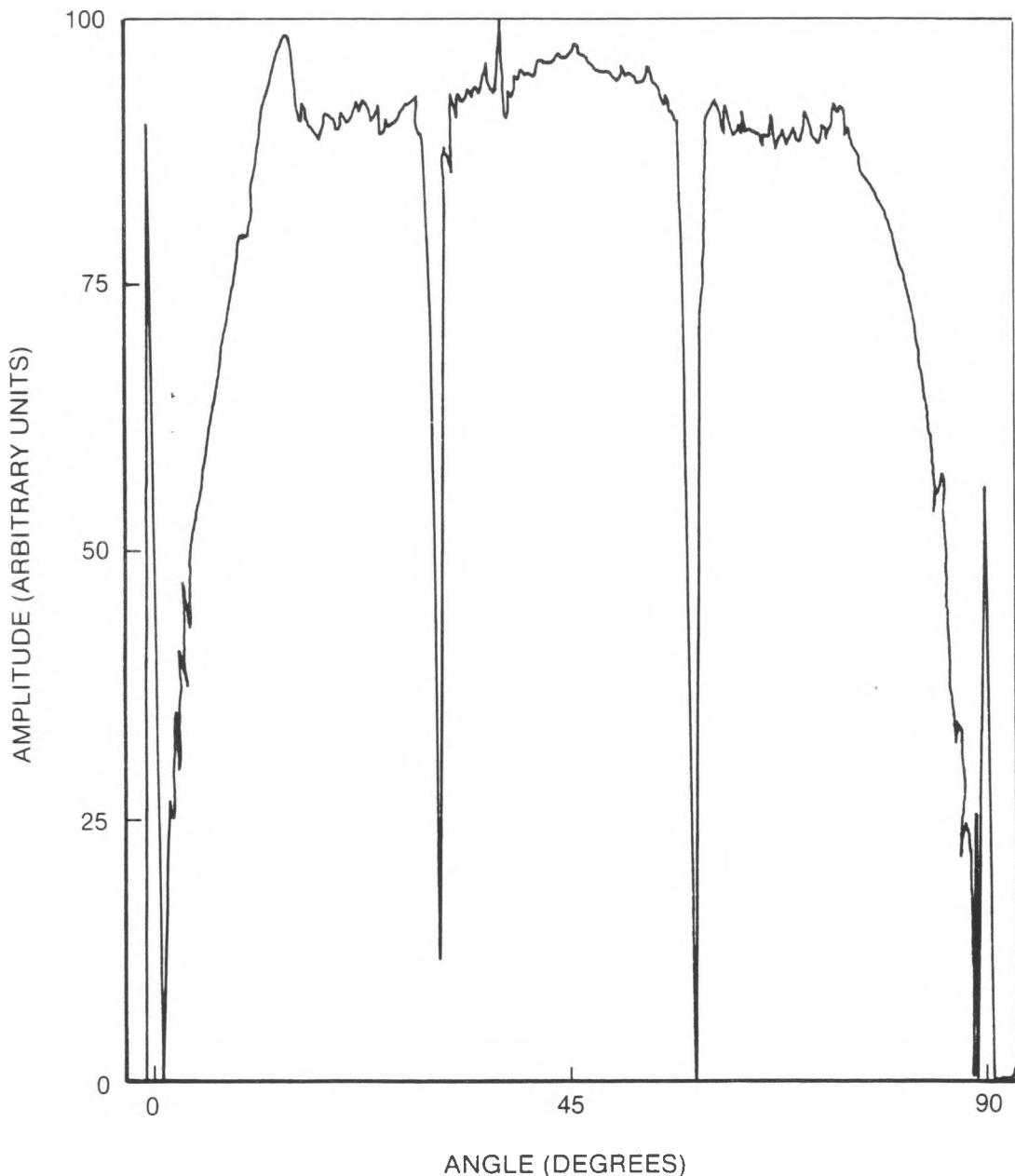


Figure 9. Typical Corner Reflector Test Response on Stainless Steel Alloy 304 Sample at 35 MHz

The signal confounding problem was solved using the two-knife-edge receiver shown at the center of Figure 4. Razor blades are used as knife edges with a length such that the second-knife-edge signal is received at the transducer before the

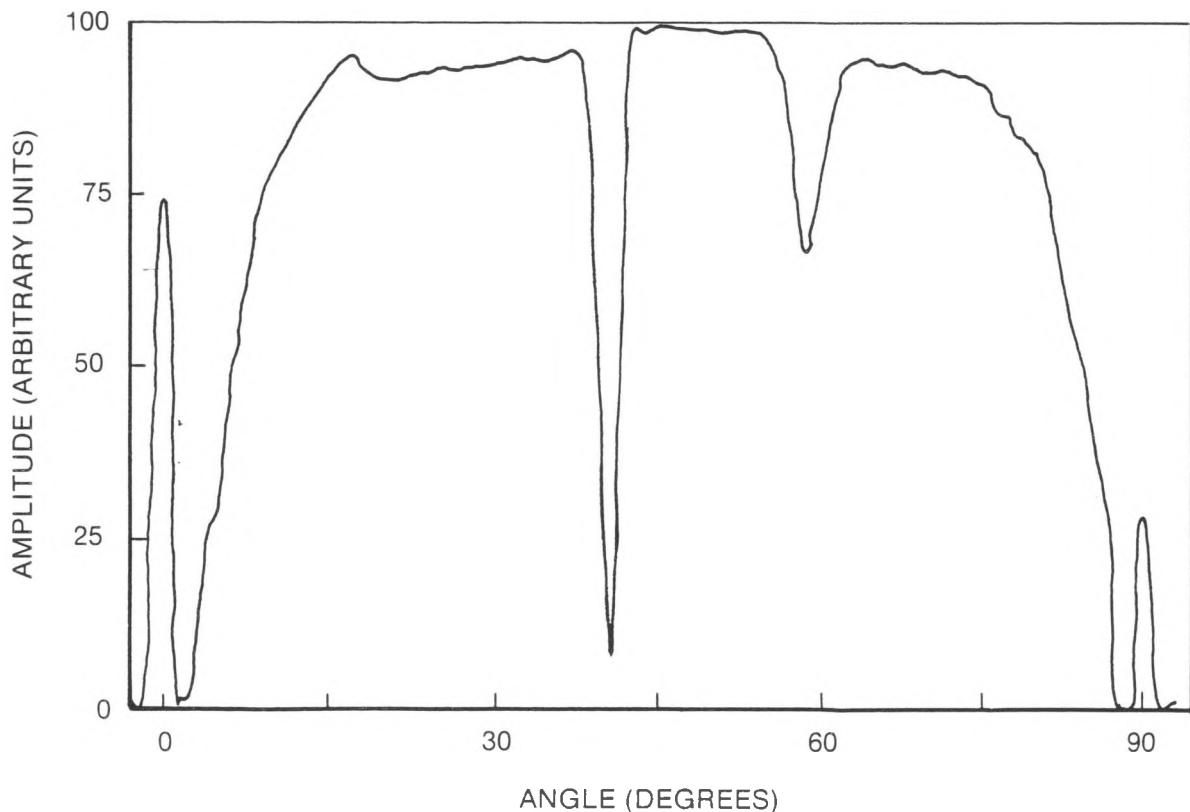


Figure 10. Typical Corner Reflector Test Response for a Copper Alloy 172 Sample at 5 MHZ

first reflected signal in the first knife edge is received at the transducer. The sample still must be thicker than the distance between knife edges, but closer knife edges are possible, if lower sensitivity is acceptable, by just using an adapting collar with closer slots. The razor blades are tapered towards the ends so that only about 12 mm of the center comes in contact with the transducer and sample. This greatly lowers the contact force required for usable signals. A force of 800 grams on the two-knife-edge transducer is more than adequate.

The contact force can be lowered even further by using needle-point contacts, as shown at the bottom right of Figure 4. Cork is used to isolate one needle from the other and to hold the

alignment. Teflon may work too and will be tried later. Flattened rear ends of the needles (thumb tack shafts were used here) contact the transducer. Acceptable response was achieved for this prototype version using a single wedge, a single knife edge, a Lucite wedge (right center in Figure 4), or a captive water column transducer (upper right in Figure 4) separately as the transmitter. Only the Lucite wedge requires a couplant. The captive water column is achieved using a Teflon tube cut at the critical angle for surface wave generation and with the angled end covered with a thin plastic film held in place by a rubber band to contain the water. Higher transmitted signal amplitude can be achieved for high attenuation samples by using a drop of water as couplant between the plastic film and the sample surface. A transit time sensitivity of 1 part in 10,000 can easily be achieved for wedge, knife, or point separations of 12 mm.

Related Methods

Birefringence and surface wave measurements have problems in separating the residual stress effect from the texture effect. For that reason, other methods and combinations of methods have been investigated recently. One method is to determine the texture by ultrasonic pole figures,⁴² X-ray diffraction, or neutron diffraction.⁴³ Another method is to include longitudinal wave velocity measurements used in combination with birefringence.⁴⁶ The temperature dependence of ultrasonic velocity^{72,73} retains the same texture separation problem. A very promising recent method is the use of shear horizontal (SH) waves generated by electromagnetic acoustic transducers (EMATs).⁷⁴⁻⁷⁸ However, EMATs need to be miniaturized considerably before a measurement can be made on a 25-mm square surface area.

One possible way to make SH wave measurements on a smaller area is to miniaturize the transmitter by using a shear wave transducer with a wedge of angle chosen to achieve critical reflection so that a SH wave propagates parallel to the sample surface. The detector may then be miniaturized using wedges, knife edges, or points as described above for surface wave testing. Higher coupled energy over a smaller surface distance may be achieved using acoustic lenses,⁷⁹ with appropriate considerations for anisotropic materials.⁸⁰ Design is in process for trying these ideas.

A combination approach has considerable merit in that measurements in addition to shear wave birefringence are readily obtained from the same sample surface. Longitudinal, surface, and SH wave measurements can be made with the same equipment by using different transducers. The wave-propagation coefficient approach of Thurston¹⁶ may be considered as a means of identifying the appropriate combination of waves and does not require evaluation of third-order elastic constants. For example, it has been noted by Allen and Sayers⁴⁶ that rolled plates of steel commonly have orthorhombic symmetry composed of cubic crystallites. The behavior for ultrasonic wave propagation is then as a material of orthorhombic symmetry.

The appropriate wave-propagation coefficients, \hat{C}_{ij} , for various crystal classes are exhibited in Table A-2 in the Appendix. The subset of coefficients that can be measured by single surface coupling and no oblique wave transmission modes are shown in Table 1. For compactness, only the subscripts are shown. For these measurements, there is no differentiation between triclinic and monoclinic and no differentiation between the two classes of tetragonal or trigonal crystals. Note that pure \hat{C}_{63} modes are only possible for triclinic and monoclinic crystals and that \hat{C}_{36}

Table 1. Wave-Propagation Coefficients for Various Crystal Classes Which Can Be Measured by Single Surface Coupling

Method*	Measurement Coefficient Subscripts	Triclinic &					Transverse		
		Monoclinic	Orthorhombic	Tetragonal	Trigonal	Hexagonal	Isotropic	Cubic	Isotropic
CRL	11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11
SH	12	2 + 66	2 + 66	2 + 66	2 + $\frac{1}{2}$ (11-12)	2 + $\frac{1}{2}$ (11-12)	2 + $\frac{1}{2}$ (11-12)	2 + 44	2 + $\frac{1}{2}$ (11-12)
BS	13	3 + 55	3 + 55	3 + 44	3 + 44	3 + $\frac{1}{2}$ (11-12)	3 + 44	3 + 44	3 + $\frac{1}{2}$ (11-12)
SH	21	1 + 66	1 + 66	1 + 66	1 + $\frac{1}{2}$ (11-12)	1 + $\frac{1}{2}$ (11-12)	1 + $\frac{1}{2}$ (11-12)	1 + 44	1 + $\frac{1}{2}$ (11-12)
CRL	22	2 + 22	2 + 22	2 + 11	2 + 11	2 + 11	2 + 11	2 + 11	2 + 11
BS	23	3 + 44	3 + 44	3 + 44	3 + 44	3 + $\frac{1}{2}$ (11-12)	3 + 44	3 + 44	3 + $\frac{1}{2}$ (11-12)
CRS	31	1 + 55	1 + 55	1 + 44	1 + 44	1 + $\frac{1}{2}$ (11-12)	1 + 44	1 + 44	1 + $\frac{1}{2}$ (11-12)
CRS	32	2 + 44	2 + 44	2 + 44	2 + 44	2 + $\frac{1}{2}$ (11-12)	2 + 44	2 + 44	2 + $\frac{1}{2}$ (11-12)
L	33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 11	3 + 11
CRS-45	36	6 + 45	6	6	6	6	6	6	6
BS-45	63	45	0	0	0	0	0	0	0
CRL-45	66	$\frac{1}{2}$ (12 + 66)	$\frac{1}{2}$ (12 + 66)	$\frac{1}{2}$ (12 + 66)	$\frac{1}{4}$ (12 + 11)	$\frac{1}{4}$ (12 + 11)	$\frac{1}{4}$ (12 + 11)	$\frac{1}{2}$ (12 + 44)	$\frac{1}{4}$ (12 + 11)

*CRL = Corner Reflector, Longitudinal Wave

CRS = Corner Reflector, Shear Wave

SH = Shear Horizontal Wave

BS = Birefringence Shear Wave

-45 = At 45° Angle To 1 Axis

modes only exist for other classes if a shear stress, T_6 , exists in the 12 plane. This will only occur when the principal axes for texture and stress are different.

The observation of MacDonald³⁶ that the difference between modes with subscripts interchanged is independent of elastic constants and yields the stress difference directly is seen to hold for all crystal classes. If one stress, for example the stress in the thickness direction, T_3 , is zero, then all of the other stresses and elastic constants in Table 1 for that class can be determined in theory. However, because the elastic constants of metals, C_{ij} , generally have magnitudes of the order of $1-10 \times 10^4$ MPa, while a stress measurement sensitivity to 1 MPa is desirable, solution in practice requires careful and highly precise measurement. This means that 4- to 5-digit wave propagation coefficient accuracy is required in order to determine stresses. For this reason, measurement techniques that require only velocity or time delay differences are usually preferred over single measurements.

The usual birefringence measurement finds the difference, $\hat{C}_{13} - \hat{C}_{23}$. Table 1 shows this difference to be zero except for triclinic, monoclinic, and orthorhombic crystal classes where the difference is $C_{55} - C_{44}$. This can be used in conjunction with corner reflector shear or surface wave difference measurement, $\hat{C}_{31} - \hat{C}_{32}$, to find the difference of the principal stresses, $T_1 - T_2$. For symmetries greater than orthorhombic, the stress difference is found directly with no need for birefringence measurements. Note that the same stress difference can always be found using shear horizontal wave modes through the difference $\hat{C}_{21} - \hat{C}_{12} = T_1 - T_2$ for all crystal classes. Any non-uniform texture through the thickness or in the plane can negate the theory and disallow the use of multiple measurement methods.

This combination of methods has been used to analyze the aluminum alloy 6061-T6 sample for which data are shown in Figure 5 and Figure 9. The measured velocities, calculated wave propagation coefficients, calculated elastic constants, and calculated stresses are shown in Table 2 under the assumption that the thickness stress, T_3 , is zero. Note that the calculated rolling direction stress, T_1 , is nearly equal to the material yield stress, probably indicating the measurement inaccuracies, especially of the corner reflector method. Surface wave measurements using a two-knife-edge receiver should be more sensitive.

The theoretical coefficients of Table 1 allow a certain amount of differentiation between materials having different symmetries. These can be very useful when measurement accuracies are high.

Conclusions

The activity described above toward identifying a universal nondestructive residual stress measurement technique using ultrasonics has produced the following conclusions. A spacer collar on a shear wave transducer arranged for motorized rotation under a measured contact force creates a very sensitive unit for shear wave birefringence measurements. Bee's honey is a good viscous couplant which is easily removable. The texture sensitivity, which is not universally removed at a particular measurement frequency, keeps birefringence from being a single universal technique, even though it shows good promise when used in combination with other measurements.

Surface wave velocity is one such other technique for combination use. Critical angle reflectometry using a corner reflector and motorized goniometer is not adequately sensitive for surface wave velocity determination. Time delay techniques using wedges,

Table 2. Measured Ultrasonic Wave Velocities and Calculated Quantities for Rolled Bar of Aluminum Alloy 6061-T6

Reduced Subscripts	Measured Velocity v_{ij} (cm/ μ s)	Wave Propagation Coefficient C_{ij} (GPa)	Elastic Constant C_{ij} (GPa)
11	0.6094	98.41	98.18
13	0.3175	26.62	-----
22	0.6163	100.70	100.36
23	0.3123	25.85	-----
31	0.3183	26.85	-----
32	0.3144	26.19	-----
33	0.6415	109.10	109.10
36	0.3174	26.70	-----
44	-----	-----	25.85
55	-----	-----	26.62
66	0.6188	101.50	-----

$$T_1 = 0.23 \text{ GPa}$$

$$T_2 = 0.34 \text{ GPa}$$

$$T_3 = 0 \text{ (by assumption)}$$

knife edges, or needle points for transduction appear adequately sensitive and eliminate the need for water immersion of the test part.

The theory of wave propagation coefficients predicts that stress differences can be determined directly using EMATs or other transducers for shear horizontal wave measurements. Even then, each stress cannot be separately determined, but only principal stress differences. At least one principal stress must be known for the other two to be determined using difference methods, whether they are birefringence, surface waves, or shear horizontal waves and whether wave propagation coefficients or third order elastic constants are used in the theory. Single

measurements can separate the stress components in theory, but measurement sensitivity, coupling repeatability, and texture independence are not adequate in practice.

ACCOMPLISHMENTS

A multiarticulated transducer and sample positioner with integral motorized transducer rotator has been fabricated and used to evaluate the shear wave birefringence method of residual stress measurement. Tension and compression samples of several metals, appropriate portable loading fixtures, and load measuring transducers have been fabricated and used for measurements in air and water. A ring collar has been used for retaining a uniform thickness layer of viscous couplant when measuring shear wave birefringence using the motorized transducer rotator for repeated measurements. Bee's honey has been verified to be a good shear wave couplant.

A corner reflector with motorized transducer goniometer and a water immersion tank have been fabricated, and Rayleigh surface wave measurements have been made on several materials, including several samples of copper alloy 172 of different heat treatments. Longitudinal and shear wave velocities also have been determined. The theory of wave-propagation coefficients has been applied to explicitly exhibit the coefficients for all crystal classes in terms of the elastic constants and stresses. Measurements on a rolled bar of aluminum alloy 6061-T6 verified its behavior as a material of orthorhombic symmetry and showed capability for complete solution of the two-dimensional stress field in the surface plane and of the associated elastic constants.

FUTURE WORK

Planned work includes refining the three-knife-edge surface wave transducer and the two-needle-point receiver. Investigation of alternate methods for compact shear horizontal wave transmitter and receivers will be carried out while following any work toward miniaturization of EMATs. Further study of theory and of combination techniques will be carried out in an attempt to measure the complete three-dimensional stress field from coupling to only a single sample surface. Stress calibration and sensitivity versus texture effect will then be studied.

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Appendix A

WAVE-PROPAGATION COEFFICIENTS FOR MATERIALS UNDER STRESS

R. N. Thurston¹⁶ points out that the full classical symmetry of effective elastic coefficients for wave propagation is lost, even in isotropic materials, for any stress state other than pure positive or negative hydrostatic pressure. A set of wave-propagation coefficients, \hat{C}_{ijkm} , is thus defined in terms of the initial elastic coefficients, C_{ijkm} , and the associated initial stresses, T_{km} :

$$\hat{C}_{ijkm} = \delta_{ij} T_{km} + \frac{1}{2} (C_{ikjm} + C_{imjk}) . \quad (A-1)$$

Here, the Kronecker delta, δ_{ij} , has the usual meaning, being zero if the subscripts are not identical and one for identical subscripts. All subscripts run over the range 1 to 3. The symmetries are such that the wave propagation coefficients can be written in an abbreviated two-subscript notation, \hat{C}_{uv} , with subscripts running from 1 to 6. The wave propagation condition then has the following form: where N_i are propagation vector components, ρ is the initial material density, V is the wave velocity, and U_j is the displacement vector,

$$(\hat{C}_{ijkm} N_k N_m - \rho V^2 \delta_{ij}) U_j = 0 . \quad (A-2)$$

The full matrix in reduced two-subscript notation for the most general triclinic crystal class is shown in Table A-1. The 27 individual components include 21 elastic constants and 6 stresses. Note that this matrix is symmetric in the absence of stress. Table A-2 shows the coefficients for other crystal classes as well. The symbols T and C are omitted, with only the subscripts shown for conciseness. The number of independent

Table A-1. Wave-Propagation Coefficients for Triclinic Crystals

$\hat{C}_{uv} = \hat{C}_{ijkm} = \delta_{ij} T_{km} + \frac{1}{2} (C_{ijkm} + C_{imjk})$					
$T_1 + C_{11}$	$T_2 + C_{66}$	$T_3 + C_{55}$	$T_4 + C_{56}$	$T_5 + C_{15}$	$T_6 + C_{16}$
$T_1 + C_{66}$	$T_2 + C_{22}$	$T_3 + C_{44}$	$T_4 + C_{24}$	$T_5 + C_{46}$	$T_6 + C_{26}$
$T_1 + C_{55}$	$T_2 + C_{44}$	$T_3 + C_{33}$	$T_4 + C_{34}$	$T_5 + C_{35}$	$T_6 + C_{45}$
C_{56}	C_{24}	C_{34}	$\frac{1}{2} (C_{23} + C_{44})$	$\frac{1}{2} (C_{45} + C_{36})$	$\frac{1}{2} (C_{46} + C_{25})$
C_{15}	C_{46}	C_{35}	$\frac{1}{2} (C_{45} + C_{36})$	$\frac{1}{2} (C_{13} + C_{55})$	$\frac{1}{2} (C_{14} + C_{56})$
C_{16}	C_{26}	C_{45}	$\frac{1}{2} (C_{46} + C_{25})$	$\frac{1}{2} (C_{14} + C_{56})$	$\frac{1}{2} (C_{12} + C_{66})$

T_i = stress component in initial state using reduced subscript notation.

C_{ij} = classical elastic constant in initial state using reduced subscript notation.

Table A-2. Crystal Classes and the Corresponding Wave-Propagation Coefficients

Coef. Sub	Particle Displacement	Propagation Direction	Triclinic			Monoclinic		Orthorhombic		Tetragonal		Trigonal		Hexagonal		Transverse		Isotropic		
			21	13	9	7	6	7	6	7	6	7	6	5	5	1 + 11	1 + 11	1 + 11	1 + 11	
11	11	11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	1 + 11	
12	11	22	2 + 66	2 + 66	2 + 66	2 + 66	2 + 66	2 + 66	2 + 66	2 + $\frac{1}{2}$ (11 - 12)	2 + 44	2 + 44	2 + $\frac{1}{2}$ (11 - 12)	2 + 44	2 + $\frac{1}{2}$ (11 - 12)					
13	11	33	3 + 55	3 + 55	3 + 55	3 + 55	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	
14	11	23	4 + 56	4	4	4	4	4	4	4 + 14	4 + 14	4 + 14	4 + 14	4	4	4	4	4	4	
15	11	31	5	5	5	5	5	5	5	5 + 15	5	5	5	5	5	5	5	5	5	
16	11	12	6 + 16	6 + 16	6	6 + 16	6	6	6	6	6	6	6	6	6	6	6	6	6	
21	22	11	1 + 66	1 + 66	1 + 66	1 + 66	1 + 66	1 + 66	1 + 66	1 + $\frac{1}{2}$ (11 - 12)	1 + 44	1 + 44	1 + $\frac{1}{2}$ (11 - 12)	1 + 44	1 + $\frac{1}{2}$ (11 - 12)					
22	22	22	2 + 22	2 + 22	2 + 22	2 + 22	2 + 22	2 + 22	2 + 22	2 + 11	2 + 11	2 + 11	2 + 11	2 + 11	2 + 11	2 + 11	2 + 11	2 + 11	2 + 11	2 + 11
23	22	33	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44	3 + 44
24	22	23	4 + 24	4	4	4	4	4	4	4 + 14	4 + 14	4 + 14	4 + 14	4	4	4	4	4	4	4
25	22	31	5 + 46	5	5	5	5	5	5	5 - 15	5	5	5	5	5	5	5	5	5	5
26	22	12	6 + 26	6 + 26	6	6 - 16	6	6	6	6	6	6	6	6	6	6	6	6	6	6
31	33	11	1 + 55	1 + 55	1 + 55	1 + 44	1 + 44	1 + 44	1 + 44	1 + 44	1 + 44	1 + 44	1 + 44	1 + 44	1 + 44	1 + 44	1 + 44	1 + 44	1 + $\frac{1}{2}$ (11 - 12)	
32	33	22	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + 44	2 + $\frac{1}{2}$ (11 - 12)	
33	33	33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 33	3 + 11	3 + 11
34	33	23	4 + 34	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
35	33	31	5 + 35	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
36	33	12	6 + 45	6 + 45	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
41	23	11	56	0	0	0	0	0	0	14	14	14	0	0	0	0	0	0	0	0
42	23	22	24	0	0	0	0	0	0	-14	-14	-14	0	0	0	0	0	0	0	0
43	23	33	34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	23	23	$\frac{1}{2}$ (23 + 44)	$\frac{1}{2}$ (23 + 44)	$\frac{1}{2}$ (23 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (12 + 44)	$\frac{1}{2}$ (12 + 44)	$\frac{1}{2}$ (12 + 11)					
45	23	41	$\frac{1}{2}$ (45 + 36)	$\frac{1}{2}$ (45 + 36)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	23	12	$\frac{1}{2}$ (46 + 25)	0	0	0	0	0	0	-15	0	0	0	0	0	0	0	0	0	0
51	31	11	15	0	0	0	0	0	0	15	0	0	0	0	0	0	0	0	0	0
52	31	22	46	0	0	0	0	0	0	-15	0	0	0	0	0	0	0	0	0	0
53	31	33	35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	31	23	$\frac{1}{2}$ (45 + 36)	$\frac{1}{2}$ (45 + 36)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	31	31	$\frac{1}{2}$ (13 + 55)	$\frac{1}{2}$ (13 + 55)	$\frac{1}{2}$ (13 + 55)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (13 + 44)	$\frac{1}{2}$ (12 + 44)	$\frac{1}{2}$ (12 + 44)	$\frac{1}{2}$ (12 + 11)						
56	31	12	$\frac{1}{2}$ (14 + 56)	0	0	0	0	0	0	14	14	14	0	0	0	0	0	0	0	0
61	12	11	16	16	0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	12	22	26	26	0	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	12	33	45	45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	12	23	$\frac{1}{2}$ (46 + 25)	0	0	0	0	0	0	-15	0	0	0	0	0	0	0	0	0	0
65	12	31	$\frac{1}{2}$ (14 + 56)	0	0	0	0	0	0	14	14	0	0	0	0	0	0	0	0	0
66	12	12	$\frac{1}{2}$ (12 + 66)	$\frac{1}{2}$ (12 + 11)	$\frac{1}{2}$ (12 + 11)	$\frac{1}{2}$ (12 + 11)	$\frac{1}{2}$ (12 + 11)	$\frac{1}{2}$ (12 + 11)	$\frac{1}{2}$ (12 + 11)	$\frac{1}{2}$ (12 + 11)	$\frac{1}{2}$ (12 + 11)	$\frac{1}{2}$ (12 + 44)	$\frac{1}{2}$ (12 + 44)	$\frac{1}{2}$ (12 + 11)						

elastic constants decreases as the crystal symmetry increases. The number of independent elastic constants is listed below each crystal class. Note that stresses T_1 , T_2 , and T_3 are the usual principal stresses, while T_4 , T_5 , and T_6 are the shear stresses.

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⁷⁶R. B. Thompson, "Effects of Microstructure on the Acoustoelastic Measurements of Stress." Nondestructive Evaluation: Application to Materials Processing, Ed. O. Buck and S. M. Wolf, ASM, 1984, pp 137-145.

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⁸⁰J. L. Rose and E. W. Deska, "New Analytical Concepts in Ultrasonic Angle Beam Analysis." Mater. Eval., Vol. 32, No. 10, October 1974, pp 223-8.