MASTER

TRANSIENT ANALYSIS OF LMFBR REINFORCED/PRESTRESSED CONCRETE CONTAINMENT

by

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Summary

The use of prestressed concrete reactor vessels (PCRVs) for LMFBR containment creates a need for analytical methods for treating the transient response of such structures, for LMFBR containments must be capable of sustaining the dynamic effects which arise in a hypothetical core disruptive accident (HCDA). These analyses require several unique features: a model of concrete which includes tensile cracking, a methodology for representing the prestressing tendons and for simulating the prestressing operation, and an efficient computational tool for treating the transient response. Furthermore, for the sake of convenience, all of these features should be available in a single computer code.

For the purpose of treating the transient response, a finite element program with explicit time integration was chosen. The use of explicit time integration has the advantage that it can easily treat the complicated constitutive model which arises from the considerations of concrete cracking and it can handle the slip between reinforcing tendons and the concrete through the use of the well known sliding interface options. However, explicit time integration programs are usually not well suited to the simulation of static processes such as prestressing. Nevertheless, explicit time integration programs can handle static processes through the introduction of damping by what is known as a dynamic relaxation procedure. For this reason, the dynamic relaxation procedure was refined through the introduction of lumped mass, viscous damping. This provision made the prestressing operation of the concrete structures by means of the explicit formulation rather convenient.

A second difficulty in the explicit transient analysis was found to be the spurious high frequency noise introduced by the sudden cracking of an element. To ameliorate these effects, the stress was gradually reduced to zero as a function of strain after the tensile limit is exceeded. It was found that this procedure reduces the high frequency noise and attendant chain reaction cracking.

For the purpose of illustrating the applicability of these techniques and the validity of the models for concrete and the prestressing tendons, several example solutions are presented and compared with experimental results. These sample problems range from simply supported beams to small scale models of PCRV's. It is shown that the analytical methods correlate quite well with experimental results, although in the vicinity of the failure load the response of the models tends to be quite sensitive to input parameters. This sensitivity of the models response in the vicinity of its failure load deserves further study.

1. Introduction

The analysis of reinforced concrete has been the topic of many investigations, beginning with the original work of Nilson [1] who duplicated the cracking pattern in a point loaded, simply supported beam. Recently, a definitive paper describing the use of a Mohr-Conlomb model was published by Argyris, et al. [2], which demonstrated the application of a finite element procedure to a large variety of static problems.

The analysis of prestressed concrete under impulsive loads such as in the safety analysis of an hypothetical core disruptive accident (HCDA), however, poses additional difficulties. An efficient solution under such conditions makes an explicit technique preferable, both because of economy and because it permits a realistic model of the interaction of prestressing tendons with the concrete through the slide line option; the latter are very difficult to program in implicit, Newton type codes. However, this necessitates an efficient solution procedure for the static prestressing. Dynamic relaxation procedures provide a natural method for obtaining static solutions by explicit, transient codes, and finite difference procedures for its implementation have been published by Otter [3], Holland [4] and the group at the Imperial College [5]. However, little was available for enhancing the efficiency in a finite element context, so we have explored this matter.

Another difficulty which has plagued our treatment of concrete models with cracking in a dynamic setting is the chain reaction of cracks introduced by cracking in a single element [6,7]. This often leads to complete failure of the structure in situations where experiments do not indicate failure. The culprit in our initial models was the complete elimination of tensile strain across the crack immediately after cracking. We have now refined this model by introducing a gradual decay in tensile stress and found experimental evidence for this phenomenon. In addition we have found literature on the strain rate dependence of the tensile and compressive strength of concrete. The incorporation of these factors has led to rather good agreement between our model and many experiments; some of these comparisons are reported here.

2. Dynamic Relaxation

Dynamic relaxation is a procedure for obtaining static solutions by solving the dynamic equations with sufficient damping to converge to the static solution. Damping can be introduced in the dynamic relaxation procedure by either viscous-damping $\underline{C}\,\dot{\underline{u}}$ or a stiffness proportional damping (which is another guise for artificial viscosity) $c_1\,\underline{K}\,\dot{\underline{u}}$. The equations of motion to be solved are then

$$\underline{M}\,\ddot{\underline{u}} + (\underline{C} + c_1\,\underline{K})\,\dot{\underline{u}} + \underline{K}\,\underline{u} = \underline{F}^{ext} \quad . \tag{1}$$

If these equations are solved by an explicit central difference method, the stiffness proportional damping must be treated by a backwards difference

$$\dot{u}^n = \frac{1}{\Delta t}\,(u^n - u^{n-1}) \quad , \tag{2}$$

where superscripts denote the time step number and Δt is the time step. However, if $\underline{C}$ is diagonal, a central difference form

$$\dot{u}^n = \frac{1}{2\Delta t}\,(u^{n+1} - u^{n-1}) \tag{3}$$

may be used for this damping. The use of the latter is important, for the stable time step of the uncoupled equation is uneffected by any damping treated by this term, whereas any damping treated by the backwards difference (2) results in a stability condition

$$\Delta t = \frac{2}{\omega_{max}} [\sqrt{1 + \mu^2} - \mu] \quad , \tag{4}$$

where ω_{max} is the maximum frequency in the mesh and μ is the fraction of critical damping in the highest frequency. Thus as μ is increased, the stable time step is reduced significantly. However, stiffness proportional damping cannot be used with the central difference velocity Eq. (3) in a strictly explicit manner for updating $\underline{u}$ since $\underline{K}$ would have to be inverted and $\underline{K}$ is never diagonal; on the other hand, a diagonal $\underline{C}$ matrix is quite natural, so Eq. (3) can be used with this damping in an explicit scheme.

With the central difference formula for the accelerations,

$$\ddot{u}^n = \frac{1}{\Delta t^2} (u^{n+1} - 2u^n + u^{n-1}) \quad , \tag{5}$$

the formula for updating the displacements is then

$$\underline{u}^{n+1} = \left(\frac{1}{\Delta t^2} \underline{M} + \frac{1}{2\Delta t} \underline{C}\right)^{-1} \left[\underline{F}^{n,ext} - \underline{K}\,\underline{u}^n + \frac{1}{2\Delta t} \underline{C}\,\underline{u}^{n-1} + \frac{1}{\Delta t^2} \underline{M}\,(2\underline{u}^n - \underline{u}^{n-1})\right] \quad . \tag{6}$$

Nonlinearities can be treated by simply replacing $\underline{K}\,\underline{u}^n$ by $\underline{F}^{n,int}$.

The $\underline{C}$-damping is primarily effective for low frequency modes, which are considerable trouble in dynamic relaxation since they require the longest time to eliminate. The artificial viscosity is used for damping the high frequency modes; for low frequency modes, artificial viscosity is quite ineffective since the fraction of critical damping, μ, in any frequency ω is given by

$$\mu = c_1 \omega \tag{7}$$

and so decreases with the frequency.

We have used two schemes for choosing the diagonal $\underline{C}$ matrix:

$$\underline{C} = c_2\, \underline{M} \quad , \tag{8}$$

$$C_{ii} = c_2 \sqrt{k_{ii}\, m_{ii}} \quad . \tag{9}$$

The scheme indicated by Eq. (8) allows Eq. (4) to be a strict stability condition and the $\underline{C}$-damping has no effect on stability. If Eq. (9) is used, the stability criterion varies slightly with c_2.

To estimate Δt for a run, we use a development of Hughes, et al. [8] that

$$\omega_{max} = \sup\, \omega_{max}^{ele} \quad , \tag{10}$$

which means that the maximum frequency of any element in the mesh bounds the maximum frequency of the system.

For a constant strain triangle

$$\omega_{max}^{ele} = \frac{2\ell}{c} \quad \text{and} \quad c^2 = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}} \quad , \tag{11}$$

where ℓ is the minimum element dimension, c is the dilatational elastic wave speed, E is Young's modulus, ν is Poisson's ratio, and ρ the density.

Since the density in the prestressing solution plays a purely fictitious role, whenever the elements vary in size significantly, convergence can be enhanced by making ω_{max}^{ele} equal for all elements by letting

$$\rho^{ele} = \frac{E\omega_{max}^2(1-\nu)}{4\ell^2(1+\nu)(1-2\nu)} \quad , \tag{12}$$

where ω_{max} can here be any convenient constant.

3. Concrete Modeling

In the treatment of concrete cracking, we have used a scheme in which cracking is triggered whenever the tensile limit is exceeded by any principal strain in the element. The direction of the crack is considered to be normal to this principal direction, and in our initial studies, the normal stress across the crack was set to zero across the crack unless the crack closed, which was indicated by a compressive normal strain across the crack. This model of concrete cracking created difficulties, because cracking in one element caused sudden load transfer to adjacent elements, and thus a chain of cracking, leading to complete failure of the model.

A basic source of this difficulty is the discrepancy between the model and crack formation in concrete. Cracking in concrete is a rather irregular process: as the tensile limit is approached, microcracks form normal to the direction of the tensile stress, but some load carrying ability is retained because of the irregularity of the crack pattern and aggregate interlock. The microcracks then coalesce into larger, isolated cracks, which further reduce the load carrying ability.

In our initial efforts to model these phenomena [6], we incorporated a time delay factor. In crack formation, the normal stress, instead of vanishing instantaneously, was reduced to zero linearly over a prescribed characteristic time. The physical motivation for this time delay was to model the limited rate of crack growth and the associated energy dissipation.

However, it was found that no single time constant for the decay time was satisfactory for even modelling the limited experimental data available to us. At the same time, we learned of the test of Evans and Marathe [9], which showed that even at slow strain rates concrete maintained tensile strength after the tensile limit was reached, so that the modelling of tensile stress decay strictly in terms of a time constant did not correlate with this experimental information. Therefore, the tensile decay was changed to a time independent phenomenon depending only on the magnitude of the tensile strain.

Another physical aspect neglected in the earlier model that was found to be of importance is the strain rate dependence of the tensile and compressive limits for concrete. McHenry and Shideler [10] and McNeeley and Lash [11] have presented data on strain rate dependence; the data is unfortunately limited to a maximum of 10 s^{-1} for compressive strength, 10^{-3} s^{-1} for tensile strength. Based on this data, the strain rate dependence of tensile and compressive strengths were fit by a function

$$f_D/f_S = A + B\,\dot{\varepsilon}^C$$

where f_D and f_S are the dynamic and static strength, $\dot{\varepsilon}$ the strain rate, and A, B and C constants.

The combination of strain rate dependence of strength and a time independent tensile strength decay have significantly improved our correlations with experiments. However, the data available for these aspects of concrete behavior are quite minute, and much additional data are needed to provide a firm foundation for these models. Our studies have clearly indicated that these factors play a significant role on the response of concrete models, so the need for this data is great.

4. Results

Some results are given here for dynamic problems for which experimental data are available. The first is a reinforced concrete beam tested at the University of Illinois [12]. The symmetrically loaded beam, its reinforcement and the finite element model are shown in Fig. 1. The yield limit of the reinforcing steel in the analytical model is taken to be 43% higher, according to the presently accepted variation for a strain rate of $0.5s^{-1}$. The history of the applied load, a record of the reactions and the resultant central deflections of the beam are given in Fig. 2, together with the analytical results of the deflection. The time of the analytical solution shown has been decreased by 0.2ms so that the initial rise of the deflection corresponds to that of the experimental data. The maximum value of the deflection is comparable to the magnitudes of the experimental data. It is noteworthy that the permanent set of deflections corresponds rather closely to the analytical results.

The second set of results given here refers to the use of viscous damping in a dynamic relaxation solution. The example used is a statically loaded cantilever beam shown in Fig. 3. This figure gives dimensions and loading of the beam, the material constants and the outline of the 12 x 4 quadrilateral finite element model used in the solution. Figure 4 shows the analytical results when a step load is used in the simulation. The effect of damping is illustrated by three different curves: one with zero damping, and two with c_2 equal to 0.05 and 0.1, respectively. The static deflection of the beam using beam theory is 0.867 cm; when this deflection is corrected for the effect of shear, it becomes 0.919 cm.

The sample illustration shows that viscous damping is very effective in reducing the oscillations present in dynamic solutions. With $c_2 = 0.1$, the vibrations are completely damped and stable equilibrium is reached almost during the first cycle of vibration. Furthermore, the equilibrium deflections reduce to the average values shown by the undamped solution.

The third set of results given here is for the PCRV model tested at Foulness [13]. The finite element model is shown in Fig. 5. This model was prestressed by the dynamic relaxation algorithm using 2300 time steps, which required 20 minutes of CPU on the IBM 370/195. The model was then loaded by a pressure time history generated from an ICECO simulation of the 27g charge. The displacement time history at point A in Fig. 5 is shown in Fig. 6. In this case the maximum experimental displacement is exceeded somewhat, but the model accurately replicates the extended period of the vessel due to crack-softening and the absence of permanent deformation brought about by crack closing.

Also shown is the elastic response using the same material constants but with no cracking. The response is markedly different. The maximum amplitude is reduced, the period is reduced substantially, and there is no hysteretic mechanism, so the vibrations persist.

5. Conclusions

Several recent advances in the solution of prestressing by dynamic relaxation and the modelling of concrete cracking phenomena have been presented. The importance of including the decreasing part of the tensile stress-strain curve of concrete after the initiation of microcracking and the strain rate dependence of concrete strength have been indicated. Sample results presented here show that current models can simulate transient experimental displacements fairly well, particularly their phenomenological aspects, but that the results are fairly sensitive to material parameters and features of the model.

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Fig. 1. Details of the Tested Reinforced Beam.

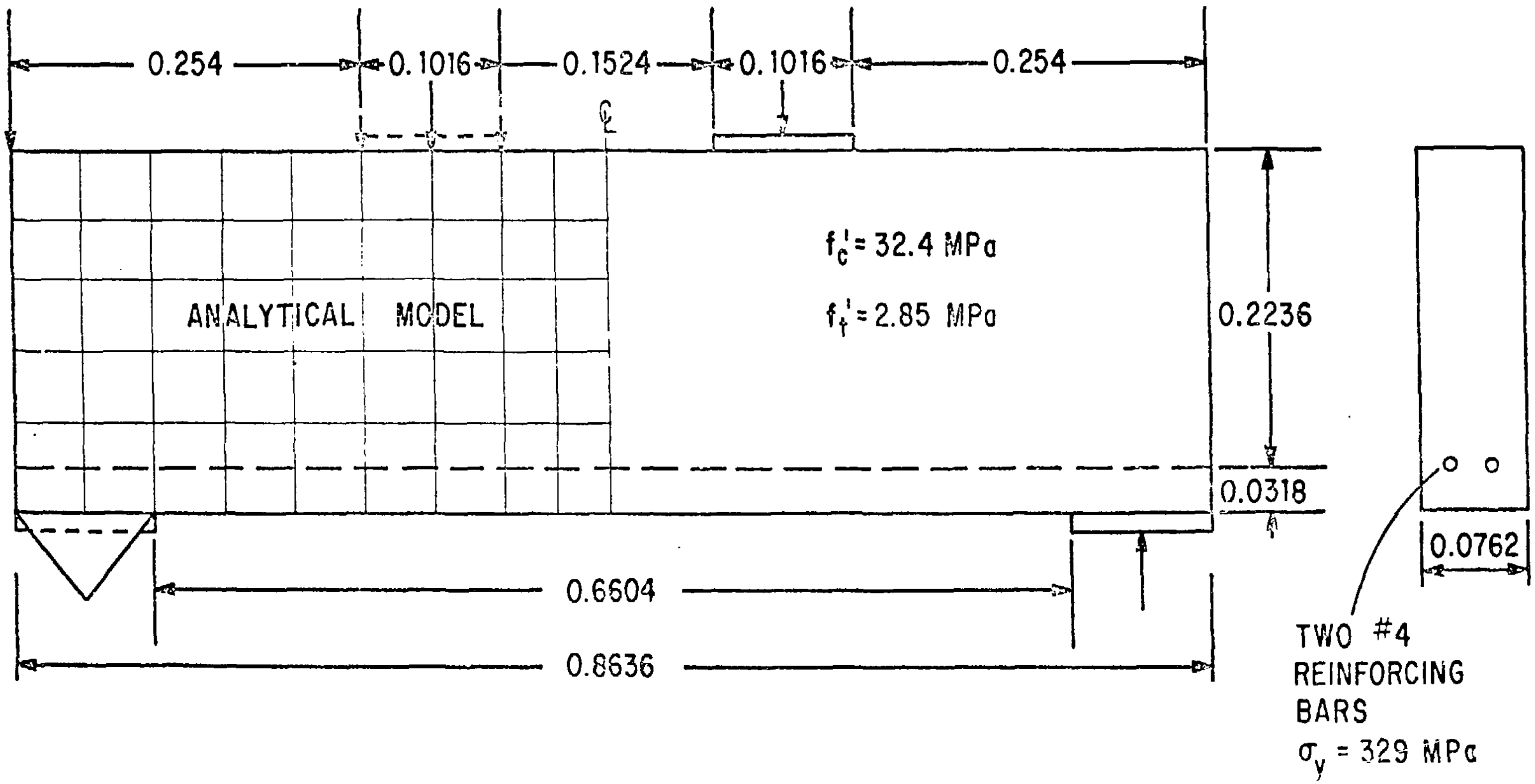
Fig. 2. History of Forces and Midspan Deflection of the Beam.

Fig. 3. Details of the Contilever Beam.

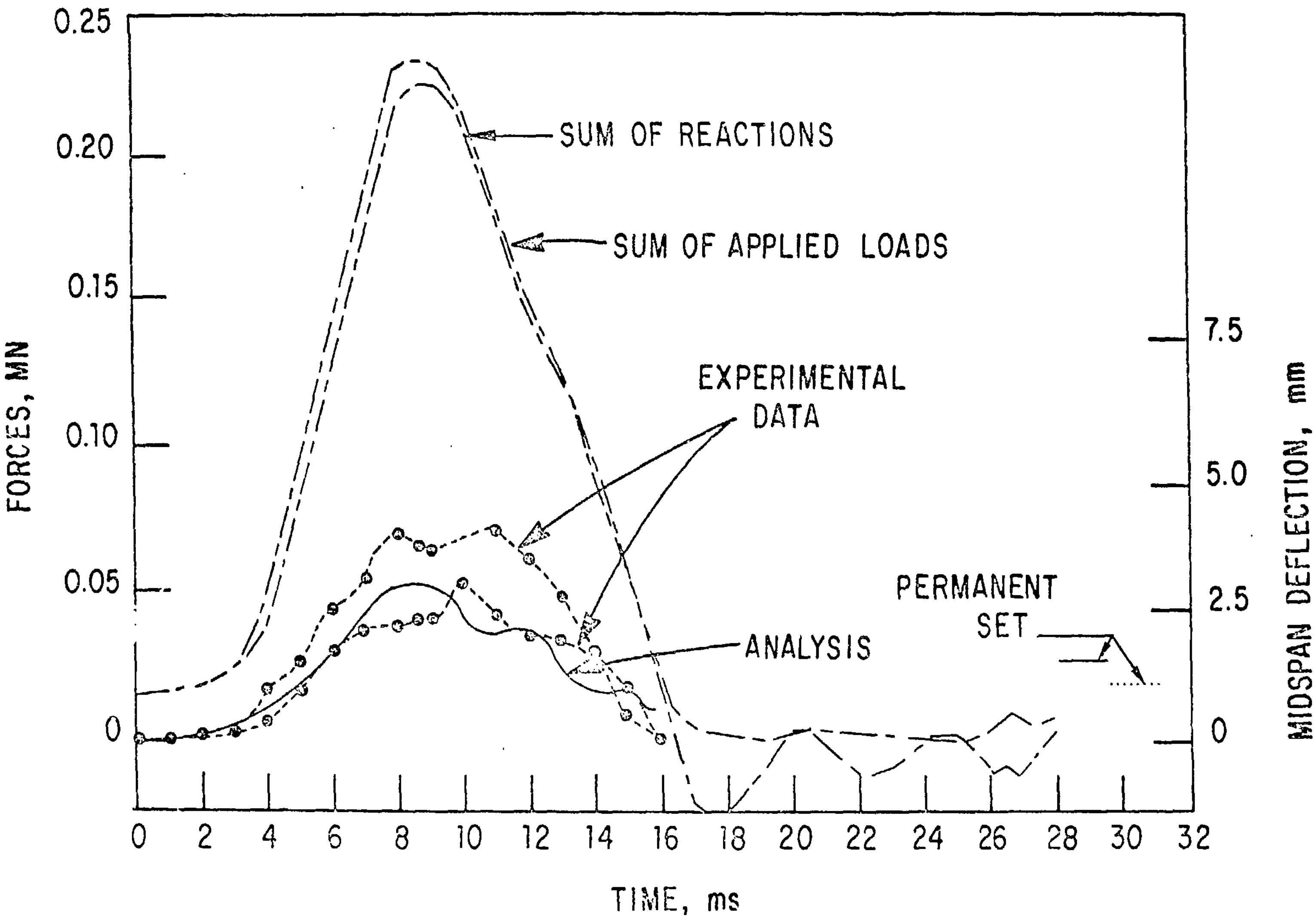
Fig. 4. End Deflection of Beam due to Step Load.

Fig. 5. Finite Element Model of the Test Container.

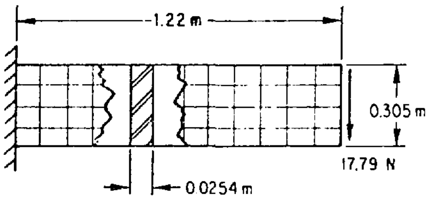
Fig. 6. Comparison of Cylindrical Wall Displacement.



0.254
0.1016
0.1524
0.1016
0.254
℄
ANALYTICAL MODEL
f'c = 32.4 MPa
f't = 2.85 MPa
0.2236
0.0318
0.6604
0.8636
0.0762
TWO #4 REINFORCING BARS
σy = 329 MPa



0.25
0.20
0.15
0.10
0.05
0
FORCES, MN
SUM OF REACTIONS
SUM OF APPLIED LOADS
EXPERIMENTAL DATA
ANALYSIS
PERMANENT SET
7.5
5.0
2.5
0
MIDSPAN DEFLECTION, mm
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
TIME, ms

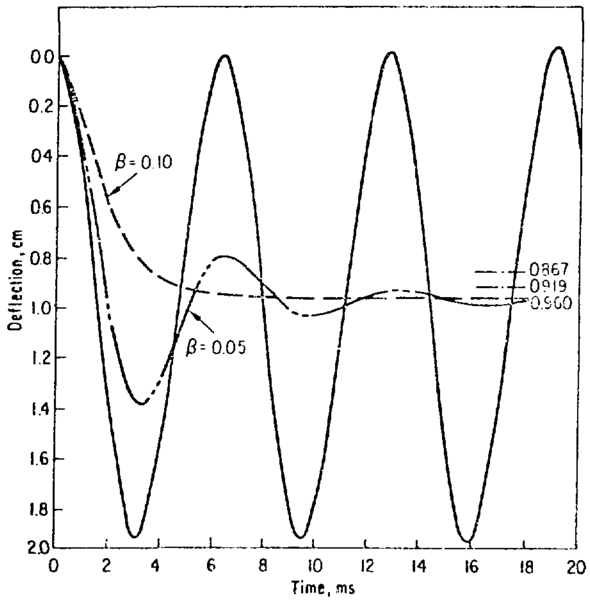


$\rho = 0.00786$ Mkg/m^3

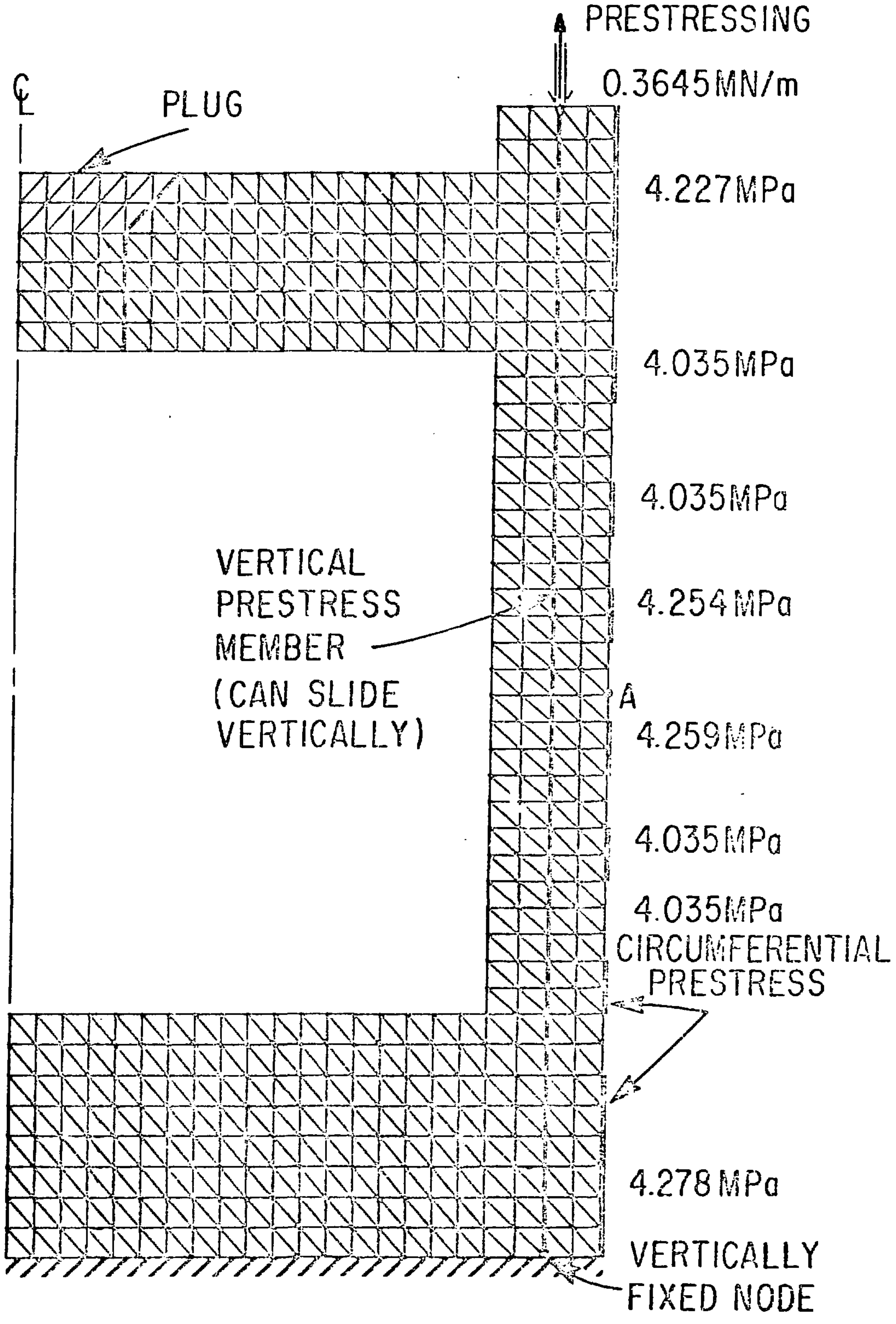
E = 2.07 MPa

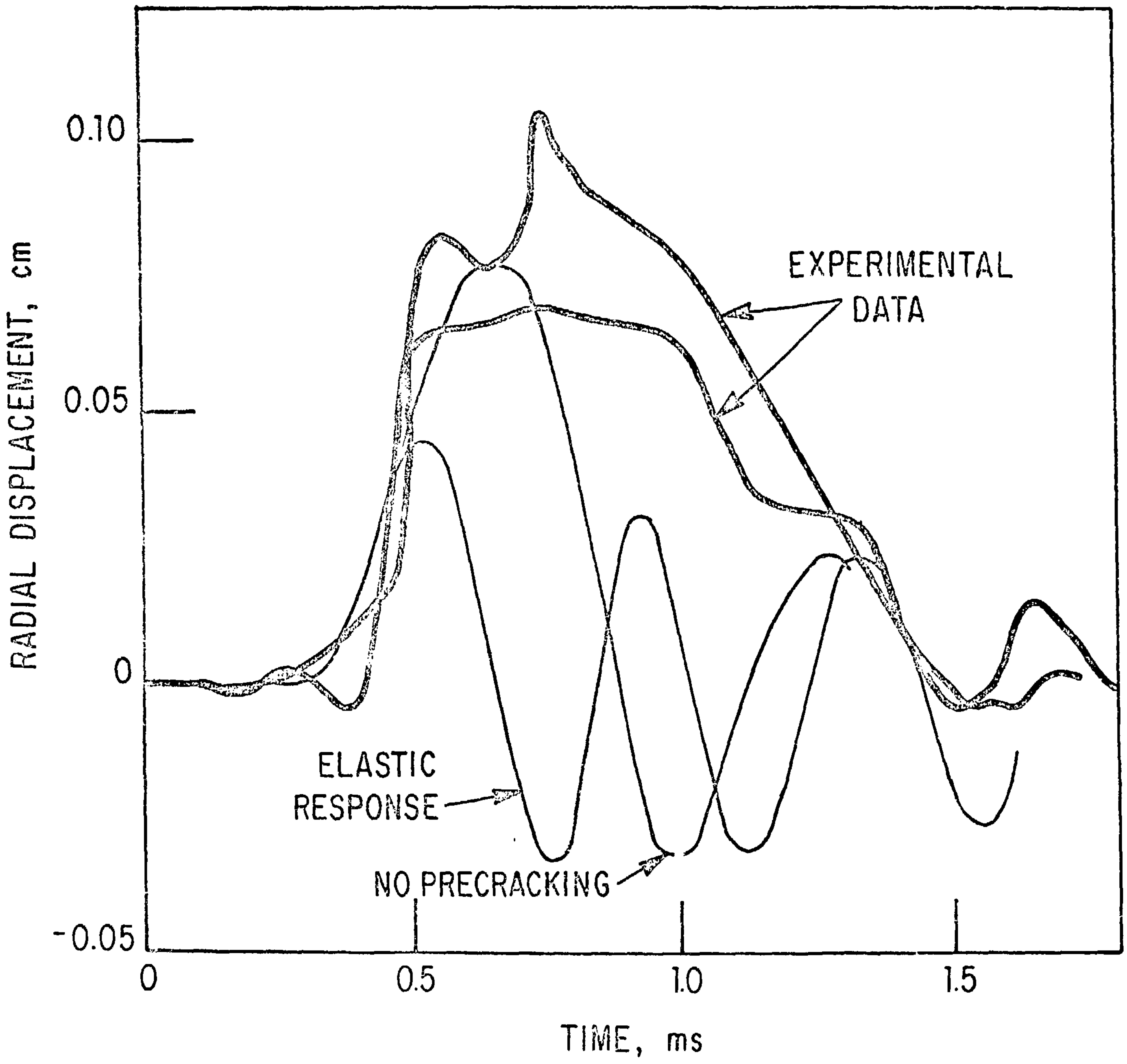
$\nu = 0.3$

$\Delta t = 1 \times 10^{-5}$ s



Deflection, cm
Time, ms
β = 0.10
β = 0.05
0.867
0.919
0.960


PRESTRESSING
0.3645MN/m
℄
PLUG
4.227MPa
4.035MPa
4.035MPa
VERTICAL
PRESTRESS
MEMBER
(CAN SLIDE
VERTICALLY)
4.254MPa
A
4.259MPa
4.035MPa
4.035MPa
CIRCUMFERENTIAL
PRESTRESS
4.278MPa
VERTICALLY
FIXED NODE



0.10
0.05
0
-0.05
RADIAL DISPLACEMENT, cm
EXPERIMENTAL
DATA
ELASTIC
RESPONSE
NO PRECRACKING
0
0.5
1.0
1.5
TIME, ms