DOE/ER/53201--2

DE89 016371

Stellarator Transport Theory

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Abstract

A program of research in Stellarator Transport Theory is described, with emphasis on kinetic modeling of divertor plasmas.

We have developed a very efficient numerical solution of the kinetic equation. It was implemented first for the ions, self-consistently with the electrostatic potential, Φ, assuming quasineutrality and a Boltzmann distribution of electron density. The presheath problem has been solved, confirming analytic work in the collisionless limit and extending the earlier work to collisional cases.

The effect of a transverse electric field on divertor flows in IMS is to redistribute the diverted flux. We have shown (experimentally) that $\vec{E} \times \vec{B}$ drifts are responsible. We thus used the 'presheath' model to examine the effect of magnetic mirrors along a diverted field line, both on the diverted fluxes and on the electrostatic potential. The calculated variation $\Delta\Phi \sim 0(T_e)$ can be expected to set up significant flows.

Numerical calculations of the stellarator ambipolar electric field and bootstrap current have been done. Further analytical and computational work on ripple transport is under way, in conjunction with IPP-Garching and as part of the US-USSR collaboration. Our earlier analytic calculations of stellarator ripple transport have also been modified to apply to tokamaks.

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I. INTRODUCTION

This document describes a program of research into stellarator transport, focusing on edge transport and ripple transport.

Control of the edge-plasma is a vital issue in all magnetic-confinement systems, and in stellarators in particular. We are principally concerned with models of the stellarator edge-plasma, and with some experimental results obtained in conjunction with the modeling activity. Since the stellarator magnetic field has strong ripples in the edge region, giving rise to multiple magnetic mirrors, a kinetic treatment is called for. We have developed a numerical solution of the kinetic equation which is very efficient and provides self-consistent solutions for the particle distributions and electrostatic potential, Φ, in a 'rippled' magnetic field.

The numerical method involves the calculation of Green's functions, or propagators, for the evolution of the distribution function during the timestep τ. This timestep can greatly exceed the timestep allowed in a finite-difference scheme. Although the use of propagators is not entirely new to transport theory, they do not appear to have been used previously for steps which are long but finite. Rees[1] used propagators for very long timesteps, in a virtually static problem. Wehner and Wolfer[2] treated a diffusion problem, but with timesteps essentially limited by the Courant-Friedrichs-Levy criterion. We have developed a method[3] which allows us to follow time evolution, using long steps, and so to obtain self-consistent distributions and electric fields. We believe that this method represents a major contribution to numerical analysis; however, we shall not dwell on this in the present proposal.

The propagator method has been applied to:

a) Study a collisional presheath. The work of Emmert et al.,[4] Scheuer and Emmert,[5] and of Bissell and Johnson,[6] for collisionless and weakly collisional presheaths has been confirmed, and extended to strongly collisional cases.

b) Investigate the effects of magnetic field ripple on the presheath, in a stellarator geometry - specifically, IMS[7] parameters for the magnetic field, mean free path, etc.

In both of these cases the ion distribution was calculated explicitly, whereas the electron density was assumed to obey a Boltzmann distribution, as is usual in presheath calculations. This is then inverted to yield Φ. The method is currently being used to calculate electron and ion distributions, simultaneously and self-consistently. This is important to study the effects of divertor biasing on plasma flows, which has been a major focus of experimental work on IMS.[8]

We have collaborated with the experimental group working on IMS in developing a model to explain the effects of applied electrostatic fields. As a result of this work it has been demonstrated that $\vec{E} \times \vec{B}$ drifts can control flows in the IMS edge plasma[8] and, by implication, it seems likely that this could occur in other stellarators. For this reason, we are particularly interested in predicting the potential set up in a stellarator edge plasma and in understanding its effects.

Other transport studies related to ripple-transport in the central plasma are being undertaken, in collaboration with groups at Garching and the Soviet Union. The 'hybrid' Monte Carlo technique which we developed is being used by Kovrizhnykh and Shasharina at the Lebedev. We are interacting with them on the use of this, and our power-series treatment of ripple transport, as a result of the 1988 US-Soviet Stellarator Workshop. In addition, the same meeting led to these two calculations being used to study a device proposed by Grekhov and Shishkin (Kharkov). This collaboration will be enhanced when C. D. Beidler, whose thesis work included the hybrid and power-series calculations, and who is currently working for H. Wobig at Garching, visits the Soviet Union as part of Garching's exchange program. Our series solution of the bounce-averaged drift-kinetic equation for ripple-trapped particles in a stellarator has also been extended to tokamaks.

W. D. D'haeseleer recently completed his Ph.D., having worked on calculating the bootstrap current in a stellarator. He did this by extending an earlier numerical calculation of Hitchon and Mynick.[9] He is currently working on NET, studying ripple effects on α-confinement. The main tool being used to describe α-particle orbits is an extension of the hybrid Monte Carlo code, which is being developed in collaboration with us and Beidler. Both he and Beidler are also involved in the edge-plasma modeling described above.

A very lengthy account of flux coordinates and their use has been written by D'haeseleer, Hitchon, Callen and Shohet. We believe that a clear and complete exposition of this subject will be very valuable for theoreticians and experimentalists alike, and so we have devoted a great deal of time to this over several years.

In the next section, we outline our divertor modeling work. Section III is devoted to 'conventional' ripple transport, Section IV concerns the flux coordinate review.

II. A KINETIC MODEL OF STELLARATOR DIVERTORS

In this section we shall discuss the approach we are taking to modeling the divertor regions of stellarators. We introduce the subject with a brief overview of the motivation, which serves to show which aspects of the problem seem most important. The various preliminary calculations we have performed will then be outlined, followed by applications to stellarators. This subject will be returned to in Section V, on future work.

As stated in the Introduction, divertor action is one of the most important topics in fusion research, since it is hoped that the use of divertors will remove the point where the bulk of the plasma-material interactions occur from inside the first wall. One of the advantages which are claimed for stellarators is the existence of 'natural' divertors in (most) stellarators. The IMS experimental program has consequently stressed studies of the diverted plasmas, and, in particular, ways of focusing the diverted flux into a subset of the 63 divertors in IMS. Control of which divertors remove the flux is very important, since stellarators such as ATF have many such 'natural' divertors, and it is probably not feasible to build a reactor with facilities for plasma exhaust through more than a small fraction of them.

Two methods have been used to try to focus the diverted fluxes. First, a vertical magnetic field B_V was applied, which had the effect of shifting the whole plasma column major-radially. This was effective in concentrating diverted plasma in the major-radially outward or inward divertors, depending on the sign of B_V. In other words, the number of divertors used was reduced by a factor of about two. It was not clear, from the experiment, whether the fluxes were affected by modifications in the magnetic field strength along field lines caused by B_V.

The disadvantage of using B_V to control the diverted fluxes, apart from the fact that a reduction in the number of divertors by a factor of two is not in itself sufficient, is that this removes a degree of freedom which is needed to optimize other aspects of the configuration. For this reason, the application of electric fields to the divertor regions was considered.

We have worked with the experimental group in developing a qualitative model for the effects of applying an electrostatic potential to the divertor plates. This model involves the structure of the magnetic field, which shows how the potential Φ is determined largely by the bias applied to a given field line, but also by shielding effects of plasma on adjacent field lines. Flow of plasma between field lines is further shown to be largely controlled by $\vec{E} \times \vec{B}$ drifts, which can be caused by the applied potentials at the plates. This in turn provides a potentially highly selective means of controlling divertor fluxes, which we shall now discuss.

A negative bias applied to the divertor plate would not be expected to lower the potential of the plasma, except in a narrow sheath region. For this reason, biasing experiments have used a positive applied potential.

It was found, experimentally, that whatever (positive) bias was applied to a divertor plate, the potential of the field line which it contacts never rose by more than about $2T_e$, where T_e is the electron temperature. However closely the plate was approached, within limitations set by the probe used, the potential of the line was nowhere more than of this order, even though the plate was at a much higher potential.

From field-line following calculations we were able to show that lines from adjacent divertor plates pass extremely close to the biased plate. As a result, electrons are able to approach the positively biased plate, but, unlike a discharge in an unmagnetized plasma, the electrons are not free to flow to the plate. (See Fig. 1). (If the electrons could reach the plate, a current would presumably flow until the line potential rose to about the bias potential, and this was originally expected to happen, before the experiment was performed.) This explains why the applied potential is shielded, to within a few times T_e.

It was found that these potentials, of about 20 V, had a significant effect on the fluxes to the biased plate and to at least two other plates, which connect to lines 'adjacent' to these contacting the biased plate. It was then necessary to determine whether the redistribution of flux was due to parallel conduction or $\vec{E} \times \vec{B}$ drifts. (Perpendicular conduction could be ruled out as being too slow.)

By reversing the magnetic field direction, it was possible to show that $\vec{E} \times \vec{B}$ drifts are the main effect.[8] The observations are explicable in terms of an $\vec{E} \times \vec{B}$ drift sweeping plasma from one bundle of field lines to another, as shown. (See Fig. 2.) The model also does an excellent job of explaining the scaling of the flows under a doubling in the field strength.

This simple qualitative picture was developed in conjunction with experiments, and explains the broad results which were obtained from them. The remainder of our theoretical program on divertors is designed to provide a detailed description of some aspects of the divertor behavior. As the experimental work implies the importance of the electric field, we emphasize the interaction of the field and transport. To date, we have concentrated on describing transport along a single field line, for which we have developed an efficient kinetic model. The potential $\Phi(l)$ which is set up is important in determining the magnitude of the potential variation both along and across the magnetic field. The latter should allow estimates of the $\vec{E} \times \vec{B}$ drifts set up in the absence of an external bias. We shall now describe these calculations briefly.

The model is based on a numerical technique which we developed, which offers an extremely efficient solution of the kinetic equation. Without this, the calculations would probably not be feasible.

The kinetic equation has been solved in a two-dimensional phase space, (l, v), in our work to date, although extensions to include a perpendicular velocity are also being made. The first application was to describe the ion distribution in a presheath. This is, by definition, a quasineutral region, so Φ is found by assuming a Boltzmann distribution of the electron density and setting $n_e = n_i$. This allows a self-consistent solution for the ion distribution f_i and Φ, which is obtained by advancing the calculation of f_i and Φ in time, updating each at each time step, until steady state is reached.

In this way, we have been able to recover the results of Emmert et al.,[4] Scheuer and Emmert,[5] and Bissell and Johnson,[6] and extend their work to high collision frequency.

A calculation of the presheath type can provide useful information on conditions in the divertors. It is straightforward to extend this calculation from a simple presheath to one where there is a magnetic field B whose strength varies along the field line. This is the simplest model of a stellarator divertor which we have considered. However, not all of the necessary effects which should ultimately be considered have been included in the model, as yet.

The first use of the model is necessarily to study a somewhat academic problem, although the conditions are derived from those in IMS. Transport of plasma through a rippled magnetic field has been treated. The particle energies ($T_i \simeq T_e \simeq T_n \simeq 4$ eV) are similar to those at the edge of IMS, as is the mean-free-path for charge-exchange, $\lambda \simeq 50$ cm. The total length of the field line, which stretches from one divertor plate to another, passing the edge of the plasma in between, is about 60 cm. The profile of the ripples and the placement of the source (which is non-zero near the plasma) relative to the ripples have been varied and their effects studied - see Figs. 3,4.

One of the first predictions which emerges is that the electrostatic potential set up by a peak in B is probably sufficient to overcome the effect of B in confining particles. As a result, the flux along a given line does not appear likely to all flow to a single plate, although a significant fraction of the flux can be directed to one plate or the other by the magnetic field. The large variation in Φ with l caused by the ripples seems likely to create strong radial and poloidal electric fields, since Φ will be different on different field lines. The formation of transverse electric fields and their effect on transport will be the subject of future work.

III. RIPPLE TRANSPORT

In the last year, we have extended our earlier work on a bounce-averaged Fokker-Planck code, to include ions and electrons with the full range of relevant energies. This allows us to self-consistently calculate transport rates along with the ambipolar electric field, and, in addition, flow velocities and the bootstrap current.

The original code FPSTEL developed by Mynick and Hitchon[10] and Hitchon and Mynick[9] used two independent variables, (y,θ). y is a pitch-angle variable, such that $y \leq 1$ for ripple-trapped particles, $y > 1$ for untrapped particles. In the last two years, we have tackled the problem of extending the description embodied in the code, which was only valid for $y \sim 0(1)$, to $y >> 1$, so that all particles can be handled. Further, the code has been structured so that a large number of different energy shells can be included. Collisions and other effects couple different energies, albeit weakly, and an iterative solution is now possible which allows for this. The contributions of the energy shells to particle fluxes, and thus to the determination of the ambipolar electric field, E_r, can straightforwardly be described in this framework. Finally, both electrons and ions are included.

In other work, we have begun modifying the code described in the previous section, for use in treating ripple transport. This is a more efficient approach than is offered by the code FPSTEL, and uses the independent variables $(v_{||}, \theta, \zeta)$, at a fixed kinetic energy, κ, and on a fixed flux surface. This removes the need for bounce-averaging, at the cost of introducing a new independent variable. The extra efficiency of this approach will largely compensate for this. This work is being done in collaboration with workers at the Max-Planck-Institut fur Plasmaphysik (Garching).

Our earlier analytic treatment of stellarator ripple transport, by solving the bounce-averaged kinetic equation by finding f in the form $f = X \cos\theta + Y \sin\theta$ and expanding X and Y as power series in k^2, has been extended to tokamaks[11].

The propagator method was also applied to bulk plasma transport in IMS. This fluid calculation used a variety of postulated flow velocities, each being consistent with experimental observations in IMS. The formation of hollow density profiles was examined. It was shown that the flow velocities employed were certainly capable of producing hollownesses in the range observed, although uncertainties in the poloidal variation of the velocity and density make detailed comparisons difficult. We attempted to show how the interaction of the measured potential Φ and particle motion in different classes of orbit could produce the type of velocity inferred from experiment.

IV. FLUX COORDINATES

Over a period of several years, a great deal of effort has been expended in preparing a review of flux coordinates which is intended to be fairly comprehensive and accessible to the entire fusion community. The work was primarily done by W. D. D'haeseleer, who was a graduate student at the time; the other authors were Hitchon, Callen and Shohet, all of whom provided a lot of effort, even though it was a small fraction of the work done by D'haeseleer. The document is several hundred pages long, so only the abstract is included with this proposal (Appendix I). A copy will be forwarded separately and others will be made available to anyone requesting them.

It was felt (primarily by J. D. Callen, who was the prime instigator of the project) that there was a real need for such a review. We hope this does indeed meet the need and will be of benefit to others.

V. SUMMARY

We have described a program of research in stellarator transport theory, with emphasis on transport in the divertors and on ripple transport. Most of this work employed numerical solutions of a kinetic equation, including:

a) A collisional presheath model was set up, where the ion distribution in (l, v) and the potential Φ were calculated self-consistently, assuming a Boltzmann distribution of electron density.

b) The presheath model was extended to include the effects of a magnetic field whose strength varies with l. Transport along a stellarator-divertor field line was studied using this formulation of the presheath problem.

c) The ambipolar field and the bootstrap current were calculated, for a reactor-scale stellarator, using a bounce-averaged code in the independent variables (θ, y, E), which treats ions and electrons simultaneously. The field $E_r \sim T_e/a$. The effect of J_{BS} on the rotational transform was found to be quite large.

Items a) and b) used our 'propagator' method. Item c) used a finite difference scheme in conservation form.

In collaboration with the experimental group, we explained observations of the effects on the diverted fluxes of applying bias to the divertor plates in IMS, in terms of $\vec{E} \times \vec{B}$ drifts.

Our earlier analytic power-series solutions to the kinetic equation, which allowed a treatment of ripple-transport at arbitrary collision frequencies in stellarators, have been extended to tokamaks. Our 'hybrid' Monte Carlo code has been modified to provide an efficient description of α-particle confinement in rippled tokamaks.

Ripple transport calculations in collaboration with Garching and the USSR were outlined.

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FIGURE CAPTIONS

Figure 1	Magnetic field geometry in the IMS edge plasma.
Figure 2	Schematic illustrating the role of $\vec{E}$ x $\vec{B}$ drifts in controlling diverted fluxes.
Figure 3	Variation of B along a field line (schematic) and location of the plasma source.
Figure 4	a) Variation of Φ with l, and b) the distribution function f_i plotted vs l and v.
Figure 5	a) Potential and b) distribution function of ions in a collisional presheath. ($\delta \equiv \lambda/L$.)

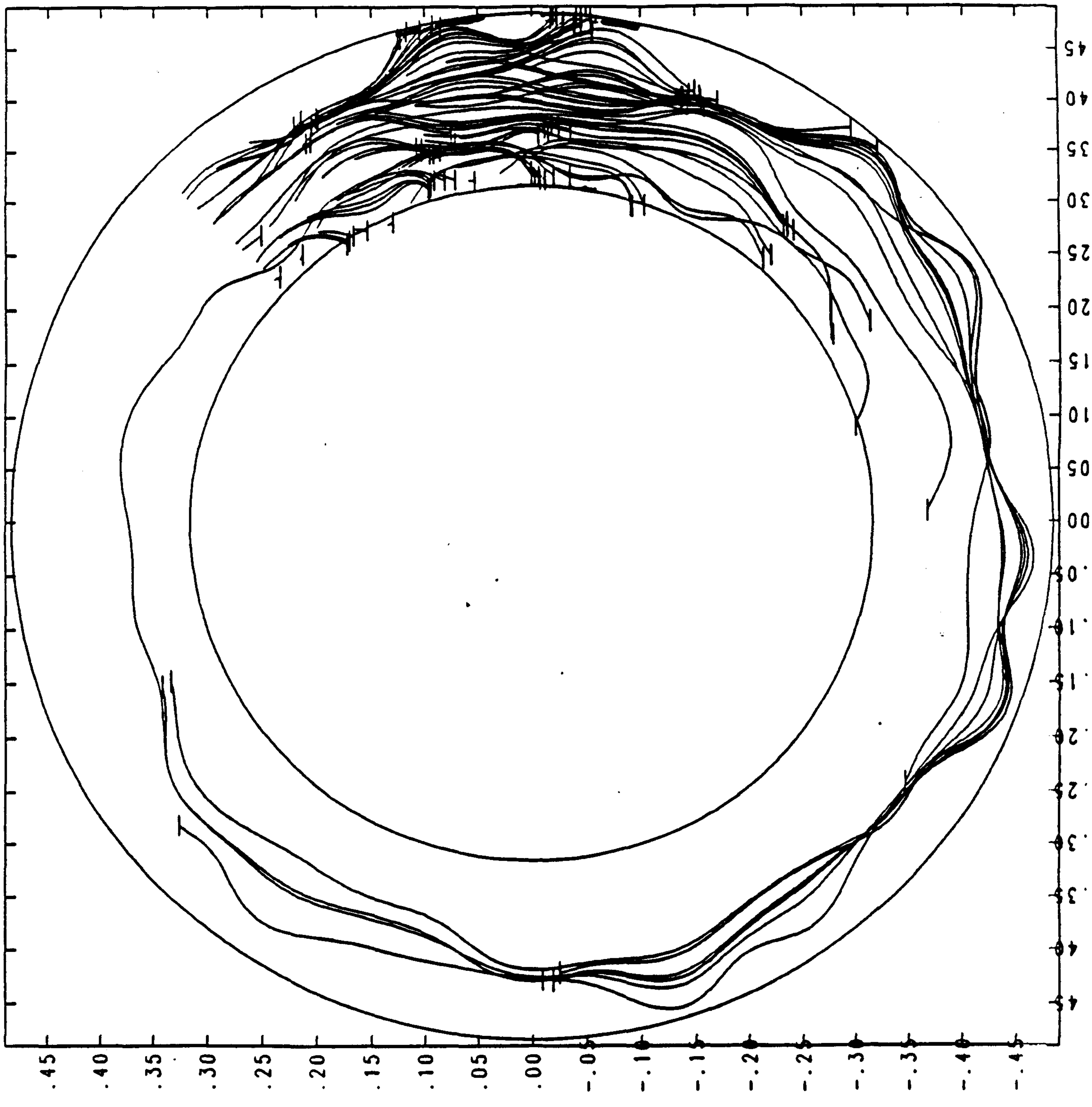
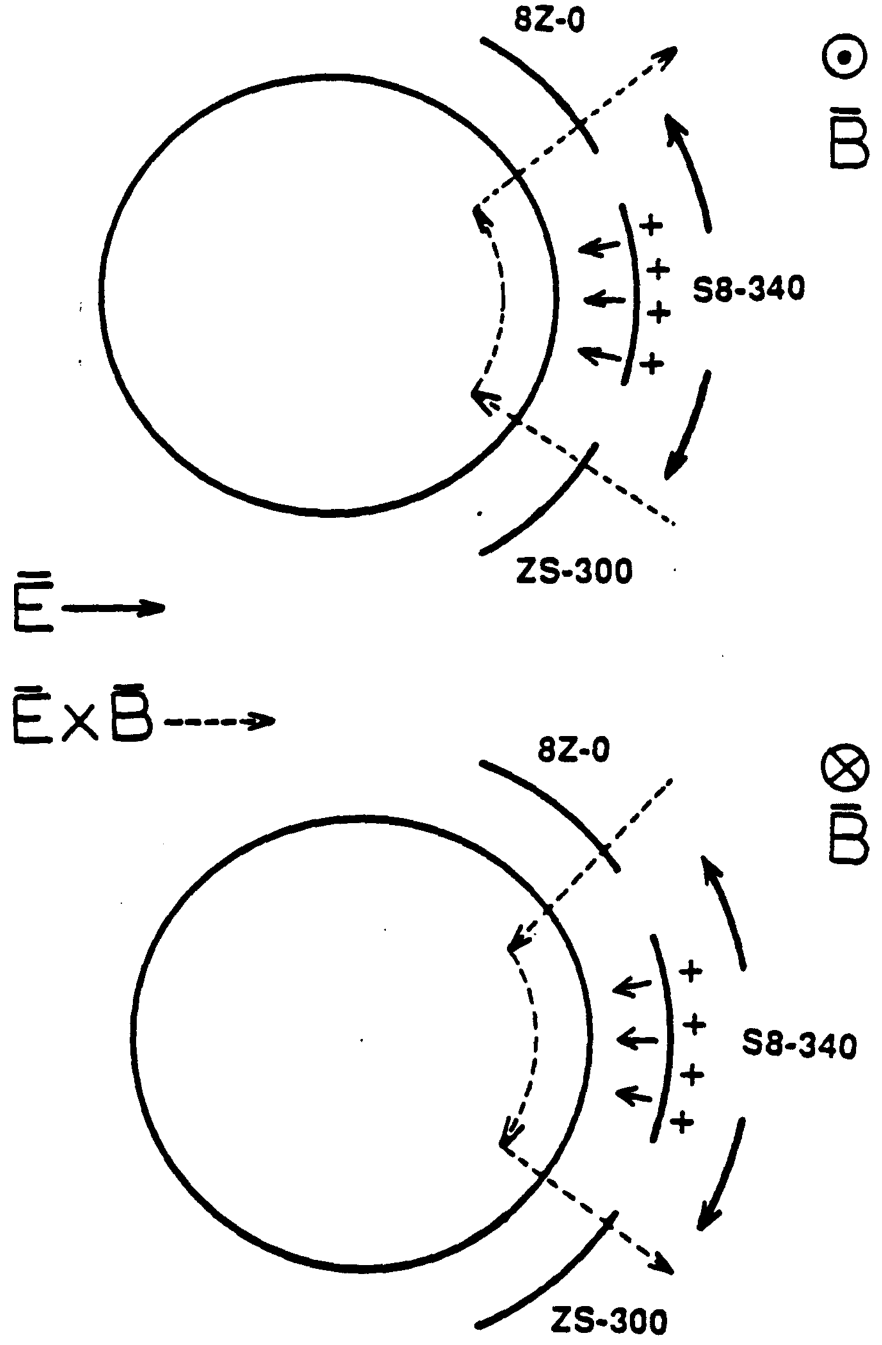


FIGURE 1



Fields and drift directions during shield-biasing experiments

FIGURE 2

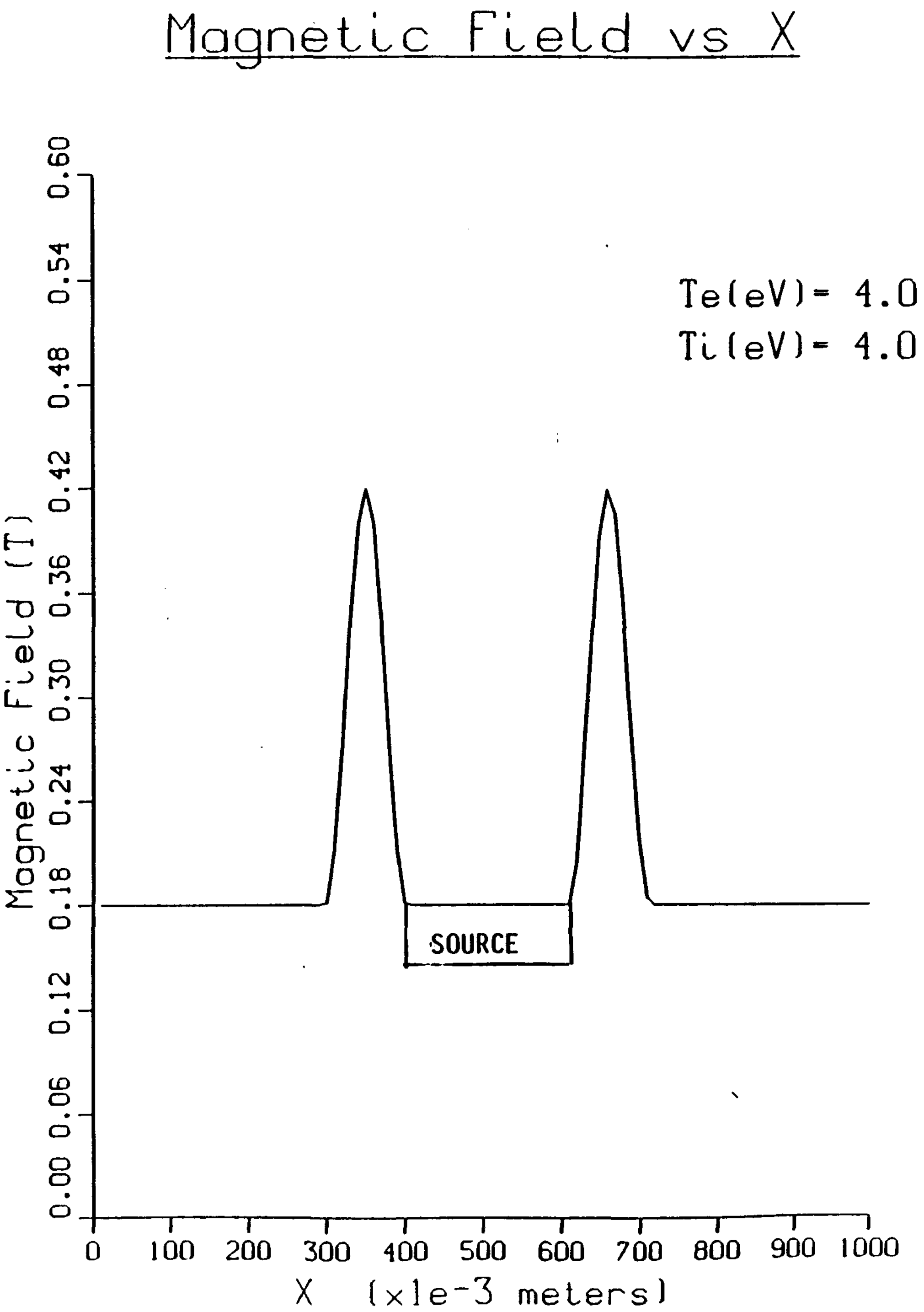

Magnetic Field vs X
Te(eV)= 4.0
Ti(eV)= 4.0
SOURCE
Magnetic Field (T)
0.00
0.06
0.12
0.18
0.24
0.30
0.36
0.42
0.48
0.54
0.60
0
100
200
300
400
500
600
700
800
900
1000
X (x1e-3 meters)

FIGURE 3

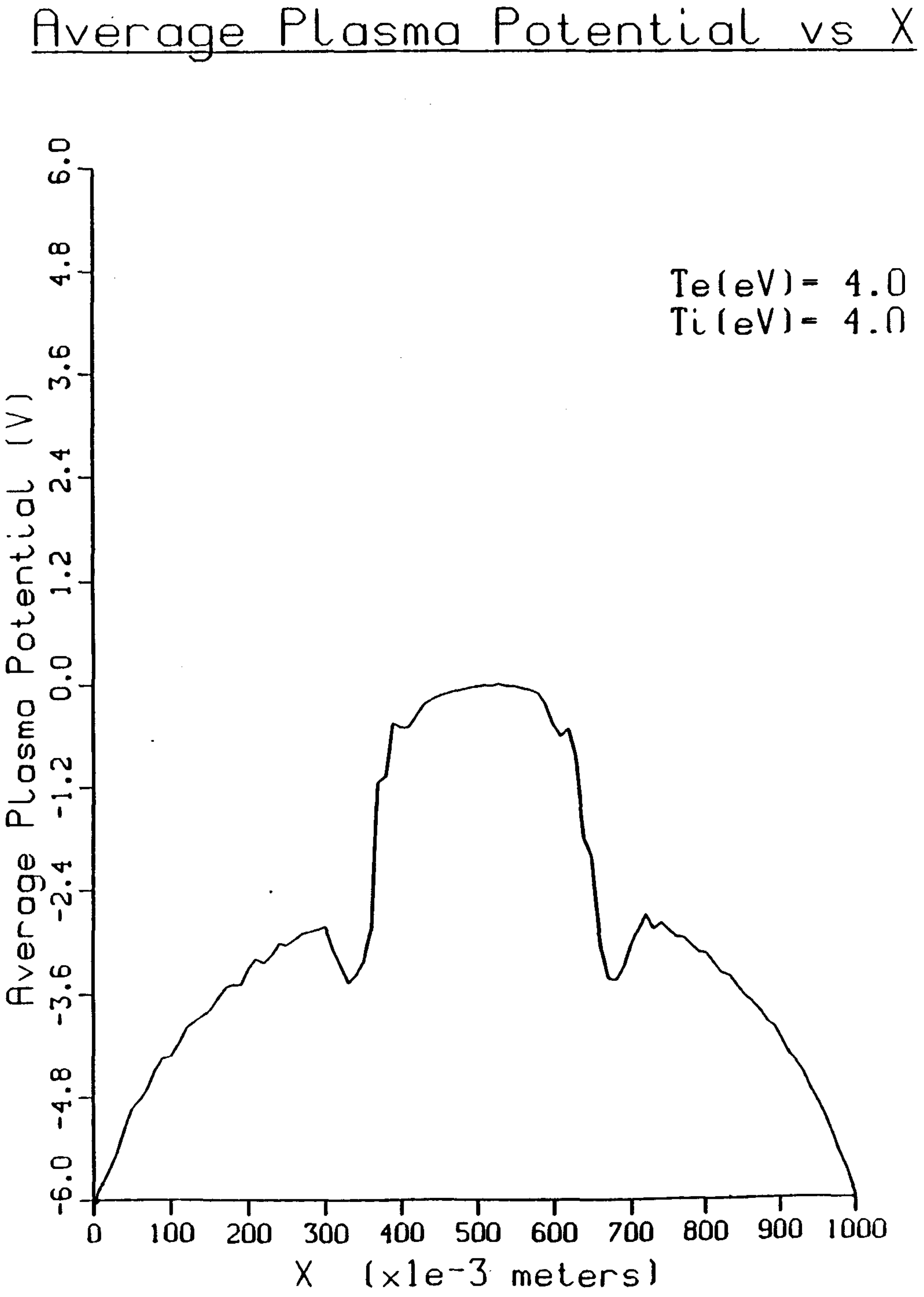

Average Plasma Potential vs X
Te(eV)= 4.0
Ti(eV)= 4.0
Average Plasma Potential (V)
6.0
4.8
3.6
2.4
1.2
0.0
-1.2
-2.4
-3.6
-4.8
-6.0
0
100
200
300
400
500
600
700
800
900
1000
X (x1e-3 meters)

FIGURE 4a

Plasma Ion Density

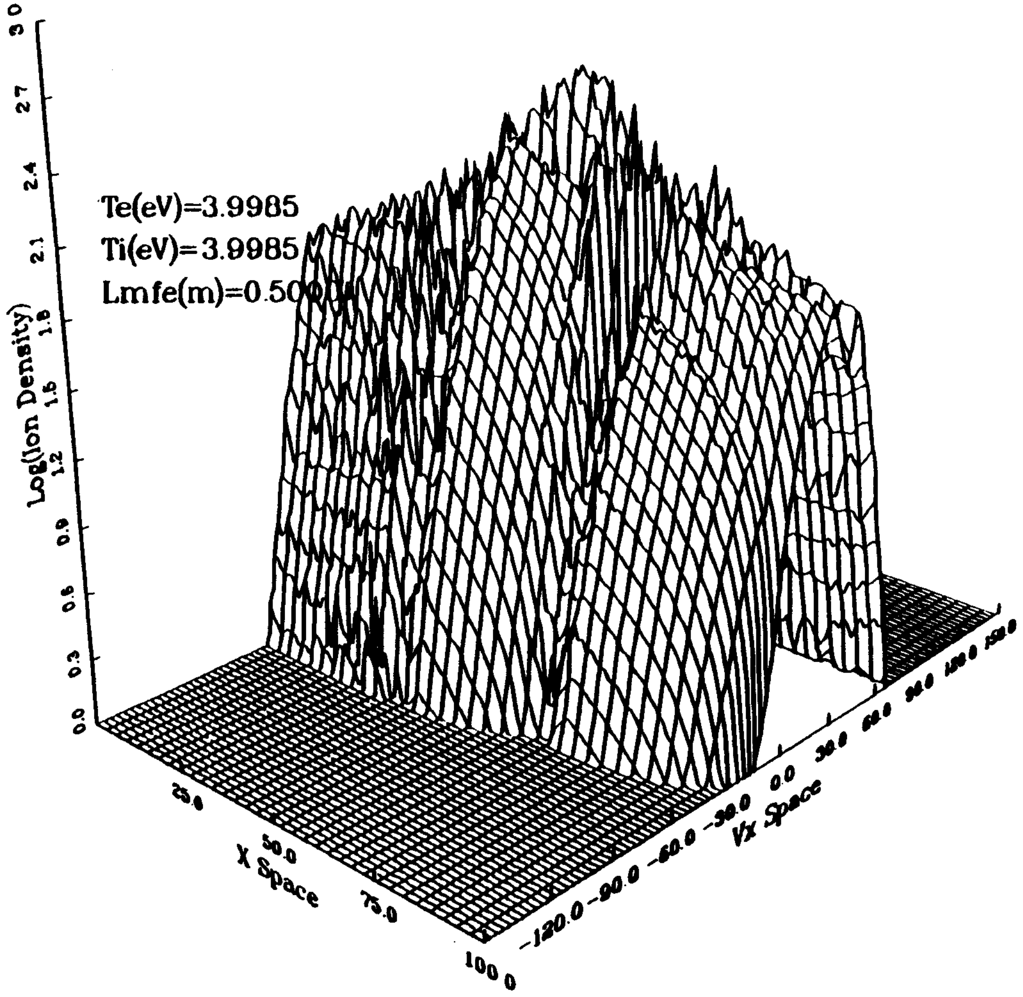


FIGURE 4b

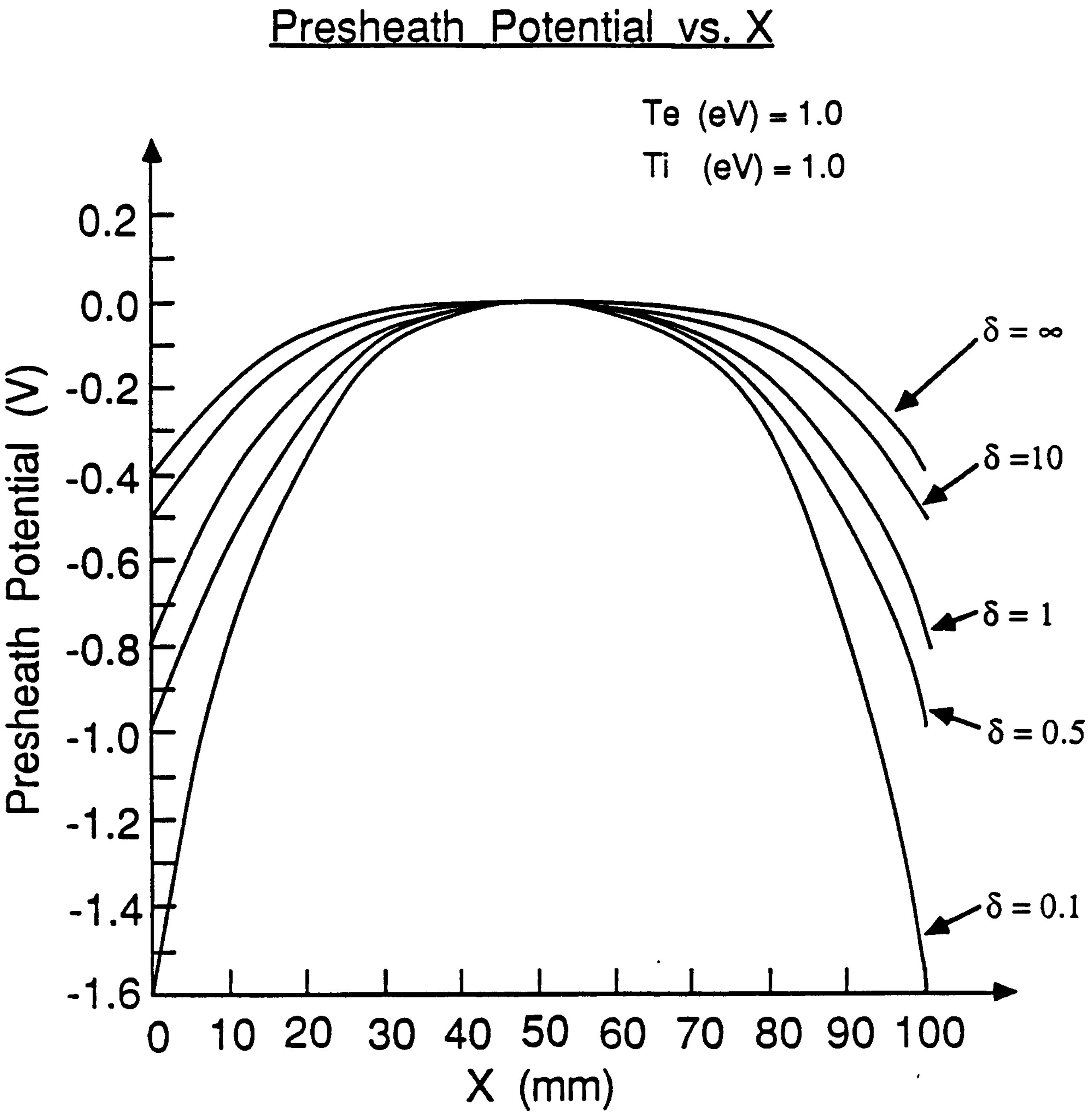

Presheath Potential vs. X
Te (eV) = 1.0
Ti (eV) = 1.0
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6
Presheath Potential (V)
0
10
20
30
40
50
60
70
80
90
100
X (mm)
δ = ∞
δ =10
δ = 1
δ = 0.5
δ = 0.1

FIGURE 5a

Presheath Ion Distribution

T_i (eV) = 1.0
T_e (eV) = 1.0
δ = 1.0

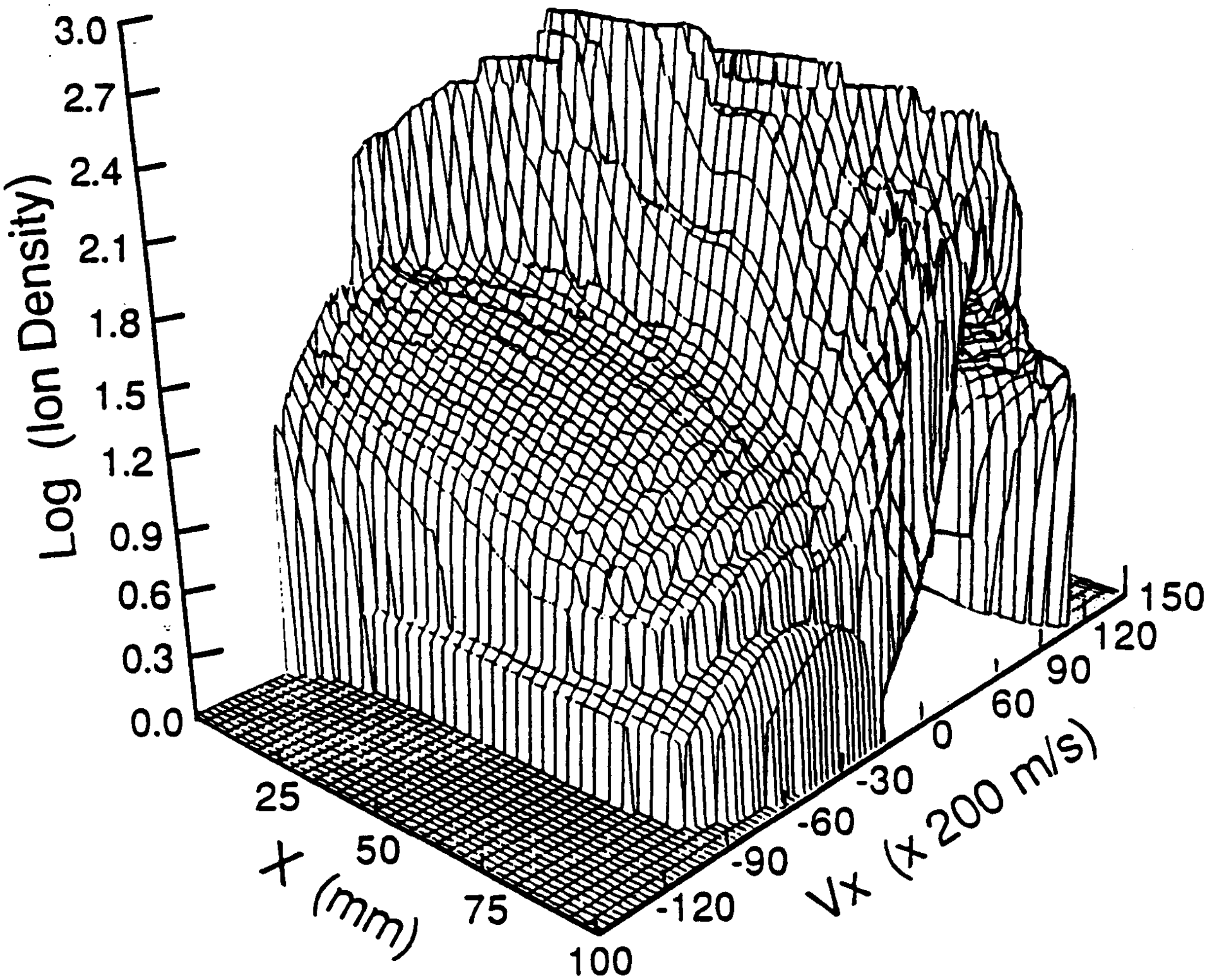


FIGURE 5b