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THEORY OF SEMICOLLISIONAL KINETIC ALFVEN MODES IN SHEARED MAGNETIC FIELDS

By

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The spectra of the semicollisional kinetic Alfvén modes in a sheared slab geometry are investigated, including the effects of finite ion Larmor radius and diamagnetic drift frequencies. The eigenfrequencies of the damped modes are derived analytically via asymptotic analyses. In particular, as one reduces the resistivity, we find that, due to finite ion Larmor radius effects, the damped mode frequencies asymptotically approach finite real values corresponding to the end points of the kinetic Alfvén continuum.

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1. INTRODUCTION

In a sheared slab configuration, resistive MHD analyses have shown that the collisional shear Alfvén mode has a discrete spectrum with real frequencies and damping rates approaching toward zero as $\nu_{ei}^{1/3}$.[1] Here, ν_{ei} is the electron-ion collisional frequency. However, with present-day magnetic confinement devices operating at such high temperatures, kinetic effects such as finite ion Larmor radius (FLR) and diamagnetic drift frequencies (ω_*) begin to play crucial roles. In previous studies, these kinetic effects have been treated separately. Although the result that the damping rates are proportional to $(\log \nu_{ei})^{-1}$ has been found, some important points, as we shall discuss below, remain unanswered.

The first issue covers the stability property. Catto, Rosenbluth, and Tsang[2] (hereafter referred to as CRT) have considered the effects of ω_{*e} and ω_{*i}, assuming the magnetized ion response; i.e., $\Gamma_o(b_i) \equiv I_o(b_i) \exp(-b_i) \approx 1 - b_i$; where $b_i = k_\perp^2\rho_i^2/2$, ρ_i the ion Larmor radius, $k_\perp$ is the perpendicular wave number, and I_o the modified Bessel function. While they have noted that the damping rates are proportional to $(\log \nu_{ei})^{-1}$, a new instability due to the finite ω_{*i} has also been claimed which later has been identified to be spurious.[3] In fact, the validity of differential formulation in the configuration space based on the magnetized ion response has been questioned in CRT. In this paper, we show that, by properly taking account of FLR effects, the semicollisional kinetic Alfvén modes are stable even in the presence of ω_{*i}. In this sense, CRT's results are valid only in the cold ion limit where $T_i/T_e \to 0$ such that ω_{*i} can be ignored.

The second issue concerns the proper description of the discrete spectrum. Drake and Kleva[4] (hereafter referred to as DK) have studied the effects of ω_{*e} using the cold ion model; where $k_\perp^2\rho_i^2 \ll 1$, but $k_\perp^2\rho_s^2$ is kept as arbitrary. Here, $\rho_s = C_s/\Omega_i$, i.e., the ion Larmor radius with electron temperature. They have also found the proper damping rate. However, by employing the WKB method, their analyses on the asymptotic behavior of the least damped eigenmode as $\nu_{ei} \to 0$ become dubious since the corresponding mode structures along the field line are relatively flat. As we shall show later, for sufficiently weak resistivities, the eigenmode frequency of the least damped mode approaches a finite real value even if $\omega_* = 0$ (instead of zero as predicted in DK) and it, therefore, could be potentially destabilized by some resonant mechanisms with energetic particles such as α particles.[5] We emphasize that this discrepancy in the asymptotic behavior is exactly due to the failure of the WKB analyses for the lowest eigenstate.

Neglecting diamagnetic frequencies, Connor, Tang, and Taylor[6] (hereafter referred to as CTT) analyzed the influence of FLR on the modes. They have found that the damping rates remain proportional to $(\log \nu_{ei})^{-1}$. Again, because of the WKB analyses employed, their results cannot be consistently applied to the least damped eigenmodes in the small resistivity limit.

In the present work, we include both the finite ω_* and FLR effects simultaneously. In order to avoid the constraints imposed by the WKB method, we employ the method of asymptotic matching. The dispersion relation thus derived not only gives the correct behaviors of the modes as $\nu_{ei} \to 0$, but also recovers the previous results in the appropriate limits.

In Sec. 2, we derive the eigenmode equation. In Sec. 3, we derive the dispersion relation for the semicollisional kinetic Alfvén modes. Various limiting cases of that dispersion relation are discussed in Sec. 4. Conclusions and discussions are presented in Sec. 5.

2. DERIVATION OF EIGENMODE EQUATION

In this section, we derive the eigenmode equation which describes the semicollisional kinetic Alfvén mode in a low-β plasma embedded in a sheared slab configuration. Here β is the ratio between plasma and magnetic pressures.

For a low-β ($m_e/m_i \ll \beta \ll 1$) plasma in a slab with sheared equilibrium magnetic field $\vec{B} = B(\hat{z} + x\hat{y}/L_s)$, the perturbed electromagnetic fields can be described by the electrostatic potential ϕ and the parallel component of the electromagnetic potential $A_\parallel$ (i.e., the compressional magnetic field perturbation $B_\parallel$ may be ignored). Meanwhile, for low frequency modes ($\omega \ll \Omega_i$) with $k_\parallel \ll k_\perp$, we can use the gyrokinetic equation for the ion response and the drift-kinetic equation for the electron response.[7,8]

The ion gyrokinetic equation for a low-β plasma in a sheared slab model is given by

$$-i(\omega - k_\parallel v_\parallel)h = -\frac{ie}{T_i} F_M \left(\omega - \omega_{*i}\right) J_0(k_\perp \rho_i)\left(\phi - \frac{v_\parallel}{c} A_\parallel\right) \quad , \tag{1}$$

where

$$f_1 = -\frac{e\phi}{T_i} F_M + he^{-iL} \quad ,$$

$L = \vec{k}_\perp \cdot \hat{b} \times \vec{v}/\Omega_i$, $L_n^{-1} = d\ln(n_0)/dx$, $k_\parallel = k'_\parallel x$, $k'_\parallel = k_y/L_s$, $\vec{k}_\perp = k_y\hat{y} - i\hat{x}(d/dx)$, $J_0(k_\perp\rho_i)$ should be properly interpreted as an operator, F_M is the Maxwellian equilibrium distribution function, f_1 is the perturbed distribution function, and h is the nonadiabatic part of it.

In Eq. (1), we have neglected the collisional effects on ions by assuming $\nu_{ii} \ll \omega$ which is generally true due to the large ion-electron mass ratio.

Assuming $k_\parallel v_{ti} << \omega$ (i.e., in the fluid-ion limit), we can neglect the ion parallel current and the ion density response can be easily obtained to be

$$\frac{n_i}{n_0} = \frac{1}{n_0} \int d^3v f_1 = -\frac{e\phi}{T_i} + \left(1 - \frac{\omega_{*i}}{\omega}\right)\Gamma_0(b_i)\,\frac{e\phi}{T_i} \quad , \tag{2}$$

where recalling $\Gamma_0(b_i) = I_0(b_i)\,\exp(-b_i)$, $b_i = k_\perp^2\rho_i^2/2$, I_0 is the modified Bessel function, and, again, note that Γ_0 is an operator.

As to the electrons, we may assume $\omega << k_\parallel v_\parallel << \nu_{ei}$ but $\omega\nu_{ei} \sim k_\parallel^2 v_\parallel^2$. Since the electron Larmor radius effects are negligible, we may adopt the drift-kinetic description for the electrons. Hence, the electron drift-kinetic equation is given by

$$-i(\omega - k_\parallel v_\parallel)h = \frac{ie}{T_e} F_M(\omega - \omega_{*e})\left(\phi - \frac{v_\parallel}{c} A_\parallel\right) + C_{ei}(h) \quad , \tag{3}$$

where

$$f_1 = \frac{e\phi}{T_e} F_M + h \quad ,$$

and C_{ei} is the number-conserving Krook-model collisional operator

$$C_{ei}(h) = -\nu_{ei}\left[h - (F_M/n_0)\int d^3vh\right] \quad .$$

Equation (3) can be solved systematically using $k_\parallel v_{\parallel e}/\nu_{ei} \sim \omega/k_\parallel v_{\parallel e} << 1$ as the small expansion parameters. Expanding $h = h^{(0)} + h^{(1)} + \ldots$ correspondingly, we obtain the following electron responses. In the zeroth order, Eq. (3) gives

$$C_{ei}\left[h^{(0)}\right] = 0 \quad ,$$

with a solution

$$h^{(0)} = n^{(0)} F_M/n_0 \quad .$$

Note that $n^{(0)}$ contains only the "nonadiabatic" part of the electron density response. In the first order, we have

$$ik_{\parallel} v_{\parallel} h^{(0)} = -\nu_{ei} h^{(1)} - i\left(\omega - \omega_{*e}\right) \frac{e}{T_e} F_M \frac{v_{\parallel}}{c} A_{\parallel} \quad , \tag{5}$$

where we have eliminated $\int d^3vh^{(1)}$ term of the collisional operator by absorbing the part of $h^{(1)}$ which contributes to $\int d^3vh^{(1)}$ into $h^{(0)}$. Taking the $\int d^3vv_{\parallel}$ moment of Eq. (5), we have

$$\frac{k_{\parallel} j_{\parallel}}{n_0 e} = \frac{i v_{te}^2 k_{\parallel}^2 n^{(0)}}{2\nu_{ei} n_0} + i\left(1 - \frac{\omega_{*e}}{\omega}\right) \frac{v_{te}^2 k_{\parallel}^2}{2\nu_{ei}} \frac{e\chi}{T_e} \quad , \tag{6}$$

where $(ck_{\parallel}/\omega)\chi = A_{\parallel}$, $v_{te}^2 \equiv 2T_e/m_e$. Now, to the second order, Eq. (3) is given by

$$ik_{\parallel} v_{\parallel} h^{(1)} - i\omega h^{(0)} = -\nu_{ei}\left[h^{(2)} - \left(F_M/n_0\right) \int d^3vh^{(2)}\right]$$

$$+ i\left(\omega - \omega_{*e}\right) \frac{e\phi}{T_e} F_M \quad . \tag{7}$$

By taking the $\int d^3v$ moment of Eq. (7), we can annihilate the unrequired information about $h^{(2)}$ via the number-conserving property and obtain

$$\frac{k_\parallel j_\parallel}{n_0 e} = -\omega \frac{n^{(0)}}{n_0} - (\omega - \omega_{*e}) \frac{e\phi}{T_e} \quad , \tag{8}$$

which is nothing but the electron density continuity equation. To see it more clearly, combining with the adiabatic response, Eq. (8) can be rewritten as

$$\omega \frac{n_e}{n_0} = \omega_{*e} \frac{e\phi}{T_e} - \frac{k_\parallel j_\parallel}{n_0 e} \quad , \tag{9}$$

where the term on the left-hand side is the partial time derivative n_e, the first term on the right-hand side is the convection of the equilibrium density by the perturbed $\vec{E} \times \vec{B}$ drift $(\vec{v}_E \cdot \vec{\nabla} n_0)$, and finally the last term comes from the divergence of the electron parallel velocity which is proportional to the parallel current.

With n and $j_\parallel$ obtained, we may proceed to Maxwell's equations and derive the desired eigenmode equation. First, noting that since we are dealing with low-frequency modes such that the wavelengths are much larger than the Debye length, the Poisson equation can be replaced by the quasi-neutrality condition, i.e., $n_e \approx n_i$ where n_i and n_e are given, respectively, by Eq. (2) and Eq. (9). The resulting relation then reads

$$-\frac{e\phi}{T_i} + \left(1 - \frac{\omega_{*i}}{\omega}\right)\Gamma_0(b_i)\frac{e\phi}{T_i} = \frac{\omega_{*e}}{\omega}\frac{e\phi}{T_e} - \frac{k_\parallel j_\parallel}{\omega n_0 e} \quad . \tag{10}$$

On the other hand, the parallel Ampere's law

$$-j_\parallel = \nabla_\perp^2 A_\parallel \quad , \tag{11}$$

together with Eq. (6) gives

$$\frac{1}{n_0 e}\frac{k_\parallel}{\omega}\nabla_\perp^2 k_\parallel \chi = -\frac{iv_{te}^2 k_\parallel^2}{2\nu_{ei}}\frac{n^{(0)}}{n_0} - \frac{iv_{te}^2 k_\parallel^2}{2\nu_{ei}}\left(1-\frac{\omega_{*e}}{\omega}\right)\frac{e\chi}{T_e} \quad . \tag{12}$$

Equations (8), (10), (11), and (12) give four equations for the four dependent variables $\phi, \chi, j_\parallel$ and $n^{(0)}$. Eliminating ϕ, χ, and $j_\parallel$, we finally obtain the following single eigenmode equation in terms of $n^{(0)}$

$$k_\parallel \frac{\nabla_\perp^2}{1-i\nabla_\perp^2/\sigma_c(\omega-\omega_{*e})} k_\parallel n^{(0)} + \frac{n_0 e^2}{T_e}\frac{(\omega-\omega_{*i})\,\omega[1-\Gamma_0(b_i)]}{1+(\omega\tau+\omega_{*e})[1-\Gamma_0(b_i)]/(\omega-\omega_{*e})} n^{(0)} = 0, \tag{13}$$

where $\sigma_c = n_o e^2/m_e \nu_{ei}$ is the Spitzer conductivity.

In order to investigate the FLR effects in sheared magnetic fields, it is more convenient to perform the analysis in the Fourier transformed space. In this space, the notations in Eq. (13) should be interpreted through their Fourier transforms, i.e., $n^{(0)}(k_x) = (2\pi)^{-1/2} \int dx \exp(-ik_x x) n^{(0)}(x)$, $k_\parallel = (ik_y/L_s)d/dk_x$, and $\nabla_\perp^2 = -k_x^2 - k_y^2$. By normalizing frequencies by v_A/L_s and inverse length by k_y, we have

$$\frac{d}{d\theta}\frac{1+\theta^2}{1+[i\lambda/(\omega-\omega_{*e})](1+\theta^2)}\frac{d}{d\theta} n^{(0)} + \frac{1}{b_0}\frac{(\omega-\omega_{*i})\omega[1-\Gamma_0(b_i)]}{1+(\omega\tau+\omega_{*e})[1-\Gamma_0(b_i)]/(\omega-\omega_{*e})} n^{(0)} = 0, \tag{14}$$

where $\lambda = \nu_{ei} v_A L_s b_0 \tau / v_{te}^2$, $b_i = b_0(1+\theta^2)$, $b_0 = (1/2)k_y^2\rho_i^2$, $\tau = T_e/T_i$, $\theta = k_x/k_y$, $\omega_{new} = \omega_{old} L_s/v_A$. Equation (14) is the desired eigenmode equation which describes the semicollisional kinetic Alfven mode.

3. DERIVATION OF DISPERSION RELATION

In this section, we derive the dispersion relation for the semi-collisional kinetic Alfvén modes starting from Eq. (14). The first term of Eq. (14) represents the bending of magnetic field lines, including the resistive correction. A local expression would look like $k_\perp^2 k_\parallel^2 v_A^2/[1+i\eta k_\perp^2/(\omega-\omega_*)]$. We can see that this term becomes smaller as we go to large θ (small x in the configuration space) due to resistive dissipation. By balancing the terms in the denominator, it is easy to show that the resistive effects become appreciable when $\theta \sim \theta_\lambda \equiv |(\omega-\omega_*)/\lambda|^{1/2}$. On the other hand, the second term in Eq. (14) can be regarded as FLR modified inertia. It is clear in the small ion Larmor radius limit where the local expressions read as $\omega(\omega - \omega_{*i})k_\perp^2$. This term contains a scale length $\theta_b \equiv b_0^{-1/2}$. When $\theta >> \theta_b$, ions are unmagnetized, while when $\theta << \theta_b$, ions are magnetized. Since we are interested in the situation where the size of the ion Larmor radius is larger than the resistive layer width, we assume $\theta_b << \theta_\lambda$, but we still assume $b_0 = (1/2)k_y^2\rho_i^2 << 1$ and, thus, exclude extremely large-k_y modes.

Before proceeding further to solve Eq. (14), it is instructive at this juncture to recall that in the usual resistive MHD limit where b_0, ω_{*e}, $\omega_{*i} \to 0$, Eq. (14) then reduces to

$$\frac{d}{d\theta}\,\frac{1+\theta^2}{1+i\lambda(1+\theta^2)/\omega}\,\frac{d}{d\theta}\,n^{(0)} + \omega^2(1+\theta^2)n^{(0)} = 0 \quad ; \tag{15}$$

which can be readily solved to yield the following discrete eigenvalues[1,6]

$$\omega = \lambda^{1/3}(2m+5)^{2/3}\left\{\begin{matrix}\exp(-i\pi/6)\\ \exp(i5\pi/6)\end{matrix}\right\} \quad ,$$

where m is a non-negative integer. Note that, as one reduces ν_{ei}, both the real frequencies and damping rates asymptotically approach zero as $\nu_{ei}^{1/3}$.

Let us now proceed to solve Eq. (14) based on the ordering $|\lambda/(\omega-\omega_{*e})| \ll b_0 \ll 1$(i.e., $\theta_\lambda \gg \theta_b \gg 1$). We can then divide the range of θ into two regions. (1) In the resistive region, $\theta \sim \theta_\lambda$, where ions are unmagnetized, we have $b_0\theta^2 \gg 1$ and Eq. (14) becomes

$$\frac{d}{d\theta} \frac{\theta^2}{1+i\lambda\theta^2/(\omega-\omega_{*e})} \frac{d}{d\theta} n_1^{(0)} + \frac{(\omega-\omega_{*e})(\omega-\omega_{*i})}{b_o(1+\tau)} n_1^{(0)} = 0 \quad . \tag{16}$$

(2) Away from the resistive region, $\theta \sim \theta_b \ll \theta_\lambda$, dissipation is negligible and the ion response can no longer be regarded as unmagnetized. Equation (14) can be approximated as

$$\frac{d}{d\theta} \theta^2 \frac{d}{d\theta} n_2^{(0)} + \frac{1}{b_0} \frac{(\omega-\omega_{*i})\omega[1-\Gamma_0(b_i)]}{1+[(\omega\tau+\omega_{*e})/(\omega-\omega_{*e})][1-\Gamma_0(b_i)]} n_2^{(0)} = 0 \quad . \tag{17}$$

To solve Eq. (16), we first make the following transformation

$$U \equiv \frac{\theta^2}{1+i\lambda\theta^2/(\omega-\omega_{*e})} \frac{d}{d\theta} n_1^{(0)} \quad ,$$

then Eq. (16) becomes

$$\frac{d^2}{d\theta^2} U + \frac{(\omega-\omega_{*i})(\omega-\omega_{*e})}{b_0(1+\tau)} \left(\frac{i\lambda}{\omega-\omega_{*e}} + \frac{1}{\theta^2}\right)U = 0 \quad , \tag{18}$$

which has a solution

$$U = \theta^{1/2} H_\alpha^{(1)}(\delta\theta) \quad .$$

Here, $H_\alpha^{(1)}$ is the Hankel function, $\delta^2 \equiv i\lambda(\omega-\omega_{*i})/b_0(1+\tau)$, and $1/4 - \alpha^2 \equiv (\omega-\omega_{*i})(\omega-\omega_{*e})/b_0(1+\tau)$, and we require $0 \leq \mathrm{Im}(\delta)$ to ensure the proper asymptotically decaying behavior of U as $\theta \to \infty$. Using the small-argument expansion of $H_\alpha^{(1)}$, the small-θ behavior of $n_1^{(0)}$ takes the form

$$\frac{d}{d\theta}\,\theta n_1^{(0)} \approx \theta^{\alpha-1/2} - \left(\frac{\delta}{2}\right)^{-2\alpha} \frac{\Gamma(1+\alpha)(1/2-\alpha)^2}{\Gamma(1-\alpha)(1/2+\alpha)^2}\, e^{\alpha\pi i}\theta^{-\alpha-1/2} \quad . \tag{19}$$

Here, we remark that it is easier to match $d[\theta n^{(0)}]/d\theta$ rather than $n^{(0)}$.

We now consider Eq. (17). Since Eq. (17) contains a transcendental function $\Gamma_0(b_i)$, we approximate it by $1/(1+b_0\theta^2)$ in order to make further analytical progress. This approximation gives the correct θ dependence in the magnetized ($\theta < \theta_b$) limit and the correct asymptotic value in the fully unmagnetized ($\theta >> \theta_b$) limit, but the transition rate from the magnetized to the unmagnetized response is faster than the actual case where $\Gamma_0(b_i) \sim (2\pi b_i)^{-1/2}$ as $b_i \to \infty$. However, as one can see from Eq. (17), all Γ_0 terms appear in the combination $1 - \Gamma_0$ and, when $b_i \geq 1$, Γ_0 is small compared to 1 such that, in the present case, the precise θ dependence of Γ_0 makes negligible difference. (This observation can, in fact, be substantiated by including the $b_i^{-1/2}$ term perturbatively.) Then Eq. (17) can be approximated as

$$\frac{d^2}{d\theta^2}\,\theta n_2^{(0)} + \frac{\omega(\omega-\omega_{*i})}{1+[\omega b_0(1+\tau)\theta^2]/(\omega-\omega_{*e})}\,\theta n_2^{(0)} = 0 \quad , \tag{20}$$

which can be rewritten as

$$\frac{d}{dt}\,(1 - t^2)\,\frac{d}{dt}\,W - \frac{(\omega-\omega_{*e})(\omega-\omega_{*i})}{(1+\tau)b_0}\,W = 0 \quad , \tag{21}$$

where we have defined $W = d[\theta n_2^{(0)}]/d\theta$ and $t = i\theta[\omega(1+\tau)b_0/(\omega-\omega_{*e})]^{1/2}$. Solution of Eq. (21), which is regular at $t = 0$, is given by

$$W = P_\nu(t) \pm P_\nu(-t) \quad , \tag{22}$$

where P_ν is the Legendre function, upper and lower signs correspond, respectively, to even and odd parity modes and $\nu = \alpha - 1/2$, i.e.,

$$\nu(\nu + 1) + \frac{(\omega-\omega_{*e})(\omega-\omega_{*i})}{(1+\tau)b_0} = 0 \quad .$$

For large θ, we have

$$\frac{d}{d\theta}\,[\theta n_2^{(0)}] \approx \theta^{\alpha-1/2} + 2^{-2\alpha}\,\frac{\Gamma(-\alpha)\Gamma(1/2+\alpha)}{\Gamma(\alpha)\Gamma(1/2-\alpha)}\,\left[\frac{(1+\tau)b_0\omega}{\omega-\omega_{*e}}\right]^{-2}\left\{\begin{matrix}\tan(\frac{\pi}{4} - \frac{\pi\alpha}{2})\\ \\ \cot(\frac{\pi}{4} - \frac{\pi\alpha}{2})\end{matrix}\right\}\theta^{-\alpha-1/2} \quad . \tag{23}$$

Matching the large-θ solution of Eq. (17), Eq. (23), and the small-θ solution of Eq. (16), Eq. (19), we obtain the desired dispersion relation

$$\frac{\Gamma(1+\alpha)\Gamma(1/2-\alpha)\Gamma(\alpha)(1/2-\alpha)^2}{\Gamma(1-\alpha)\Gamma(1/2+\alpha)\Gamma(-\alpha)(1/2+\alpha)^2}\,\exp(\alpha\pi i)\,2^{2\alpha}\left[\frac{\omega(1+\tau)b_0}{\omega-\omega_{*e}}\right]^{\alpha} + \left[\frac{i\lambda(\omega-\omega_{*i})}{4(1+\tau)b_0}\right]^{\alpha}\left\{\begin{matrix}\tan(\frac{\pi}{4} - \frac{\pi\alpha}{2})\\ \\ \cot(\frac{\pi}{4} - \frac{\pi\alpha}{2})\end{matrix}\right\} = 0 \quad . \tag{24}$$

Equation (24) is the dispersion relation for the semicollisional kinetic Alfvén modes including the effects of FLR and ω_*'s. This expression in its present form, however, is too complicated to be illuminating. Therefore, in the next section, we shall discuss various limiting cases and identify their relations to previous works.[2,4,6]

4. LIMITING CASES OF DISPERSION RELATION

The weak resistivity limit is of particular interest as discussed in Sec. 1. This limit can be obtained by taking the $\alpha \to 0$ limit of Eq. (24). We then get

$$\frac{(\omega-\omega_{*e})(\omega-\omega_{*i})}{(1+\tau)b_0} = \frac{1}{4} + \frac{n^2\pi^2}{\{(1/2)\log[-i\lambda(\omega-\omega_{*e})(\omega-\omega_{*i})/\omega(1+\tau)^2 b_0^2]+4+c-4\log 2\mp\pi/2\}^2}, \tag{25}$$

where $n = 1,2 \ldots$ is a positive integer, $c = 0.577215\ldots$ is the Euler's constant. Equation (25) shows that not only positive-frequency $[\mathrm{Re}(\omega) > 0]$ modes but also negative-frequency $[\mathrm{Re}(\omega) < 0]$ modes are damped with rates proportional to $(\log \nu_{ei})^{-1}$. This differs from CRT's result which predicts instability for $\omega_{*i} < \mathrm{Re}(\omega) < 0$. Here, we note that the assumption $|\alpha| \ll 1$ restricts the validity of Eq. (25), i.e., $n\pi \ll |(1/2)\log[-i\lambda(\omega-\omega_{*e})(\omega-\omega_{*i})/\omega(1+\tau)^2 b_0^2] + 4 + c - 4\log 2 \mp \pi/2|$, which is certainly satisfied for the lowest eigenmode.

Now we discuss the reason why CRT's unstable solution is not consistent with the assumptions about the ion response. CRT's model corresponds to the magnetized ion response where $\Gamma_0(b_i) \approx 1 - b_i$ has been used even in the resistive region. That assumption can be satisfied if $\theta_\lambda^2 \ll b_0^{-1} = \theta_b^2$. This is the (highly) collisional regime. But CRT's solution (especially logarithmic dependence on ν_{ei}) shows that b_i tends to be larger than 1 and, thus, violating the condition for expanding $\Gamma_0(b_i)$ for small b_i. Mathematically, this comes from the spurious secularity in the expression $1 - b_0(1 + \theta^2)$ when θ becomes very large. Interestingly, the cold ion model (such as DK) does not have this problem. Since $\tau = T_e/T_i \gg 1$, one can have a situation where $b_0\theta_\lambda^2 \ll 1$ such that the magnetized ion response is justified and meanwhile, $b_\theta\theta_\lambda^2 \gg 1$ such that the effective ion Larmor radius at the electron temperature is greater than the resistive layer width, i.e., the semicollisional situation.

Let us come back to Eq. (25). As $\nu_{ei} \to 0$, the eigenfrequencies of the modes approach to

$$\omega_\pm = \frac{1}{2}\left\{(\omega_{*e} + \omega_{*i}) \pm \left[(\omega_{*e} - \omega_{*i})^2 + (1 + \tau)b_0\right]^{1/2}\right\} \quad , \tag{26}$$

which is finite even when ω_*'s $\to 0$. We note that by examining the $\lambda = 0$, large-θ solutions of Eq. (14), it is straightforward to show that $\omega_- < \omega < \omega_+$ corresponds to a gap in the kinetic Alfven continuum, i.e., $\omega_\pm$ are the end points of the continuum. It is also interesting to note that, if we neglect ω_*'s, Eq. (26) is similar to the first order FLR (magnetized) solution of CTT [Eq. (16) thereof] without dissipation. The only difference is the numerical factor [they obtained $(3/4) + \tau$ instead of $1 + \tau$]. However, when they considered the full FLR effects together with resistivity in the same paper, they employed a WKB procedure to calculate the eigenvalues. For $\lambda \to 0$, this approach cannot be consistently applied to the least damped eigenstate. In this limit, their dispersion relation would give $\omega \to 0$, instead of our result, $\omega \to \pm(1/2)[(1+\tau)b_0]^{1/2}$, obtained from our more general asymptotic matching analysis. The fact that the mode has a finite real frequency in the small resistivity limit could have potentially important practical implications such as destabilization via resonance with α or energetic particles.[5]

We now demonstrate that Eq. (24) can recover the WKB solutions of DK and CTT. For the WKB method to be valid, the eigenfunction should vary faster than the effective potential. Therefore, strictly speaking, we can use the WKB method for modes with high quantum numbers n. The WKB limit of Eq. (24) can be obtained by taking $|\alpha| \gg 1$, then Eq. (24) reduces to

$$\frac{(\omega-\omega_{*e})(\omega-\omega_{*i})}{(1+\tau)b_0} = \{\frac{2n\pi}{\log[16i\omega(1+\tau)b_0/\lambda]-2}\}^2 \quad . \tag{27}$$

Here, expressions of Γ functions with large imaginary arguments have been used in deriving Eq. (27). If we take the cold-ion limit ($\omega_{*i}, b_0 \to 0$, $\tau \to \infty$ such that τb_0 is kept finite), Eq. (27) becomes

$$\omega(\omega - \omega_{*e}) = [\frac{2n\pi b_\theta^{1/2}}{\log(16i\omega b_\theta/\lambda)-2}]^2 \quad , \tag{28}$$

where $b_\theta = \tau b_0$. Equation (28) is exactly the same as Eqs. (41) and (46) of DK in the limit that their solutions are valid, i.e., $2n\pi \gg |\log[16i\omega(1+\tau)b_0/\lambda] - 2|$. On the other hand, if we neglect ω_*'s from Eq. (27), we get

$$\omega = \frac{2n\pi[(1+\tau)b_0]^{1/2}}{\log[16i\omega(1+\tau)b_0/\lambda]-4} \approx \frac{2n\pi[(1+\tau)b_0]^{1/2}}{\log(\omega/\lambda)+\pi i/2} \quad , \tag{29}$$

where $\log[16(1+\tau)b_0] - 4$ is ignored compared to the larger term $\log(\omega/\lambda)$. If one expands the denominator of Eq. (29), noting that the imaginary term is smaller than the logarithmic term, one obtains the same expression as Eq. (48) of CTT.

Finally, we comment on the self-consistency of our ordering $\theta_\lambda \gg \theta_b$. Since $\theta_\lambda \approx |(\omega-\omega_{*e})/\lambda|$, the most stringent case corresponds to the limit $|\alpha| \ll 1$ where Eq. (25) and, approximately, Eq. (26) is applicable. In this case, we require $b_0 \gg \lambda^{2/3}$ which can be easily satisfied due to the small resistivity.

5. CONCLUSIONS

In this paper, we have derived an eigenmode equation which describes the semicollisional kinetic Alfvén modes including FLR and ω_* effects. The corresponding dispersion relation is then obtained via asymptotic analyses in the semicollisional regime.

The results show that the eigenmodes are stable with damping rates proportional to $(\log \nu_{ei})^{-1}$ as previously found.[2,4,6] The semicollisional damping rates are, thus, stronger than those of the resistive MHD (collisional) regime where the damping rates scale as $\nu_{ei}^{1/3}$.

The asymptotic-matching procedure employed here, furthermore, allows us to apply our results to the important regime where the WKB approach used in previous studies[4,6] becomes inappropriate. In particular, this has led to the result that as $\lambda \to 0$, the eigenmode frequencies (for the least damped eigenstates) approach real values corresponding to the end points of the FLR-modified Alfvén continuum. Therefore, this mode could be destabilized via resonances with energetic particles. Analyses dealing with this interesting possibility, however, lie beyond the scope intended for the present work and need to be examined in the future.

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