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A SIMPLIFIED SIZING AND MASS MODEL FOR AXIAL FLOW TURBINES

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ABSTRACT

An axial flow turbine mass model has been developed and used to study axial flow turbines for space power systems. Hydrogen, helium-xenon, hydrogen-water vapor, air, and potassium vapor working fluids have been investigated to date. The impact of construction material, inlet temperature, rotational speed, pressure ratio, and power level on turbine mass and volume has been analyzed. This paper presents the turbine model description and results of parametric studies showing general design trends characteristic of any axial flow machine. Also, a comparison of axial flow turbine designs using helium-xenon mixtures and potassium vapor working fluids, which are used in Brayton and Rankine space power systems respectively, is presented.

INTRODUCTION

An axial flow expansion turbine model was developed for use in our space power system studies to provide consistent mass and performance comparisons between alternate turbine powered concepts. This model determines total turbine mass and dimensions of any axial flow turbine as a function of turbine operating parameters, working fluid conditions, and material constraints. The model uses principles from thermodynamics, fluid dynamics, and strength of materials to iteratively determine each stage outlet flow area and size. The combined stages result in the complete turbine. The turbine model is programmed for use on an IBM PC, uses little computer time for most working fluids, and is incorporated into our system-level space power codes. This allows optimization studies to be performed on multi-parameter space power systems in a consistent and reasonable manner [1]. The model output identifies basic design trends and limiting design constraints as functions of the system operating conditions and specified turbine parameters. The working fluid may be any gas or condensing vapor provided that the appropriate equation of state is used to calculate the fluid's thermodynamic conditions. Despite this level of detail, the turbine model is considered simplified because it does not account for effects such as boundary layer interactions, shock waves, stress concentrations, hub seal and blade tip leakage, three-dimensional flows, bending and thermal stresses, or aerodynamically induced blade vibrations.

TURBINE MODEL DESCRIPTION

The axial flow turbine model determines the blade length and disk radius for each stage of a turbine by calculating: (1) the energy transfer from the working fluid to the rotating blades for each stage of the turbine; (2) the flow area required by the working fluid as it exits each stage; and (3) the limiting stage blade speed due to material strength considerations. The process is iterative because of the interdependence of these calculations. Blade length and disk radius are then used to define the turbine stage size because they are the primary size limiting dimensions. The model calculates a stage mass based on these size criteria and sums the individual stage masses to give the complete turbine mass. This provides a realistic basis for the turbine model and allows a consistent comparison between axial flow turbines for the many system design parameters and working fluid conditions.

Energy Transfer Considerations

Energy transfer occurs at each stage of an axial flow turbine when high velocity fluid imparts a force, due to the momentum change of the working fluid, to the moving blades and does work on them. This interaction is represented by the two-dimensional velocity diagram in Figure 1. The working fluid enters the stage at absolute velocity c_1, is accelerated through the nozzles to absolute velocity c_2, impinges on the rotor blades that are traveling at a tangential velocity U, and leaves the stage at absolute velocity c_3. The stage work per unit mass of working fluid, W_s, is:

$$W_s = (c_{2z} - c_{3z})U \quad (1)$$

Note that the stage outlet tangential velocity, c_{3z}, may be opposite in direction (and negative) relative to the inlet fluid velocity, c_{2z}, so that W_s is always a positive quantity. The momentum change of the working fluid is calculated at the mean blade radius for each stage of the turbine. Thus, the tangential blade velocity used in Equation (1) is that velocity which corresponds to the mean blade radius, R_m, or:

$$U = R_m N \quad (2)$$

where N represents the angular rotational speed of the turbine.

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A dimensionless stage work coefficient, ψ, can be defined as the ratio of change in tangential fluid velocities to the stage tangential blade velocity [2], or:

$$\psi = (c_{2z} - c_{3z})/U \ . \qquad (3)$$

Therefore, the turbine stage work may be expressed as a function of the stage mean blade speed and design work coefficient by combining Equations (1) and (3):

$$W_s = \psi\, U^2 \ . \qquad (4)$$

The stage work is also the working fluid's change in energy across each stage of the turbine. For adiabatic steady flow and no change in potential energy, the first law of thermodynamics results in:

$$W_s = h_1 - h_3 + (c_1^2 - c_3^2)/2 \qquad (5)$$

where h_1 and h_3 are the static enthalpies of the turbine working fluid at each stage inlet and exit, respectively.

Stage Flow Area Considerations

The annular flow area required by the working fluid at each turbine stage exit is determined by dividing the stage volume flow rate by the absolute axial exit velocity, c_{3x}. Thus:

$$2\,\pi\, R_m\, L = m\, v_3/c_{3x} \qquad (6)$$

where L is the stage blade length, m is the stage mass flow rate, and v_3 is the specific volume of the working fluid at the stage exit. The specific volume is obtained from the stage outlet static enthalpy (calculated from Equation (5)) and outlet static pressure. The outlet static pressure is developed from the definition of total-to-total stage efficiency, n_s, and the working fluid equation of state. For example, the stage exit static pressure, P_3, for an ideal gas working fluid with constant specific heats can be derived based on an isentropic expansion from the stage inlet pressure, P_1, as:

$$P_3 = P_1[T_3T_{1t}/(T_1T_{3t}) - W_sT_3/(c_pn_sT_1T_{3t})]^{k/(k-1)} \qquad (7)$$

where total-to-total stage efficiency is defined as:

$$n_s = W_s/(h_{1t} - h_{3ts}) \ . \qquad (8)$$

In the above equations T_1 and T_3 are the stage inlet and exit static temperatures respectively, T_{1t} (which equals $T_1 + c_1^2/2c_p$) and T_{3t} are the stage inlet and exit total temperatures respectively, c_p is the working fluid constant pressure specific heat, k is the ratio of constant pressure specific heat to constant volume specific heat, h_{1t} is the stage inlet total enthalpy (which equals $h_1 + c_1^2/2$), and h_{3ts} is the stage outlet total enthalpy for an isentropic expansion of the working fluid. For a non-ideal gas or condensing vapor working fluid, P_3 is that pressure corresponding to the state point defined by h_{3ts} (from Equation (8) and using a specified n_s) and the stage inlet entropy.

The specific volume, v_3, may then be obtained from P_3 and used to calculate the stage exit annular flow area from Equation (6). For an ideal gas, $v_3 = RT_3/P_3$ (where R is the gas constant), but for other working fluids v_3 may be a complex expression involving temperature and numerical constants [3]. The total-to-total stage efficiency is either specified or based on a default value in the program that accounts for all stage losses (including wall friction, secondary flow, and leakage losses) of large, subsonic 50% reaction turbine designs [4,5]. Lower stage efficiencies are specified for special conditions of short blade lengths or near sonic fluid velocities. In the special case of condensing vapors, the stage efficiency is reduced in direct proportion to the average stage vapor quality [6].

To determine the working fluid velocities required in Equations (5) and (6) two restrictions are applied to the turbine design. First, restriction of a 50% reaction turbine (defined as equal enthalpy change of the working fluid across the nozzles and blades) at the mean blade radius, R_m, implies that absolute velocity c_2 equals relative velocity w_3. The second restriction of constant axial fluid velocity through the stage requires that c_{2x} equals c_{3x}. These restrictions result in similar velocity triangles as shown in Figure 1 and allow determination of the stage outlet velocity, c_3, with knowledge only of the work coefficient, ψ, nozzle-to-blade angle, α_2, and stage blade speed, U. Thus, from geometry:

$$c_{2x} = c_{3x} = (\psi\, U + U)/(2\tan\alpha_2) \qquad (9)$$

and

$$c_3 = U\,\{[(\psi - 1)/2]^2 + [(\psi + 1)/(2\tan\alpha_2)]^2\}^{1/2}. \qquad (10)$$

These restrictions are common design practice in the gas turbine industry [2] and allow consistent comparative turbine designs to be obtained without requiring a large amount of input design data.

Mass flow rate is the last parameter required in Equation (6) so that the stage annular flow area can be calculated. This parameter is determined from the input turbine design information of working fluid selection, turbine inlet and exhaust temperatures, power output, and stage efficiency (if the turbine working fluid is a condensing vapor). For non-extraction turbines the mass flow rate through each stage is identical. For extraction turbines (generally used in regenerative Rankine systems), the mass flow through each subsequent stage decreases as fluid is removed from the turbine. This latter condition requires iteration through the entire turbine to compute the required mass flow through each stage. If the turbine working fluid is a condensing vapor, exhaust temperature alone does not specify the outlet conditions. In this case, the overall turbine efficiency determines the outlet fluid enthalpy and corresponding mass flow rate. A condensing vapor case requires iteration through the entire turbine because stage efficiency decreases with decreasing stage quality. The mass flow rate is calculated for all cases within the program with no additional inputs required.

Turbine Material Strength Considerations

The limiting stage blade speed is derived from the allowable blade stress at the blade root and the maximum allowable disk stress at the circumference of the disk center bore. If these stresses, caused by the disk and blade centrifugal forces, are exceeded then the turbine rotational speed must be correspondingly reduced. For each stage the program determines the largest possible disk radius based on material allowable stress and turbine rotational speed by [7]:

$$(R_m - L/2) = \{(\sigma_d /[0.9 \rho_d])^{1/2}\}/N \quad (11)$$

where the quantity $(R_m - L/2)$ is the disk radius, σ_d is the disk stress, and ρ_d is the disk density. If the resultant stage blade length is less than that specified as an input parameter the disk radius is automatically reduced. Also, the disk stress is that allowable stress corresponding to the specified operational temperature at the disk center bore. This may require disk cooling for some turbine designs, but this is standard practice in the gas turbine industry [8]. The maximum blade stress is calculated from [9]:

$$\sigma_b = \rho_b \, 0.7 \, N^2 R_m L \quad (12)$$

and occurs at the blade root. In Equation (12) σ_b is the blade stress (also temperature dependent), ρ_b is the blade density, and 0.7 is a factor for tapered blades. If the allowable blade stress is exceeded for any stage, the turbine operating speed must be reduced.

Turbine Mass Considerations

The overall turbine volume and mass is the sum of the individual stage volumes and mass. Stage volumes are obtained by multipling the cross-sectional area of each stage (disk and blade swept areas) by the stage thickness, which is determined from a specified aspect ratio (the ratio of blade length to blade axial thickness). The individual stage volumes include the disk, blade, and nozzle volumes. The turbine blade swept volume and nozzle volume is then given a mass based on an average density 30% that of the blade material density. The disk volume (the circular disk area without deduction for the shaft center bore times the blade axial thickness) is assigned an average density 100% that of the disk material density. The remaining stage volume between disks is provided a mass based on 20% of the disk material density to account for seals, shaft and connections. The casing mass is determined from hoop stress calculations and the resultant disk material required thickness. Although no specific mass is allotted for some turbine mechanical components such as bearings, ducting, seals, and cooling passages, the gross density estimates indicated above are intended to account for them in providing the overall turbine mass.

PARAMETRIC STUDY RESULTS

Turbine Rotational Speed, Pressure Ratio, and Power

The axial flow turbine model provides a realistic and consistent approach to estimating turbine size and mass for nearly all combinations of working fluids, materials of construction, and operating conditions. Thus, the effect of system parameter variations, such as power level, pressure ratio, and rotational speed, on turbine designs may be readily investigated with trends and limiting conditions identified.

For example, a simple parametric study of axial flow turbines reveals that turbine mass and size vary dramatically with operating speed. Turbine mass decreases more than an order of magnitude as rotational speed increases by a factor of only two or three as shown in Figure 2. This mass reduction occurs for all axial flow turbines and indicates the futility of specifing turbine mass or turbine specific mass (i.e., Kg/KW) without a knowledge of turbine speed. The 10 MWe air working fluid turbines of Figure 2 are of nickel superalloy construction, have inlet temperatures of 1350 K, inlet pressures of 3 MPa, work coefficients of about one, and stage efficiencies of 0.9. The disk centers for all stages of these turbines are cooled to 900 K to provide increased strength capability and the minimum allowed blade lengths are 0.01 m. It is important to note that due to material strength limitations the curves in Figure 2 can not be extrapolated. Also, higher speed turbines than those indicated would only be possible with increased blade or disk stresses.

The variation of turbine mass with pressure ratio is also indicated in Figure 2. For any fixed pressure ratio, turbine mass decreases significantly with increased speed. However, for a constant speed, turbine mass increases with increasing pressure ratio because of the greater number of stages required. Note that the higher pressure ratio turbines have higher allowable maximum speeds before blade stress is exceeded.

Finally, the dotted curves in Figure 2 show the effects of turbine speed on turbine mass for 5 MWe and 20 MWe machines. These curves are for a pressure ratio of 12 with all other design constraints the same as indicated above. As expected, for similar operating conditions lower power machines have higher allowable rotational speeds. Thus, if speed is not constrained, a turbine application may be less massive with several low-power, high-speed machines rather than one much larger power, low-speed turbine. For instance, the minimum specific mass of the 20 MWe, 10 MWe, and 5 MWe turbines each operating at its maximum allowable speed of 8700, 12300, and 17400 rpm respectively, is 0.0453, 0.0358, and 0.0278 Kg/KWe as determined from the information in Figure 2. However, for a fixed speed the less massive turbine arrangement is with a single large turbine as shown in Figure 2. At a fixed speed of 8000 rpm a single 20 MWe turbine might have a mass of 1020 Kg, while two 10 MWe turbines would have a combined mass of 1340 Kg, and four 5 MWe turbines a mass of 1870 Kg. Thus, the speed of the connected load effects the turbine specific mass.

Figure 3 indicates the variation of turbine mass with rotational speed for potassium vapor working fluid turbines. These potassium turbines have the same design parameters as the air turbines in Figure 2 except that the turbine power output is 1 MWe, the disks are cooled to 950 K, the minimum stage blade length is 0.007 m, and the inlet pressure is saturated vapor at 0.963 MPa. The significant decrease in turbine mass with increasing speed is again readily visible from Figure 3. As indicated, turbine mass decreases from over 2000 Kg to about 100 Kg for the 900 K outlet temperature turbine when rotational speed is allowed to increase from 4000 rpm to nearly 22000 rpm.

The turbine outlet temperature indicated in Figure 3 is also the condensing temperature of a potassium Rankine cycle because of the two-phase mixture at the turbine exhaust. Thus, the turbine pressure ratio can be determined from the outlet saturation pressure corresponding to the specified turbine outlet temperature. The turbine pressure ratios in Figure 3 vary from a little less than 2.5 for the 1200 K outlet temperature up to nearly 38 for the 900 K outlet temperature. The variation of turbine mass with pressure ratio shown in Figure 3 is similar to the trend shown previously for the air working fluid turbines. For either working fluid, higher pressure ratio turbines have greater mass at fixed speeds. However, higher pressure ratio turbines can operate at greater speeds without exceeding material allowable stress limits.

Turbine Inlet Pressure and Work Coefficient

Although turbine rotational speed, power output, and pressure ratio are primary mass impacts they are not the only design variables that effect turbine mass. For example, Figure 4 shows the effect of inlet pressures on turbine mass. The results of Figure 4 are for the turbine design constraints and conditions of Figure 2 at a pressure ratio of 12. Note that the higher inlet pressure turbines may operate at much higher maximum speeds, approximately 18000 rpm for the 6 MPa inlet pressure turbines versus 7000 rpm for the 1 MPa turbines. This higher speed results in correspondingly less massive and more compact machines. Even at the same speed, however, higher turbine inlet pressure contributes to reduced turbine mass because the higher pressure working fluid is more dense and results in shorter required blade lengths at each stage.

Figure 4 also shows the turbine mass reduction possible as the stage work coefficient is increased. As can be seen in Figure 4, doubling the stage work coefficient nearly reduces the turbine mass at any constant speed by a factor of two because doubling the stage work coefficient nearly doubles the stage work provided by the working fluid (see Equation (4)) and results in approximately half the number of stages. However, increased stage work coefficients must be balanced against the increased fluid velocities which cause reduced stage efficiencies or the concern for sonic velocities within the turbine.

Brayton and Rankine Turbine Comparison

The realistic and consistent turbine dimensions predicted with this turbine model allow direct comparisons to be made of axial flow turbines for nearly any system operating conditions. The axial flow turbines of both Rankine and Brayton continuous space power systems are compared in Table 1 for 1 MWe and 5 MWe net power output systems. Note that the turbine power outputs must also include any system parasitic loads such as pumps and compressors. For the same sized power systems, Rankine turbines are 3 to 4 times more massive than the corresponding Brayton turbines, providing the turbines are constrained to operate at the same speed and are made of the same nickel superalloy materials. The associated system operating conditions are shown in Table 2.

Both the potassium vapor Rankine and helium-xenon mixture Brayton turbines require cooling to operate at the conditions indicated in Tables 1 and 2. The Rankine turbine disks are cooled to 950 K while the Brayton turbine disks and blades are cooled to 900 K and 1350 K respectively. In addition, the blades of the 5 MWe Rankine turbine must be constructed of higher strength material than nickel superalloy because blade cooling in the two-phase environment of the Rankine turbine is much more difficult than for gas turbines. Thus, the 5 MWe Rankine turbine in Table 1 has TZM blades. If these temperature limits prove unacceptable, the turbine speeds could be reduced but only at the expense of increased turbine mass.

The mass fraction of helium in the Brayton cycle turbines is selected that provides a minimum mass turbine with realistic blade lengths at the required rotational speed. For the systems shown the mass fraction of helium varied between 0.05 and 0.35. At a fixed turbine speed and work coefficient, increasing the helium mass fraction increases the number of stages and mass of the turbine while simultaneously decreasing the individual stage blade lengths. However, helium may be necessary in other parts of the Brayton system for heat transfer and pressure drop benefits.

CONCLUSION

A simplified axial flow turbine sizing and mass model is available that is easy to use and can be applied to nearly any combination of turbine materials and working fluids, including gases and two-phase vapors. The model provides realistic and consistent mass estimates of axial flow turbines for most system operating conditions and has been used to investigate the effect of turbine parameters on space power system mass. Parametric studies reveal that turbine mass decreases significantly with increased rotational speed for all axial flow turbines. Other conditions that effect turbine mass are power level, pressure ratio, work coefficient, and inlet pressure. A comparison of Rankine and Brayton space power systems of 1 MWe to 5 MWe shows that Rankine system potassium vapor axial flow turbines are 3 to 4 times more massive than the corresponding Brayton system helium-xenon turbines operating at the same speed.

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REFERENCES

[1] M.W. Edenburn, "The Effect of Operating Temperature on Open, Multimegawatt Space Power Systems," Trans. 4th Sym. on Space Nuclear Power Systems, CONF-870102-Summs., pp. 427-430, Jan. 1987.

[2] D.G. Wilson, The Design of High-Efficiency Turbomachinery and Gas Turbines, MIT Press, Cambridge, Mass., 1984.

[3] W.C. Reynolds, Thermodynamic Properties in SI, Dept. of Mech. Engr., Stanford Univ., Stanford, CA, 1979.

[4] T. Baumeister and L.S. Marks, eds., Standard Handbook for Mechanical Engineers, 7th ed., McGraw-Hill Book Co., New York, NY, 1967.

[5] R.A. Budenholzer, "Turbine," Encycl. Brit., W.E. Preece et al., eds., William Benton Pub., Chicago, IL, v. 22, pp. 340-357, 1970.

[6] M.J. Moore and C.H. Sieverding, Two-Phase Steam Flow in Turbines and Separators, Hemisphere Pub. Corp., Washington, DC, 1976.

[7] R.J. Roark and W.C. Young, Formulas for Stress and Strain, McGraw-Hill Book Co., New York, NY, 1975.

[8] H. Cohen, G.F.C. Rogers, and H.I.H. Saravanamuttoo, Gas Turbine Theory, 3rd ed., Longman Scientific & Technical, Essex, UK, 1987.

[9] J.H. Horlock, Axial Flow Turbines, Robert E. Krieger Pub. Co., Malabar, FL, 1966.

Table 1. Comparison of Axial Flow Turbines in Rankine and Brayton Space Power Systems.

	1 MWe Systems/15000 rpm Turbines				5 MWe Systems/10000 rpm Turbines			
System Type	Turbine Power MWe	Number of Stages	Average Dia. m	Turbine Mass Kg	Turbine Power MWe	Number of Stages	Average Dia m	Turbine Mass Kg
Rankine	1.03	5	0.39	95	5.15	5	0.63	441
Brayton (Simple)	2.0	3	0.24	21	10.0	3	0 60	148
Brayton (Recuperated)	1.67	4	0.25	28	8.33	3	0 62	157

Table 2. Space Power System Operating Conditions.

Parameter	Potassium Rankine	Simple Brayton	Recuper. Brayton
Inlet Temperature, K	1350	1500	1500
Inlet Pressure, MPa	0.963 (Sat.)	4.0	4.0
Pressure Ratio	12.6	3.5	2.0
Work Coefficient, ψ	1.0	2.2	2.0
Turbine Efficiency, %	85	90	90

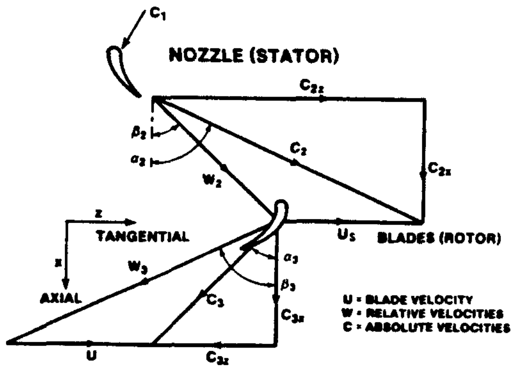

Figure 1. 50% Reaction Axial Flow Turbine Stage Velocity Diagram.

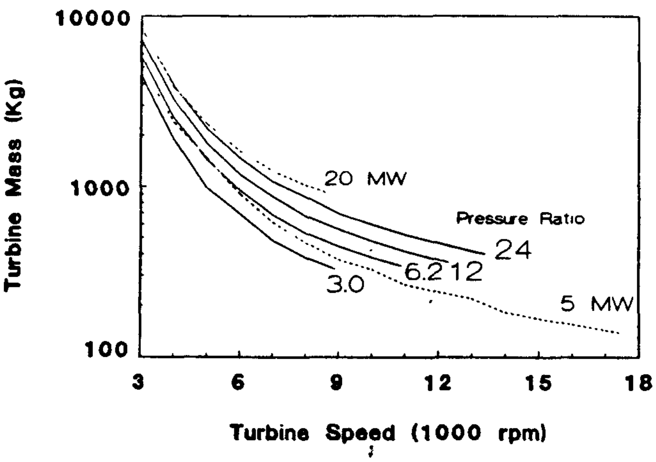

Figure 2. Effect of Rotational Speed on 10 MWe Turbine Mass (Air Working Fluid).

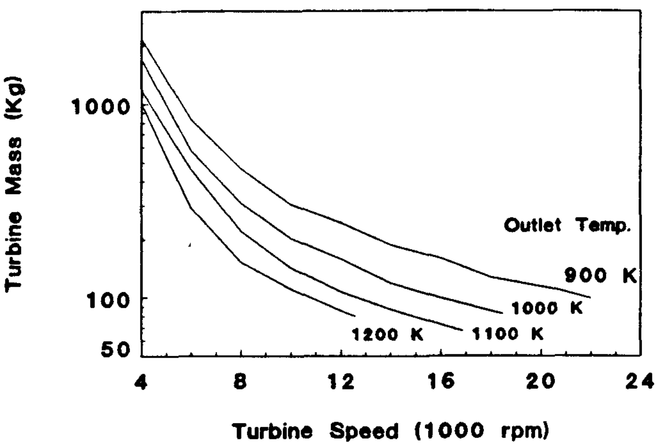

Figure 3. Effect of Rotational Speed on 1 MWe Turbine Mass (Potassium Vapor Working Fluid).

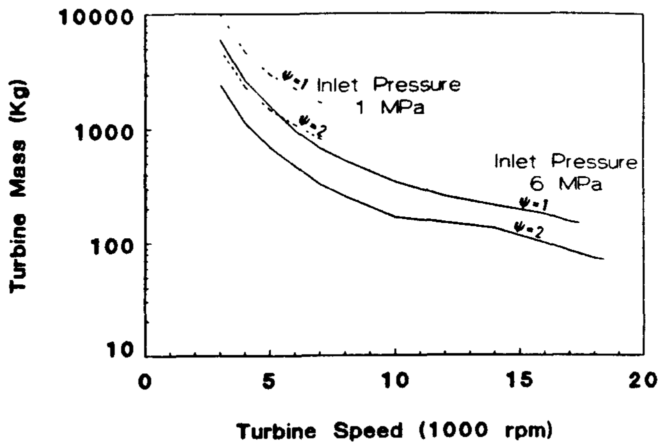

Figure 4. Effect of Inlet Pressure on 10 MWe Turbine Mass (Air Working Fluid).