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NEW INFINITE-DIMENSIONAL ALGEBRAS, SINE BRACKETS, AND SU(∞)

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Abstract

We investigate the infinite dimensional algebras we have previously introduced, which involve trigonometric functions in their structure constants. We find a realization for them which provides a basis-independent formulation, identified with the algebra of sine brackets. A special family of them, the cyclotomic ones, contain SU(N) as invariant subalgebras. In this basis, it is evident by inspection that the algebra of SU(∞) is equivalent to the centerless algebra of SDiff_0 on two-dimensional manifolds. Gauge theories of SU(∞) are thus simply reformulated in terms of surface (sheet) coordinates. Spacetime-independent configurations of their gauge fields describe strings through the quadratic Schild action.

Recently, we have introduced infinite-dimensional algebras involving trigonometric functions in their structure constants[1]. We shall discuss some of their intriguing properties and relevance to large N and string physics. The generators of the algebras we have introduced are indexed by 2-vectors $\mathbf{m} = (m_1, m_2)$. The components of these vectors do not have to be integers to satisfy the Jacobi identities, but we take them to be integral in what follows for the sake of interpreting them as Fourier modes:

$$[K_{\mathbf{m}}, K_{\mathbf{n}}] = r \sin k(\mathbf{m}\times\mathbf{n})\; K_{\mathbf{m}+\mathbf{n}} + \mathbf{a}\cdot\mathbf{m}\; \delta_{\mathbf{m}+\mathbf{n},0}\ , \tag{1}$$

where $\mathbf{m}\times\mathbf{n} = m_1 n_2 - m_2 n_1$, r and k are arbitrary (complex) constants, and $\mathbf{a}$ is an arbitrary 2-vector .

These algebras include as a special case that of $\mathrm{SDiff}_0(T^2)$, the infinitesimal area-preserving diffeomorphisms of the torus[2,3], which emerges as a residual symmetry of the

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membrane when gauge-fixed on the light-cone. Taking $r = 1/k$ in the limit $k \to 0$ yields the algebra

$$[L_{\mathbf{m}}, L_{\mathbf{n}}] = (\mathbf{m} \times \mathbf{n}) L_{\mathbf{m}+\mathbf{n}} + \mathbf{a} \cdot \mathbf{m} \ \delta_{\mathbf{m}+\mathbf{n},0} \ . \tag{2}$$

The representation theory of these algebras is an interesting open problem.

The supersymmetric extension of our algebra (1) is

$$\begin{aligned} [K_{\mathbf{m}}, K_{\mathbf{n}}] &= r \ \sin k(\mathbf{m} \times \mathbf{n}) \ K_{\mathbf{m}+\mathbf{n}}, \\ [K_{\mathbf{m}}, F_{\mathbf{n}}] &= r \ \sin k(\mathbf{m} \times \mathbf{n}) \ F_{\mathbf{m}+\mathbf{n}}, \\ \{F_{\mathbf{m}}, F_{\mathbf{n}}\} &= s \ \cos k(\mathbf{m} \times \mathbf{n}) \ K_{\mathbf{m}+\mathbf{n}}, \end{aligned} \tag{3}$$

where the $F_{\mathbf{n}}$ are fermionic generators. The Jacobi identity which involves two F's and one K dictates that no non-trivial center be present. If a pure Grassmann c-number ϵ were available, however, then there might be a center $\epsilon \mathbf{a} \cdot \mathbf{m} \ \delta_{\mathbf{m}+\mathbf{n},0}$ in the $[K_{\mathbf{m}}, F_{\mathbf{n}}] = -[K_{\mathbf{n}}, F_{\mathbf{m}}]$ commutator, as occurs in other algebras[3,5].

Choosing $s = 1$, $r = 1/k$, and taking the $k \to 0$ limit yields the supersymmetric generalization of (3) given in ref.[6]:

$$[L_{\mathbf{m}}, L_{\mathbf{n}}] = (\mathbf{m} \times \mathbf{n}) L_{\mathbf{m}+\mathbf{n}} \ , \quad [L_{\mathbf{m}}, G_{\mathbf{n}}] = (\mathbf{m} \times \mathbf{n}) G_{\mathbf{m}+\mathbf{n}}, \quad \{G_{\mathbf{m}}, G_{\mathbf{n}}\} = L_{\mathbf{m}+\mathbf{n}} \ . \tag{4}$$

The algebra (2) is, in a particular basis optimal for the torus, that of the generic area-preserving (symplectic) reparameterizations of a 2-surface. Taking x and p to be local (commuting) coordinates for the surface, and f and g to be differentiable functions of them, a basis-independent realization for the generators of the centerless algebra is[2]:

$$L_f = \frac{\partial f}{\partial x} \frac{\partial}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial}{\partial x} \quad \Longrightarrow \quad [L_f, L_g] = L_{\{f,g\}} \ , \quad [L_f, g] = \{f, g\} \ , \tag{5}$$

where

$$\{f, g\} \equiv \frac{\partial f}{\partial x} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial x} \ , \tag{6}$$

the Poisson bracket of classical phase-space. The generator L_f transforms (x, p) to $(x - \partial f/\partial p \ , \ p + \partial f/\partial x)$. Infinitesimally, this is a canonical transformation[7] which preserves the phase-space area element $dxdp$. For a small patch of 2-surface, we may expand the functions $f(x, p)$ in any coordinate basis we choose. If the surface is a torus, we shall prefer a globally adequate coordinate system, such as $\exp(inx + imp)$; if it is a sphere, spherical harmonics; if it is a plane, powers; and so on. Nevertheless, for the infinitesimal transformations effected by the algebra generators in a patch, *any* coordinate basis will do, and may be transformed to other ones. (When such transformations are singular, however, a number of generators may be lost, leading to a subalgebra, as noted by Pope and Stelle, and Hoppe[8].)

Choosing the torus basis, $f = -e^{i(m_1x+m_2p)}$ and $g = -e^{i(n_1x+n_2p)}$, $0 \leq x, p \leq 2\pi$, yields

$$L_f = L_{(m_1,m_2)} = -ie^{i(m_1x+m_2p)}(m_1\partial/\partial p - m_2\partial/\partial x)\ , \tag{7}$$

which obey the centerless algebra in the basis (2). Conversely, given the basis (2), any function $f(x,p)$ can be reconstituted through

$$f(x,p) = -\sum_{m_1,m_2} F(m_1,m_2)e^{i(m_1x+m_2p)}\ , \tag{8}$$

and thus the linear combinations

$$L_f = \sum_{m_1,m_2} F(m_1,m_2)L_{(m_1,m_2)} \tag{9}$$

are seen to obey the Poisson-bracket algebra (5).

We have found an analogous realization for the torus-basis algebra (1) generators:

$$\begin{aligned} K_{(m_1,m_2)} &= (ir/2)\exp(im_1x + km_2\frac{\partial}{\partial x} + im_2p - km_1\frac{\partial}{\partial p}) \\ &= (ir/2)\exp(im_1x + im_2p)\ \exp(km_2\frac{\partial}{\partial x} - km_1\frac{\partial}{\partial p})\ , \end{aligned} \tag{10}$$

somewhat analogous to the one-variable realization found by Hoppe[3]. A corresponding realization for the fermionic generators of (3) is

$$F_{\mathbf{m}} = K_{\mathbf{m}}(\theta + \frac{is}{r}\frac{\partial}{\partial\theta})\ , \tag{11}$$

for a Grassmann coordinate θ and the above realization (10) for K.

To Fourier-compose this to a basis-independent realization, we first define, in analogy to (9),

$$K_f \equiv \sum_{m_1,m_2} F(m_1,m_2)K(m_1,m_2) \equiv \frac{r}{2i}f(x + ik\frac{\partial}{\partial p}, p - ik\frac{\partial}{\partial x})\ , \tag{12}$$

where the last side of the equation is a formal expression to evoke (5): the "normal ordering" of its derivatives is specified in its Fourier-series definition, in which they stand to the right of all coordinates, by virtue of eq. (10). For a superfield $V(x,p,\theta) = f(x,p) + \theta\, g(x,p)$, the above extends to

$$V(x+ik\frac{\partial}{\partial p}, p-ik\frac{\partial}{\partial x}, \theta+\frac{is}{r}\frac{\partial}{\partial\theta}) = f(x+ik\frac{\partial}{\partial p}, p-ik\frac{\partial}{\partial x})+(\theta+\frac{is}{r}\frac{\partial}{\partial\theta})\, g(x+ik\frac{\partial}{\partial p}, p-ik\frac{\partial}{\partial x}). \tag{13}$$

This then yields (10) and (11) for components of f and g, respectively, in the above exponential basis. Moreover, assembling bosonic and fermionic coordinates on the same footing and denoting all by z^i, this generalizes suggestively to $V(z^i + A_{ij}\partial/\partial z^i)$, where the matrix A_{ij} is antisymmetric for both i,j corresponding to bosonic coordinates, and symmetric for fermionic coordinates.

What is the analogue of the Poisson bracket in this case? It turns out to be the *sine,* or *Moyal, bracket* $\{\{f,g\}\}$, namely the extension of the Poisson brackets $\{f,g\}$ to statistical distributions on phase-space, introduced by Weyl[7] and Moyal[9b], and explored by several authors[9] in an alternative formulation of quantum mechanics, regarded as a deformation of the algebra of classical observables. It is a generalized convolution which reduces to the Poisson bracket as $\hbar$, replaced by $2k$ in our context, is taken to zero:

$$\{\{f,g\}\} = \frac{-r}{4\pi^2k^2}\int dp'dp''dx'dx''\; f(x',p')g(x'',p'')\sin\frac{1}{k}\Big(p(x'-x'')+x(p''-p')+p'x''-p''x'\Big)\;. \tag{14}$$

The argument of the sine above is

$$\frac{1}{k}\det\begin{pmatrix} 1 & p & x \\ 1 & p' & x' \\ 1 & p'' & x'' \end{pmatrix} = \frac{1}{k}\int p\cdot dq\;, \tag{15}$$

i.e. $2/k$ times the area of the equilateral phase-space triangle with vertices at $(x,p), (x',p')$, and (x'',p''). The antisymmetry of f with g is evident in the determinant. A similar entity is the cosine bracket, introduced by Baker[9c],

$$(f,g) = \frac{-r}{4\pi^2k^2}\int dp'dp''dx'dx''\; f(x',p')g(x'',p'')\cos\Big(\frac{1}{k}(p(x'-x'')+x(p''-p')+p'x''-p''x')\Big)\;. \tag{16}$$

The sine and cosine brackets together satisfy the graded Jacobi identities[9d], just as their Fourier components (3) (see the next paragraph) do, and thus specify a graded Lie algebra. These brackets help reformulate quantum mechanics in terms of Wigner's phase-space distribution[9].

The Fourier transform of the sine bracket results from substitution in (14) of the exponential basis used in (7):

$$\{\{f,g\}\} = \frac{-ir}{8\pi^2k^2}\int dp'dp''dx'dx''\ e^{i(m_1x'+n_1x'')+i(m_2p'+n_2p'')}\left(e^{\frac{i}{k}(p(x'-x'')+x(p''-p')+p'x''-p''x')}-(k\leftrightarrow -k)\right)$$

$$= -r\sin k(\mathbf{m}\times\mathbf{n})\ e^{i(m_1+n_1)x+i(m_2+n_2)p}\ . \tag{17}$$

As in (9), it then follows through the linearity of the operators defined in (12), and (1), that these indeed obey the algebra

$$[K_f,K_g] = r\sum_{m_1,m_2,n_1,n_2} F(m_1,m_2)G(n_1,n_2)\sin k(\mathbf{m}\times\mathbf{n})\ K_{\mathbf{m}+\mathbf{n}} = K_{\{\{f,g\}\}}\ . \tag{18}$$

Our algebra is thus identified with that of sine brackets. You might wish to expand it in other bases, such as spherical harmonics, so as to specify the corresponding generalizations of $\mathrm{SDiff}_0(S^2)$, and so on. The corresponding anticommutation relation for the supersymmetric extension built out of Fourier-composing (11) naturally involves the cosine bracket.

Focus now on an interesting centerless family of the algebras (1), namely the *cyclotomic* family: the one for which $k=2\pi/N$, for integer $N>2$. In this family, there is an additional $\mathbb{Z}\times\mathbb{Z}$ algebra isomorphism

$$K_{(m_1,m_2)} \longmapsto K_{(m_1,m_2)+(Nt,Nq)} \tag{19}$$

for arbitrary integers t and q. Since the structure constants $\sin\frac{2\pi}{N}(m_1n_2-n_1m_2)$ are only sensitive to the modulo-N values of the indices, the 2-dimensional integer lattice separates into $N\times N$ cells, each of which may be referred to some fundamental cell, e.g. around the coordinate center of the lattice, by proper N-translations. The fundamental $N\times N$ cell contains N^2 index points, but the operator $K_{(0,0)}$, like its lattice translations $K_{N(t,q)}$, factors out of the algebra: it commutes with all K's and cannot result as a commutator of any two such. Thus the fundamental cell involves only N^2-1 generators, and there are no more structure constants than those occurring in this cell. In consequence, the infinite-dimensional centerless cyclotomic algebras, with the $K_{N(t,q)}$'s factored out, possess the following finite-dimensional ideal of "lattice average" operators $\mathcal{K}$:

$$\mathcal{K}_{(m_1,m_2)} \equiv \sum_{s,v} K_{(m_1+Ns,m_2+Nv)}, \qquad [\mathcal{K}_{\mathbf{m}},\mathcal{K}_{\mathbf{n}}] = r\sin\frac{2\pi}{N}(\mathbf{m}\times\mathbf{n})\ \mathcal{K}_{\mathbf{m}+\mathbf{n}}, \tag{20}$$

where $\mathbf{m}$,$\mathbf{n}$,$\mathbf{m}+\mathbf{n}$ are indices in the fundamental cell, and an infinite normalization has been absorbed in r.

This (N^2-1)-dimensional ideal specifies, in fact, a basis for SU(N) which may be thought of as a generalization of the Pauli matrices[10]. For brevity, consider odd N's, and consult our paper for even ones. A basis for SU(N) algebras, odd N, may be built from two unitary unimodular matrices:

$$g \equiv \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & \omega & 0 & \dots & 0 \\ 0 & 0 & \omega^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \omega^{N-1} \end{pmatrix} , \quad h \equiv \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & \dots & 0 \end{pmatrix} , \quad g^N = h^N = \mathbf{1} , \qquad (21)$$

where ω is a primitive N'th root of unity, i.e. with period no smaller than N, here taken to be $e^{4\pi i/N}$. They obey the identity

$$hg = \omega\, gh. \qquad (22)$$

You also encounter these matrices in the context of representations of quantum SU(2) – cf. the talk by D.B.F. The complete set of unitary unimodular $N \times N$ matrices

$$J_{(m_1,m_2)} \equiv \omega^{m_1 m_2/2}\, g^{m_1} h^{m_2} , \qquad (23)$$

where

$$J^\dagger_{(m_1,m_2)} = J_{(-m_1,-m_2)} \; ; \qquad \mathrm{Tr} J_{(m_1,m_2)} = 0 \quad \text{except for} \quad m_1 = m_2 = 0 \bmod N , \qquad (24)$$

suffice to span the algebra of SU(N). Like the Pauli matrices, they close under multiplication to just one such, by virtue of (22):

$$J_{\mathbf{m}} J_{\mathbf{n}} = \omega^{\mathbf{n}\times\mathbf{m}/2} J_{\mathbf{m}+\mathbf{n}} \, . \qquad (25)$$

They therefore satisfy the algebra

$$[J_{\mathbf{m}}, J_{\mathbf{n}}] = -2i \sin \frac{2\pi}{N}(\mathbf{m}\times\mathbf{n})\; J_{\mathbf{m}+\mathbf{n}} \, . \qquad (26)$$

Consequently, in this convenient two-index basis with the above simple structure constants, SU(N) describes the algebra (20) of the ideal $\{K\}$.

The symmetric d-coefficients in this basis also follow simply from (25):

$$\{J_{\mathbf{m}}, J_{\mathbf{n}}\} = 2 \cos \frac{2\pi}{N}(\mathbf{m}\times\mathbf{n})\; J_{\mathbf{m}+\mathbf{n}}, \qquad (27)$$

and consequently the same matrices may represent the lattice averages of both boson and fermion operators $\mathcal{F}_{\mathbf{n}}$, when these considerations are applied to the obvious corresponding ideal of the supersymmetric algebra (3). (It is the (f,d) subalgebra of SU($N|N$)).

The 2-index SU(N) basis we have considered has a particularly simple large N limit. As N increases, the fundamental $N \times N$ cell covers the entire index lattice; the operators $\mathcal{K}$ supplant the K's and, in turn, since $k \to 0$, the operators L of eq.(2).

More directly, you immediately see by inspection that, as $N \to \infty$, the SU(N) algebra (26) goes over to the centerless algebra (2) of SDiff$_0(T^2)$ through the identification:

$$\frac{iN}{4\pi} J_{\mathbf{m}} \to L_{\mathbf{m}} \, . \tag{28}$$

An identification of this type was first noted by Hoppe[3] in the context of membrane physics: he connected the infinite N limit of the SU(N) algebra in a special basis to that of SDiff$_0(S^2)$, i.e. the infinitesimal symplectic diffeomorphisms in the sphere basis. A discussion of the group topology of SU(N), or SDiff$_0(T^2)$ versus SDiff$_0(S^2)$, or other 2-dimensional manifolds for that matter, exceeds the scope of this type of local analysis, important as it may be for membrane physics applications; such a discussion has been initiated by Pope and Stelle[8], who consider central extensions that are sensitive to global features of the 2-surface.

Floratos et al.[4] utilized Hoppe's identification to take the limit of SU(N) gauge theory. Their results are immediately reproduced, again by inspection, on thee basis of the orthogonality condition dictated by (24) and (25):

$$\mathrm{Tr} J_{\mathbf{m}} J_{\mathbf{n}} = N \delta_{\mathbf{m}+\mathbf{n},0} \;\; \to \;\; \mathrm{Tr} L_{\mathbf{m}} L_{\mathbf{n}} = -\frac{N^3}{(4\pi)^2} \delta_{\mathbf{m}+\mathbf{n},0} \, . \tag{29}$$

As a result, for a gauge field A_μ in an SU(N) matrix normalization with trace 1, the analog of eq. (9) is

$$A_\mu \equiv A_\mu^{\mathbf{m}} \frac{J_{\mathbf{m}}}{\sqrt{N}} \;\; \to \;\; \frac{4\pi}{iN^{3/2}} A_\mu^{\mathbf{m}} L_{\mathbf{m}} = \tilde{A}_\mu^{\mathbf{m}} L_{\mathbf{m}} \, , \tag{30}$$

where summation over repeated $\mathbf{m}$'s is implied, and we have defined $\tilde{A}_\mu^{\mathbf{m}} \equiv (4\pi/iN^{3/2}) A_\mu^{\mathbf{m}}$. As $N \to \infty$, the indices $\mathbf{m}$ cover the entire integer lattice, and hence we may define

$$a_\mu^{(x,p)} \equiv -\sum_{\mathbf{m}} \tilde{A}_\mu^{\mathbf{m}} e^{i(m_1 x + m_2 p)} \, . \tag{31}$$

By eq. (5),

$$[A_\mu, A_\nu] \;\; \to \;\; [L_{a_\mu}, L_{a_\nu}] = L_{\{a_\mu, a_\nu\}} \, . \tag{32}$$

Hence, by virtue of the linearity of L in its arguments,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \;\rightarrow\; L_{f_{\mu\nu}}$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu + \{a_\mu, a_\nu\} \;. \tag{33}$$

The group trace defining the Yang-Mills lagrangian density is thus

$$\mathrm{Tr} F_{\mu\nu} F_{\mu\nu} \rightarrow -\frac{N^3}{(4\pi)^2} \tilde{F}^{\mathbf{m}}_{\mu\nu} \tilde{F}^{-\mathbf{m}}_{\mu\nu} = \frac{-N^3}{64\pi^4} \int dx dp \sum_{m_1, m_2, n_1, n_2} e^{ix(m_1+m_2)+ip(m_2+n_2)} \tilde{F}^{(m_1,m_2)}_{\mu\nu} \tilde{F}^{(n_1,n_2)}_{\mu\nu}$$

$$= (-N^3/64\pi^4) \int dx dp \; f^{(x,p)}_{\mu\nu} f^{(x,p)}_{\mu\nu} \;. \tag{34}$$

What emerges is a gauge theory whose group indices are surface (torus) coordinates, and the fields are rescaled Fourier transforms of the original SU(N) fields; the group composition rule for them is given by the Poisson bracket, and the trace by surface integration.

Further note that a connection to strings emerges: for gauge fields independent of x^μ (e.g. vacuum configurations), this lagrangian density reduces to $\{a_\mu, a_\nu\}\{a_\mu, a_\nu\}$, the quadratic Schild-Eguchi action density for strings[11], where the a_μ now serve as string variables, and the surface serves as the world-sheet. Whether a superstring follows analogously from super-Yang-Mills is an interesting question.

The lagrangian (34) with the sine bracket supplanting the Poisson bracket is also a gauge invariant theory, provided that the gauge transformation involves the sine instead of the Poisson bracket. This is provable through the identities

$$\int dx dp \; \{\{f, g\}\} = 0, \qquad \int dx dp \; (f, g) = \int dx dp \; f g \;, \tag{35}$$

for arbitrary functions f and g, and use of the Jacobi identities. It is not however clear, at the moment, what system is described by the corresponding spacetime-independent lagrangian density $\{\{a_\mu, a_\nu\}\}\{\{a_\mu, a_\nu\}\}$.

This compact formulation of SU(∞) gauge theory ought to be of use in large-N model calculations, or various "master-field" efforts; membrane physics[2,3,6], as covered in this conference; and the exploration of connections between gauge theory and strings[11], as demonstrated above.

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