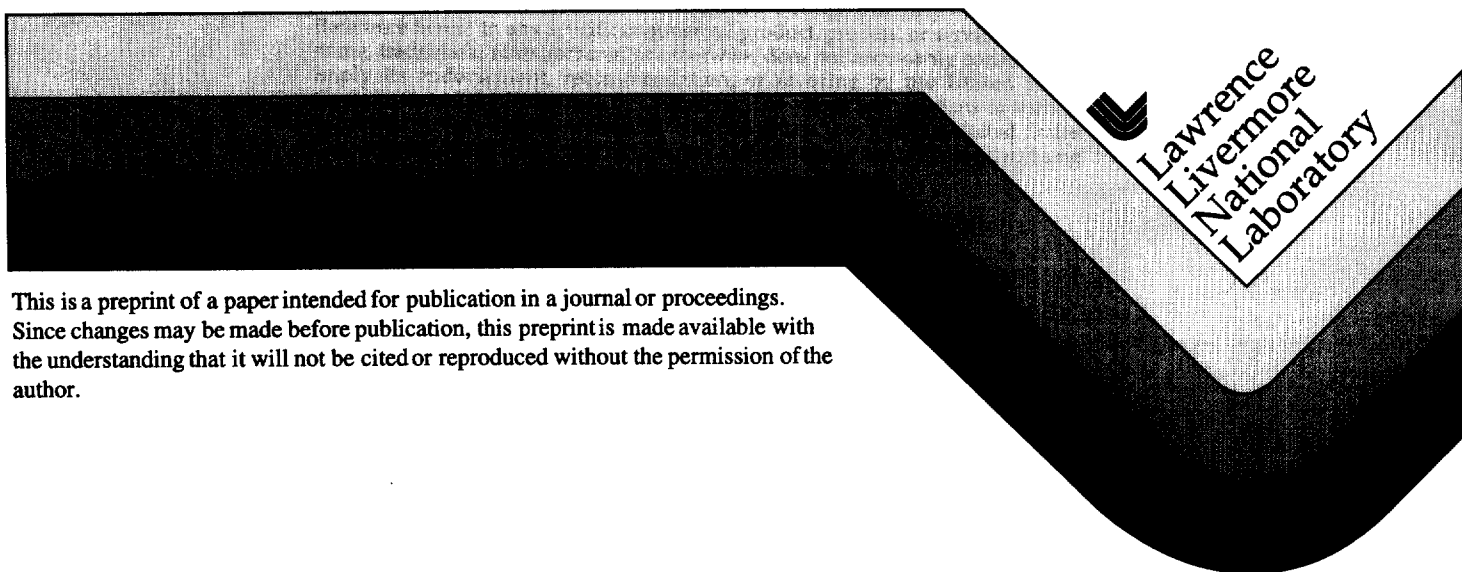# Integration of Geologic Interpretation into Geostatistical Simulation

Steven F. Carle

This paper was prepared for submittal to the
International Association for Mathematical Geology
Barcelona, Spain
September 22-27, 1997

June 1997

DISCLAIMER

INTEGRATION OF GEOLOGIC INTERPRETATION INTO GEOSTATISTICAL SIMULATION

Steven F. Carle
Lawrence Livermore National Laboratory, L-206
Livermore, California, 94551, USA

SUMMARY. Embedded Markov chain analysis has been used to quantify geologic interpretation of juxtapositional tendencies of geologic facies. Such interpretations can also be translated into continuous-lag Markov chain models of spatial variability for use in geostatistical simulation of facies architecture.

1. INTRODUCTION

Traditional indicator geostatistical approaches rely on empirical curve-fitting of indicator variograms to develop models of spatial variability from detailed data sets or "training images." However, most real-world data sets characterize spatial variability adequately in the vertical direction at best. Direct means are needed for infusing subjective knowledge of facies architecture into the conditional simulation process, so that the resulting "realizations" are indeed realistic.

Alternatively, indicator geostatistics can be recast in a conceptually simple yet theoretically powerful transition probability/Markov framework. Model parameters directly relate to fundamental properties of proportions, mean length, and juxtapositional tendencies. Mathematically, Markov chains consist of linear combinations exponential functions, although a spectrum of model shapes can be produced, including "hole effect" and Gaussian-like structures. Asymmetric juxtapositional patterns such as fining-upward tendencies common to fluvial depositional systems can be considered. An example is developed for a hypothetical fluvial system with *no data* to demonstrate possibilities for implementing geostatistics in situations of sparse data.

2. EMBEDDED MARKOV CHAIN ANALYSIS

Markov chain analysis has been used by geologists since Vistelius in 1949 [13] for quantitative interpretation of juxtapositional tendencies in vertical stratigraphic successions and, more specifically, to address various questions arising in geologic interpretation, for example:

- Does a vertical sequence exhibit a fining-upward tendency?
- Are lateral juxtapositional tendencies similar to those in the vertical (i.e., Walther's Law)?
- What is the degree of order (vs. disorder) in the juxtapositional tendencies?

Now consider a fluvial system consisting of five facies:

1 = lower channel (*lc*)
2 = upper channel (*uc*)
3 = levee/crevasse splay (*lcs*)
4 = floodplain (*fl*)
5 = peat (*pt*)

Embedded Markov chain analysis [5,6,9,10] evaluates the probabilities of one geologic unit occurring adjacent to another in a particular direction, say vertical (z), in terms of

transition frequencies $\phi_{jk,z}$

$$\phi_{45,z} = \Pr\{fl \text{ occurs below and } pt \text{ occurs above}\}$$

or transition (conditional) probabilities $\pi_{jk,z}$

$$\pi_{45,z} = \Pr\{pt \text{ occurs above} \mid fl \text{ occurs below}\}$$

For example, a classical fining-upward tendency of

$$lc \rightarrow uc \rightarrow lcs \rightarrow fl \rightleftarrows pt \qquad (1)$$

would be evident in a vertical embedded transition probability matrix $\mathbf{\Pi}_z$ of

$$\mathbf{\Pi}_z = \begin{bmatrix} \pi_{11,z} & \cdots & \pi_{1K,z} \\ \vdots & \ddots & \vdots \\ \pi_{K1,z} & \cdots & \pi_{KK,z} \end{bmatrix} = \begin{bmatrix} - & 1.0 & 0 & 0 & 0 \\ 0.005 & - & 0.600 & 0.395 & 0 \\ 0.005 & 0.020 & - & 0.975 & 0 \\ 0.071 & 0.036 & 0.570 & - & 0.324 \\ 0.020 & 0 & 0 & 0.980 & - \end{bmatrix}$$

3. MAXIMUM DISORDER

The juxtapositional tendencies in a geologic system reflect some degree of order (or disorder) in the stratigraphy [6]. Indeed, the disorder in a particular direction, say z, can be measured by the entropy S_z of the transition frequencies $\phi_{jk,z}$

$$S_z = -\sum_j \sum_k \phi_{jk,z} \ln \phi_{jk,z} \qquad (2)$$

Given the frequencies of the embedded occurrences $\phi_{j,z}$

$$\phi_{j,z} = \sum_k \phi_{jk,z} = \sum_k \phi_{kj,z} \qquad (3)$$

a system of maximally disordered juxtapositional tendencies can be found by maximizing (2) subject to (3) using iterative proportion fitting (IPF) [7]. A "maximum entropy" transition probability matrix $\mathbf{\Pi}_z^{(S\max)}$ can then be obtained by dividing the maximum entropy transition frequencies by their row sums

$$\mathbf{\Pi}_z^{(S\max)} = \begin{bmatrix} - & 0.028 & 0.143 & 0.763 & 0.066 \\ 0.016 & - & 0.145 & 0.773 & 0.066 \\ 0.018 & 0.032 & - & 0.874 & 0.075 \\ 0.062 & 0.111 & 0.566 & - & 0.260 \\ 0.016 & 0.030 & 0.150 & 0.804 & - \end{bmatrix}$$

The fining-upward tendency is evident in $\mathbf{\Pi}_z$ because the $\pi_{j(j+1)}$ entries are greater than in $\mathbf{\Pi}_z^{(S\max)}$.

4. CONTINUOUS-LAG MARKOV CHAIN MODELS

Markov chain models can also be constructed with a spatial dependency to formulate a geostatistical model of spatial variability [3,8,9]. A transition probability matrix $\mathbf{T}(h)$ is constructed as a function of time or distance separation or "lag" h. Under a Markov assumption, the probability of a category k occurring at a location x depends on the transition probability matrix $\mathbf{T}(\Delta h)$ for the nearest datum located at $x - \Delta h$. Spatial Markov chains are usually formulated in the discrete form by successive multiplication of $\mathbf{T}(\Delta h)$, which has limited applicability because it depends on a regular data spacing.

A more general mathematical expression of a Markov chain model is given by the continuous-lag form

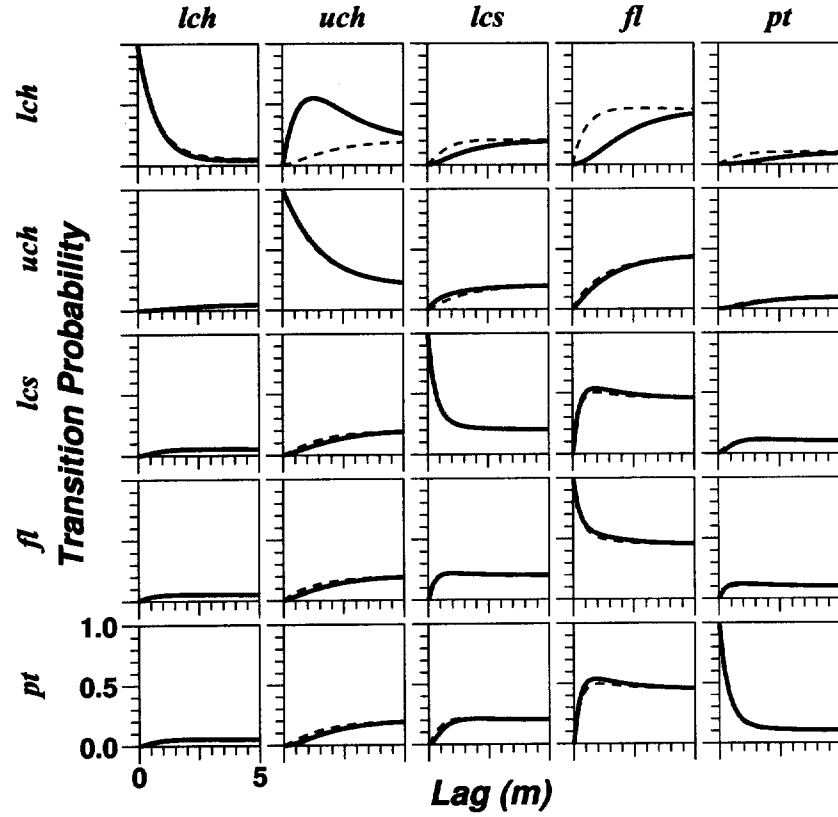$$\mathbf{T}(h) = \exp[\mathbf{R}h] \qquad (4)$$

Figure 1: Matrix of transition probabilities for vertical direction showing interpreted (solid lines) and maximum entropy (dashed lines) continuous-lag Markov chain models.

a vertical transition rate matrix $\mathbf{R}_z$ can be established from $\mathbf{\Pi}_z$ and (7) as

$$\mathbf{R}_z = \begin{bmatrix} -1.2500 & 1.2500 & 0 & 0 & 0 \\ 0.0025 & -0.5000 & 0.3000 & 0.1975 & 0 \\ 0.0125 & 0.0500 & -2.5000 & 2.4375 & 0 \\ 0.1211 & 0.0611 & 0.9778 & -1.7156 & 0.5556 \\ 0.0500 & 0 & 0 & 2.4500 & -2.500 \end{bmatrix} \ \mathrm{m}^{-1}$$

which yields the continuous-lag Markov chain model shown in Figure 1. Similarly, the maximum entropy embedded transition probability matrix can be transformed to obtain a "maximum entropy" continuous-lag Markov chain model, also shown in Figure 1.

The column and row summing constraints (5) and (6) eliminate the need to specify entries for one row and column. If symmetry is assumed for a particular cross-relationship, then the relationship

$$r_{jk} = \left(\frac{p_k}{p_j}\right) r_{kj} \tag{8}$$

holds, so that only one of the opposing upper or lower off-diagonal entries needs to be specified. For example, a strike (x)-direction $\mathbf{\Pi}_x$ could be developed conceptually as

$$\mathbf{\Pi}_x = \begin{bmatrix} - & 0.87 & 0.07 & (6) & 0.01 \\ (8) & - & (8) & (6) & 0 \\ (8) & 0.50 & - & (6) & 0 \\ (5) & (5) & (5) & (5) & (5) \\ (8) & (8) & (8) & (6) & - \end{bmatrix}$$

where the entries in parentheses indicate the equations applied. Assuming proportions

as above and x-direction mean lengths of (10, 20, 20, *, 50) meters, the corresponding transition rate matrix $\mathbf{R}_x$ becomes

$$\mathbf{R}_x = \begin{bmatrix} -0.1000 & 0.0870 & 0.0070 & 0.0050 & 0.0010 \\ 0.0218 & -0.0500 & 0.0200 & 0.0083 & 0 \\ 0.0018 & 0.0200 & -0.0400 & 0.0183 & 0.0000 \\ 0.0006 & 0.0037 & 0.0081 & -0.0167 & 0.0433 \\ 0.0005 & 0 & 0 & 0.0195 & -0.0200 \end{bmatrix}$$

Two- or 3-D Markov chain models can then be developed by ellipsoidally interpolating transition rates [4] so that spatial variability in any one direction is modeled by a 1-D Markov chain [11].
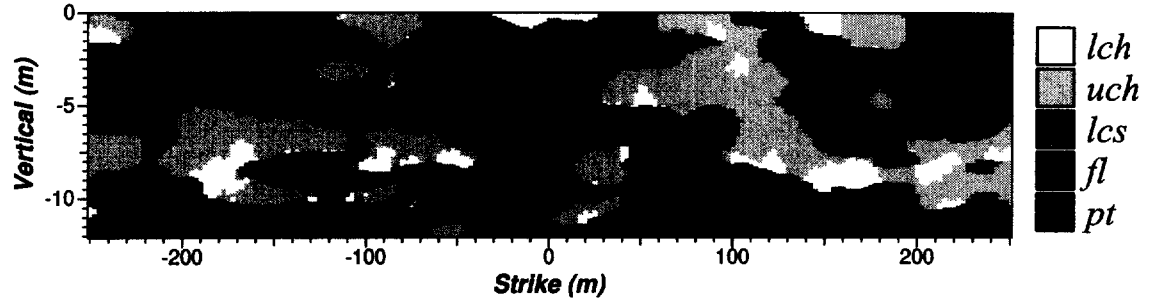

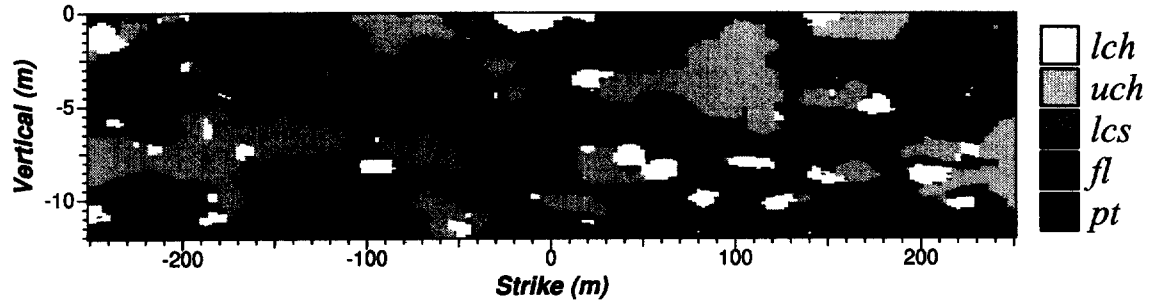Figure 2. Geostatistical simulation based on interpreted Markov chain models.

Figure 3. Geostatistical simulation based on maximum entropy juxtapositional tendencies.

6. GEOSTATISTICAL SIMULATION

The 3-D Markov chain model can be used to formulate cokriging estimates [2,3] and objective functions used in the implementation of sequential indicator simulation and simulated quenching (zero-temperature annealing) geostatistical simulation algorithms [4], respectively. Figure 2 shows a perspective view of a 3-D "realization" resulting from the interpreted 3-D Markov chain model. Fining upward tendencies as per (1) are clearly evident, as are juxtapositional tendencies of

- lcs occurring laterally adjacent to uch
- lch occurring below uch

This geologically-plausible facies architecture was originally prescribed in the embedded transition probability matrix and carried through to end result of the geostatistical simulation process. As a comparison, Markov chains with "maximum entropy" juxtapositional tendencies were used to implement the geostatistical simulation procedure to create the realization shown in Figure 3. The main difference between the two realizations appears to be the location of lch, which is reflected in the difference between the interpreted and

as above and x-direction mean lengths of (10, 20, 20, *, 50) meters, the corresponding transition rate matrix $\mathbf{R}_x$ becomes

$$\mathbf{R}_x = \begin{bmatrix} -0.1000 & 0.0870 & 0.0070 & 0.0050 & 0.0010 \\ 0.0218 & -0.0500 & 0.0200 & 0.0083 & 0 \\ 0.0018 & 0.0200 & -0.0400 & 0.0183 & 0.0000 \\ 0.0006 & 0.0037 & 0.0081 & -0.0167 & 0.0433 \\ 0.0005 & 0 & 0 & 0.0195 & -0.0200 \end{bmatrix}$$

Two- or 3-D Markov chain models can then be developed by ellipsoidally interpolating transition rates [4] so that spatial variability in any one direction is modeled by a 1-D Markov chain [11].
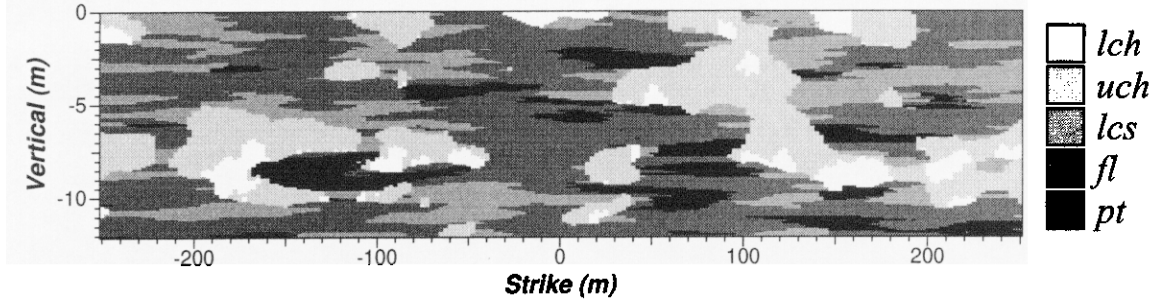

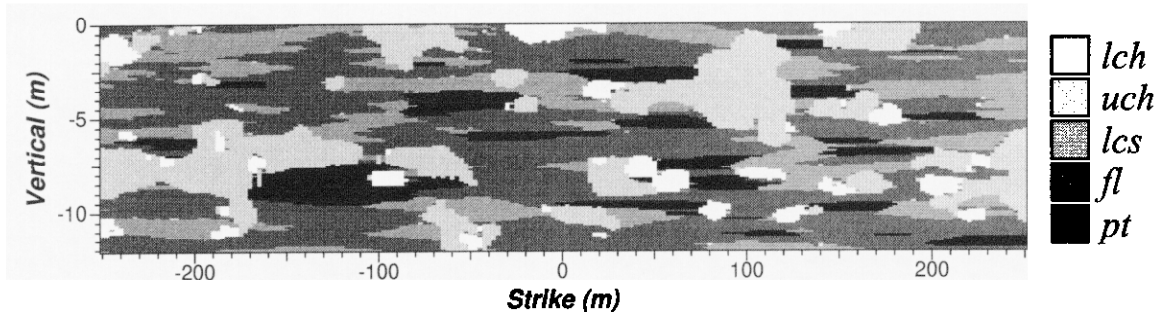Figure 2. Geostatistical simulation based on interpreted Markov chain models.

Figure 3. Geostatistical simulation based on maximum entropy juxtapositional tendencies.

6. GEOSTATISTICAL SIMULATION

The 3-D Markov chain model can be used to formulate cokriging estimates [2,3] and objective functions used in the implementation of sequential indicator simulation and simulated quenching (zero-temperature annealing) geostatistical simulation algorithms [4], respectively. Figure 2 shows a perspective view of a 3-D "realization" resulting from the interpreted 3-D Markov chain model. Fining upward tendencies as per (1) are clearly evident, as are juxtapositional tendencies of

- *lcs* occurring laterally adjacent to *uch*
- *lch* occurring below *uch*

This geologically-plausible facies architecture was originally prescribed in the embedded transition probability matrix and carried through to end result of the geostatistical simulation process. As a comparison, Markov chains with "maximum entropy" juxtapositional tendencies were used to implement the geostatistical simulation procedure to create the realization shown in Figure 3. The main difference between the two realizations appears to be the location of *lch*, which is reflected in the difference between the interpreted and

maximum entropy vertical Markov chain models of Figure 1. Comparison of flow and transport modeling results for both geologically-ordered and maximally disordered systems could add to understanding of the role of facies architecture in hydrogeologic and petroleum reservoir system behavior.

7. CONCLUSIONS

The Markov chain approach ensures a consistency with probability laws and geologic interpretation, which demands a rigorous understanding of the model parameters. The resulting interpretability facilitates interplay and feedback between the spatial variability modeling procedure, geology, and geostatistical simulation results as compared to the more prevalent empirical approaches.

ACKNOWLEDGMENTS

REFERENCES

[1] AGTERBERG, F.P. - "*Geomathematics*". Elsevier Scientific Publishing Company, Amsterdam, New York, 1974.

[2] CARLE, S.F., and FOGG, G.E. - Transition Probability-based Indicator Geostatistics, *Math. Geology*, **28**, 453-477 (1996).

[3] CARLE, S.F. - A Transition Probability-based Approach to Geostatistical Characterization of Hydrostratigraphic Architecture, Ph.D. dissertation, University of California, Davis (1996).

[4] DEUTSCH, C.V., and JOURNEL, A.G. - "*Geostatistical Software Library and User's Guide*". Oxford University Press, New York, 1992.

[5] DOVETON, J.H., An Application of Markov Chain Analysis to the Ayrshire Coal Measures Succession: *Scott. J. Geol.*, 7, 11-27 (1971).

[6] HATTORI, I. - Entropy in Markov Chains and Discrimination of Cyclic Patterns in Lithologic Successions, *Math. Geology*, **8**, 477-497 (1976).

[7] JOHNSTON, R.J., and PATTIE, C.J. - Entropy-Maximizing and the Iterative Proportional Fitting Procedure, *Professional Geographer*, **45**, 317-322 (1993).

[8] KRUMBEIN, W.C. - 'Fortran IV Computer Program for Simulation of Transgressive and Regression with Continuous-Time Markov Models', Kansas State Geological Survey Computer Contribution 26, 1968, p. 1-38.

[9] KRUMBEIN, W.C., and DACEY, M.F. - Markov Chains and Embedded Markov Chains in Geology, *Math. Geology*, **1**, 79-96 (1969).

[10] MIALL, A.D. - Markov Chain Analysis Applied to an Ancient Alluvial Plain Succession: Sedimentology, **20**, 347-364 (1973).

[11] POLITIS, D.N. - Markov Chains in Many Dimensions, *Adv. Appl. Prob.*, **26**, 756-774 (1994).

[12] SCHWARZACHER, W. The Use of Markov Chains in the Study of Sedimentary Cycles, *Math. Geology*, **12**, 213-234 (1969).

[13] VISTELIUS, A.B. - On the Question of the Mechanism of Formation of Strata, *Dokl. Akad. Naug SSSR*, **65**, 191-194 (1949).