SAND--89-0359C

DE89 007420

Measurement Errors for Thermocouples Attached to Thin Plates: Application to Heat Flux Measurement Devices

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Abstract

Unsteady Surface Element (USE) methods are applied to a model of a thermocouple wire attached to a thin disk. Green's functions are used to develop the integral equations for the wire and the disk. The model can be used to evaluate transient and steady state responses for many types of heat flux measurement devices including thin skin calorimeters and circular foil (Gardon) heat flux gages. The model can accomodate either surface or volumetric heating of the disk. The boundary condition at the outer radius of the disk can be either insulated or constant temperature. Effect on the errors of geometrical and thermal factors can be assessed. Examples are given.

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Introduction

The operation of a variety of heat flux sensors and calorimeters involves contact temperature measurements on thin plates. Thermocouples are often used for this purpose. Estimating and/or correcting the errors involved in making these measurements is an important problem in experimental heat transfer. Numerous papers have been written on this subject.

For thin skin calorimeters, Burnett (1961) and Larson and Nelson (1969) developed approximate models for estimating the magnitude of the errors. Henning and Parker (1967) and Keltner (1973,1974) developed analytical models for the transient response of intrinsic thermocouples. Keltner and Bickle (1976) and Wally (1977) used these response models to correct measurement errors. Cassagne et al. (1980), Keltner and Beck (1983), and Litkouhi and Beck (1985) developed more accurate transient response models. Kidd (1985,1986) developed numerical models and used them for sensitivity analyses.

For the circular foil heat flux gages, which are generally called Gardon gages after the developer, Gardon (1953) described the response in terms of a 1st order or exponential response. Analyses by Ash (1969) and Kirchoff (1972) indicated that the exponential response model was not sufficient for rapid transients. Malone (1967) found that accounting for heat transfer to the center thermocouple wire could significantly affect the shape of the transient response. Keltner and Wildin (1974,1975) developed a response model for the gages and used it to estimate measurement errors. Borell and Diller (1987) analyzed the response to convective heating and developed convective calibration methods.

The errors involved in making temperature measurements with thermocouples attached to thin plates may be transient, steady state, or both. The errors may result from the thermocouple installation altering the local surface temperature distribution or the effects of heat transfer in the thermocouple/plate combination. This paper will deal with the latter problem. There are many sources of this type of error, but the most significant are:

1. Thermal constriction effects within the plate to which the thermocouple is attached
2. Thermal inertia of the thermocouple
3. Imperfect contact between the thermocouple and the surface
4. Heat loss from the thermocouple to the ambient
5. The effective junction location being displaced from the surface.

Keltner and Beck (1983) developed the Unsteady Surface Element (USE) methods that are applied to a model of a thin disk attached to a wire. Green's functions are used to develop the integral equations describing the temperature of the wire and the disk. The model can accommodate either surface or volumetric heating of the disk. The boundary condition at the outer radius of the disk can be either insulated or constant temperature. The model can be used to evaluate transient and steady state responses for many types of heat flux measurement devices including thin skin calorimeters and circular foil heat flux gages. The effect on the errors of geometrical factors, such as the disk to wire radius ratio or the ratio of disk thickness to wire radius, and thermal factors, such as contact resistance between the wire and the disk or heat loss from the wire, can be assessed.

A sketch of the model is shown in Figure 1. The disk portion of the model is two dimensional. The thermocouple is modeled as one-dimensional; heat conduction occurs only in the axial direction. A fin correction can be used to allow for heat loss from the thermocouple. Imperfect thermal contact at the interface of the disk and the wire is modeled by a contact heat transfer coefficient, h; for perfect contact, h goes to infinity.

The response models are developed for a step change in either the initial temperature or the surface heat flux. For surface heating, the initial temperature of the disk is the same as that of the wire. For volumetric heating, the initial temperature of the disk is different from that of the wire and the surface heat flux is zero. The response to a time varying condition can be obtained from the step response via convolution.

Mathematical Formation

The heat transfer at the interface of the wire and the disk can be expressed:

$$q_{0,1} = h(T_2(t) - T_1(t)) \quad (1)$$

where h is the contact heat transfer coefficient. For perfect contact, h is infinite.

By energy conservation, the area averaged heat flux entering body 1 at the interface is equal to that leaving body 2, or:

$$q_{0,1} = -q_{0,2} \quad (2)$$

The temperature at $x=0$ for the disk is given by (Beck, et. al. 1988):

$$T_1(r,0,t) = 2\pi\alpha_1\int_{\tau=0}^{t}\int_{r'=0}^{b} (q_L/k_1)G_{R0J}(r,t/r',\tau)G_{X22}(0,t/0,\tau)r'dr'd\tau$$

$$+2\pi\alpha_1\int_{\tau=0}^{t}\int_{r'=0}^{a}(q_{0,1}(\tau)/k_1)G_{R0J}(r,t/r',\tau)G_{X22}(0,t/0,\tau)r'dr'd\tau \quad (3)$$

$$+2\pi\int_{x=0}^{L}\int_{r'=0}^{b}T_{1,i}\, G_{R0J}(r,t/r',0)G_{X22}(x,t/0,0)r'dr'dx$$

The numbering system utilized for the Greens function is that developed by Beck and Litkouhi (1988). G_X represents the x-direction Greens function; whereas G_R represents the radial direction Greens function. The numeral subscripts indicate the boundary conditions: $J=0$ is an infinite boundary, $J=1$ indicates a prescribed temperature boundary condition, and $J=2$ indicates prescribed heat flux boundary condition.

For the purposes of this paper either $q_L=0$ (impulsive, volumetric heating) or $T_{1,i}=0$ (surface heating). Without loss of generality, $T_{2,i}$ can be set equal to zero. For the insulated boundary case, the third term is equal to the initial temperature of body 1 ($T_{1,i}$).

The average temperature over the disk/wire interface is the concern of this paper. The average temperature over the area $0 \le r \le a$ can be expressed as:

$$\bar{T}_1(t) = \frac{1}{\pi a^2} \int_{r=0}^{a} T_1(t) 2\pi r \, dr \tag{4}$$

The average non-dimensionalized temperature for the case of impulsive, volumetric heating is given by:

$$\begin{aligned} \bar{T}_1^{+}(t_a^{+}) &= -1 \\ &+ \frac{4\pi}{a}\int_{\tau_a^{+}=0}^{t_a^{+}}\int_{r=0}^{a}\int_{r'=0}^{b} q_0^{+}(\tau_a^{+}) G_{ROJ}(r,t_a^{+}/r',\tau_a^{+}) G_{X22}(0,t_a^{+}/0,\tau_a^{+}) r' r dr' dr d\tau_a^{+} \\ &+ \frac{4\pi}{a^2}\int_{x=0}^{L}\int_{r=0}^{a}\int_{r'=0}^{b} G_{ROJ}(r',t_a^{+}/r,0) G_{X22}(x,t_a^{+}/0,0) r' r dr' dr dx \end{aligned} \tag{5a}$$

Whereas that for surface heating can be expressed as:

$$\begin{aligned} \bar{T}_1^{++}(t_a^{+}) &= \frac{4\pi}{a}\int_{\tau_a^{+}=0}^{t_a^{+}}\int_{r=0}^{a}\int_{r'=0}^{b} G_{ROJ}(r',t_a^{+}/r,\tau_a^{+}) G_{X22}(L,t_a^{+}/0,\tau_a^{+}) r' r dr' dr d\tau_a^{+} \\ &+ \frac{4\pi}{a}\int_{\tau_a^{+}=0}^{t_a^{+}}\int_{r=0}^{a}\int_{r'=0}^{b} q_o^{++}(\tau_a^{+}) G_{ROJ}(r',t_a^{+}/r,\tau_a^{+}) G_{X22}(0,t_a^{+}/0,\tau_a^{+}) r' r dr' dr d\tau_a^{+} \end{aligned} \tag{5b}$$

where $t_a^{+} = \alpha_1 t/a^2$ (6a)

$T_1^{+} = (T_1 - T_{1,i})/(T_{1,i} - T_{2,i})$ (6b)

$q_0^{+} = q_{0,1} a/\{k_1(T_{1,i} - T_{2,i})\}$ (6c)

$T_1^{++} = T_1/(q_L a/k_1)$ (6d)

$q_0^{++} = q_{0,1}/q_L$ (6e)

The wire is considered to have conduction in the axial direction only. The nondimensional temperature of the wire at x=0 can be expressed (Beck, et. al. 1988):

$$T_2^{+}(t_a^{+}) = -a\,\frac{A}{K}\int_{\tau_a^{+}=0}^{t_a^{+}} q_0^{+}(\tau_a^{+}) G_{X21}(0,t_a^{+}/0,\tau_a^{+}) d\tau_a^{+} \tag{7a}$$

or

$$T_2^{++}(t_a^{+}) = -a\,\frac{A}{K}\int_{\tau_a^{+}=0}^{t_a^{+}} q_0^{++}(\tau_a^{+}) G_{X21}(0,t_a^{+}/0,\tau_a^{+}) d\tau_a^{+} \tag{7b}$$

where $K = k_2/k_1$ and $A = \alpha_2/\alpha_1$ (8)

The fin approximation is used to allow for heat loss from the wire (Beck, et. al., 1988).

$$T_2(t_a^+) = T_{2,nl}\exp(-BiAt_a^+) \quad (9)$$

where: $Bi = 2ha/k_2$ (10)

Many of the Greens functions are in the form of infinite series; as time approaches zero, a very large number of terms are necessary for accurate evaluation. Time partitioning allows the use of semi-infinite solutions at early times. A more detailed explanation of time partitioning is found in Keltner and Beck (1987). The Greens functions are (Beck, et al. 1988):

$$G_{x21}(0,t_a^+/0,\tau_a^+) = A^{-1/2}\ \frac{1}{a}\{\pi(t_a^+-\tau_a^+)\}^{-1/2} \quad \text{for } t_a^+-\tau_a^+\le .022(\ell^+)^2 \quad (11a)$$

$$= \frac{2}{\ell}\sum_{m=1}^{8}\exp\{-c_m^2(t_a^+-\tau_a^+)/(\ell^+)^2\} \quad \text{for } t_a^+-\tau_a^+ > .022(\ell^+)^2 \quad (11b)$$

where: $c_m = \pi(m-1/2)$ (11c)

$$G_{x22}(0,t_a^+/0,\tau_a^+)$$

$$= \frac{1}{a}\{\pi(t_a^+-\tau_a^+)\}^{-1/2} \quad \text{for } t_a^+-\tau_a^+\le .1(L^+)^2 \quad (12a)$$

$$= \frac{1}{L}\{1+2\exp[-\pi^2(ta^+-\tau_a^+)/(L^+)^2]+2\exp[-4\pi^2(t_a^+-\tau_a^+)/(L^+)^2]\} \quad \text{for } t_a^+-\tau_a^+ > .1(L^+)^2$$

$$G_{x22}(L,t_a^+/0,\tau_a^+)$$

$$= 0 \quad \text{for } t_a^+-\tau_a^+\le .02(L^+)^2 \quad (13a)$$

$$= \frac{1}{L}\{1+2\sum_{m=1}^{7}(-1)^m\exp[-m^2\pi^2(t_a^+-\tau_a^+)/(L^+)^2]\} \quad \text{for } t_a^+-\tau_a^+ > .02(L^+)^2 \quad (13b)$$

$$\int_{r=0}^{a}\int_{r'=0}^{b} G_{R01}(r,t_a^+/r',\tau_a^+)r'rdr'dr \quad (14a)$$

$$= \frac{a^2}{4\pi} \quad \text{for } t_a^+-\tau_a^+\le(b^+-1)^2/12 \quad (14b)$$

$$= \frac{a^2b^+}{\pi}\sum_{m=1}^{8}\frac{J_1(\beta_m/b^+)}{\beta_m^2J_1(\beta_m)}\exp[-(\beta_m/b^+)^2(t_a^+-\tau_a^+)] \quad \text{for } t_a^+-\tau_a^+ > (b^+-1)^2/12 \quad (14c)$$

where: $J_0(\beta_m) = 0$

$$\int_{r'=0}^{a}\int_{r=0}^{b} G_{R01}(r, t_a^+/r', \tau_a^+) r' r dr' dr$$

$$= \frac{a^2}{\pi}\left\{1-\exp\left(\frac{1}{2(t_a^+-\tau_a^+)}\right)\left[I_0\left(\frac{1}{2(t_a^+-\tau_a^+)}\right)+I_1\left(\frac{1}{2(t_a^+-\tau_a^+)}\right)\right]\right\} \quad (15a)$$

for $t_a^+-\tau_a^+ \le (b^+-1)^2/12$

$$= \frac{a^2}{\pi}\sum_{m=1}^{8}\left[\frac{J_1(\beta_m/b^+)}{\beta_m J_1(\beta_m)}\right]^2 \exp[-(\beta_m/b^+)^2(t_a^+-\tau_a^+)] \quad \text{for } t_a^+-\tau_a^+>(b^+-1)^2/12 \quad (15b)$$

$$\int_{r=0}^{a}\int_{r'=0}^{b} G_{R02}(r, t_a^+/r', \tau_a^+) r' r dr' dr$$

$$= \frac{a^2}{4\pi} \qquad \text{for all } t_a^+-\tau_a^+ \quad (16)$$

$$\int_{r=0}^{a}\int_{r'=0}^{a} G_{R02}(r, t_a^+/r', \tau_a^+) r' r dr' dr$$

$$= \frac{a^2}{\pi}\left\{1-\exp\left(\frac{1}{2(t_a^+-\tau_a^+)}\right)\left[I_0\left(\frac{1}{2(t_a^+-\tau_a^+)}\right)+I_1\left(\frac{1}{2(t_a^+-\tau_a^+)}\right)\right]\right\} \quad (17a)$$

for $t_a^+-\tau_a^+ \le (b^+-1)^2/12$

$$= \frac{a^2}{\pi}\left\{\frac{1}{b^{+2}} + 4\sum_{m=1}^{6}\left[\frac{J_1(\gamma_m/b^+)}{\gamma_m J_0(\gamma_m)}\right]^2 \exp[-(\gamma_m/b^+)^2\ (t_a^+-\tau_a^+)]\right\} \quad (17b)$$

for $t_a^+-\tau_a^+>(b^+-1)^2/12$ (17c)

where: $J_1(\gamma_m) = 0 \qquad m = 1, 2, 3, 4 \ldots$

The non-dimensional form of Eq. 1 is:

$$q_0^+ = 1/B\{T_2^+-T_1^+-1\} \quad (18a)$$

or

$$q_0^{++} = 1/B\{T_2^{++} - T_1^{++}\} \quad (18b)$$

The Laplace transforms of Eqs. 5, 7, and 18 are taken; Eqs 5 and 7 are substituted into Eq 18. The resulting equation can be solved for heat flux at the interface. From this solution and Eq 7, $T_2^+(t_a^+)$ or $T_2^{++}(t_a^+)$ can be determined. The Gaver-Stehfest method of numerical inversion is used (Stehfest, 1970).

A Fortran model was developed using this formulation. Variables effecting the behavior of the response of the thermocouple are geometric parameters (b^+, L^+, and ℓ^+), thermophysical properties (K and A) and heat transfer characteristics (Bi, and 1/B). The effect of varying these parameters can be examined with the model.

Case 1 - Volumetrically heated, insulated boundary gage

A gage which undergoes impulsive, volumetric heating or a step change of the disk temperature with an insulated boundary at r=b and a very long wire is a good model of a flash x-ray calorimeter. Using such a model the effects of different parameters can be determined. The error associated with the measurement is for this gage is the difference between the value of T_2^+ and unity.

By using very large values of b^+ and L^+, this model can also be used for a wire attached to a semi-infinite body which undergoes a step change in temperature. The semi-infinite response for the present model is compared with the response for the same conditions (K=A=1, 1/B=Bi=0) from Keltner and Beck (1983) in Figure 2. The maximum difference between the responses from the two models is 3% which occurs at $t_a^+=0.5$.

Figure 3 shows the effect of the ratio of the disk thickness to the wire radius on the response of the thermocouple. These responses are for the similar metals (K=A=1) with no heat loss from the wire and perfect contact at the interface. The response for L^+ values ranging from 0.2 to 5 are compared to the response for an ideal intrinsic thermocouple attached to a semi-infinite body. The boundary at x=L begins to affect the response of the wire at $t_a^+=0.1L^{+2}$. The response does not vary significantly from the semi-infinite response until approximately an order of magnitude longer, however. For large disk-to-wire radius ratios and L^+ values greater than 10, the response approaches the case of an ideal intrinsic thermocouple attached to a semi-infinite body; the maximum difference between the response for $L^+=10$ and the semi-infinite response is 0.15%.

Eventually, energy conducted from the disk into the wire will affect the response. This effect is dependent upon the combination of L^+ and b^+. One method of examining this effect is to hold L^+ constant and vary b^+. For $L^+=2$ and b^+ values ranging from 20 to 1000, the heat loss from the disk begins to have an effect at approximately $t_a^+=(b^+-1)^2$. Figure 4 shows the response for disks with these geometric parameters, with K=A=1, no contact resistance or heat loss from the wire. The temperature of the disk would become equal to that of the wire at very late times.

Material property effects are shown in Figure 5 for an ideal intrinsic thermocouple (1/B=0) with no heat loss from the wire (Bi=0) attached to a disk with the following geometric properties: $L^+=2$, $b^+=1000$, ℓ^+ approaching infinity. The response is much slower for larger values of $K/\ell A$. The very long time response is unity for all values of $K/\ell A$, however

Heat lost from the thermocouple will also drive the response to zero. The effect of varying rates of heat loss (values of Bi) from the wire for a gage with $L^+=2$, $b^+=100$, no contact resistance (1/B=0), and made from similar materials (K=A=1) is shown in Figure 6. At early times, the heat loss has little effect. As the wire heats, this loss becomes more important and the response falls below the zero loss case.

Case 2: Surface Heated, Insulated Boundary Gage

A gage which is heated by a constant flux at its surface ($L^+=1$) and is insulated at the radial boundary ($r=b$), represents a thin skin calorimeter. Such calorimeters are frequently used in wind tunnel testing. They have the advantage of being easy and inexpensive to construct. The ideal response of such is a gage is a linear increase of temperature following a short transient.

One design for a thin skin calorimeter consists of a 36 gage (.127 mm) type K thermocouple (chromel/alumel) intrinsically attached to a 1 mm 304 stainless steel plate. The wire is very long compared to its diameter. (This design was taken from Keltner and Bickle (1976).) The resulting value of L^+ is 15.7 with b^+ and ℓ^+ very large. For the chromel wire, $K=1.13$ and $A=1.27$; whereas for the alumel wire $K=1.75$ and $A=1.88$. The gage is considered to have no interfacial resistance to heat flux ($1/B=0$) or heat loss from the wire ($Bi=0$).

The resulting responses are shown in Figure 7. Also given is the ideal temperature or the average non-dimensional temperature for the substrate over $0 \leq r \leq a$ if no wire was present. A value of t_a^+ of 1000 represents a real time of approximately 1 second for this gage design.

The ratio of the actual response to the ideal response is considered to be the difference between the error and unity. This value is given in Figure 8. At $t_a^+=1000$, the error is 3% for the chromel wire and 4% for the alumel wire.

Case 3: Surface Heated Constant Temperature Boundary Gage

A thin foil (Gardon) heat flux gage can be represented by a gage which experiences surface heating and has a constant temperature at the radial boundary ($r=b$). Such gages often consist of a copper wire attached to a constantan disk ($K=16.1$, $A=17.0$). Typical geometric parameters are $L^+=1.875$, $b^+=45$, and $\ell^+=90.6$ for a wire radius of 0.0016 in. The response for such a gage is compared to the ideal response in Figure 9. The gage achieves a steady state response at $t_a^+=3000$ which corresponds to an real time of 0.75 seconds. For a 30 W/cm^2 flux, the steady state value of 215 represents a 120 °C temperature difference between the center of the disk and its edge. The ratio of the response to the ideal response is shown in Figure 10. The ratio of the steady state response to the ideal steady state response is 0.794.

Keltner and Wildin (1974) analyzed a gage with the same parameters. The normalized responses (the response divided by the steady state response) are compared in Figure 11. Although the normalized are similar, the present model predicts a ratio of the steady state response to the ideal steady state response of 0.794 compared to a value of 0.830 for Keltner and Wildin (1974). The difference in steady state values of 4.3% may be due to the fact that Keltner and Wildin (1974) used the centerline temperature the average over the interfacial area.

Summary

Using the unsteady surface element method and Greens function integral equations, a model has been of a thermocouple attached to a thin disk has been developed. The model can be adapted to a variety of heat flux gages by varying flux and boundary conditions. Varying a few geometric, thermophysical, or heat transfer properties allows to model to be applied to many different situations.

Nomenclature

a = wire radius
A = thermal diffusivity ratio
= α_2/α_1
b = disk radius
b^+ = disk radius to wire radius ratio
= b/a
B = contact Biot modulus
= ha/k_1
Bi = lateral surface Biot modulus
= $2h_c a/k_1$
c_m = $\pi(m-1)$
G_R = radial Greens function
G_X = x-direction Greens function
h = contact heat transfer coefficient
h_c = lateral heat transfer coefficient
J = Bessel function
k = thermal conductivity
K = thermal conductivity ratio
= k_2/k_1
ℓ = wire length
ℓ^+ = ratio of wire length to wire radius
= l/a
L = disk thickness
L^+ = ratio of disk thickness to wire radius
= L/a
q_0 = heat flux at the disk/wire interface
q^+ = dimensionless heat flux, Eq ??
q^{++} = dimensionless heat flux, Eq ??
q_L = heat flux at surface $x=L$
r = radial coordinate
s = Laplace transform coordinate
T^+ = non-dimensional temperature, Eq. ?
T^{++} = non-dimensional temperature, Eq. ?

Greek Symbols

α = thermal diffusivity
β_m = roots of $J_0(\beta_m)=0$
γ_m = roots of $J_1(\gamma_m)=0$

Subscripts

1 = related to the disk
2 = related to the wire
j,i = initial value of a parameter for body j
nl = no heat loss from the wire
nw = value of a parameter if wire is not present
ss = steady state value of a parameter

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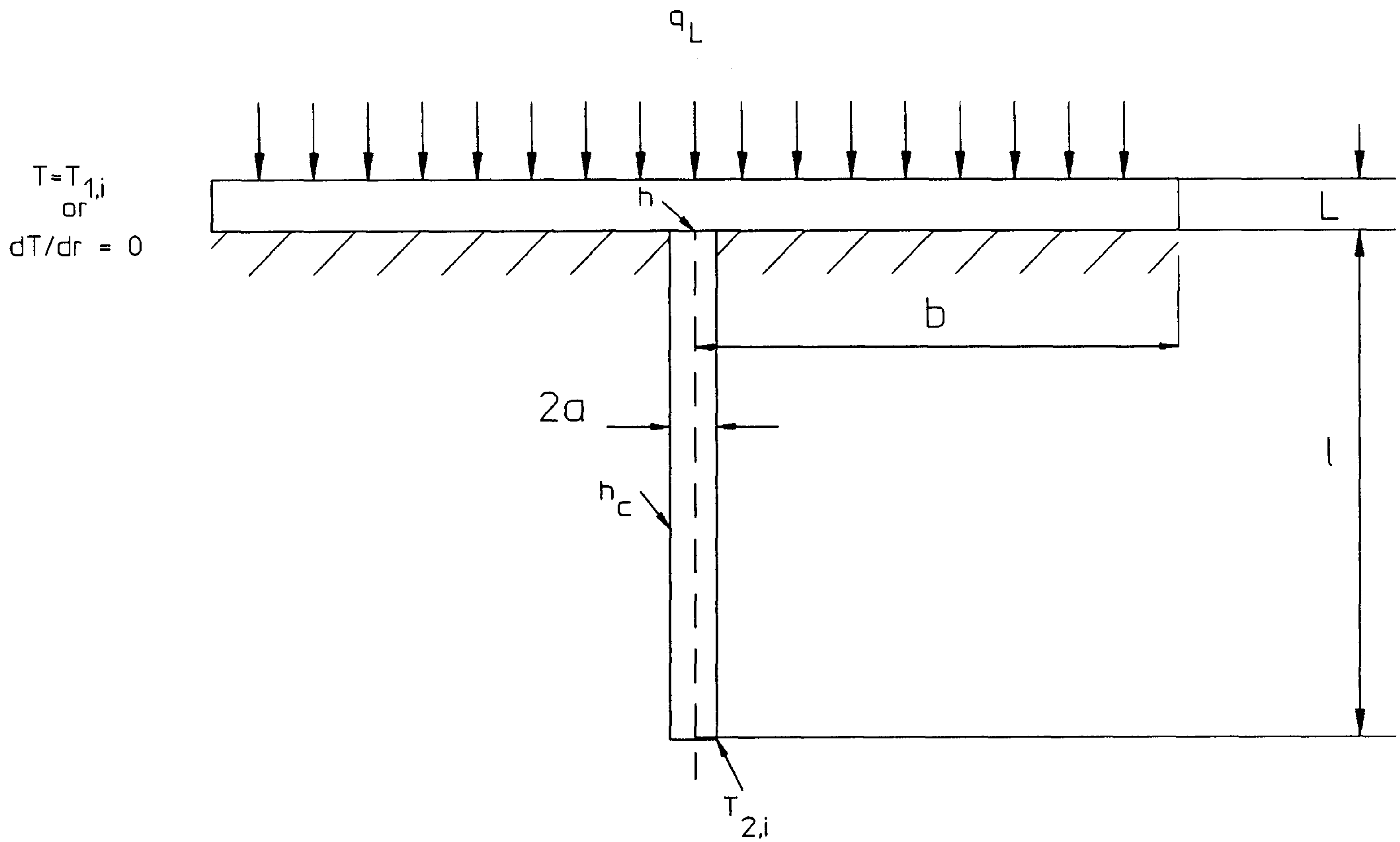
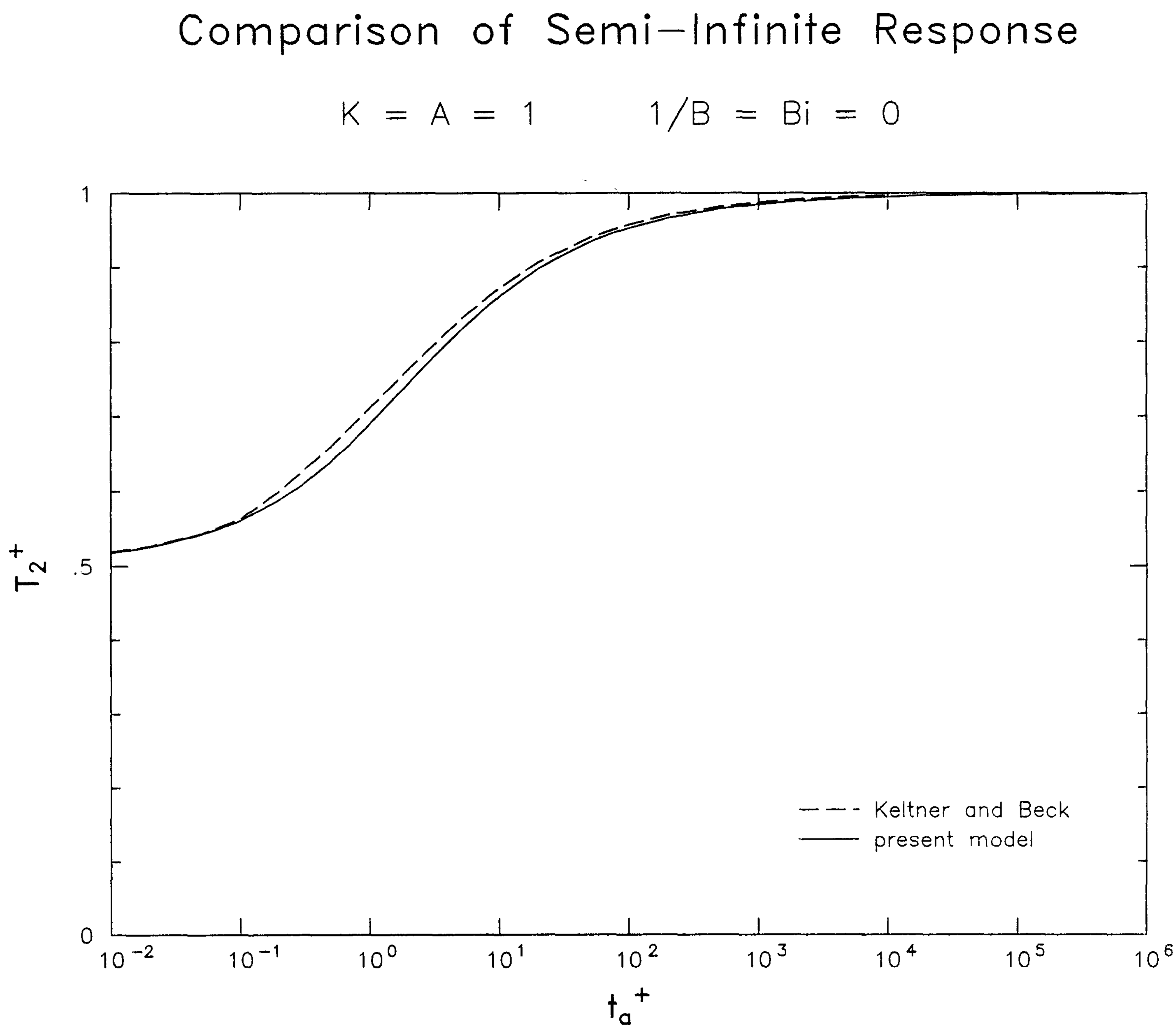
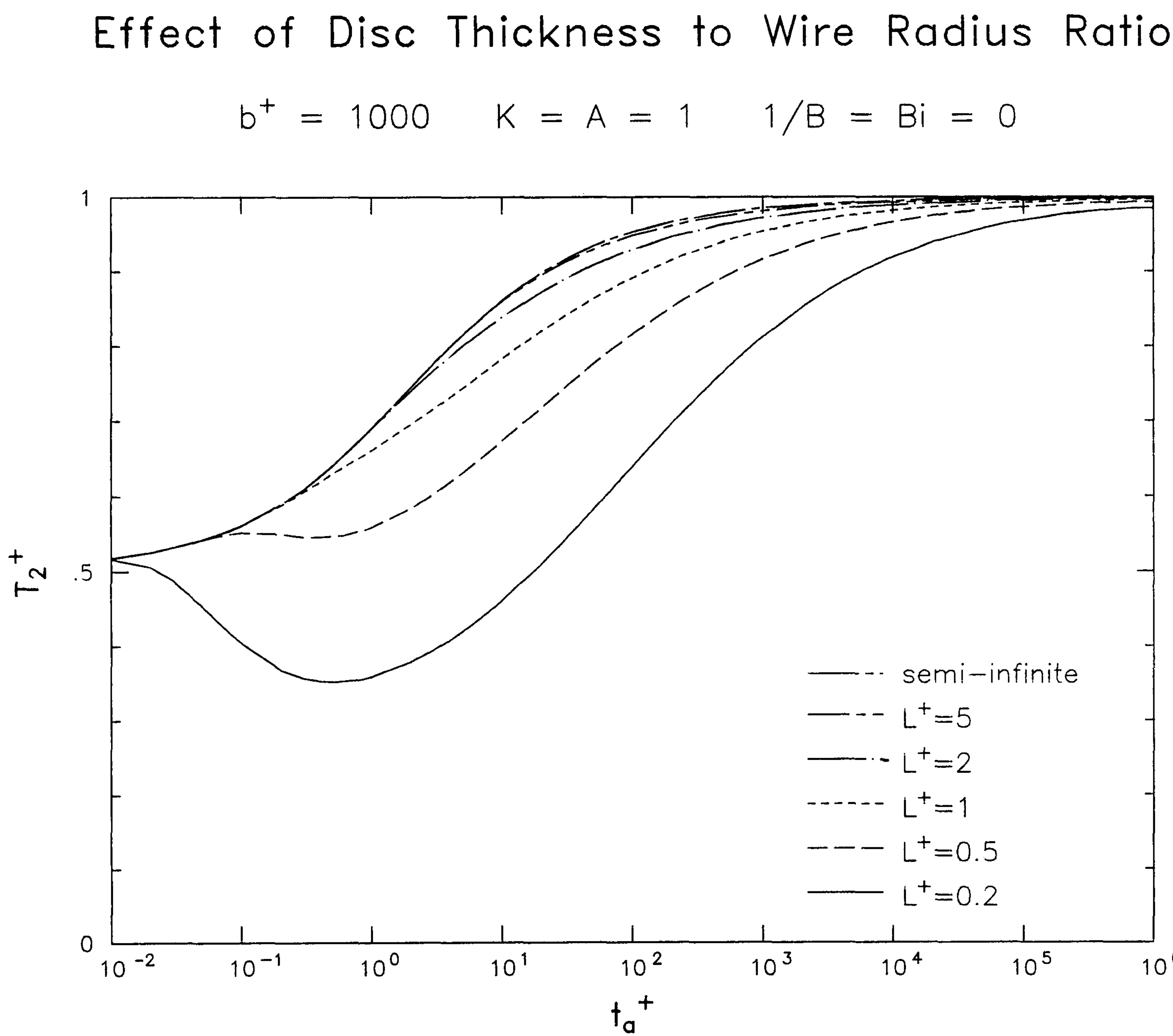


q_L
T=T_1,i
or
dT/dr = 0
h
L
b
2a
h_c
l
T_2,i

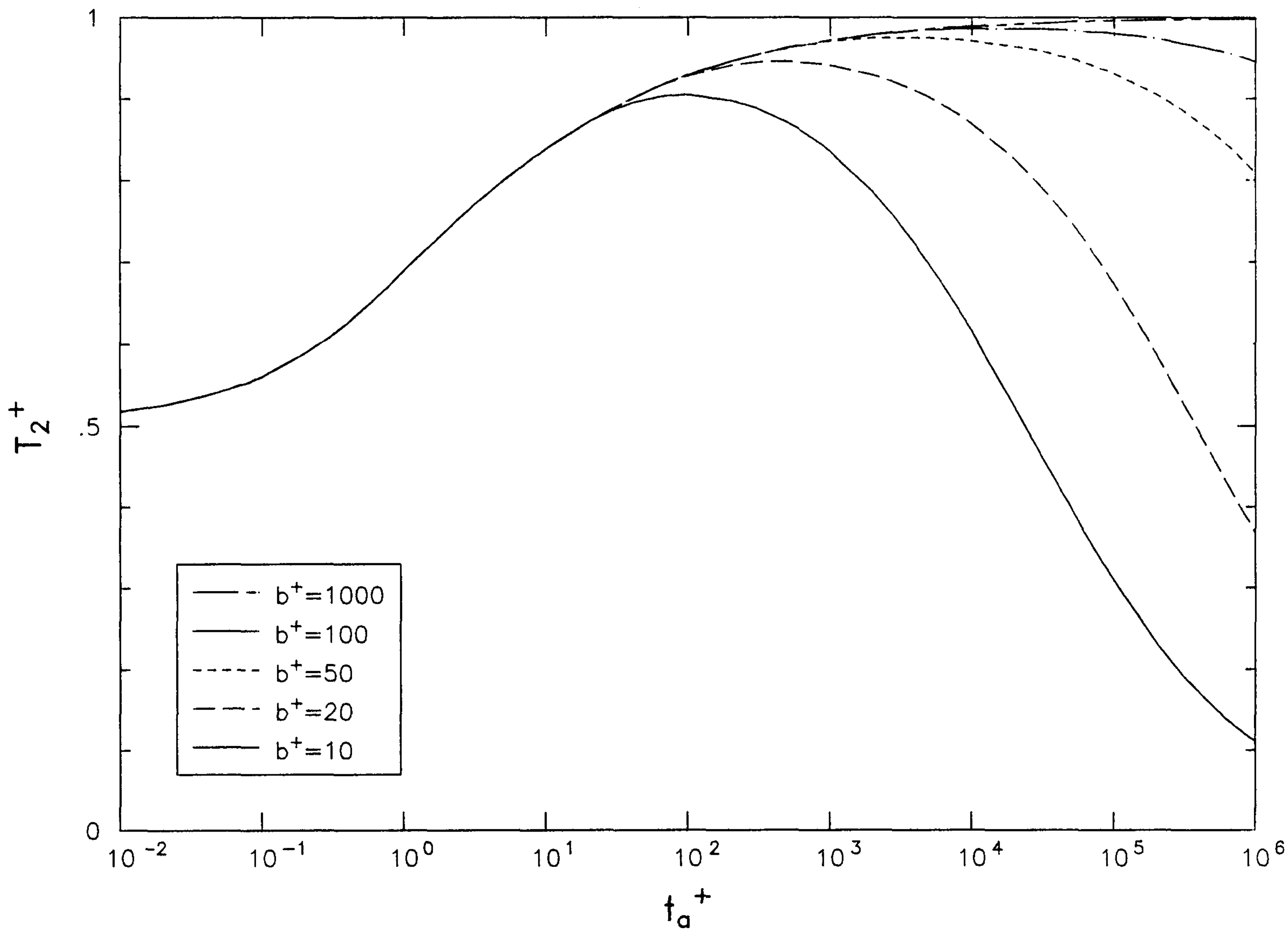


Comparison of Semi-Infinite Response
K = A = 1   1/B = Bi = 0
1
.5
0
T_2^+
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
t_a^+
Keltner and Beck
present model


Effect of Disc Thickness to Wire Radius Ratio
b+ = 1000 K = A = 1 1/B = Bi = 0
1
.5
0
T2+
10 -2
10-1
10 0
10 1
10 2
10 3
10 4
10 5
10 6
ta+
semi-infinite
L+=5
L+=2
L+=1
L+=0.5
L+=0.2

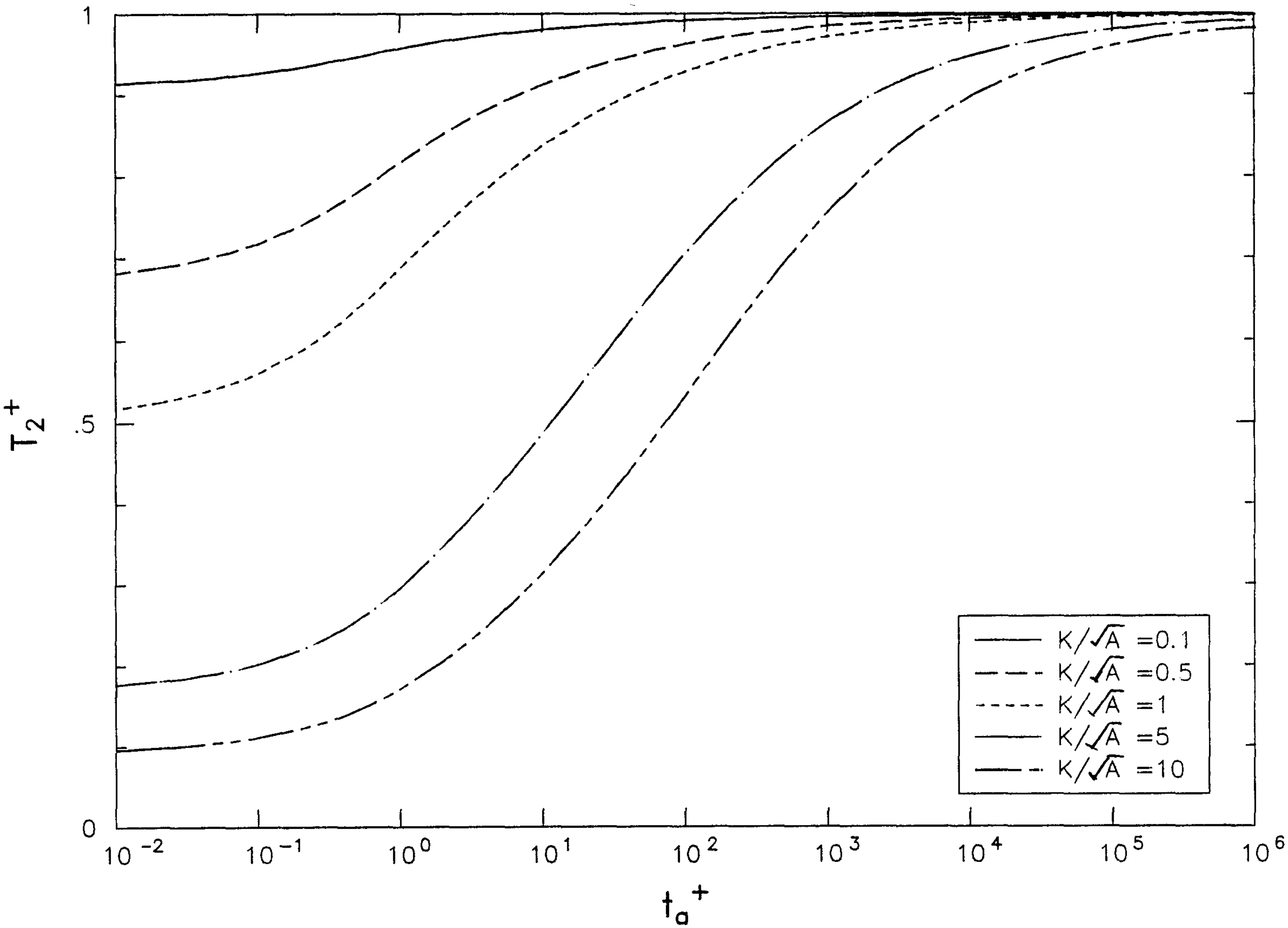
Effect of Disc to Wire Radius Ratio

$L^+ = 2 \quad K = A = 1 \quad 1/B = Bi = 0$



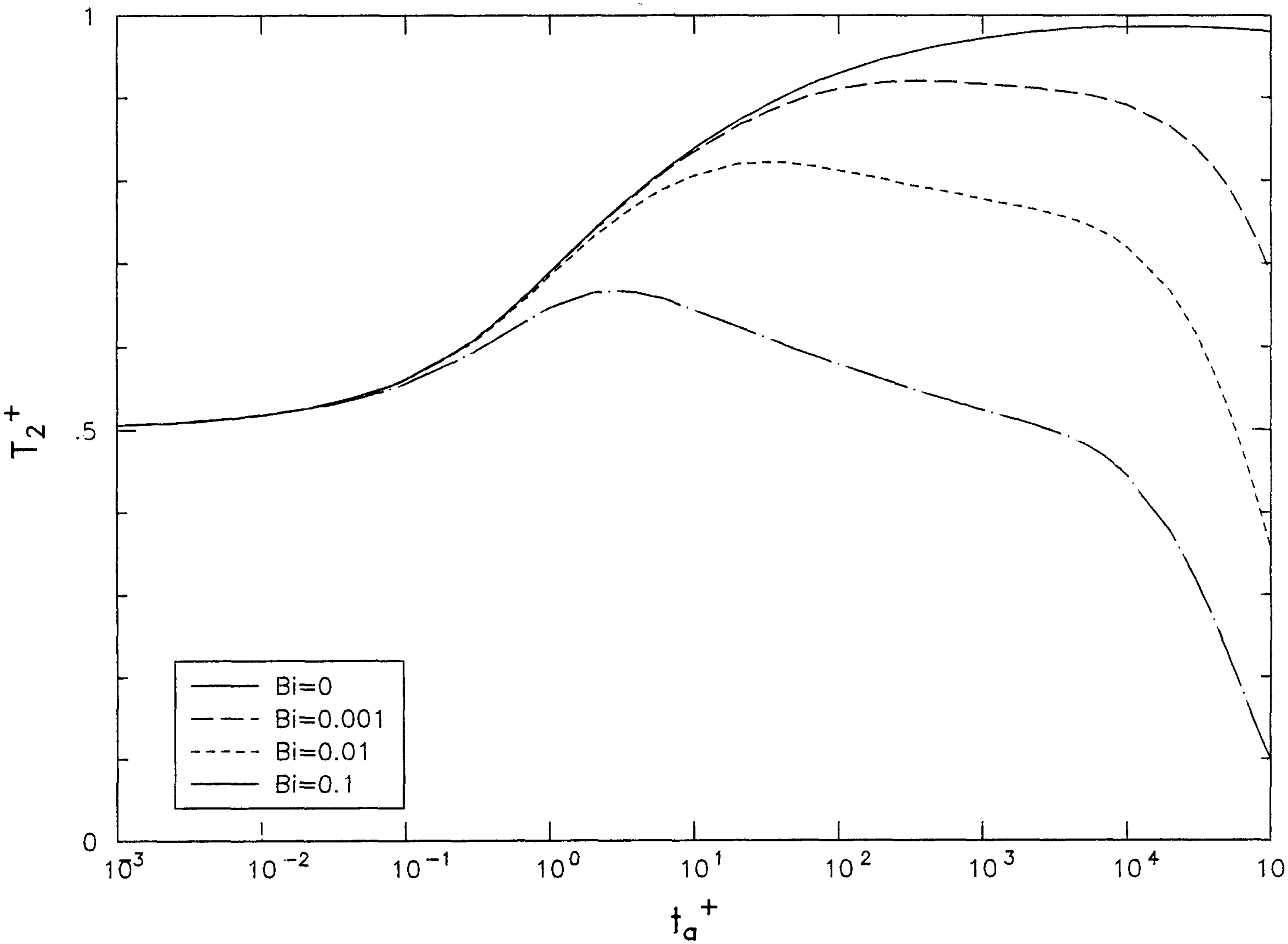
Effect of $K/\sqrt{A}$

$L^+ = 2 \quad b^+ = 1000 \quad 1/B = Bi = 0$



Effect of Heat Loss from Wire

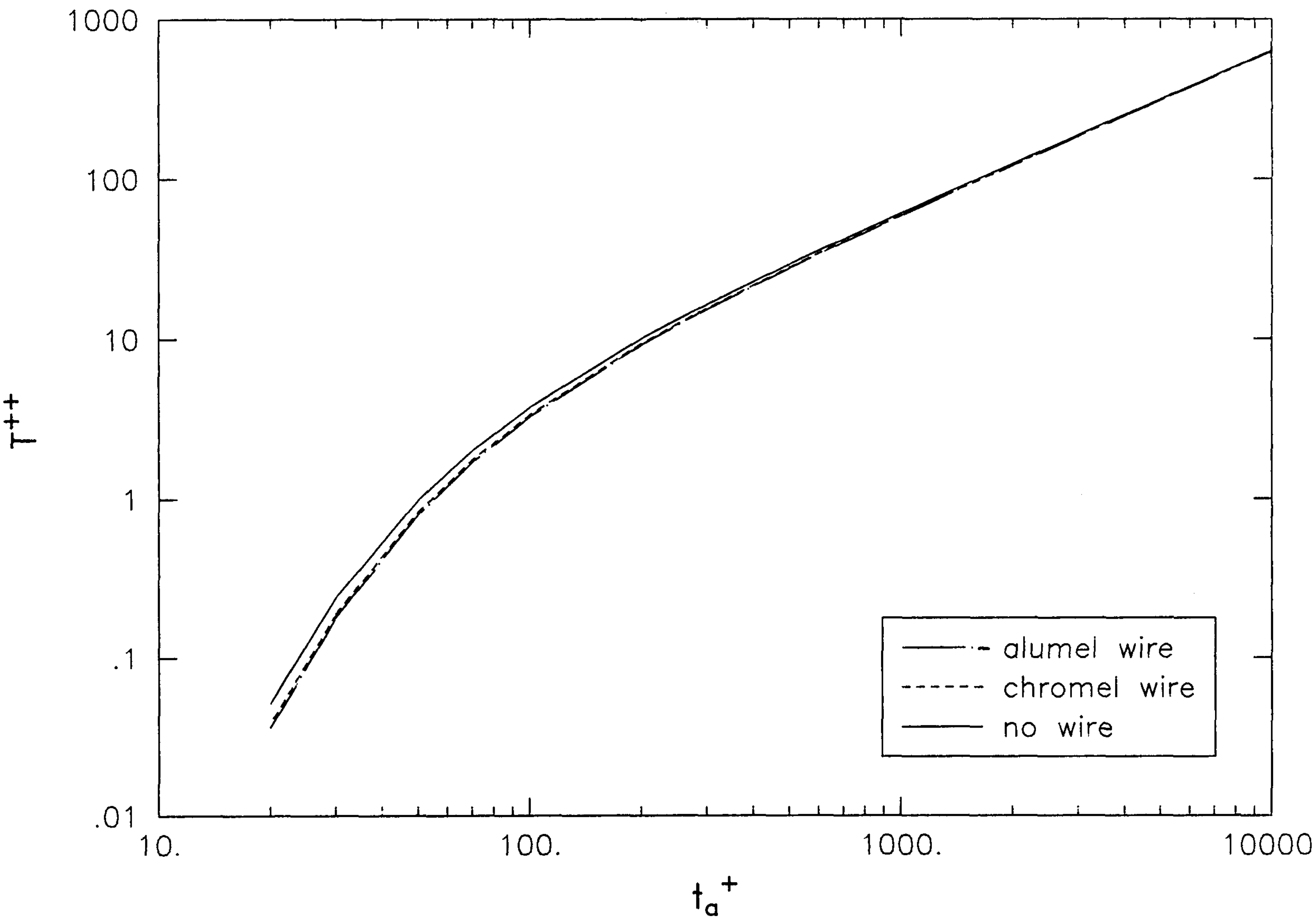
$L^+ = 2 \quad b^+ = 100 \quad K = A = 1 \quad 1/B = 0$

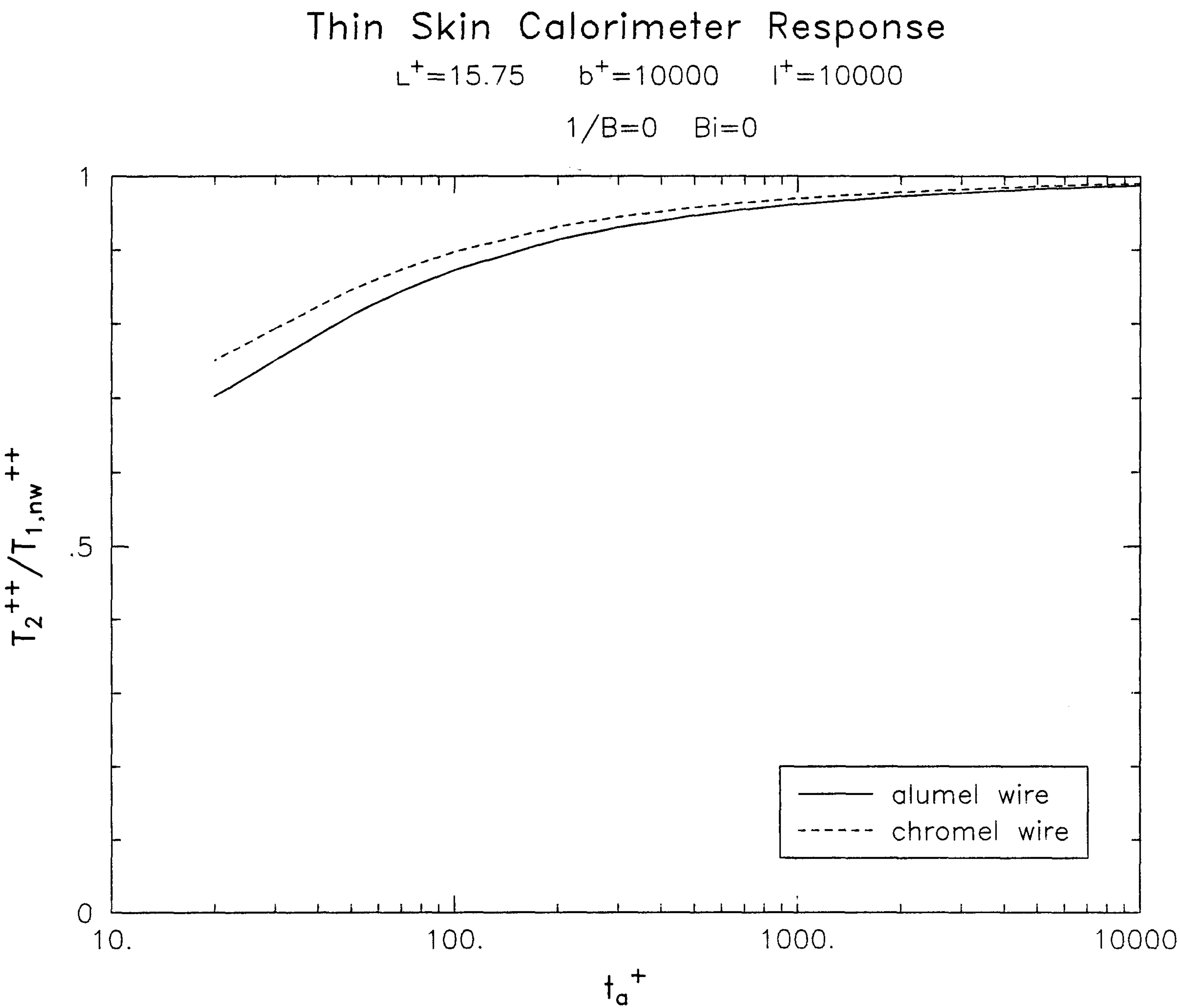


Thin Skin Calorimeter Response

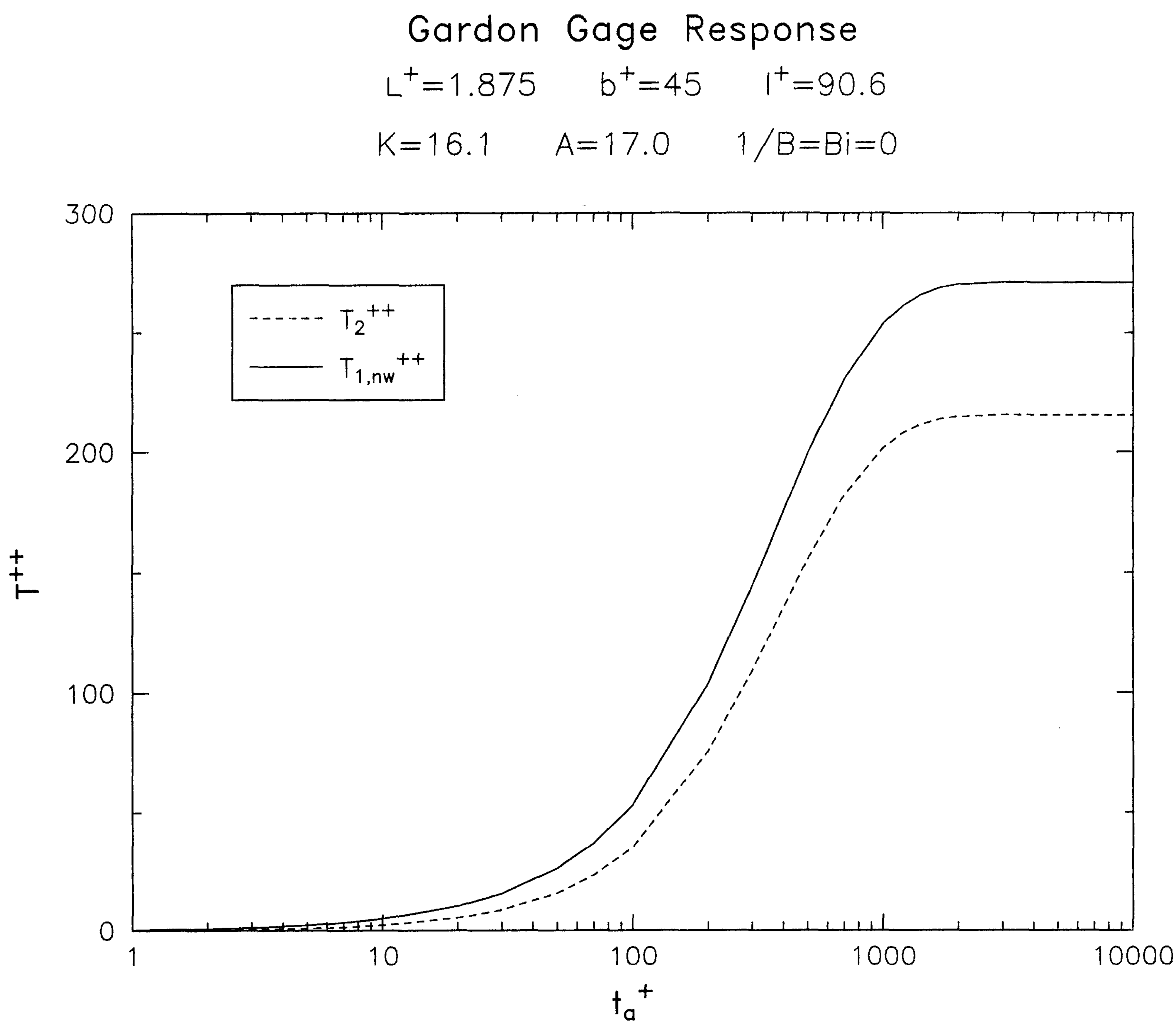
$L^+=15.75$ $b^+=10000$ $l^+=10000$

$1/B=0$ $Bi=0$

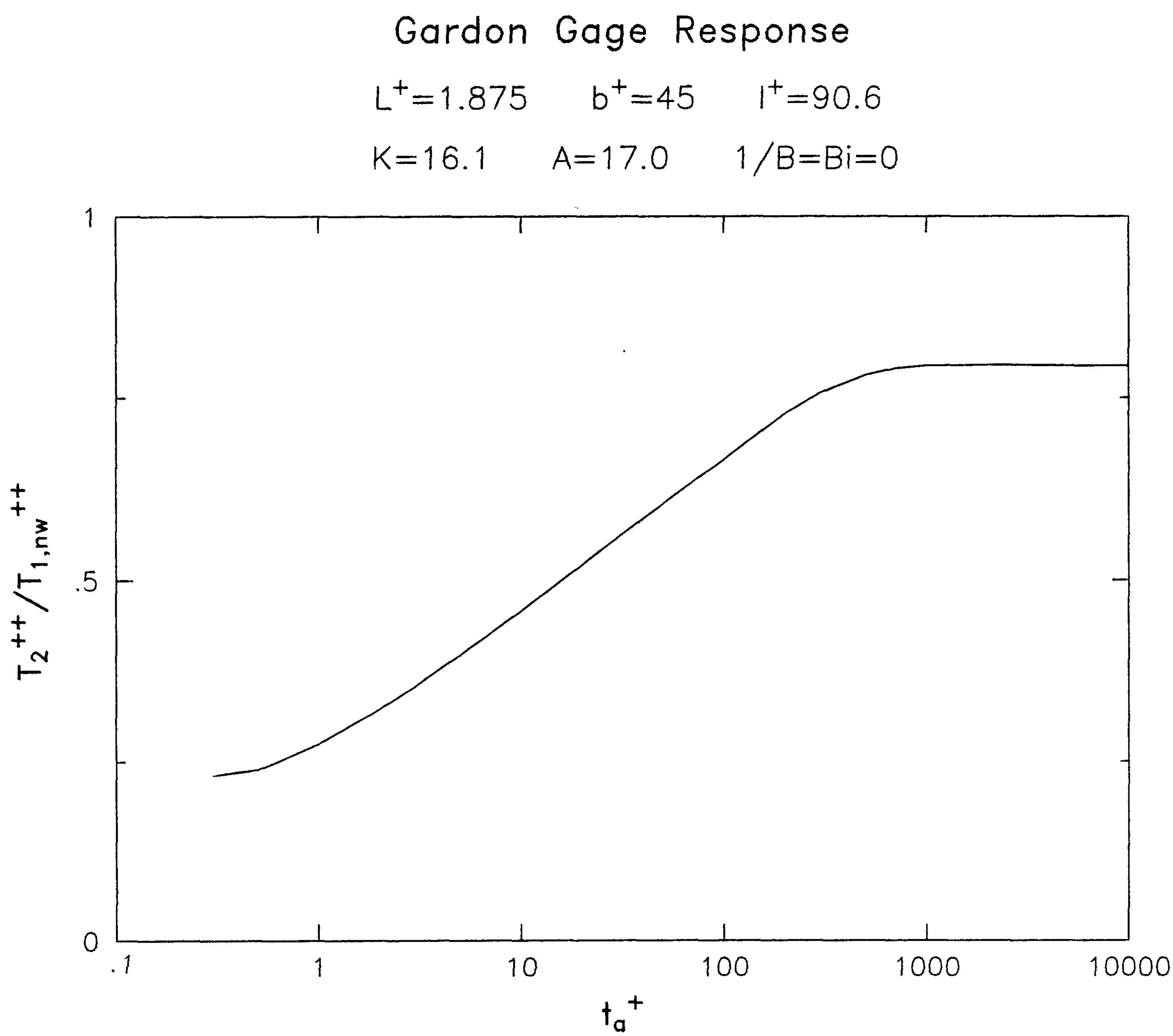




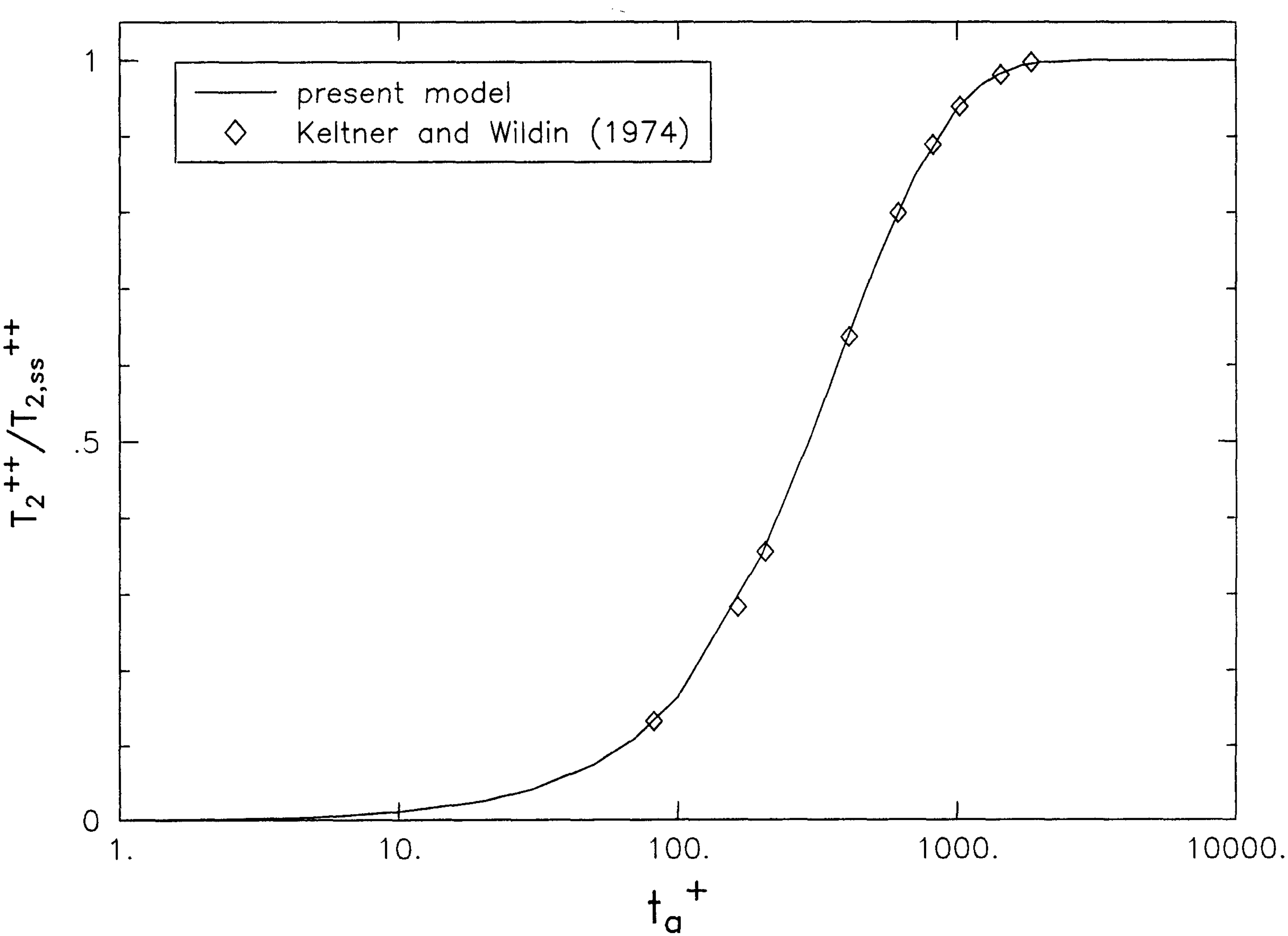
Thin Skin Calorimeter Response
L+=15.75 b+=10000 l+=10000
1/B=0 Bi=0
1
.5
0
10.
100.
1000.
10000.
T2++/T1,nw++
ta+
alumel wire
chromel wire



Gardon Gage Response
L+=1.875 b+=45 l+=90.6
K=16.1 A=17.0 1/B=Bi=0
T2++
T1,nw++
T++
ta+
0
100
200
300
1
10
100
1000
10000



Gardon Gage Response
L+=1.875 b+=45 l+=90.6
K=16.1 A=17.0 1/B=Bi=0
1
.5
0
.1
1
10
100
1000
10000
t_a+
T_2++/T_1,nw++



present model
Keltner and Wildin (1974)
1
.5
0
1.
10.
100.
1000.
10000.
$T_2^{++}/T_{2,ss}^{++}$
t_a^+