30 12/14/87 w.B. ① ⑤ I-12759 Cat ## DR c

The "Phase Velocity" of Nonlinear Plasma Waves in the Laser Beat-Wave Accelerator*

SLAC-PUB--3641

COF DE88 003612

W. L. SPENCE

Stanford Linear Accelerator Center

Stanford University, Stanford, California, 94305

The suggested plasma-laser accelerator[1] is an attempt to achieve a very high energy gradient by resonantly exciting a longitudinal wave traveling at close to the speed of light in a cold plasma by means of the beat-wave generated by the transverse fields in two laser beams. Nonlinearities enter through the usual advective and relativistic increase-of-inertia effects in fluid motion, quantitatively modify the form of the plasma wave, and eventually stop its growth by lengthening its oscillation period so that it no longer resonates with the beat-wave.[2] Previous calculations[3,4] to all orders in v_x have been done essentially from the 'laboratory frame' point of view and have treated the plasma wave as having a sharply defined phase velocity equal to the speed of light. However a high energy particle beam undergoing acceleration sees the plasma wave from a nearly light-like frame of reference and hence is very sensitive to 'small' deviations in its phase velocity. Here we introduce a calculational scheme that includes all orders in v_x and in the plasma density, and additionally takes into account the influence of plasma nonlinearities on the wave's phase velocity. The main assumption is that the laser frequencies are very large compared to the plasma frequency—under which we are able to in essence formally sum up all orders of forward Raman scattering. We find that the nonlinear plasma wave does not have simply a single phase velocity—it is really a superposition of many—but that the beat-wave which drives it is usefully described by a non-local "effective phase velocity" function. The following sections follow a time-space domain approach (rather than (ω, k) space) which leads naturally to the latter notion. It should be pointed out that although we explicitly discuss only the 'simple' beat-wave accelerator there does exist a phase-stabilization scheme, the "surfatron",[5] in which an additional magnetic field is applied to move the particle beam transversely across a broad plasma wave. The particle path length over which the gradient can be used is thereby substantially increased due to the enhanced phase-locking, the precise calculation of which would be of the same form as described here.

The Adiabatic Approximation: We treat the case of incident laser beams which are colinear infinite plane waves as the simplest starting point—the results obtained will then apply sufficiently deep inside the laser beam envelope.[6] We furthermore assume that turbulence does not occur, *i.e.*, that perturbations that would break translation invariance in the transverse plane do not grow. Thus the problem is 'one dimensional' and $\nabla \to \hat{\mathbf{x}}\partial_x$. We additionally assume that to a good approximation the plasma is, and remains for the time span of interest, a cold fluid. In reality this assumption can break down if the fluid velocity approaches the phase velocity of the plasma wave ('wave breaking') frequently enough so that an appreciable fraction of the plasma electrons are cumulatively trapped in the wave, thereby heating the plasma.[7] Our final assumption is that the laser frequencies are large compared to their difference, which is tuned to resonate with small amplitude plasma oscillations, *i.e.*

$$\omega_1, \omega_2 \gg |\omega_1 - \omega_2| \to \omega_{p0} \equiv \sqrt{\frac{e^2 n_0}{m_e}} \qquad (1)$$

where n_0 is the equilibrium value of the plasma electron density.[8] This assumption will be more precisely posed as a limit below. Thus the laser-plasma system is completely described by the boundary conditions that the transverse electromagnetic field per laser be ($\omega = \omega_1$ or ω_2)

$$\mathbf{E}_\perp \sim \hat{\epsilon}\sin[\omega(t - x)]; \ \mathbf{B}_\perp \sim \hat{\mathbf{x}}\times\mathbf{E}_\perp \qquad (2)$$

outside the plasma, and the coupled fluid momentum conservation - Maxwell system of equations. The latter are conveniently classified into the transverse equations:

$$(\partial_t + v_x\partial_x)\gamma\mathbf{v}_\perp = -\frac{e}{m_e}(\mathbf{E}_\perp + v_x\hat{\mathbf{x}}\times\mathbf{B}_\perp)$$
$$\hat{\mathbf{x}}\times\partial_x\mathbf{E}_\perp = -\partial_t\mathbf{B}_\perp \qquad \hat{\mathbf{x}}\times\partial_x\mathbf{B}_\perp = \partial_t\mathbf{E}_\perp - en\mathbf{v}_\perp \qquad (3)$$

and the *longitudinal* equations:

$$(\partial_t + v_x\partial_x)\gamma v_x = -\frac{e}{m_e}[E_x + (\mathbf{v}_\perp\times\mathbf{B}_\perp)_x]$$
$$\partial_x E_x = -e\delta n \qquad \partial_t E_x - env_x = 0 \qquad (4)$$

where $\delta n \equiv n - n_0$ is the (not necessarily small) departure of the electron density n from n_0. To proceed we seek to implement (1) in the *formal* solution to (3) which can then be substituted into (4) to provide a nonlinear system closed in E_x and v_x. In accordance with (2) we take $\mathbf{E}_\perp$ and $\mathbf{B}_\perp$ to be rapidly varying in both t and x and are informed by the first equation in (3) that $\gamma\mathbf{v}_\perp$ should be rapidly varying as well. On the other hand we assume—and can show *a posteriori*—that n/γ changes only adiabatically. Since n may presumably be eliminated in favor of v_x through (4) this entails that v_x contains rapid variations. Except for the terms having manifest v_x dependence the set (3) is then seen to be linear and homogeneous in the rapidly changing variables, with adiabatically varying coefficients. Its solutions satisfying (2) would then be of the form

$$\mathbf{E}_\perp \propto \hat{\epsilon}\sin[\omega(t - x\beta_\perp^{-1})]; \ \mathbf{B}_\perp = \beta_\perp^{-1}\hat{\mathbf{x}}\times\mathbf{E}_\perp \qquad (5)$$

where $\beta_\perp$ is an adiabatically varying function of t and x representing the "effective phase velocity" of the transverse laser fields in the plasma. Boldly substituting directly into (3) reveals that in fact (5) is a solution with

$$\beta_\perp^{-1} = \sqrt{1 - \frac{e^2 n}{m_e\gamma\omega^2}} \cong 1 - \frac{1}{2}\frac{e^2 n}{m_e\gamma\omega^2}$$
$$\partial_t(\gamma\mathbf{v}_\perp) \cong -\beta_\perp^{-1}\partial_x(\gamma\mathbf{v}_\perp) \cong \frac{-e}{m_e}\mathbf{E}_\perp \qquad (6)$$

This is of course precisely the form (with $n \to n_0$, $\gamma \to 1$) that would be obtained by completely linearizing (3) —but it is important to realize that no assumption has been made as to the magnitude of v_x and amusing to witness its cancellation.

* Work supported by the US Department of Energy, contract DE – AC03 – 76SF00515.

Contributed to the 1985 Particle Accelerator Conference
Vancouver, British Columbia, May 13 - 16, 1985

The beat-wave force in (4) may now be calculated—it arises in the 'difference' term having the z, t dependence

$$(\omega_1 - \omega_2)t - (\omega_1\beta_{\perp 1}^{-1} - \omega_2\beta_{\perp 2}^{-1})z$$
$$\cong (\omega_1 - \omega_2)\left[t - z\left(1 + \frac{1}{2}\frac{e^2 n}{m_e \gamma}\frac{1}{\omega_1\omega_2}\right)\right]$$

which we write on resonance as

$$-\frac{e}{m_e}(\mathbf{v}_\perp \times \mathbf{B}_\perp)_z \cong \frac{\omega_{P0}}{\pi r_0}\frac{1}{\gamma}\sin[\omega_{P0}(t - z\beta^{-1}) + \phi] \tag{7}$$

where

$$\beta \cong 1 - \frac{1}{2}\frac{\omega_{P0}^2}{\omega_1\omega_2}\frac{n}{n_0\gamma} \tag{8}$$

is the "effective phase velocity" of the beat-wave, and ϕ the relative phase of the fields of the two lasers. Its strength is parameterized by r_0, which is normalized to represent the number of oscillations required for the plasma wave to grow to the point where $|v_z| = 1$ in the linear approximation (which is generally not the actual saturation time), and in terms of which the total laser flux is

$$S_{lasers} = \frac{4}{\pi}\frac{m_e^2}{e^2}\frac{\omega_1\omega_2\omega_{P0}}{|\omega_1 - \omega_2|}\frac{1}{r_0}$$

Note that (7) incorporates an averaging out of frequencies $\gg \omega_{P0}$ in expectation of their negligibility relative to the resonantly coupled adiabatic oscillations. Similarly we must also supply the $\gamma\mathbf{v}_\perp$ dependence in γ which is needed in (4). Again averaging over the rapid variations, we obtain

$$\gamma^2(1 - v_z^2) \cong 1 + \frac{2}{\pi r_0}\{1 + \cos[\omega_{P0}(t - z\beta^{-1}) + \phi]\}\ . \tag{9}$$

Fluid Mechanics on the Light Cone: Now that the beat-wave force and γ are known as explicit functions of the 'adiabatic' variables, the longitudinal equations (4) constitute a closed system. The beat-wave's complete space-time dependence is implicit—indicating that in actuality it is a superposition of traveling plane waves of diverse frequencies, wave numbers, and phase velocities; which may however usefully be pictured as a simple harmonic wave, different locations in which move with different phase velocities dependent on the local values of n and v_z —hence our usage "effective phase velocity". The same may be said of the plasma wave in E_z with the revision that the latter is distorted by nonlinearities from the simple harmonic form. By assumption (1) the deviations from the speed of light in the effective phase velocity of the beat-wave, and hence the plasma wave, are small—but, in practice as we envision it, are still considerably larger than those of a relativistic particle beam undergoing acceleration in the plasma. An essentially light-like particle slipping across an appreciable fraction of a plasma wave half-cycle then moves a distance proportional to $(1-\beta)^{-1}$, hence of order (see[9] for the treatment of the linear case) $\Delta z \sim 2\pi\omega_1\omega_2/\omega_{P0}^3$. The difference between β and 1 in (7) for such 'large' variations in z, including its dependence on fluctuations in n/γ in the nonlinear regime, must *ipso facto* be taken into account. To calculate the E_z seen by such a particle one must evidently in general integrate the partial differential equations in (4) along a line $z = t +$ constant (*cf.* assuming $\beta =$ constant, *i.e.*, a single phase velocity in the fluid, which would allow immediate reduction to an ordinary differential equation via the similarity variable $t - z\beta^{-1}$; see[4]). The natural variables

$$t_- \equiv t - z, \text{ the light} - \text{cone variable, and}$$
$$\hat{z} \equiv z\frac{\omega_{P0}^2}{\omega_1\omega_2} \tag{10}$$

suggest themselves, whereupon the differential operators then become

$$\partial_t = \partial_-$$
$$\partial_z = -\partial_- + \frac{\omega_{P0}^2}{\omega_1\omega_2}\partial_{\hat{z}} \tag{11}$$

Using these variables the only explicit $\omega_{P0}^2/\omega_1\omega_2$ dependence in the equations occurs in the above derivative expressions—hence in the desired limit

$$\frac{\omega_{P0}^2}{\omega_1\omega_2} \to 0\ ,\ \hat{z} = \text{ fixed}, \tag{12}$$

$\partial_{\hat{z}}$ drops out entirely. Further simplification may be achieved by using the charge continuity relation implicit in (4) to deduce that in the limit (12)

$$n \cong \frac{n_0}{1 - v_z}$$

which leads immediately to the final form of the equations

$$(1 - v_z)\partial_-\gamma v_z = -\frac{e}{m_e}E_z + \frac{\omega_{P0}}{\pi r_0\gamma}\sin\theta \quad ; \quad \partial_- E_z = en_0\frac{v_z}{1 - v_z}$$
$$\gamma^2(1 - v_z^2) = 1 + \frac{2}{\pi r_0}(1 + \cos\theta)$$
$$\theta = \omega_{P0}\left(t_- - \frac{\hat{z}}{2\gamma(1 - v_z)}\right) + \phi \tag{13}$$

valid in the high laser frequency limit (12). They are readily solved numerically by standard initial value techniques.[10] It is convenient to choose the point $t_- = \hat{z}/2$ as the front of the beat-wave, for which $v_z = 0$ and $E_z = 0$, and assume that the plasma occupies the half-space $\hat{z} \geq 0$. The plasma wave seen by a light-like test particle traveling on a trajectory specified by $t_- = t_{-b} > \hat{z}/2$ at a particular $\hat{z}$ is then obtained by integrating from $t_- = \hat{z}/2$ to $t_- = t_{-b}$, *i.e.*, tracking the evolution of the plasma at $\hat{z}$ from the time it first experiences the beat-wave force until it is encountered by the particle.

Illustrative Results: The results we now present aim at exposing the phenomena described by (13) and illustrating their typical ramifications for a beat-wave accelerator. Given the beat-wave strength r_0^{-1} and phase ϕ the parameters necessary for a calculation are the initial phase of the plasma wave relative to the particle, specified implicitly by t_{-b}, and the range in $\hat{z}$ in which we are interested. t_{-b} may be thought of as signifying the fact that at the time the laser beams enter the plasma the particle is at $\hat{z} = -t_{-b}$ and that the particle trajectory overtakes the front of the wave when it reaches $\hat{z} = 2t_{-b}$. Thus if the plasma is not too nonlinear the trajectory traverses roughly $t_{-b}\omega_{P0}/\pi$ plasma oscillations, which is also the number of oscillations that have occurred at $\hat{z} = 0$ by the time the particle enters the plasma. Inasmuch as the plasma *is* nonlinear the precise 'phase' of the wave a particle with a given t_{-b} sees upon entering the plasma must be found by solving (13). For purposes of acceleration it is desired that it be in the saturated regime and where E_z has the proper sign—and of course the particle is extracted or the plasma terminated after traversing at most one half-period. Many considerations may ultimately enter into the choice of an optimal plasma accelerating 'phase' and stage length[11] —in the accompanying figures (calculated using an earlier version of (13) restricted to $r_0 \gtrsim 1$) we simply follow the test particle through several oscillations beginning at $\hat{z} = 0$ for $\phi = 0$ and a t_{-b} such that the plasma wave is essentially saturated. There is a striking amount of 'decay' in the amplitude as it 'ages' in traveling through the plasma —plainly attributable to the diversion of beat-wave amplitude away from the resonant frequency and into sidebands as the plasma wave

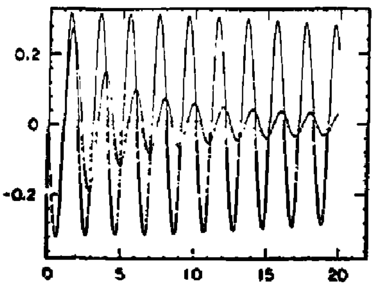

FIG. 1. $-eE_z/m_e\omega_{P0}$ vs. $\xi\omega_{P0}/2\pi$ for $\tau_0 = 100$ and $t_{-\frac{1}{2}} = 40(2\pi/\omega_{P0})$. The usually larger amplitude curve, shown for comparison, results from assuming the effective phase velocity =constant, *i.e.*, fixing $\gamma(1 - v_s) = 1$ in θ in (13).

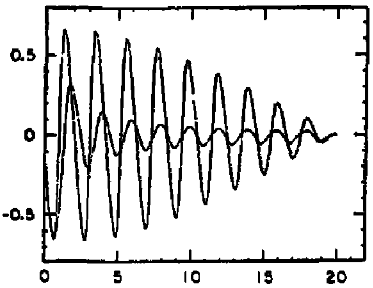

FIG. 2. Likewise for $\tau_0 = 10$ and $t_{-\frac{1}{2}} = 10(2\pi/\omega_{P0})$.

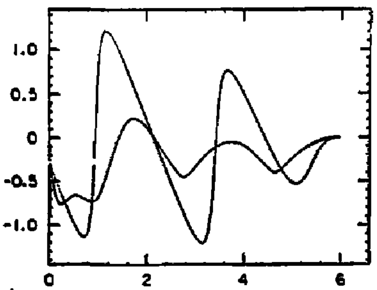

FIG. 3. Likewise for $\tau_0 = 1$ and $t_{-\frac{1}{2}} = 3(2\pi/\omega_{P0})$.

becomes nonlinear. There is also a notable amount of 'phase shifting' and 'period shifting' (both lengthening and shortening) *vis-à-vis* what would appear if the fluctuations in the effective phase velocity were neglected and clearly distinct from the period lengthening with amplitude due to the plasma wave's direct self-interactions.[12] Of prime importance to beat-wave accelerators is however the evident fact that the effects on the first half-period under realistic circumstances are qualitatively not radical. For $\tau_0 = 100$ the effect on the amplitude is very small and the period's deviation from the linear estimate less than 10%; for ten times as much excitation the effect on the average gradient for almost any staging scheme is incidental and the period is affected (possibly advantageously lengthened) by 30%—calculations like those described here would be needed in a serious application. In the example of very large excitation given last (which may be straining somewhat the criterion just mentioned) we see a substantial effect on the peak amplitude which may however be offset by the possibility of a longer high-gradient stage.

It is a pleasure to thank Pisin Chen, especially Bob Noble, and Ron Ruth for many useful discussions.

---

1. T. Tajima and J. M. Dawson, *Phys. Rev. Letters* **43** (1979) 267. See P. Channell, ed., *Laser Acceleration of Particles (Los Alamos, 1982)*, American Institute of Physics, New York, 1982; for reviews of early work; and C. Joshi *et al.*, *Nature* **311** (1984) 525; for an up to date summary.

2. M. N. Rosenbluth and C. S. Liu, *Phys. Rev. Letters* **29** (1972) 701.

3. C. M. Tang, P. Sprangle and R. N. Sudan, *Appl. Phys. Letters* **45** (1984) 375.

4. R. J. Noble, SLAC-PUB-3463(1984), to be published in *Phys. Rev. A.*

5. T. Katsouleas and J. M. Dawson, *Phys. Rev. Letters* **51** (1983) 392; R. Sugihara and Y. Midzuno, *J. Phys. Soc. Japan* **47** (1979) 1290; J. D. Lawson, Rutherford-Appleton Laboratory Report RAL 84-059(1984).

6. The transverse particle beam focusing thereby neglected is nevertheless of critical importance as shown for linear plasma waves by R. D. Ruth, A. W. Chao, P. L. Morton, and P. B. Wilson, SLAC-PUB-3374 (1984), to be published in *Particle Accelerators*. Nonlinear effects remain to be studied.

7. J. Dawson, *Physics of Fluids* **11** (1968) 1506.

8. *Heaviside units and $c = 1$ are employed.*

9. R. D. Ruth and A. W. Chao, in P. Channell, ed., *op. cit.*

10. They have a tendency to be numerically stiff on only portions of the integration path, so it is useful to have an integrator which can switch between stiff and non-stiff methods such as *ODEPACK*, for a copy of which I thank Alan Hindmarsh.

11. See[4] for, *e.g.*, constraints imposed by transverse focusing in the linear regime.

12. R. J. Noble, *Proc. 12th Intl. Conf. on High Energy Accelerators*, Fermilab, 1983.