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AUTHOR(S) U. F. Kocks
P. Franciosi
M. Kawai

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Los Alamos National Laboratory
Los Alamos, New Mexico 87545

A FOREST MODEL OF LATENT HARDENING AND ITS APPLICATION TO POLYCRYSTAL DEFORMATION†

U. F. KOCKS[1], P. FRANCIOSI[2], AND M. KAWAI[3]
[1]Center for Materials Science, Los Alamos National Laboratory, Mail Stop K765, Los Alamos, NM 87545, USA
[2]Lab.PMTM, Université Paris Nord, Villetaneuse, FRANCE
[3]Institute of Engineering Mechanics, University of Tsukuba, Sakura, Nihari, Ibaraki, 305 JAPAN

INTRODUCTION

The term "latent hardening" is commonly used for the effect of slip on one system on the flow stress of another. One of the early surprises of the crystallographic slip theory of plasticity was that such a hardening effect exists at all, and later, that the hardening of the "latent" system is actually usually larger than (or at least equal to) that on the primary system.[1-4] These observations relate primarily to the deformation of free monocrystals, in which single slip is the rule, not the exception. In single crystals of special orientations, or under more severe straining conditions, or in grains of a polycrystal, slip generally occurs on many systems simultaneously. The question is still relevant, for the purpose of a general theory of strain hardening, whether the hardening of each system is indepenent from slip on all the others or not. If it is not, we say that there is "latent hardening", and we now mean this term to refer primarily to the rate of strain hardening in the different systems, and only by consequence to the flow stresses.

In the grains of a polycrystal, there is another consequence of latent hardening. Since any change in the flow stress ratios between different slip systems (active or not) causes a change in shape of the single-crystal yield surface (SCYS), the distribution of slips on the different systems, for a given macroscopic strain increment, may change, and cause different orientation changes in this grain. This

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could lead to different deformation textures in the same class of material (say, single-phase FCC metals slipping on the octahedral slip system only) if, for some reason, the latent-hardening behavior is different (e.g., because of different stacking-fault energies). Such a mechanism has been invoked, qualitatively, to explain differences in rolling texture between, say, copper and brass.[5] If one understands latent hardening, one can make predictions about the direction of possible texture changes due to it, and about the maximum conceivable effect.

This paper means to assess latent hardening effects on large-scale polycrystal deformation on the basis of the dislocation theory of plastic flow and of a wide variety of experimental observations on single crystals. The qualifier "large-scale" is introduced to eliminate effects during the earliest stages of flow, where mono- and polycrystals behave differently: we will not consider "easy glide" in single crystals[4,6]; and we will not consider the gradual transition, over perhaps ten times the elastic strain, to polyslip in polycrystals[6]; or, for that matter, initial Lüders-type deformation in polycrystals[7]. In terms of dislocation theory, we will consider the "flow stress" to mean the percolation limit of dislocations in areal glide[8].

Dislocation theory will be used at a rather gross level. Experience has shown that many of the details of the interactions between dislocations and of their arrangement[9] are of surprisingly little influence on macroscopic plastic behavior.[8] We will present a latent hardening model that is based on the well established "forest" model of flow stress and strain hardening. We will then assess various pieces of experimental and theoretical evidence that limit the number and magnitude of the parameters and predict the direction in which they might depend on stacking-fault energy. Finally, we will summarize results obtained by polycrystal simulation in which the model was implemented. They show only marginal effects, and only in certain cases.

FLOW STRESS AND DISLOCATION ACCUMULATION

Perhaps the most fundamental relation of the dislocation theory of plastic flow is

$$\tau = \alpha \mu b \sqrt{\rho} \qquad (1)$$

where μ is an appropriate shear modulus, b the magnitude of the Burgers vector, ρ some measure of the dislocation

density, and α a proportionality constant. Equation (1) has been derived on the basis of various specific mechanisms, with various precise meanings of ρ, and values of the interaction strength α between 0.1 and 1.0; we are here not concerned with temperature and rate effects and assume α to be the appropriate value for the given temperature and strain rate. The term τ is the contribution to the flow stress from dislocation interactions only, to which other contributions may be added or superposed in some other way. (For background and older references, see ref.8.)

The theory of flow stress that has been, overall, the most successful and is now widely accepted is the forest theory. Here, ρ has the specific meaning of the number of ("forest") dislocations that intersect the slip plane on which τ is measured. One of the many advantages of the forest theory, which is of particular interest here, is that the flow stress in different slip systems of the same slip plane is perforce the same: an observation that has been made, at least approximately, in all experiments on latent hardening[1-4]. Equation (1) can then be expressed (in differential form), specifically for the flow stress on plane P, as

$$d\tau^P = \frac{(\alpha\mu b)^2}{2\,\tau^{(P)}}\, d\rho_f^P \qquad (2)$$

The increase of the forest density ρ_f on plane P is equal to the increase in dislocation length Λ (per unit volume) in all slip systems s not contained in P:

$$d\rho_f^P = \begin{vmatrix} 0\,0\,0 & 1\,1\,1 & 1\,1\,1 & 1\,1\,1 \\ 1\,1\,1 & 0\,0\,0 & 1\,1\,1 & 1\,1\,1 \\ 1\,1\,1 & 1\,1\,1 & 0\,0\,0 & 1\,1\,1 \\ 1\,1\,1 & 1\,1\,1 & 1\,1\,1 & 0\,0\,0 \end{vmatrix} d\Lambda^s = Y^{Ps}\, d\Lambda^s \qquad (3)$$

where the "yield matrix" Y has been introduced. If this matrix is inserted into eq.(2), and α is taken to be a constant, this means that all "trees" are treated equal, regardless of the specific interaction between the forest dislocation and the mobile one. This represents a considerable simplification: the "uniform forest" theory. Attempts at more detailed treatments are hampered by the question of the appropriate superposition and averaging rules

The length of dislocations belonging to each slip system can increase in essentially two ways. The first is

"direct storage": the dislocation length left behind by the dislocations that are moving in this slip system (in areal glide). It is proportional to the strain increment $d\gamma$ in the system. The proportionality constant is usually related to a "mean free path" L of the dislocations, which must be assumed proportional to (say, K times) $1/\sqrt{\rho}$ in order obtain the linear strain hardening observed. (This is tantamount to assuming the "principle of similitude"[10]; dynamic recovery will be discussed below.) Then, we have

$$d\Lambda_D^s = \frac{d\gamma^s}{b\,L} = \frac{\sqrt{\rho_f^S}}{b\,K}\,d\gamma^s = \frac{\tau^S}{\alpha K\,\mu b^2}\,d\gamma^s \qquad (4)$$

The subscript D signifies the "direct" storage, and the superscript S relates to the slip plane of system s.

Now comes the major mechanistic assumption of this paper, drawn from the "statistical theory of flow stress and strain-hardening": the "primary" dislocations accumulated, from the pool of mobile dislocations by some statistical mechanisms, cause various forest dislocations in the neighborhood to move (and perhaps multiply) in response to the internal stresses set up by the primarily stored dislocations, and such as to lower the energy of the arrangement and stabilize it ("plastic relaxation").[8] This may cause an "indirect" (subscript I) increase in dislocation line length in all slip systems. It is reasonable to assume that the total length of each set of indirectly stored dislocations is proportional to the directly stored ones:

$$d\Lambda_I^t = R^{ts}\,d\Lambda_D^s \qquad (s \neq t) \qquad (5)$$

The "relaxation matrix" R expresses the strength of the interaction between the two sets of dislocations. Before we discuss it in detail, let us *define* each $R^{ss}=1$; then, the *total* increase in dislocation length can be written as

$$d\Lambda^t = d\Lambda_D + d\Lambda_I = R^{ts}\,d\Lambda_D^s \qquad (6)$$

In general, one might wish to include other causes for dislocation generation, such as products of dislocation reactions, or the remnants of dynamic pile-ups[9]. Here, we consider only "primary" dislocation accumulation (from the mobile pool) and plastic relaxation.

Inserting eq.(6) into eq.(2) and using eq.(3), we get

$$d\tau^P = \frac{\alpha\mu}{2K} \frac{\tau^{(S)}}{\tau^{(P)}} Y^{Pt} R^{ts} d\gamma^s \qquad (7)$$

This is the increase in flow stress from dislocation accumulation only; from it should be subtracted any effect of dynamic recovery, which acts globally on all previously stored dislocations[8]; this will be done in eq.(9).

Equation (7) is similar in form to the commonly assumed hardening law

$$d\tau^t = h^{ts} d\gamma^s \qquad (8)$$

except for these major differences: the ratio of the current flow stresses makes eq.(7) nonlinear, nonsymmetric (as h has often been assumed to be) and dependent on strain. It is worth recapitulating where these flow stresses came from. The one in the numerator came from assuming that the rate of dislocation accumulation in a slip plane is proportional to the current forest density in that slip plane; this is reasonable if strain hardening is due to dislocation interactions only (and the forest theory is accepted). In other cases, such as a dominance of second-phase particles as causes for dislocation accumulation, $\tau^{(S)}$ in eq.(7) should be replaced by a constant. On the other hand, the $\tau^{(P)}$ in the denominator is unavoidable: it is a direct consequence of the fundamental relation (1).

The major task now is to estimate the quantitative nature of the relaxation matrix R. Then, it will be multiplied with the matrix Y; we will normalize this product matrix with its first member and then call it H. The final form of eq.(7) then becomes

$$d\tau^P = \theta_1 \frac{\tau^{(S)}}{\tau^{(P)}} H^{Ps} d\gamma^s \qquad (9)$$

where θ_1 is the single-slip hardening rate; it may incorporate all dynamic recovery effects.

3. THE RELAXATION MATRIX AND THE HARDENING MATRIX

There are many possible types of interaction between dislocations: elastic interaction, reactions at nodes, the formation of jogs or kinks in one or the other are just some of these. All interactions between dislocations on differ-

ent slip systems are, however, restricted by symmetry considerations[1]. For example, there are two "conjugate" slip systems to each "primary" slip system, and these two must react to a primary dislocation in the same way. (They may have different applied stresses on them, which may cause an orientation dependence, but should be a minor influence on plastic relaxation under primarily internal stresses.)

Thus, one can draw up a "dislocation interaction matrix" that has only a few independent parameters. Table I shows it for FCC metals, in a slip system nomenclature that allows immediate identification of the crystallographic relation between two slip systems[1]. For example, when the two letters are interchanged, the two systems have collinear Burgers vectors (we label this situation by the letter q); when no letter is in common, the Burgers vectors are perpendicular (u); when the first letter is the same, they are coplanar (p); and when the second letter is the same, they are "conjugate" (c). After these examples, there are only two possible interaction types left: in every case, one dislocation receives a kink, the other a jog when they intersect (k and j). For elastic and junction interactions, k = j.

Table I: The Relaxation Matrix R

	PK	PQ	PU	QU	QP	QK	KP	KU	KQ	UQ	UK	UP
PK	1	p	p	u	j	c	q	k	k	u	c	j
PQ	p	1	p	k	q	k	j	u	c	c	u	j
PU	p	p	1	c	j	u	j	c	u	k	k	q
QU	u	j	c	1	p	p	u	c	j	q	k	k
QP	k	q	k	p	1	p	c	u	j	j	u	c
QK	c	j	u	p	p	1	k	k	q	j	c	u
KP	q	k	k	u	c	j	1	p	p	u	j	c
KU	j	u	c	c	u	j	p	1	p	k	q	k
KQ	j	c	u	k	k	q	p	p	1	c	j	u
UQ	u	c	j	q	k	k	u	j	c	1	p	p
UK	c	u	j	j	u	c	k	q	k	p	1	p
UP	k	k	q	j	c	u	c	j	u	p	p	1

Table I displays the general, logical relation between two slip systems, or two sets of dislocations. For the present purposes, we wish to use it to describe plastic relaxation, which we envisage to occur due to the elastic

interactions. For these, we assume that the number of secondary dislocations generated is proportional to the degree of elastic interaction between the two dislocations. Then, Table I becomes the relaxation matrix R introduced in the last section, and p, q, etc. become coefficients in a linear relation.

Multiplication of the relaxation matrix R with the yield matrix Y shows that there are, under the existing symmetry restrictions, only 3 independent parameters in the product matrix, one of which we use for normalization. Then,

$$H = \begin{vmatrix} 1 & 1 & 1 & h & g & h & g & h & h & h & h & g \\ h & g & h & 1 & 1 & 1 & h & h & g & g & h & h \\ g & h & h & h & h & g & 1 & 1 & 1 & h & g & h \\ h & h & g & g & h & h & h & g & h & 1 & 1 & 1 \end{vmatrix} \qquad (10)$$

with

$$h = \frac{1 + q + c + u + j + 2\,(p + k)}{q + 2\,(c + u + k + j)}$$

$$g = \frac{1 + 2\,(p + c + u + j)}{q + 2\,(c + u + k + j)} \qquad (11)$$

QUANTITATIVE ESTIMATES

A quantitative derivation of all parameters in the interaction matrix on the basis of dislocation theory could only be obtained on the basis of very particular assumptions about the interactions and their superpositions, and about the actual arrangement of dislocations. One can, however, derive a number of equalities and inequalities that hold under many assumptions. One of these is j=k, as mentionad above. In general, we must keep two criteria in mind: all parameters must be positive, because the primary dislocation can always be relaxed by one sign of secondary dislocation; and no parameter should be much greater than 1, because then one primary dislocation would cause many times its length of a particular secondary dislocation set to be generated.

From single-crystal experiments, we know that the difference between different non-coplanar systems is hard to tell. Let us, therefore, assume $h \approx g$, which demands

$$q = c + u - j.$$

This relation seems not unreasonable from the point of view of dislocation theory. Combining, in a qualitative way, long-range elastic interactions and the energy to be gained from a reaction of parallel dislocations, one would expect plastic relaxation to be particularly effective for q and u, less for c and j. (Another effect of setting h = g is that H can be written as a 4x4 matrix, rather than a 12x4, if one introduces the algebraic sum of shears *in each slip plane*.)

Finally, let us set p ≈ 1/2, characterizing the effectiveness of relaxation between coplanar dislocations. Then,

$$h = g = \frac{2 + 2q + 4j}{3q + 6j} \approx \frac{2 + 2q}{3q} \qquad (12)$$

where the last approximation comes from neglecting j in comparison to q/2. It is, of course, easy to make other sensible assumptions; but eq. (12) gives the following set of circumstances, which are in rough agreement with various bits of experience. First, the last expression in eq.(12) is 4/3 for q=1, 2 for q= 1/2. These values may be appropriate, for example, for aluminum and copper, respectively: the effectiveness of cross-slip relaxation should be almost perfect for full dislocations, less for extended ones.

Second, experimental single-crystal flow stress ratios reflect approximately $\sqrt{h}$, not h (because of the integration over all the previous hardening, with $\tau^{(p)}$ in the denominator, eq. (9)); this would give "latent hardening ratios" of 1.15 for Al, 1.41 for Cu: in good accord with observations.

Third, from TEM observations, one knows that the total length of "secondary" dislocations stored after single slip in stage II strain hardening is of the same order as that of "primary" dislocations (or those of the primary Burgers vector, or those in the primary slip plane): that is true for q between 1/2 and 1, as can be derived from Table I.

Finally, let us make an estimate of the highest value of h that may be expected under these assumptions: q = 1/4 seems rather ineffective: it gives h = 10/3; q= 1/9 seems ridiculously small: it gives h = 20/3. Thus, we assume that h and g are likely to be larger than 4/3 and smaller than 4, giving rise to latent hardening ratios between 1.15 and 2.

Incidentally, *isotropic* behavior would demand q = 2 which is, under the present set of assumptions, almost

impossible: it would demand one primary dislocation to attract *two* cross-slip dislocations for its neutralization.

In multiple slip, eq.(9) predicts strain-hardening rates that are higher than for single slip (if they are expressed in the usual terms of the algebraic shear sum $d\Gamma$): in <111> tension, for example,

$$d\tau/d\Gamma = (2 + 4h)/6 \cdot \theta_1$$

where θ_1 is the single-slip value defined in eq.(9). For Cu, the ratio would be about 5/3; in <100> crystals, it would be a bit higher yet–both in accord with observations.

The difference in flow stress between different active systems would be small when many systems are active. When there are only 6 systems, on 3 planes, the inactive plane has a higher flow stress: for <111> tension, as above, the hardening rate would be $h \cdot \theta_1$, and the flow stress ration $\sqrt{h}$, i.e. about 1.15 in Al – which has been observed[7].

APPLICATION TO POLYCRYSTAL DEFORMATION

In polycrystals, under general deformation and with its attendant orientation changes in every grain, it is unlikely that any slip plane will be inactive for any substantial strain increment. Thus, the evolution of the flow stress should be almost isotropic. However, small differences in flow stress could be important under certain circumstances. Figure 1 shows a schematic single-crystal yield surface (SCYS) section in which four facets, corresponding to four slip systems, meet at one vertex in three dimensions. This means that these four slip systems have one dependency relation between them: only three are necessary to accommodate an arbitrary prescribed strain increment within this subspace. Within the framework of rate independent Taylor theory, this leads to an ambiguity in slip system selection (which disappears when a finite, even though perhaps very small, rate sensitivity is used). Now let us assume we have prescribed a deformation with $d\varepsilon_1=0$. Then, the "front" and "back" slip systems are not needed: the deformation will presumably be carried by the "right" and "left" ones. If the active systems harden some, but the inactive more, the 4-vertex decomposes into two 3-vertices. This alters the slip system selection procedure for the next step. Thus, texture development may be affected.

For a quantitative estimate of the effects, we have used the Los Alamos polycrystal plasticity (LApp) code (version 6.4a). Since the SCYS continually changes, we had to run the simulation in a strictly viscoplastic mode; the rate sensitivity was set to 0.03, which was previously shown to give the same results as any smaller (though nonzero) value. Relaxed Constraints were incorporated gradually when grains became sufficiently flat. Strain hardening within the grains was incorporated using a Voce law, with the parameters for aluminum, copper, and silver as determined from previous experiments[11]. The latent hardening parameters were chosen in accordance with the above discussion: h=g=4/3 for aluminum, 2 for copper and, arbitrarily, 4 for silver (to demonstrate the maximum conceivable effect under this model).

Tension, compression, and plane-strain rolling were simulated, to von Mises strains of up to 3.0. The first and most striking result was that both the tension and the rolling textures showed only very subtle differences against a calculation in which all flow stress ratios were kept equal to 1 ("isotropic hardening"). In compression, however, there were marked differences—and they depended not only on the latent hardening parameters chosen, but also on the precise scalar strain hardening law. This is presumably due to the fact that when the scalar hardening rate (θ) goes to

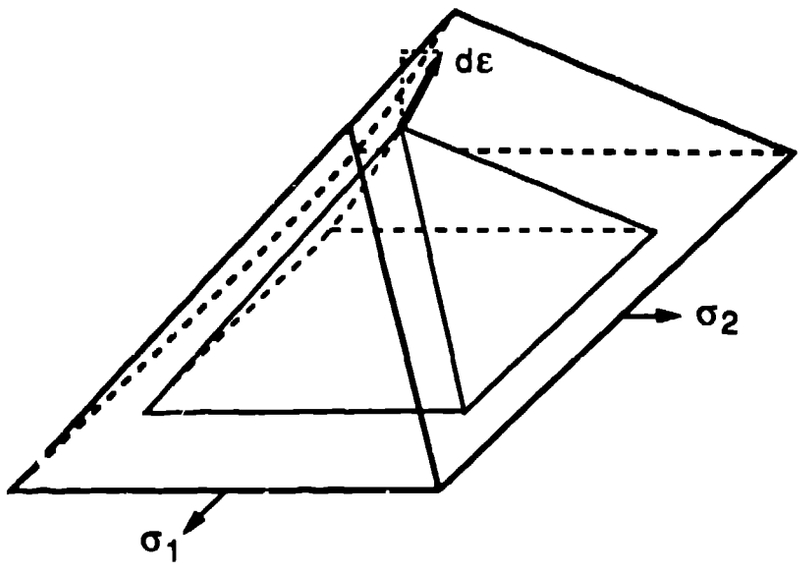

Fig. 1 - Schematic single-crystal yield surface section showing decomposition of a vertex through latent hardening.

zero, the flow stress ratios cannot change any more either. Figure 2 shows the results for the three experimentally determined hardening laws, with the three assumed latent hardening parameters. It is perhaps important that the <110> texture that develops in FCC compression has a strong tendency for plane-strain deformation and can be forced into uniaxial deformation only by a 50% increase in stress. (It is for this reason that "grain curling" may occur, as in BCC tension.) This is a particularly sensitive case. The textures exhibited in Fig. 2 are not observed quantitatively. It is, however, true that a tendency for a spread from <110> toward <111> develops as one progresses from Al to Cu to Ag. The second question to be asked, in addition to texture development, is the effect on macroscopic plastic anisotropy. This would be affected not only by the texture, but also by the flow stress ratios themselves. We have followed, in the simulations, the average over all grains of the greatest flow stress (normalized by the average): after an initial decrease, the approximate values (after a strain of about 0.5) are 1.03, 1.09, and 1.15, respectively, for

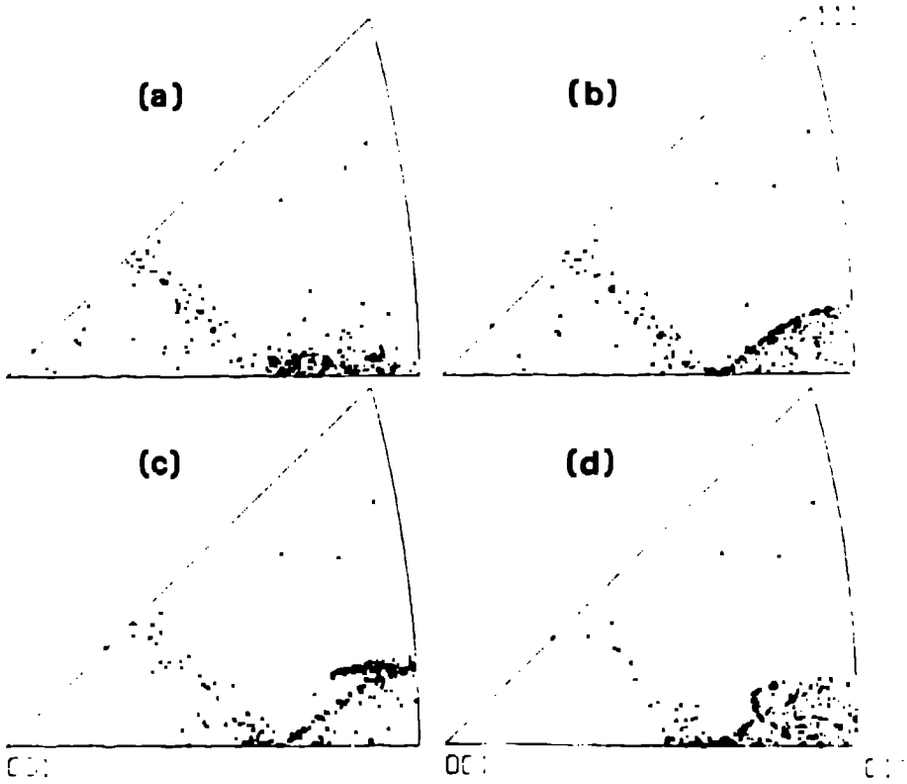

Fig. 2 - Compression textures at a strain of 1.0, simulated by LApp for (a) isotropic hardening, (b) Al, (c) Cu, (d) Ag.

h-g-4/3, 2, and 4. The real test is the polycrystal yield surface after deformtion: in principle, one might expect a somewhat stronger expansion in all stress space directions except the loading direction. This was in fact demonstrated by the simulations - but the effect is only a few percent. In the worst case (which we find at best marginally realistic: h-g-4), the additional lateral spread was 6% after rolling to a von Mises strain of 2.0. Experimental observations[12,13] of effects of the order of 10% in aluminum can therefore not be explained on the present basis. A perhaps more important effect is the observation that the "vertices" (or rather regions of high curvature) may, in some cases, get sharper and the "flat spots" more extensive. This may alter, for example, the "R-value" in certain cases.

SUMMARY

Based on the forest theory and the modified Taylor model of polycrystal plasticity, both of which are well established, the effects of latent hardening on the development of texture and plastic anisotropy are expected to be quite small, though they may be of interest in some cases.

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