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S. P. Hirshman

OAK RIDGE NATIONAL LABORATORY

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ABSTRACT

The nonlinear motion of a charged particle in a uniformly magnetized, two-dimensional plasma is analyzed in the presence of finite amplitude electrostatic waves traveling in the plane perpendicular to the magnetic field. In a strongly magnetized plasma, no particle trapping occurs in a single electrostatic traveling wave of arbitrary amplitude, since the induced $\vec{E} \times \vec{B}$ motion is orthogonal to the direction of wave propagation. However, particle orbits can be trapped when there are at least two mutually orthogonal components of the electrostatic field with finite wave amplitude $E_\perp/B \geq \omega/k_\perp$ such that a particle drifts one wavelength in a wave period. This threshold amplitude for strong nonlinear behavior coincides with and helps to explain the many-wave turbulence criterion for the onset of particle diffusion in a two-dimensional plasma.

It is well known that a charged particle in an unmagnetized plasma can be trapped (i.e., swept along in a wave trough) by a single electrostatic wave whenever the wave potential energy $e\Phi$ exceeds the particle kinetic energy in the wave frame, $1/2\ m(v - v_w)^2$. Here, $v_w = \omega/k$ is the wave phase speed and v is the particle speed in the $\vec{k}$ direction. In a strongly magnetized plasma the two-dimensional particle motion in the plane perpendicular to a uniform magnetic field $\vec{B} = B\hat{z}$ is determined by the $\vec{E} \times \vec{B}$ drift:

$$\frac{d\vec{r}_\perp}{dt} = \frac{c\vec{B} \times \nabla\Phi(\vec{r},t)}{B^2} \quad , \tag{1}$$

where $\vec{E} = -\nabla\Phi$. Since there is no energy exchange with the wave, $\vec{E} \cdot d\vec{r}_\perp/dt = 0$, it is apparent that trapping due to a perpendicularly propagating potential wave will be quite different than the trapping mechanism along the magnetic field. Indeed, for a single wave $\Phi = \phi \cos(k_\perp x - \omega t)$ traveling in the x direction, Eq. (1) predicts

$$\frac{dx}{dt} = 0; \quad \frac{dy}{dt} = -\frac{ck_\perp\phi}{B} \sin(k_\perp x - \omega t) \quad . \tag{2}$$

Thus, there is no particle trapping, even for arbitrary wave amplitude ϕ, since $x = x_0$ is a constant of motion for this single wave.

To achieve particle trapping requires, by definition, induced motion in the direction of the propagating wave. For a magnetized plasma, such induced motion is absent for a single electrostatic wave, or even for a spectrum of waves propagating in the same direction, since $\vec{E} \times \vec{B}$ is always orthogonal to the wave vector $\vec{k}$. Hence, at least two waves with wave vectors $\vec{k}_1$ and $\vec{k}_2$ such that $\vec{k}_1 \times \vec{k}_2 \neq 0$ are necessary for $\vec{E} \times \vec{B}$ trapping.

We now show that two electrostatic waves of sufficient amplitude do, indeed, result in trapping. Consider the potential field

$$\begin{aligned}\Phi(\vec{r},t) &= \frac{\phi}{2}\ [\sin(\vec{k}_1 \cdot \vec{r} - \omega_1 t) + \sin(\vec{k}_2 \cdot \vec{r} - \omega_2 t)] \\ &= \phi \sin(\vec{k}_+ \cdot \vec{r} - \omega_+ t) \cos(\vec{k}_- \cdot \vec{r} - \omega_- t) ,\end{aligned} \tag{3}$$

which consists of two traveling waves with wave vectors $\vec{k}_1$ and $\vec{k}_2$ perpendicular to $\vec{B}$. Here, $\vec{k}_\pm = (\vec{k}_1 \pm \vec{k}_2)/2$ and $\omega_\pm = (\omega_1 \pm \omega_2)/2$. It is useful to define unit vectors $\underline{\vec{e}_\pm} = \vec{k}_\pm/|\vec{k}_\pm|$, which are generally not orthogonal (Fig. 1).

To solve Eq. (1) with this potential, it is convenient to transform to the wave frame in which the potential is static:

$$\vec{r}_w = \vec{r}_\perp - \vec{v}_w t \quad . \tag{4a}$$

Here, the wave frame velocity $\vec{v}_w$ is completely determined by its contravariant vector components, $\vec{v}_w \cdot \underline{\vec{e}_\pm} = \underline{\omega_\pm}/\underline{k_\pm}$:

$$\vec{v}_w = \frac{\omega_+}{k_+}\vec{e}^{\,+} + \frac{\omega_-}{k_-}\vec{e}^{\,-} , \tag{4b}$$

where $\vec{e}^{\,+} = \vec{e}_- \times \hat{z}/(\vec{e}_+ \times \vec{e}_- \cdot \hat{z})$ and $\vec{e}^{\,-} = \hat{z} \times \vec{e}_+/(\vec{e}_+ \times \vec{e}_- \cdot \hat{z})$ are the vectors adjoint to $\underline{\vec{e}_\pm}$, i.e., $\underline{\vec{e}_\pm} \cdot \vec{e}^{\,\mp} = 0$ and $\underline{\vec{e}_\pm} \cdot \vec{e}^{\,\pm} = 1$ (Fig. 1). The condition $\vec{k}_1 \times \vec{k}_2 \neq 0$ guarantees the existence of the wave frame, $\vec{e}_+ \times \vec{e}_- \cdot \hat{z} \neq 0$. In this frame, Eq. (1) becomes

$$\frac{d\vec{r}_w}{dt} = \frac{c\vec{B} \times \nabla\Phi_w(\vec{r}_w)}{B^2} , \tag{5a}$$

where

$$\Phi_w(\vec{r}_w) = \Phi(\vec{r}_w,0) - \vec{r}_w \cdot \left(\frac{\vec{v}_w}{c} \times \vec{B}\right) \tag{5b}$$

is the potential in the wave frame. Note that the last term in Eq. (5b) corresponds to a spatially uniform electric field $\vec{E}_{in} = \vec{v}_w \times \vec{B}/c$ induced by motion of the wave across the uniform magnetic field.

From Eq. (5a), note that in the wave frame $\nabla\Phi_w \cdot d\vec{r}_w/dt = d\Phi_w/dt = 0$. Thus, particles move along contours of constant wave frame potential

$$\Phi_w(\vec{r}_w) = \Phi_0 \quad , \tag{6}$$

where the constant of motion Φ_0 is determined by the value of Φ_w at an arbitrary initial phase point $\vec{r}_{w0}$.

The position vector in the wave frame, $\vec{r}_w = x_w\vec{e}_x + y_w\vec{e}_y$ (here, $\vec{e}_x$ and $\vec{e}_y$ are the orthogonal cartesian unit vectors), can also be decomposed along $\vec{e}^{\pm}$:

$$\vec{r}_w = (x'\vec{e}^{+} + y'\vec{e}^{-}) \quad , \tag{7a}$$

where the transformation between the orthogonal (x_w and y_w) and contravariant (x' and y') coordinates is given by

$$x' = \vec{e}_+ \cdot \vec{r}_w = x_w\vec{e}_+ \cdot \vec{e}_x + y_w\vec{e}_+ \cdot \vec{e}_y \quad , \tag{7b}$$

$$y' = \vec{e}_- \cdot \vec{r}_w = x_w\vec{e}_- \cdot \vec{e}_x + y_w\vec{e}_- \cdot \vec{e}_y \quad . \tag{7c}$$

In the representation Eq. (7a), $\vec{k}_+ \cdot \vec{r}_w = k_+x'$ and $\vec{k}_- \cdot \vec{r}_w = k_-y'$. The following normalized variables are conveniently defined:

$$X = k_+x'/2\pi \quad , \tag{8a}$$

$$Y = k_-y'/2\pi \quad , \tag{8b}$$

$$Y_0 = \frac{c}{B} \frac{(\vec{k}_+ \times \vec{k}_- \cdot \hat{z})\Phi_0}{2\pi\omega_+} \quad , \tag{8c}$$

$$\varepsilon_0 = -Y_0\phi/\Phi_0 \quad , \tag{8d}$$

$$\alpha = \omega_-/\omega_+ \quad ,$$

where $|\alpha| < 1$ can be chosen without loss of generality. Then the particle trajectories in Eq. (6) in the X-Y phase plane become

$$Y - Y_0 - \alpha X = \varepsilon_0 \sin 2\pi X \cos 2\pi Y \quad . \tag{9}$$

The remaining integration constant characterizing the original system in Eq. (1) is associated with the time to traverse the level curves of Eq. (9). Using the $\vec{k}_-$ component of Eq. (5a), noting $\nabla = \vec{e}_+\partial/\partial x' + \vec{e}_-\partial/\partial y'$, yields

$$\omega_+ t = \int_{-Y(0)}^{Y} \frac{dY}{\alpha/2\pi + \varepsilon_0 \cos 2\pi X(Y) \cos 2\pi Y} \quad , \tag{10}$$

where X(Y) is determined from Eq. (9).

In order to study the onset of particle trapping, it is sufficient to analyze the level curves $\varepsilon_0(X,Y;\, Y_0,\alpha) = \varepsilon_0$ determined by Eq. (9). For analytic simplicity, we shall subsequently consider the special case $\alpha = 0$, corresponding to $\omega_1 = \omega_2 = \omega_+$. The main conclusion regarding particle trapping is not qualitatively altered for $\alpha \neq 0$, cf. Eq. (12). From Eq. (3), note that the potential for $\alpha = 0$ is a standing wave in the $\vec{e}_-(y')$ direction with wavelength $2\pi/k_-$ and a traveling wave with phase velocity ω_+/k_+ in the $\vec{e}_+(x')$ direction.

For $\varepsilon_0 \ll 1$ and $\alpha = 0$, $Y-Y_0 \simeq \varepsilon_0 \sin 2\pi X \cos 2\pi Y_0$ is a periodic open curve in the X,Y phase plane corresponding to particles circulating in the X direction in the wave frame (Fig. 2). Thus, for small wave amplitudes, all particles are untrapped and undergo finite oscillations about a point in the laboratory frame.

There is a threshold value of $\varepsilon = \varepsilon^*(Y_0)$ above which stable elliptic fixed points of the mapping in Eq. (9) begin to appear (Fig. 3). The phase space trajectories around these points are closed curves corresponding to bounded excursions in the wave frame and hence to particles which are trapped by the traveling $\vec{E} \times \vec{B}$ wave. The emergence of the first (lowest order) fixed point corresponds to the onset of nonlinear trapping.

To determine the fixed points and hence the criterion for trapping in this model, the system of simultaneous equations $\partial\varepsilon_0/\partial X = \partial\varepsilon_0/\partial Y = 0$, together with Eq. (9), must be solved. Two sets of (nontrivial) fixed points are found in the periodic interval $0 \leq X \leq 1$:

$$X = \frac{1}{4}\,; \quad Y_n^+ = \frac{\xi_{2n+1}}{2\pi}\,; \text{ for } \varepsilon_0 = \varepsilon_n^+ = [1 + (\xi_{2n+1} - \xi_0)^2]^{\frac{1}{2}}/2\pi \qquad (11a)$$

$$X = \frac{3}{4}\,; \quad Y_n^- = \frac{\xi_{2n}}{2\pi}\,; \quad \text{ for } \varepsilon_0 = \varepsilon_n^- = [1 + (\xi_{2n} - \xi_0)^2]^{\frac{1}{2}}/2\pi\,. \qquad (11b)$$

Here, $\xi_0 = 2\pi Y_0$ and ξ_n are the roots of

$$\tan \xi_n = -\frac{1}{(\xi_n - \xi_0)}\,. \qquad (11c)$$

The roots are uniquely characterized by $\mathrm{sgn}(\sin \xi_n) = (-1)^n$.

Figure 3 shows the level curves of $\varepsilon_0(X,Y)$ for the fixed phase $Y_0 = 0$. The predicted fixed points are ($X = 1/4$, $Y_n^+ \doteq 0.97, 1.99, n + 1$), ($X = 3/4$, $Y_n^- \doteq 0.44, 1.48, n + 1/2$), for $n \geq 2$, with $\varepsilon_n^{\pm} \doteq Y_n^{\pm}$. These are in agreement with the numerically computed values.

From Eq. (11) it is clear that the choice $\xi_n = \xi_0 = \pi/2 + n\pi \equiv \xi_n^*$ (n is an integer) yields the minimum value $\varepsilon_0 = \varepsilon^* = 1/2\pi$ for the appearance of a fixed point. When $\alpha \neq 0$, it can be shown that $\varepsilon^* = (1 + |\alpha|)/2\pi$. This corresponds to the relation $cE_\perp/B = \omega/k_\perp$ or, more precisely,

$$1 = \frac{c}{B} \frac{|\vec{k}_+ \cdot (\vec{k}_- \times \hat{z})\phi|}{|\omega_+| + |\omega_-|} \quad . \tag{12}$$

Thus, particle trapping will first occur (for those particles with the proper initial phase) when the electric field amplitude is sufficient to convect particles one wavelength in a wave period. This threshold behavior contrasts with trapping in an unmagnetized plasma, where there are always some trapped particles for any finite amplitude electric field. Also note that the two-dimensional dynamics considered here are quite different from the three-dimensional motion in a sheared magnetic field, for which there is also no threshold for trapped particle island formation.[1]

At the value $\varepsilon_0 = \varepsilon^*$ and $\xi_n = \xi_0 = \xi_n^*$ (i.e., $Y = 2n + 1/4 = Y_0$), the fixed point root of Eq. (11c) exists only in a limiting sense. In fact, for $\varepsilon > \varepsilon^*$, three characteristic points emerge from this single coalesced root. One is the fixed point $X = 3/4$, $Y = 1/4$ (or $X = 1/4$, $Y = 3/4$) calculated above. The remaining two are hyperbolic points which migrate away from the fixed point as ε increases along the separatrix $Y = 1/4$ (or $Y = 3/4$) and are located at the roots of $\sin 2\pi X = \pm\varepsilon^*/\varepsilon$.

The phase space trajectories for two values of wave amplitude $\varepsilon_0 = (0.5, 2.0)$ are shown in Figs. 4 and 5. As ε_0 increases, the island shapes tend to become rectangular and the hyperbolic points on the separatrix $Y = 1/4$, $3/4$ move away from the fixed points toward half-integer values of X. For $\varepsilon_0 \to \infty$, the separatrices tend toward the lines $X = n/2$, $Y = 1/4 + m/2$ for integer m and n. Thus, the limiting phase space for large wave amplitude is a lattice delineated by the separatrices which enclose bounded island trajectories (Fig. 6). For this two-wave system, the nonlinear island orbits never tend to overlap, even for arbitrarily large wave amplitudes. Therefore, there is no possibility for stochastic particle motion,[2] a fact that may be inferred from Eq. (5a), which represents an integrable dynamical system. The persistence of islands in the two-dimensional, two-wave system is in marked contrast with magnetic braiding[1] which occurs in a sheared magnetic field and stochastic particle orbits which appear in two-dimensional, single wave plasmas when finite Larmor radius effects are retained.[3]

The analysis presented here has been for the so-called "single wave" case for which a transformation of the form in Eq. (4) can be found to make Φ_W independent of time. In the many-wave turbulent limit, a wave frame in which Φ is static exists only approximately for time intervals comparable to the autocorrelation time. For longer times, a statistical treatment of the particle orbit is required. Nevertheless, the salient nonlinear feature of the single wave analysis (trapping threshold) has a turbulent analogue. Dupree[4] has obtained a threshold criterion for the onset of diffusion due to random $\vec{E} \times \vec{B}$ fluctuations which is identical to Eq. (12), with $\vec{E}_k$ and $\omega/k_\perp$ appropriately averaged over the spectrum. Thus, the <u>coherent</u> $\vec{E} \times \vec{B}$ <u>trapping</u> threshold becomes the criterion for finite <u>turbulent diffusion</u>.

As the wave spectrum broadens, there is thus apparently a continuous transition from trapped particle to turbulent motion. It is thus possible to infer a diffusion coefficient for two-dimensional $\vec{E} \times \vec{B}$ turbulence from the present two-wave analysis. In the transition between coherent and random particle motion, the autocorrelation time and trapping time (time to complete an island orbit) become comparable. From Eq. (10), $\Delta t_{Trap} \sim (\omega\varepsilon_0)^{-1}$. The maximum correlated particle step size Δx is an island width, which from Fig. 6 is found to be a wavelength, $\Delta x = 2\pi/k_\perp$. Note that as the electric field amplitude increases, the island width remains constant whereas the trapping time decreases. The diffusion coefficient is estimated to be $D \equiv (\Delta x)^2/\Delta t = (cE_\perp/B)k_\perp^{-1}$, which is in agreement with the many-wave results when appropriate spectrum averages are taken.[4] Physically, for island orbits that are near the separatrices of Fig. 6, small perturbations of the particle orbit due to other waves can produce "diffusion" in the X-Y phase plane as the particle moves along the lattice of separatrices from island to island.

Finally, we note that the existence of self-consistent, large amplitude $\vec{E} \times \vec{B}$ waves needs to be investigated in the future.

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FIGURE CAPTIONS

Fig. 1 Wave frame coordinate system.

Fig. 2 Phase space trajectories for $\varepsilon_0 = 0.1$.

Fig. 3 Level curves of the mapping in Eq. (9) for $Y_0 = 0$ and various values of ε_0.

Fig. 4 Phase space trajectories for $\varepsilon_0 = 0.5$.

Fig. 5 Phase space trajectories for $\varepsilon_0 = 2.0$.

Fig. 6 Limiting trajectories for $\varepsilon_0 \rightarrow \infty$.

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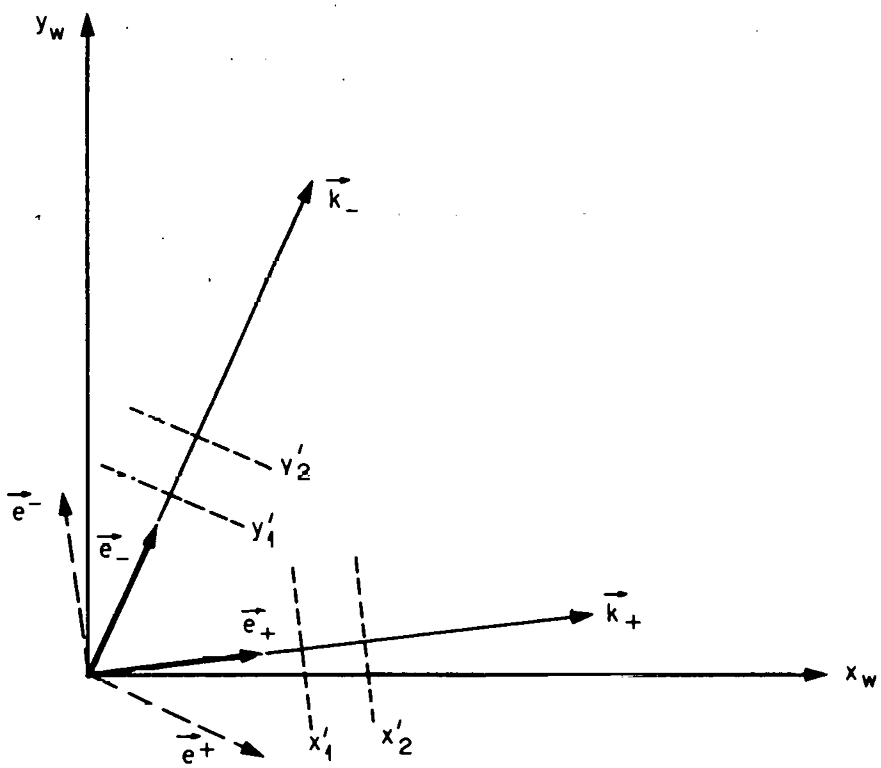

Fig. 1.

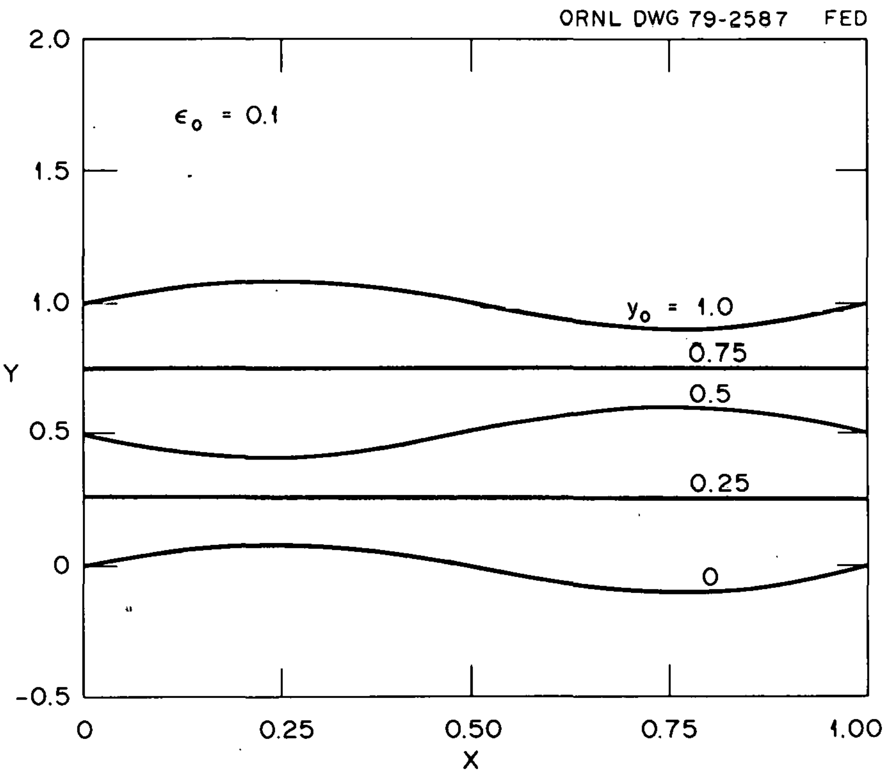
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Fig. 2.

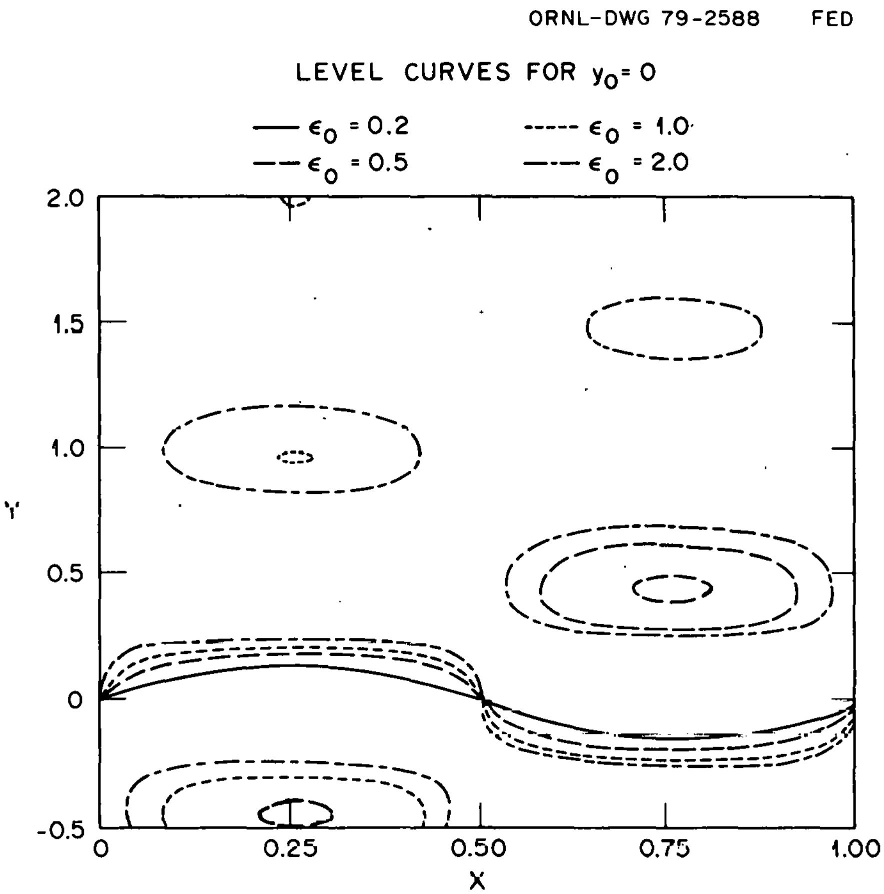
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Fig. 3.

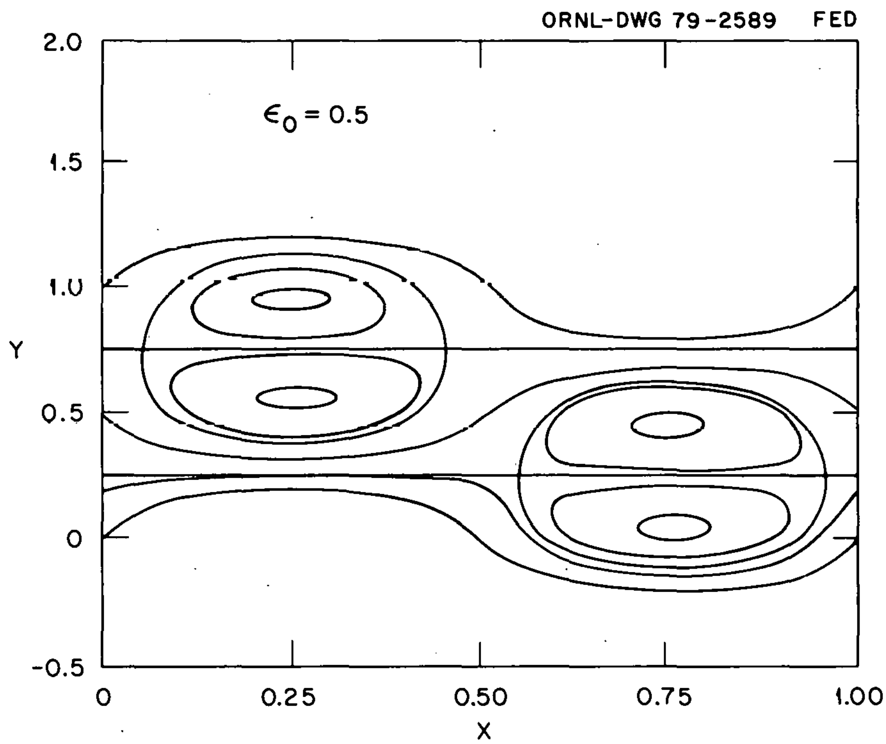
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Fig. 4.

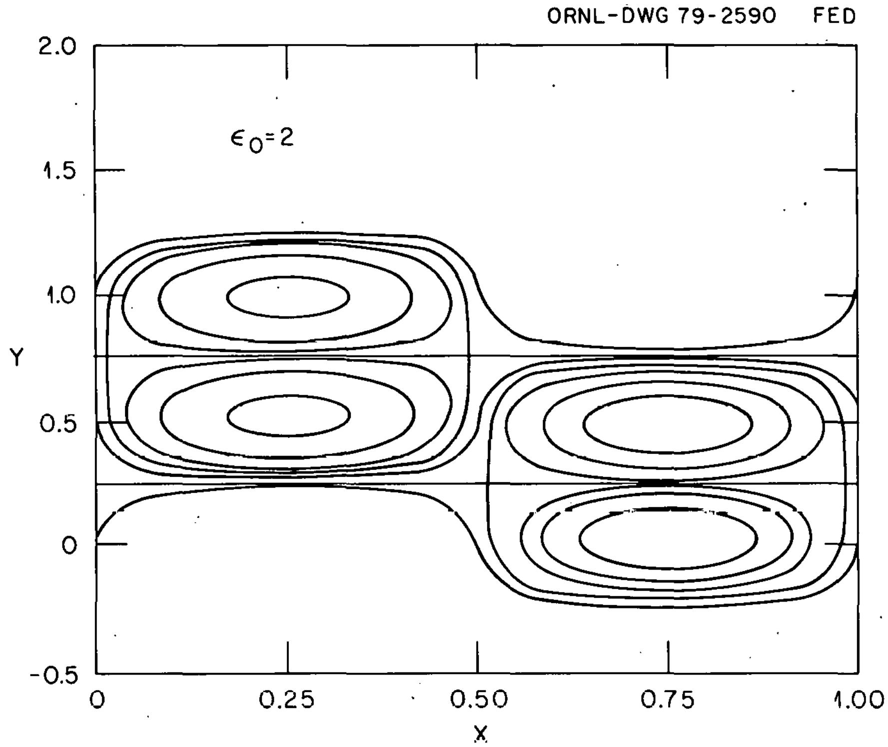
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Fig. 5.

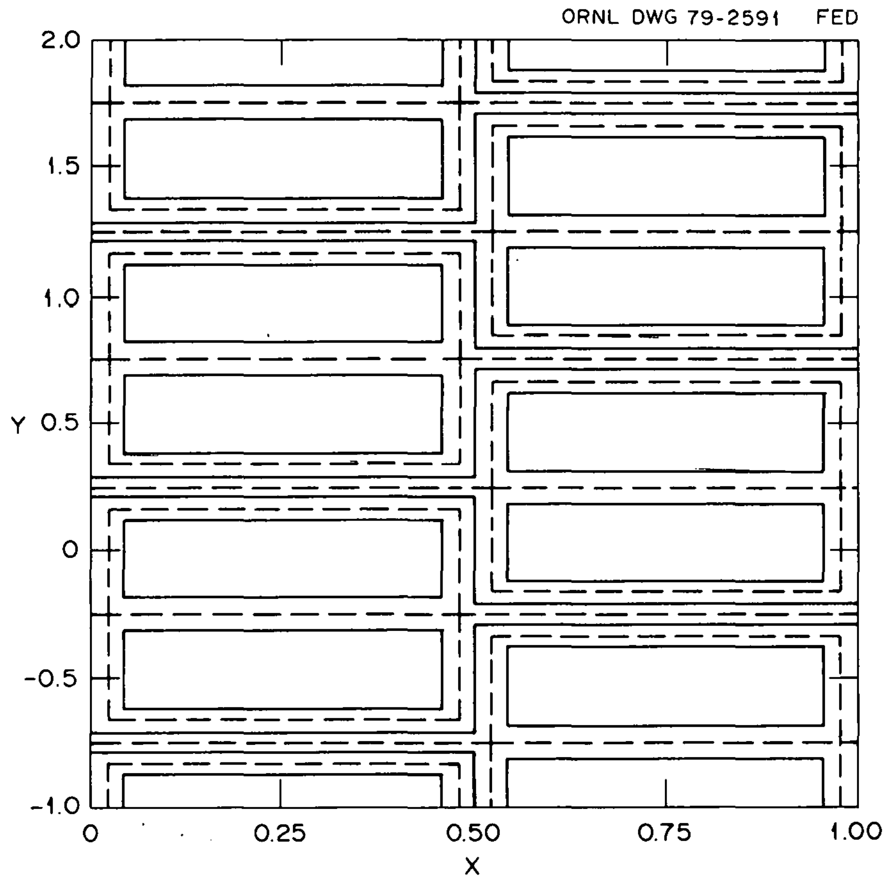

Fig. 6.

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