

This is a brief summary of the work performed over the last year, as well as the work in progress to be continued under this grant. For details publications and preprints are appended.*

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Torus Structure in Higher Dimensional Hamiltonian Systems.

This work was already reported in last year's Annual Progress Report as work in progress. This has been completed and published in Physical Review A, with co-authors G. Gyorgyi and F. Ling¹.

Statistical Thermodynamics is based on the notion that mechanical systems of sufficient complexity perform quasi ergodic motion in phase space. On the other hand systems of low phase space dimensionality (3), are ergodic only in parts of phase space their spread being impeded by KAM tori. In higher dimensional systems Arnold Diffusion enables particle orbits to circumnavigate many of these tori, making a larger part of phase space ergodic.

Tori surrounding stable periodic orbits (island tori) seem to be an exception. We find using analytical as well as numerical methods that within islands robust invariants exist preventing Arnold Diffusion. We developed a new method the "quasi surface of section" method (QSS) to study the structure and topology of higher dimensional tori generated by these invariants.

We give an analytical estimate of the measure of regions inaccessible to ergodic motion as the number of degrees of freedom (dimensionality of phase space) increases. It is found that the measure of these regions decreases very fast with increasing degrees of freedom, so in the limit of infinite degrees of freedom ergodicity holds overall.

Particle Heating and Stochastic Web Diffusion.

In the last few years a number of papers appeared to analyze stochastic
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web diffusion, and claims have been made as to the applicability to charged particle heating. To analyze this phenomenon I investigated a realistic physical model, where the particle in a magnetic field is subject to a standing wave. This is a realizable situation, contrary to the models investigated by others, who typically use unrealistic δ function time dependence. It was found that the stochastic web fades away with increasing particle energy, so this scheme does not lend itself to particle heating in this form. A paper was published² and a reprint is enclosed.

Scaling Behavior of Coupled Conservative Nonlinear Systems.

This work is an outgrowth of our work on Torus Structure in Higher Dimensional Hamiltonian Systems. Here instead of maps, we investigate coupled undamped (Hamiltonian) Duffing oscillators. We find well defined scaling laws using analytical expansion methods for normal modes, as we expected based on our more general work on coupled maps. This work has been submitted for publication³ and a copy is enclosed.

Box Counting Algorithm and Dimensional Analysis of a Pulsar.

Time series of signals are frequently used to get information (fractal dimensions), of complex objects. A pulsar is an ideal object for such investigations since it gives a very precise time series, where the pulse strength varies over several orders of magnitude. We obtained data courtesy of Dr. J. Cordes of Cornell of pulsar 0950 + 08 as detected in Arecibo and subjected it to analysis. We find a large embedding dimension (≈ 14) and a correlation dimension of about 4.5. Trying several methods of data analysis we found that the box counting algorithm is far more efficient than the commonly used Grassberger-Procaccia algorithm. We analyze the efficiency of these

method, on a number of different systems. This work is completed and submitted for publication⁴.

Universality of Coupled Nonlinear Systems.

We are in the process of understanding properties of coupled nonlinear systems, on deeper level of universality.

Coupled systems have recently been investigated by a large number of people, since it holds the clue to understanding turbulence, Belousov-Zhabotinsky reactions, pattern formation, and a host of other problems described by nonlinear partial differential equations. It usually involves of picking some system of coupled maps and investigate some of its properties in some part of parameter space numerically. This process reveals a host of complexity, order, chaos, period doublings, Hopf bifurcations, circle map type behavior etc., in a seemingly random manner. The trouble is that it tells us nothing about what to expect if another system of equations is used, or a different part of parameter space is explored. If we had some framework of organizing these systems into classes with universal characteristics one could make predictions about the expected behavior of complex systems.

The first step in this direction was taken by Kuznetsov in the USSR, who applied renormalization group treatment to two coupled quadratic maps, found universal scaling behavior, well confirmed by numerical analysis. we have found recently that such treatment can be vastly extended over a wide variety of coupled systems, long chains, or planar formations, and structures in three spatial dimensions.

We identified several universality classes with different scaling properties, and confirmed our analytical results numerically. The first phase of this work is nearing completion and submittal for publication. It will also

be presented as invited papers on several meetings next Summer.

Additional Publications.

My co-worker D. F. H. Ling has completed various projects here that he started abroad. He credited DOE with support. Copies are enclosed ⁵⁻⁸.

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Publications:

1. Torus Structure in Higher Dimensional Hamiltonian Systems, G. Gyorgyi, F. H. Ling and G. Schmidt, Phys. Rev. A., 40, 5311 (1989).
2. Comments on Particle Heating and Stochastic Web Diffusion, G. Schmidt, Comments on Plasma Physics and Controlled Fusion, 13, 77 (1989).
3. Scaling Behavior of Coupled Conservative Nonlinear Oscillators, F. H. Ling and G. Schmidt, submitted to J. of Sound and Vibrations.
4. Box Counting Algorithm and Dimensional Analysis of a Pulsar, F. H. Ling and G. Schmidt, submitted to Phys. Lett. A.
5. Bifurcations in a Hard Spring Duffing Oscillator, F. H. Ling and Y. M. Cheung, IUTAM Symposium on Nonlinear Dynamics in Engineering Systems, to appear. Also submitted to J. of Sound and Vibrations.
6. On the Fractal Dimensions of a Combined Fractal Set, F. H. Ling, Phys. Lett. A., 138, 25 (1989).
7. A Numerical Study of Different Attractors in Parameter Space, F. H. Ling, Z. Phys. B., 75, 539 (1989).
8. Dynamic Stability Analysis of the Geophone Spurious Frequency Phenomenon, F. H. Ling, W. M. Wang and R. J. Mao, J. of Sound and Vibration, 138, 1 (1990).

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