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BLOWUP OF A WEAK BEAM DUE TO INTERACTION WITH A STRONG BEAM IN AN ELECTRON STORAGE RING

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The theoretical description of the beam-beam interaction presented here takes into account all the important features of the beam-beam phenomenon: the nonlinear beam-beam force and its dependence on both transverse coordinates, damping of the oscillations, presence of noise in the particle motion, in particular the quantum noise in its synchrotron radiation, actual machine functions, layout and the number B of interaction points, and to some extent imperfections present in the machine. The model deals not with a separate particle, but with the beam as a whole using phase space distribution functions and the average (unperturbed and perturbed) characteristics of the bunch such as its emittances, space charge parameters, etc.

The calculations are done by a perturbation method[1] using the Green's function of the Fokker-Planck equation. This limits the applicability of the method in at least two ways: First, the current of the strong beam (or its space charge parameter, ξ) should not be too large.

The beam blowup is presented roughly speaking as a series in ratio $\xi/2\pi r$, where $2\pi r$ is the betatron phase advance between adjacent interaction points. This ratio is usually smaller than 1. Second, there are regions in the tune diagram where approximation breaks down even for small current (resonance regions). The method treats all resonances simultaneously. The blowup curve is a result of the action of an infinite number of resonances positioned at the same place

At the present stage of development of the theory presented in this work, the longitudinal particle motion is not implemented in the model. The following assumptions and limitations are used in the course of the calculations.

1. The weak beam – strong beam interaction. The particle distribution of the strong bunch is assumed to be unaffected by its interaction with the counter-rotating weak beam.
2. The bunch is assumed to be short in comparison to the value of the beta function at the interaction point β. This might become a serious restriction, especially when considering dynamic β (i.e., perturbed by the linear part of the beam-beam force).
3. The collisions are assumed to occur head-on. This assumption makes the beam-beam force antisymmetric, thus eliminating all odd order resonances.
4. The aspect ratio of the strong beam is assumed to be very small (flat beam).

Under these assumptions the beam-beam interaction produces two effects in the motion of a weak beam particle. First, the linear part of the force with which the strong bunch acts on such a particle changes the effective machine parameters for the weak beam. The tunes, the values of the amplitude beta function at the interaction point, and consequently the values of space charge parameters and beam emittances are changed. I will refer to the new (dynamic) tunes, beta functions, and emittances as perturbed machine parameters.

Second, from the rest portion of the force (i.e., its nonlinear part) a transverse component of the particle velocity experiences an instantaneous change ('kick'), the magnitude of which depends on the particle coordinates at the moment of interaction. Between the subsequent interactions a particle performed damped betatron oscillations in both lateral planes. The motion may be influenced by noise (such as the quantum noise in synchrotron radiation, for example) and this noise should be taken into account. Strictly speaking, the particle motion is influenced also by other nonlinearities in the machine lattice (the most important of which are sextupole fields). I neglect here all nonlinear forces apart from the beam-beam force. The evaluation of the sextupole magnets influence on the particle motion is done in Ref. 2.

The result of all the subsequent interactions should then be averaged over the particle distribution in the four dimensional phase space of coupled transverse motion. All these tasks are achieved here by using the Green's function method.

I restrict myself here to evaluation of the vertical emittance of the weak beam. Indeed, for the flat beam, the beam blowup is observed mainly in the vertical plane. The reason for this is of course that the vertical component of the interaction force is in this case much larger than the horizontal one for the vast majority of the particles.

The vertical beam blowup is presented here as a function of the tunes, ν, r, the damping rates, α, δ, the values of the β functions, β_x, β_y, at interaction point (IP), and the space charge parameters ξ_x, ξ_y, of the perturbed machine for the horizontal and vertical planes, respectively. It also depends on the number and distribution around the ring of the interaction points and the aspect ratio of the strong beam, σ_y/σ_x. The remarkable feature of the result is that for the machine staying away from the resonance, the vertical beam blowup is actually independent of the value of the damping rates.

The derivation of the formula for the vertical beam blowup, i.e., the ratio of the vertical rms size Σ_y of the weak beam perturbed by the interaction to the unperturbed value σ_y of the same parameter, can be found in Ref. 3. The result is:

$$\frac{\Sigma_y}{\sigma_y} = \sqrt{\frac{E_y}{\epsilon_y}} \cdot \sqrt{\frac{\beta_y}{\beta_{y0}}} \qquad \text{where} \qquad (1)$$

$$\frac{E_y}{\epsilon_y} = 1 - \frac{1}{B}\sum_{j=1}^{B}\frac{\hat{\xi}_{yj}}{4\pi r_j}\left(1-\frac{\sigma_y}{\sigma_x}\right) + \sum_{j=1}^{B}\hat{\xi}_{yj} \qquad (2)$$

$$\times \left\{\frac{1}{4\pi\delta B}\sum_{i=1}^{B}\left(\hat{\xi}_{yj+i-1}W_{1j,j+i-1} + \hat{\xi}_{xj+i-1}W_{2j,j+i-1}\right) + \frac{1}{4\pi B r_j}\sum_{i=1}^{B}\left(\hat{\xi}_{yj+i-1}W_{3j,j+i-1} + \hat{\xi}_{xj+i-1}W_{4j,j+i-1}\right)\right\},$$

where

$$\hat{\xi}_x = \frac{r_0 N_b \beta_x}{\gamma\sigma_x^2} \qquad (3)$$

$$\hat{\xi}_y = \frac{r_0 N_b \beta_y}{\gamma\sigma_x\sigma_y} \,. \qquad (4)$$

are the zeroth order terms of the expansion of the space charge parameters of the perturbed machine in the power series in $\sqrt{p} = \sigma_y/\sigma_x$. The factors W have the following meaning

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$$W_{1j,j+i-1} = \sum_{\ell=0}^{\infty}{}' e^{-2\ell\delta}$$

$$\times \left\{ \frac{2}{\sqrt{z_1}} \left[z_3(1+\cos 2r_i\theta) - \frac{1-\cos 2r_i\theta}{\sqrt{3+z_2}} \right] - \frac{4}{z_1}\sqrt{p}\ \cos 2r_i\theta \right\} \quad (5)$$

$$W_{2j,j+i-1} = -\sqrt{p} \sum_{\ell=0}^{\infty} e^{-2\ell\delta - 2\alpha\phi}\ \frac{z_3}{z_1^{3/2}}\ \sin 2r_i\theta\ \sin 2\nu_i\phi \quad (6)$$

$$W_{3j,j+i-1} = \sum_{\ell=0}^{\infty} e^{-2\ell\delta}$$

$$\times \sin 2r_i\theta \left\{ \frac{2z_3\sqrt{3+z_2}+1}{\sqrt{z_1}\sqrt{3+z_2}} + \frac{1}{2} - \frac{3}{z_1}\sqrt{p} - \frac{1}{2}\sqrt{p} \right\} \quad (7)$$

$$W_{4j,j+i-1} = \sum_{\ell=0}^{\infty} e^{-2\alpha\phi}$$

$$\times \sin 2\nu_i\phi \left\{ \frac{1}{2} - \frac{1}{z_1} + \sqrt{p} \left[\frac{2z_3(1-z_2)+\sqrt{3+z_2}}{z_1^{3/2}} - \frac{3}{2} \right] \right\} . \quad (8)$$

Here

$$z_1 = 4 - e^{-2\alpha\phi} \cos^2 \nu_i\phi , \quad (9)$$

$$z_2 = 1 - e^{-2\ell\delta} \cos^2 r_i\theta , \quad (10)$$

$$z_3 = \frac{1}{\sqrt{1-z_2}}\ \arctan \sqrt{\frac{1-z_2}{3+z_2}} , \quad (11)$$

and $(i,j = 1,2\ldots B)$

$$\theta \equiv \theta_{0j,j+i-1} + 2\pi B\ell , \quad (12)$$

$$\phi \equiv \phi_{0j,j+i-1} + 2\pi B\ell , \quad (13)$$

where $\theta_{0j,j+i-1}$ and $\phi_{0j,j+i-1}$ are initial vertical and horizontal betatron phases of the i-th interaction point if the j-th interaction point is considered to be a starting one. The prime on the sum sign in (5) means that the value of the zeroth term in it should be taken with the weight 1/2.

Discussion and Numerical Illustration There are several points which are worth being mentioned here.

1. Apart from damping each term in the infinite sums (5) through (8) depends on θ and ϕ only through the functions $\sin 2r\theta$ or $\cos 2r\theta$ and $\sin 2\nu\phi$ or $\cos 2\nu\phi$ correspondingly. This is a consequence of the antisymmetric beam-beam force, which is assumed in the present work.
2. The nonlinear character of the dependence of the terms in the sums W on sines and/or cosines, produces all kinds of the nonlinear resonance enhancements in the beam blowup. The condition for nonlinear resonance of the $(m+k)$th order for an imperfect ring is as follows:

$$2\nu Bm + 2rBk = \ell \quad (14)$$

where m, k, and ℓ are any positive or negative integers. An infinite number of these resonances are positioned on each of the resonance lines (14). The sums W represent the result of simultaneous action of all such resonances.
3. An ideal symmetric lattice with B identical superperiods and B interaction points does not differ from a lattice built out of one superperiod and with only one interaction point. Hence for the symmetric ring without errors formula (2) should be (and indeed is) invariant under the following transformation:

$$\frac{\Sigma_y}{\sigma_y}(\nu,\alpha,r,\delta,B) = \frac{\Sigma_y}{\sigma_y}(B\nu,B\alpha,Br,B\delta,1) . \quad (15)$$

4. Due to the damping of the oscillations, the blowup appears to be finite even when the perturbed tunes ν and r are exactly on one of the resonance lines (14). Still the magnitude of the blowup at such a point should not be considered to be strictly correct since here breaks the validity of the perturbation theory.
5. Formula (2) explicitly depends on the damping rates α and δ, but its construction (especially the form of the sums W_1 and W_2) is such that the result for tunes away from any resonance (at least for an ideal ring) does not depend on the values of α and δ separately but only on their ratio α/δ). The reason for this is the following. Summations in formulae (5) through (8) result effectively in the appearance of resonant denominators, in which the damping constants enter as quadratic terms. Away from any resonance such terms are negligibly small in comparison to the term depending on the distance to the resonance, since usually $\delta \ll r$ and $\alpha \ll \nu$.

   Since the zeroth term in (5) is taken with the weight factor 1/2, the sums W_1 and W_2 are proportional to δ. A more detailed discussion of such behavior of the sums like (5) can be found in Ref. 4. In regions around the resonance lines (14), where the approximation is not valid anyway, the magnitude of the blowup does depend on the damping rates.
6. Formula (14) once more implies the importance of the machine imperfections in the problem of the beam-beam instability – the fact understood as the result of computational studies.[5] Indeed, the resonant structure of the beam-beam blowup is richer for the machine with imperfections. Expression (2) allows one to take into account several causes of the breaking of the symmetry of the storage ring: differences in betatron phase advances per superperiod, β-asymmetries and asymmetries in bunch currents.

As an illustration of the derived formula, I present here the results of calculation for the current PEP configuration. No imperfections of the machine are included here. The calculations for storage rings with imperfections and comparison with experiment are presented in an accompanying paper.[6]

Table 1. The unperturbed nominal PEP parameters

Particle energy	14.5	GeV
Strong beam current	24.0	ma
Horizontal β_x	3.0	m
Vertical β_y	0.11	m
Ratio of vertical to horizontal emittances	0.01	
Number of interaction points	6	
Horizontal tune per superperiod	3.545	
Vertical tune per superperiod	3.032	

Figure 1 presents the beam blowup in function of the unperturbed vertical tune. One can clearly see the resonant regions where the blowup actually occurs. Several main resonances are identified by comparing the horizontal and vertical perturbed tunes, dependence of which on the unperturbed values are presented in Figs. 2 and 3, correspondingly. Remember though, that the beam blowup at each point is the result of the simultaneous action of the infinite number of resonances which appear at the same place.

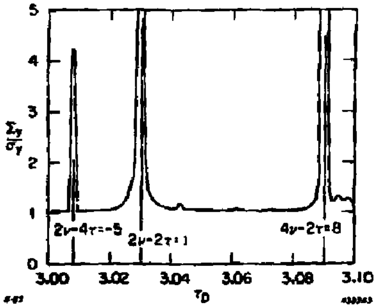

Fig. 1. Ratio of the perturbed rms vertical size of the bunch Σ_y to the unperturbed one σ_y versus the unperturbed vertical tune τ_0 per one superperiod of PEP. The strongest resonances are identified by the lowest order integers for perturbed vertical τ and horizontal ν tunes (for example, $2\nu - 2\tau = 1$ represents all resonances $m(2\nu - 2\tau) = m$, where m is any integer). The widths of the resonance curves represent the estimate for the upper boundary, i.e., the step size in the increment of the independent viariable. The actual resonance curve might be narrower.

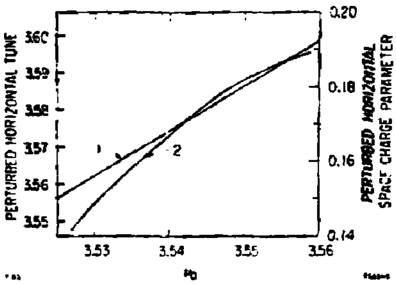

Fig. 2. The dependence of the perturbed tune (curve 1) and the space charge parameter (curve 2) on the unperturbed tune for the horizontal plane.

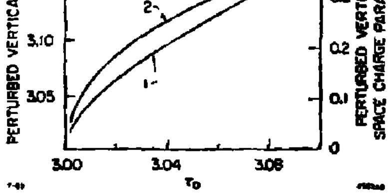

Fig. 3. The dependence of the perturbed tune (curve 1) and space charge parameter (curve 2) on the unperturbed tune for the vertical plane.

Next, Fig. 4 illustrates the dependence of the beam blowup on the beam current for one particular point of the tune diagram. The rising branch of the curve is a natural one and it is easy to understand. The falling branch needs an explanation since it is never observed in real life. One can attribute absence of the blowup decrease with the current increase to several reasons. The most obvious one is the negligence of the coherent beam-beam instability.[7] It produces two main effects: a) creates additional unstable regions for the tune values, depending in particular on the number of bunches and b) offsets the bunches at the interaction points breaking thus the assumption of the head-on collision. The machine imperfections neglected here should produce much more dense mash of the resonances, especially close to the half-integer to where the tune is shifted with the increase of the current. That also can eliminate the falling branch of the blowup curve.

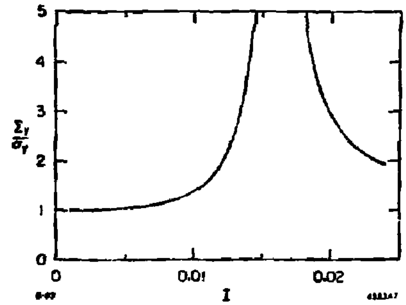

Fig. 4. The beam blowup Σ_y/σ_y as a function of the strong beam current I in amp.

At last it is not excluded that the decrease in the blowup might be connected to the failure of the perturbation treatment used in present work. Indeed, both space charge parameters grow with the increasing current.

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