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MODEL REFINEMENT USING TRANSIENT RESPONSE

C. R. Dohrmann and T. G. Carne

Sandia National Laboratories
P.O. Box 5800, Mail Stop 0439
Albuquerque, NM 87185

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1 Abstract

A method is presented for estimating uncertain or unknown parameters in a mathematical model using measurements of transient response. The method is based on a least squares formulation in which the differences between the model and test-based responses are minimized. An application of the method is presented for a nonlinear structural dynamic system. The method is also applied to a model of the Department of Energy armored tractor trailer. For the subject problem, the transient response was generated by driving the vehicle over a bump of prescribed shape and size. Results from the analysis and inspection of the test data revealed that a linear model of the vehicle's suspension is not adequate to accurately predict the response caused by the bump.

2 Nomenclature

t	time
$x(t)$	state vector
$\dot{x}(t)$	state vector time derivative
$h(t)$	input vector
p	model parameter vector
p_0	initial estimate of p
$\hat{p}$	current estimate of p
y	vector of measurements
c	model-based vector corresponding to y
W_p	parameter weighting matrix
W	measurement weighting matrix
D	sensitivity matrix
M	mass matrix

3 Introduction

Model parameter estimation for structural dynamic systems is commonly done using modal test data [1-4]. The basic idea is to adjust uncertain or unknown parameters in the model so to improve the correlation between the analysis and test. An advantage of using modal test data (frequencies, damping ratios, modes shapes, frequency response functions) is that the amount of test data used for comparison with the model is reduced significantly. For example, parameter estimates for a model can often be obtained using just a limited number of characteristic frequencies.

Although parameter estimation based on modal test data has many advantages, it has its limitations as well. It is assumed implicitly in a modal test that a structure's response is linear. Consequently, the use of modal test data for parameter estimation is restricted to linear models. The ability of such models to produce accurate simulations requires that the response of the system be predominantly linear. For many systems this requirement is satisfied, but for others it is not. An illustration of some limitations of linear models is given by a simple example problem in Section 6. Other situations that would require the use of transient response data include: a) blast loading on a structure, b) one-time transient events, and c) shock inputs to a system.

The basis for the proposed method is a least squares formulation in which the differences between the model and test-based responses are minimized [5]. An example problem is presented to demonstrate the use of the method to nonlinear systems. Results are also presented for an application involving the Department of Energy armored tractor trailer.

4 Problem Formulation

The governing equations for many systems can be expressed in standard first-order form as

$$\dot{x}(t) = g(x(t), h(t), p, t) \tag{1}$$

where $x \in R^n$ is the state, $h \in R^m$ is the input, $p \in R^r$ is a vector of model parameters, and t is time. Given the input $h(t)$, Eq. (1) can be integrated in time to yield

$$x_i = x(p, t_i) \tag{2}$$

$$\dot{x}_i = \dot{x}(p, t_i) \tag{3}$$

for $i = 1, \ldots, N$. The mathematical formulation of the parameter estimation problem is stated as follows. Determine the parameter values which minimize the function

$$G = (p - p_0)^T W_p (p - p_0) + (c - y)^T W (c - y) \tag{4}$$

where the matrices W_p and W are symmetric, positive semi-definite, p_0 is a vector of initial parameter estimates, y is a vector obtained from the measured

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transient response, and c is a vector function relating x_i and $\dot{x}_i$ to y. For example, y could be a vector containing accelerometer signals taken at specific points in time at various locations on a structure. The vector c would then contain the corresponding accelerations generated by the model. The formulation presented above allows for various transformations of the measured and modeled transient responses for purposes of comparison; one could even use transformed quantities like principal vectors for the comparison.

5 Problem Solution

Minimization of the function G can be accomplished using a general-purpose optimization code or, as in this analysis, with a simple gradient-based scheme. Let

$$\Delta p = p - \hat{p} \tag{5}$$

where $\hat{p}$ is the current estimate of p. The function c is approximated by

$$c = \hat{c} + D\Delta p \tag{6}$$

where $\hat{c} = c(\hat{p})$ and D is the sensitivity matrix, $\partial c/\partial p$, evaluated at $p = \hat{p}$. Substituting Eqs. (5) and (6) into Eq. (4) yields

$$\begin{aligned} G &= (\Delta p + \hat{p} - p_0)^T W_p (\Delta p + \hat{p} - p_0) + \\ & \quad (D\Delta p + \hat{c} - y)^T W (D\Delta p + \hat{c} - y) \end{aligned} \tag{7}$$

The function G is minimized by setting the gradient of the right hand side of Eq. (7) equal to zero. The result is

$$(W_p + D^T W D)\Delta p = -[W_p(\hat{p} - p_0) + D^T W(\hat{c} - y)] \tag{8}$$

Equation (8) can be solved for Δp provided the coefficient matrix on the left hand side of the equation is positive definite. Once Δp is obtained, $\hat{p}$ is updated via $\hat{p} \leftarrow \hat{p} + \alpha\Delta p$ where $\alpha \in (0, 1]$. The process of updating $\hat{p}$ is repeated until the desired level of convergence is achieved.

In the context of structural dynamics, the governing equations can be expressed as

$$M\ddot{u} + g(u, \dot{u}, t) = 0 \tag{9}$$

where M is the mass matrix, g is a vector of internal and external forces, and u is the displacement vector. For linear problems, one has $g(u, \dot{u}, t) = C\dot{u} + Ku - F$ where C and K and are the damping and stiffness matrices and F is a vector of generalized forces.

Newmark's method is a standard technique for the time integration of structural dynamic systems which is applicable to both linear and nonlinear problems. Given the force input $F(t)$, one can use this method to determine the responses $u_i = u(p, t_i)$, $\dot{u}_i = \dot{u}(p, t_i)$ and $\ddot{u}_i = \ddot{u}(p, t_i)$ for $i = 1, \ldots, N$. In the context of Eq. (2), one has

$$x_i = \begin{bmatrix} u_i & \dot{u}_i \end{bmatrix}^T \tag{10}$$

The terms $\hat{c}$ and D appearing in Eq. (8) can be calculated using Newmark's method together with finite difference calculations.

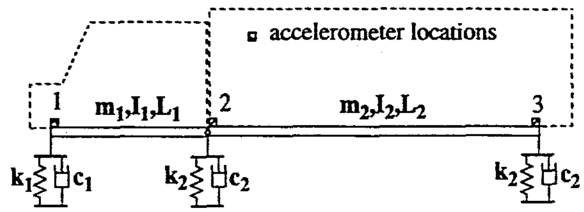

Figure 1: Simplified model of tractor trailer.

6 Example Problem

Consider the simplified three degree of freedom model of a tractor trailer shown in Figure 1. In this example, L_1 = 20 feet, L_2 = 40 feet, m_1 = 20000 lbm, m_2 = 40000 lbm, $I_1 = m_1 L_1^2/12$ and $I_2 = m_2 L_2^2/12$. The spring/damper pair k_j and c_j exerts a vertical force f_j on the vehicle given by

$$f_j = -(k_j u + k_{jn} u^3 + c_j \dot{u}) \tag{11}$$

where u is the extension of the spring/damper pair. The problem is to estimate the values of the coefficients k_1, k_{1n}, c_1, k_2, k_{2n} and c_2 using acceleration measurements of vertical motion at the three stations shown in Figure 1.

Simulated test data was generated by driving the vehicle at a speed of 55 mph over a 2-inch vertical half sine wave bump that is 4 feet long. The data was generated using parameter values of k_1 = 5000 lbf/inch, k_2 = 20000 lbf/inch, c_1 = 40 lbf-sec/inch, c_2 = 80 lbf-sec/inch, k_{1n} = 500 lbf/inch^3 and k_{2n} = 2000 lbf/inch^3. To this data was added normally distributed random numbers with a mean of zero and standard deviation of 5 inch/sec^2.

Initial estimates of the suspension parameters were given by k_1 = 6000 lbf/inch, k_2 = 17000 lbf/inch, c_1 = 0, c_2 = 0, k_{1n} = 0 and k_{2n} = 0. Comparisons of acceleration time series at station 1 (front end of tractor) are shown in Figure 2 after 0, 2, 4 and 6 iterations of the approach presented in Section 5. Very close agreement between the "test data" and the simulation is evident after 6 iterations. These results were obtained by including in the vector c the simulated accelerations at all three stations over the range of times shown in the figure. The matrix W_p in Eq. (4) was set equal to zero (no penalty for parameter changes). In addition, the weighting matrix W was set equal to the identity and α was set equal to 1 for all iterations. Values of k_1,

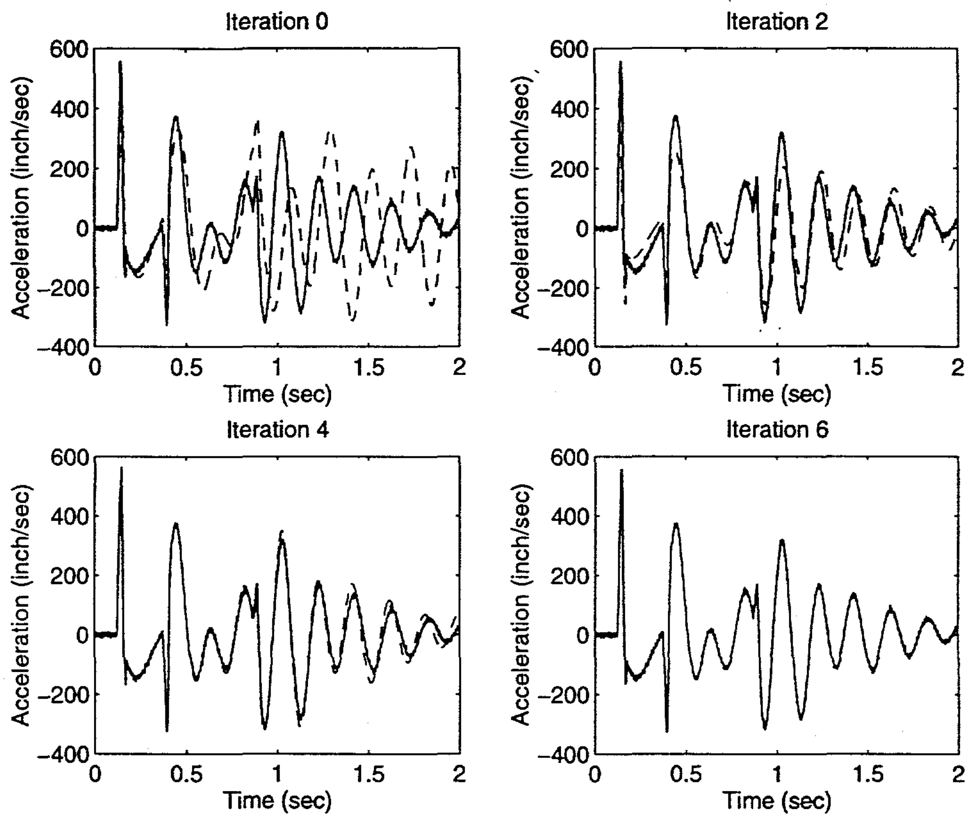

Figure 2: Comparison of simulated data (solid lines) and proposed approach (dashed lines).

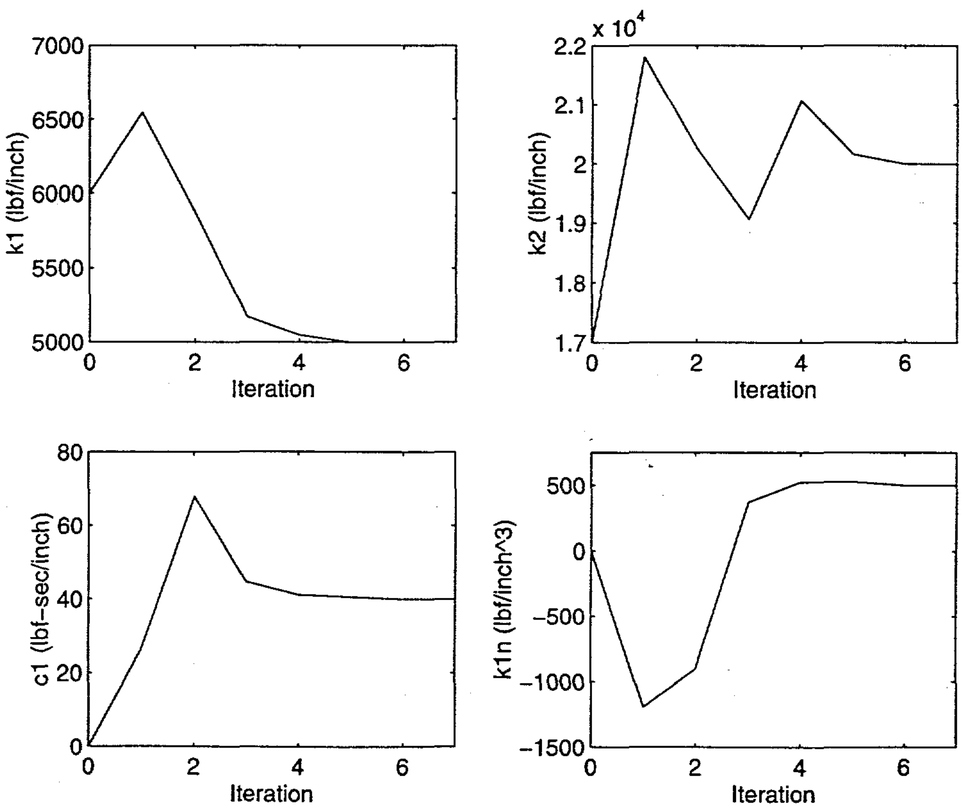

Figure 3: Convergence of parameters for example problem.

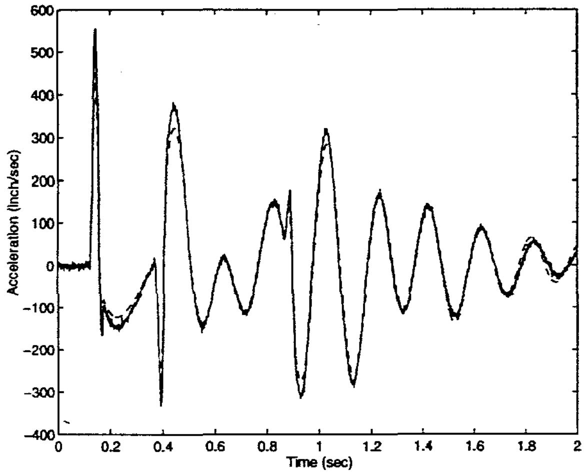

Figure 4: Linear model fit (dashed line) to simulated nonlinear response (solid line).

k_2, c_1 and k_{1n} at each iteration are shown in Figure 3. Convergence of the parameters to their known values is evident.

It is noted that the parameters were estimated successfully in this example for two primary reasons. First, the form of the model used in the simulations is adequate for predicting the system response. Indeed, in this example the model form is exact. Second, the "test data" contains enough information about all the different parameters. The singular values of the matrix $D^T D$ provide a simple measure of the amount of information in the data. In this example, the singular values of $D^T D$ at the solution ranged from a minimum value of 1 to a maximum value of 5×10^4.

Attempts were also made to fit a linear model ($k_{1n} = 0$ and $k_{2n} = 0$) to the simulated test data. Results from this analysis lead to converged parameter estimates of $k_1 = 5186$ lbf/inch, $k_2 = 21090$ lbf/inch, $c_1 = 35.8$ lbf-sec/inch and $c_2 = 70.4$ lbf-sec/inch. Comparisons of acceleration time series at station 1 are shown in Figure 4 for these parameter values. Although the agreement between these results is very good, some limitations of using a linear model to simulate the transient response of a nonlinear system are evident, particularly at the high values of the response.

It is noted that the nonlinear effects in the simulated data are relatively small for this example. Clearly, the limitations of the linear model will be more pronounced as the nonlinear effects become more significant. We believe that the nonlinear behavior of the leaf springs and/or shock absorbers in the tractor trailer makes it very difficult to simulate the response of rolling over a 2-inch bump at 20 mph with a linear model. This point is discussed further in the next section.

7 Tractor Trailer Problem

A modal-data based system identification of the Department of Energy tractor trailer was performed prior to this analysis to estimate several tire and suspension stiffnesses. The results of this system identification provided valuable information to the modeling process, but they also suffered from two limitations. First, the vehicle's shock absorbers were not connected during the modal tests. Consequently, a characterization of the shock absorbers could not be made based on modal test data. Second, we suspected that the suspension stiffnesses calculated from the modal tests would only be meaningful for low-level responses. It was thought that the estimated leaf spring stiffnesses are smaller at in-service response levels due to sliding between the individual leaves that was not present in the modal tests.

The data used in this analysis was generated by driving the vehicle over a 2-inch vertical bump at a nominal speed of 20 mph. The bump was in the shape of a half sine wave with a length of 48 inches. Both sides of the vehicle were subjected to the bump input. Displacement and acceleration time histories were recorded at several locations along the passenger side of the vehicle. The displacement time histories used in the analysis included those from relative displacement gauges mounted between the five axles and corresponding points above them on the tractor or trailer. High frequency content from the accelerometer gauges often swamped the signals, thus making it difficult to use this data directly. In addition, the acceleration time histories were often much more sensitive to ambient road roughness than those for displacement.

In addition to stiffness and damping parameters of the tires and suspensions, it was necessary to estimate the instants in time when the tires on the five axles first made contact with the bump. This was necessary because the speed of the vehicle during the test was only known approximately and varied slightly over the test duration. Parameter estimates were obtained by setting W_p equal to zero and W equal to the identity. Only transient responses between the times of 3.2 and 6.0 seconds were used in the analysis. This time window included the time just before first contact with the bump to a time after the rear trailer axle tires roll over the bump. The step length parameter α used to update the nominal values of the parameters was set equal to 1 for all iterations.

The process of integrating the equations of motion for the tractor trailer finite element model was expedited using modal superposition. This involved first calculating the mode shapes and frequencies of a model in which the estimated parameters (e.g. springs and dampers) were set equal to zero. A modal model of the system was then constructed which accounted for the presence

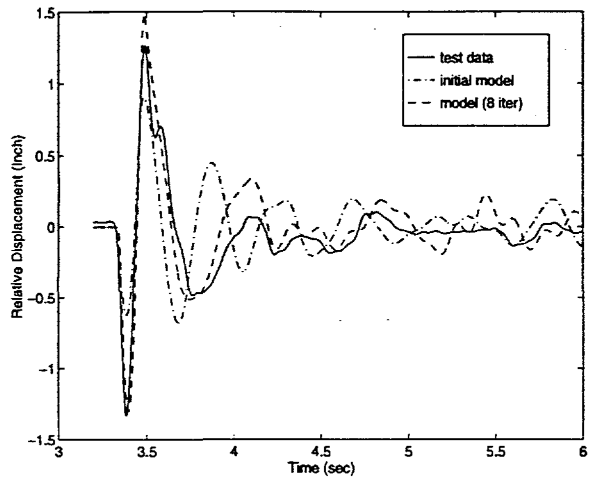

Figure 5: Relative displacement time series at steer axle.

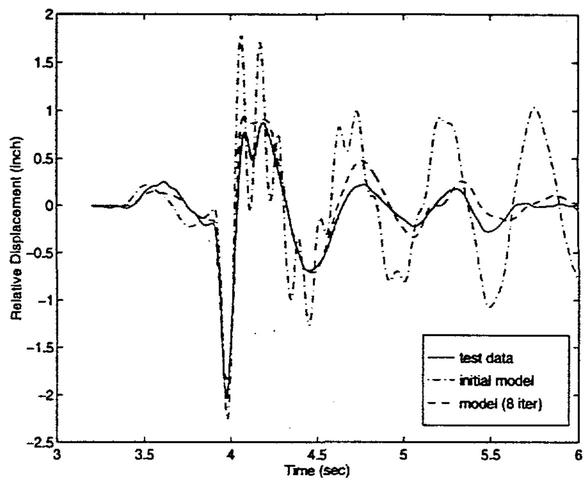

Figure 6: Relative displacement time series at front drive axle.

of the parameters to be estimated and the road input. A total of 36 modes were included in the modal model spanning a frequency range from 0 to 25 Hz. All calculations for this analysis were done using the Matlab [6] computing environment.

Comparisons of relative displacement time series are shown in Figures 5-8 for the test data (solid lines), initial model (dot-dashed lines) and the model after 8 iterations (dashed lines). The agreement between the test and the model-based results is clearly improved after 8 iterations, but the values obtained for the stiffness and damping coefficients of the tires are unrealistic. A major difficulty that arose in the analysis was the very poor conditioning of the matrix $D^T D$. As a result, different "converged" parameter values could be obtained if different initial parameter estimates were used. This situation indicates that there is not enough information in the displacement time series to independently identify all the unknown parameters in the model and/or the model form is significantly in error.

In order to investigate potential causes for these difficulties, additional measurements were included in the vector c. Specifically, the acceleration time series at the steer axle was integrated twice to obtain the absolute displacement of the steer axle. These displacements were then included in the analysis. It was found with the existing linear model of the tractor trailer that it is not possible to reasonably match both the absolute displacement of the steer axle and the relative displacement between the steer axle and the frame. We believe the main reason for this is that the leaf springs in the steer axle and trailer axle suspensions respond nonlinearly to the bump input.

To show why we believe a linear model of the leaf spring suspensions is inadequate, consider the time series shown in Figure 9 and 10. These plots show the

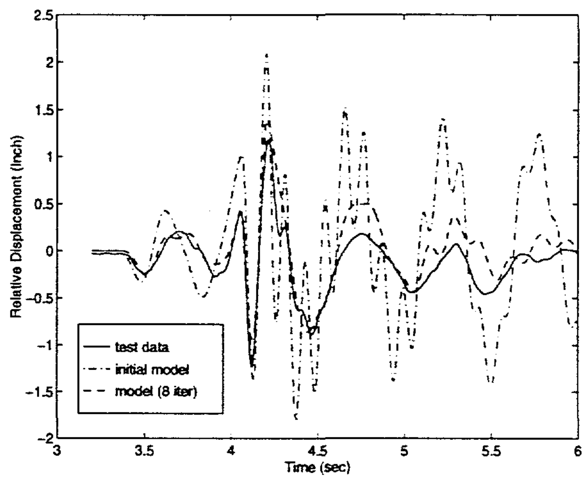

Figure 7: Relative displacement time series at rear drive axle.

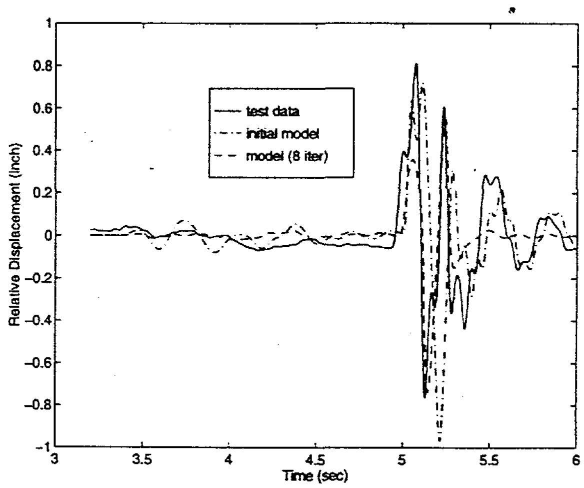

Figure 8: Relative displacement time series at rear trailer axle.

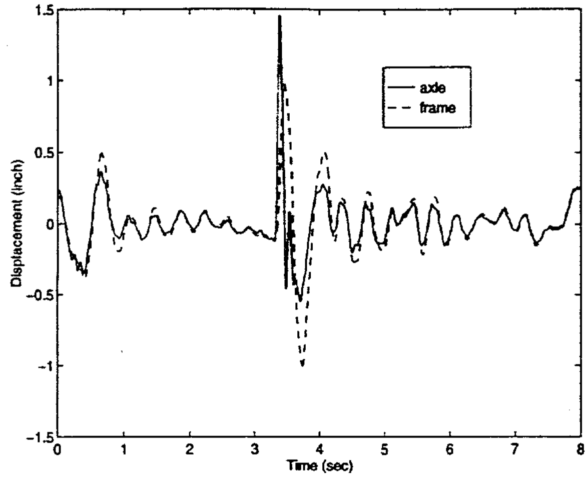

Figure 9: Displacement time series at steer axle.

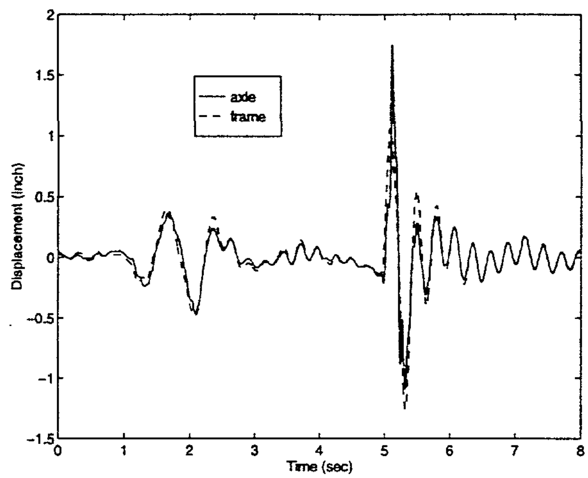

Figure 10: Displacement time series at rear trailer axle.

absolute displacements of the axle and the frame above the axle for the steer and rear trailer axles. Notice in the figure that there is very little relative motion between the axles and the frames shortly after the bump is rolled over. This indicates that the leaf spring stiffnesses are very high for normal highway inputs. In contrast, it is evident that the suspension stiffnesses must decrease significantly when the bump is encountered. A single value of stiffness clearly cannot be used to model the behavior of the suspensions for both high and low amplitude inputs.

Because of time constraints and other considerations, the leaf spring suspension models were not modified to include nonlinearities caused by stiction in the leaves. The test data presented in Figures 9 and 10, however, suggests that it is reasonable to use linear models of the suspensions to simulate vehicle response over relatively smooth surfaces such as those encountered in normal highway driving. The natural excitation technique (NExT) [7] was recently used to determine the mode shapes and frequencies of the DOE tractor trailer using test data collected while driving the vehicle over a stretch of Interstate 40 east of Albuquerque, New Mexico. In the test, the shock absorbers were connected and the vehicle was subjected to normal highway road inputs. A recommendation was made to use a linear model identified using the results of the NExT analysis in the design of a cab isolation system for the tractor trailer.

8 Conclusions

A method was presented for estimating uncertain or unknown parameters in a mathematical model using transient response measurements. The method is applicable to either linear or nonlinear models and was applied successfully to a simple test problem. As with other estimation techniques, an adequate model form is essential to the success of the method. In addition, the measured transient response must include enough information to estimate all of the parameters in the model. Useful measures in this regard are the singular values of the sensitivity matrix D. The method can also be used with various transformations of the transient response.

Several insights were gained as a result of a real-life application involving the Department of Energy tractor trailer. Based on an analysis of the test data and unsuccessful application of the method, it was concluded that a single linear model of the suspension system is inadequate for both high and low amplitude inputs. Examination of the test data also revealed that direct use of accelerometer signals may cause difficulties for the method because of large amplitude responses at frequencies above the range of interest. Other potential sources of difficulty are time series with a large number of cycles. Various transforms of the time series may be required to address these two issues.

9 Acknowledgments

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