166
12/5/79

LA-8097-MS

Informal Report

Hi 362

Endochronic Viscoplasticity Model

MASTER

University of California

LOS ALAMOS SCIENTIFIC LABORATORY

Post Office Box 1663 Los Alamos, New Mexico 87545

LA-8097-MS
Informal Report

UC-71
Issued: October 1979

Endochronic Viscoplasticity Model

William A. Cook

NOMENCLATURE

σ_{ij} - Stress

s_{ij} - Deviatoric stress ($s_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk}$, $s_{kk} = 0$)

ε_{ij} - Strain

e_{ij} - Deviatoric stress ($e_{ij} = \varepsilon_{ij} - \frac{1}{3} \delta_{ij} \varepsilon_{kk}$, $e_{kk} = 0$)

μ - Elastic shear modulus, one of Lame's constants

k - Elastic bulk modulus

G_1 - Relaxation function for shear

G_2 - Relaxation function for dilatation

J_1 - Creep function for shear

J_2 - Creep function for dilatation

t - Time

$\bar{t}$ - Intrinsic time

F - Force

A - Cross-sectional area

E - Young's modulus (elastic)

ν - Poisson's ratio

V - Volume

S - Surface area

λ - $\lambda = \frac{3k - 2\mu}{3}$, one of Lame's constants

Repeated Latin indices indicate summation 1-3 in Secs. II and IV.

ENDOCHRONIC VISCOPLASTICITY MODEL

by

William A. Cook

ABSTRACT

The endochronic viscoplasticity model is presented, and the criteria for general problem analyses are discussed. Two approaches are then developed for inclusion of this model in nonlinear finite element codes. One approach includes reformulating the stiffness matrix for solution by iteration, and the other approach does not. Also, the uniaxial tension problem is studied, and the problems encountered with the use of this model are stated. Finally, recommendations are presented to check the basic postulates used to develop this model.

I. INTRODUCTION

The US Department of Energy, Assistant Secretary for Environment, Division of Environmental Control Technology is directing computer code development for analyzing nuclear material shipping containers that have been subjected to severe impact conditions. The endochronic viscoplasticity model was chosen as a possible model for the materials used in shipping containers. Argonne National Laboratory (ANL) was charged with the responsibility of developing this model for finite element codes. Los Alamos Scientific Laboratory (LASL) was charged with adding this model to a finite element code being developed for shipping container analysis.

The endochronic viscoplasticity model was introduced by Professor K. C. Valamis in 1971 with Ref. 1. The purpose of this model was to avoid the use of yield surfaces and thus simplify viscoplasticity analysis.

This report describes the work done at LASL with this model. In Sec. II, the model is described and discussed and in Sec. III, two approaches are presented for using this model in a finite element code. In Sec. IV, the uniaxial test problem is discussed along with the difficulties that are encountered in its use; and in Sec. V, recommendations are described for testing this model.

II. ENDOCHRONIC MODEL

In this section, the endochronic viscoplasticity model is developed. This development is extended from that presented in Ref. 1 to include a variable bulk modulus. We start with the basic viscoelastic constitutive relations and four basic definitions and produce an endochronic viscoplasticity model that may be used in theoretical analysis or included in a finite element analysis. This section is concluded with a discussion of the information that is required on a particular material for use in the model and on the tests that might be performed to obtain this information.

The shear viscoelastic constitutive relations[2] are

$$s_{ij}(x,t) = \int_0^t G_1(t - \tau) \frac{\partial e_{ij}(x,\tau)}{\partial \tau} d\tau \quad , \text{ and} \tag{1}$$

$$e_{ij}(x,t) = \int_0^t J_1(t - \tau) \frac{\partial s_{ij}(x,\tau)}{\partial \tau} d\tau \quad . \tag{2}$$

The shear elastic relation is

$$s_{ij}(x,t) = 2\mu e_{ij}(x,t) \quad . \tag{3}$$

The dilatation viscoelastic constitutive relations[2] are

$$\sigma_{kk}(x,t) = \int_0^t G_2(t - \tau) \frac{\partial \varepsilon_{kk}(x,\tau)}{\partial \tau} d\tau \quad , \text{ and} \tag{4}$$

$$\varepsilon_{kk}(x,t) = \int_0^t J_2(t - \tau) \frac{\partial \sigma_{kk}(x,\tau)}{\partial \tau} d\tau \quad . \tag{5}$$

The dilatation elastic relation is

$$\sigma_{kk}(x,t) = 3k\varepsilon_{kk}(x,t) \quad . \tag{6}$$

An endochronic viscoplasticity model can be defined as follows.

(1) Replace the time (t) in the viscoelastic constitutive relation with a new variable, intrinsic time ($\bar{t}$).

(2) Define the intrinsic time ($\bar{t}$) as

$$d\bar{t} = \frac{d\zeta}{1 + \beta\zeta} \quad , \tag{7}$$

where β is a constant and

where $(\beta d\zeta)^2 = K_1 \, d\varepsilon_{ii} \, d\varepsilon_{jj} + K_2 \, d\varepsilon_{ij} \, d\varepsilon_{ij}$ and $d\zeta \geq 0$.

K_1 and K_2 are either constants or functions of strain invariants. If K_1 and K_2 are functions of strains and not strain invariants, a change in coordinate definition would alter the calculated behavior of the material.

(3) Assume a relaxation function for shear of

$$G_1(\bar{t}) = 2\mu \, e^{-\alpha\bar{t}} \quad , \tag{8}$$

where α is a constant.

(4) Assume a creep function for dilatation of

$$J_2(\bar{t}) = \frac{1}{3k} e^{-\gamma\bar{t}} \quad , \tag{9}$$

where γ is a constant.

From endochronic viscoplasticity definitions (1) and (3),

$$s_{ij}(x,\bar{t}) = \int_0^{\bar{t}} 2\mu \, e^{-\alpha(\bar{t}-\tau)} \frac{\partial e_{ij}(x,\tau)}{\partial \tau} \, d\tau \quad . \tag{10}$$

Differentiating this equation with respect to $\bar{t}$ (see Ref. 3, page 312, 313),

$$\frac{\partial s_{ij}(x,\bar{t})}{\partial \bar{t}} = -\alpha s_{ij}(x,\bar{t}) + \frac{2\mu \, \partial e_{ij}(x,\bar{t})}{\partial \bar{t}} \quad . \tag{11}$$

This equation can be written in incremental form as

$$\Delta s_{ij} = -\alpha \, s_{ij} \, \Delta\bar{t} + 2\mu \, \Delta e_{ij} \quad . \tag{12}$$

From endochronic viscoplasticity definition (2), the intrinsic time can be written incrementally as

$$\Delta\bar{t} = \frac{\Delta\zeta}{1 + \beta\zeta} \quad . \tag{13}$$

Thus Eq. (12) can be written as

$$\Delta s_{ij} = -\alpha \, s_{ij} \frac{\Delta\zeta}{1 + \beta\zeta} + 2\mu \, \Delta e_{ij} \quad . \tag{14}$$

When the definitions for deviatoric stress and strain are substituted into Eq. (14),

$$\Delta\sigma_{ij} = -\left(\frac{\alpha \, \Delta\zeta}{1 + \beta\zeta}\right)\left(\sigma_{ij} - \frac{1}{3}\delta_{ij}\,\sigma_{kk}\right) + 2\mu\left(\Delta\varepsilon_{ij} - \frac{1}{3}\delta_{ij}\,\Delta\varepsilon_{kk}\right)$$
$$+ \frac{1}{3}\delta_{ij}\,\Delta\sigma_{kk} \quad . \tag{15}$$

From endochronic viscoplasticity definition (1) and (4),

$$\varepsilon_{kk}(x,\bar{t}) = \int_0^{\bar{t}} \frac{e^{-\gamma(\bar{t}-\tau)}}{3k} \frac{\partial\sigma_{kk}(x,\tau)}{\partial\tau} d\tau \quad . \tag{16}$$

Differentiating this equation with respect to $\bar{t}$ (see Ref. 3, p. 312, 313),

$$\frac{\partial\varepsilon_{kk}(x,\bar{t})}{\partial\bar{t}} = -\gamma\, \varepsilon_{kk}(x,\bar{t}) + \frac{1}{3k} \frac{\partial\sigma_{kk}(x,\bar{t})}{\partial\bar{t}} \quad . \tag{17}$$

This equation may be written in incremental form as

$$\Delta\varepsilon_{kk} = -\gamma\, \varepsilon_{kk}\, \Delta\bar{t} + \frac{1}{3k} \Delta\sigma_{kk} \quad . \tag{18}$$

Equation (13) can be used to eliminate the intrinsic time in Eq. (18) and give

$$\Delta\sigma_{kk} = 3k\, \Delta\varepsilon_{kk} + 3k\left(\frac{\gamma\, \Delta\zeta}{1 + \beta\zeta}\right)\varepsilon_{kk} \quad . \tag{19}$$

Using Eq. (19), $\Delta\sigma_{kk}$ may be eliminated in Eq. (15) to give the endochronic viscoplasticity relation

$$\Delta\sigma_{ij} = -\frac{\alpha\, \Delta\zeta}{1 + \beta\zeta}\left(\sigma_{ij} - \frac{1}{3}\delta_{ij}\,\sigma_{kk}\right) + 2\mu\, \Delta\varepsilon_{ij} + \delta_{ij}\left(\frac{3k - 2\mu}{3}\right)\Delta\varepsilon_{kk}$$

$$+ \delta_{ij}\, k\left(\frac{\gamma\, \Delta\zeta}{1 + \beta\zeta}\right)\varepsilon_{kk} \quad . \tag{20}$$

For α and γ defined as

$$\alpha = 3\,[\alpha_1 + \delta_{ij}\,(\alpha_2 - \alpha_1)]\,\beta \text{ and } \quad \gamma = 3\gamma_1\beta \quad , \tag{21}$$

Eq. (20) can be written as

$$\Delta\sigma_{ij} = -3\,[\,\alpha_1 + \delta_{ij}(\alpha_2 - \alpha_1)\,]\,\frac{\beta\,\Delta\zeta}{1+\beta\zeta}\left(\sigma_{ij} - \frac{1}{3}\,\delta_{ij}\,\sigma_{kk}\right) + 2\mu\,\Delta\varepsilon_{ij}$$

$$+\ \delta_{ij}\left(\frac{3k-2\mu}{3}\right)\Delta\varepsilon_{kk} + \delta_{ij}\,3k\,\gamma_1\left(\frac{\beta\,\Delta\zeta}{1+\beta\zeta}\right)\varepsilon_{kk}\quad . \tag{22}$$

The term $\frac{\beta\,\Delta\zeta}{1+\beta\zeta}$ is obtained from the incremental equation (see endochronic viscoplastic definition (2) or Eq. (7))

$$(\beta\,\Delta\zeta)^2 = K_1\,(\Delta\varepsilon_{kk})^2 + K_2\,\Delta\varepsilon_{ij}\,\Delta\varepsilon_{ij}\quad . \tag{23}$$

Equation (19) can be written in terms of γ_1 as

$$\Delta\sigma_{kk} = 3k\,\Delta\varepsilon_{kk} + 9k\,\gamma_1\left(\frac{\beta\,\Delta\zeta}{1+\beta\zeta}\right)\varepsilon_{kk}\quad . \tag{24}$$

To solve any particular problem, we need to know the following material properties for Eqs. (22), (23), and (24): elastic properties k and μ, and endochronic viscoplasticity properties α_1, α_2, γ_1, K_1, and K_2.

For this endochronic viscoplasticity model to be a general viscoplasticity model, these endochronic viscoplasticity properties must be independent of any problem, dependent only on material. An inspection of Eqs. (22), (23), and (24) suggests the following:

(1) a hydrostatic compression test,

(2) a tensile test,

(3) a shear test, and

(4) a triaxial test.

The hydrostatic test would determine γ_1, the tensile test would determine α_1, the shear test would determine α_2, the tensile and shear tests would determine K_1 and K_2, and the triaxial test would be a check to determine whether the endochronic viscoplasticity model is a general model.

III. IMPLEMENTING THE ENDOCHRONIC MODEL IN A FINITE ELEMENT CODE

In this section I develop two possible approaches for using the endochronic viscoplasticity model in a finite element code. Both approaches use iteration for the solution of the nonlinearity caused by this nonlinear material model. Model 1 does not reform the master stiffness, whereas Model 2 reforms the stiffness but probably converges more rapidly in the iteration scheme.

The endochronic model, Eq. (22) of Sec. II, can be written in matrix notation as

$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon\} - f(\zeta)\,[G]\{\sigma\} + f(\zeta)\,[H]\{\varepsilon\} \quad , \tag{25}$$

where

$$f(\zeta) = \frac{\beta\ \Delta\zeta}{1 + \beta\zeta} \quad ,$$

$$\{\Delta\sigma\}^T = (\Delta\sigma_{11}\ \Delta\sigma_{22}\ \Delta\sigma_{33}\ \Delta\sigma_{12}\ \Delta\sigma_{23}\ \Delta\sigma_{31}) \quad ,$$

$$\{\Delta\varepsilon\}^T = (\Delta\varepsilon_{11}\ \Delta\varepsilon_{22}\ \Delta\varepsilon_{33}\ \Delta\varepsilon_{12}\ \Delta\varepsilon_{23}\ \Delta\varepsilon_{31}) \quad ,$$

$$\{\sigma\}^T = (\sigma_{11}\ \sigma_{22}\ \sigma_{33}\ \sigma_{12}\ \sigma_{23}\ \sigma_{31}) \quad ,$$

$$\{\varepsilon\}^T = (\varepsilon_{11}\ \varepsilon_{22}\ \varepsilon_{33}\ \varepsilon_{12}\ \varepsilon_{23}\ \varepsilon_{31}) \quad ,$$

$$[D] = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu \end{bmatrix} ,$$

$$
[G] = \alpha_1 \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} , \text{ and}
$$

$$
[H] = 3k\gamma_1 \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} .
$$

Let the index j indicate the jth time, and let k indicate the finite element increment (repeated indices do not indicate summation in this section). Then the virtual work relation for finite element analysis may be written as

$$
\sum_{k=1}^{K} \left(\int_{V_k} \{\Delta\sigma\}_{jk}^{T} \delta\{\Delta\varepsilon\}_{jk} \, dV - \int_{V_k} \{\Delta F\}_{jk}^{T} \delta\{\Delta u\}_{jk} \, dV - \int_{S_k} \{\Delta T\}_{jk}^{T} \delta\{\Delta u\}_{jk} \, dS \right) = 0 , \tag{26}
$$

where $\{\Delta F\}$ are the body and or inertia loads, $\{\Delta T\}$ are the incremental traction loads, and $\{\Delta u\}$ are the incremental displacements.

Equation (1) can be written as

$$\{\Delta\sigma\}_j = [D]\ \{\Delta\varepsilon\}_j - f(\zeta_j)\ [G]\{\sigma\}_j + f(\zeta_j)\ [H]\{\varepsilon\}_j \quad . \tag{27}$$

Two methods were investigated for including the endochronic model in the NONSAP code.[4]

Method 1: Substitute Eq. (27) into Eq. (26)

$$\sum_{k=1}^{K} \Bigg(\{\Delta\bar{u}\}_{jk}^T \int_{V_k} [B]^T\ [D]\ [B]\ dV - \int_{V_k} \{\sigma\}_{jk}^T\ [G]\ f(\zeta_{jk})\ [B]\ dV$$
$$+ \int_{V_k} \{\varepsilon\}_{jk}^T\ [H]\ f(\zeta_{jk})\ [B]\ dV - \int_{V_k} \{\Delta F\}_{jk}^T\ [N]\ dV \tag{28}$$
$$- \int_{S_k} \{\Delta T\}_{jk}^T\ [N]\ dV \Bigg) \quad \delta\{\Delta\bar{u}\}_{jk} = 0 \quad ,$$

where [N] is the shape function. Thus

$$\{\varepsilon\}_{jk} = [B]\{\Delta\bar{u}\}_{jk} \quad , \qquad \text{and}$$

$$\{\Delta u\}_{jk} = [N]\ \{\Delta\bar{u}\}_{jk} \quad .$$

The first term in Eq. (28) is the stiffness matrix; note that it is independent of j, whereas the next four terms that compose the force vector are dependent on j. However, the last two terms are independent of stress $\{\sigma\}_j$, strain $\{\varepsilon\}_j$, or intrinsic time ζ_j. Thus Eq. (28) can be written as

$$\left\{F_{1j}\ (\ \{\varepsilon\}_j, \{\sigma\}_j, \zeta_j) + \Delta F_{2j}\right\} = [K]\ \{\Delta U\}_j \quad , \tag{29}$$

where $\{\Delta U\}$ is the matrix of all nodal values, $\left\{\Delta F_{1j}(\{\varepsilon\}_j, \{\sigma\}_j, \zeta_j) + \Delta F_{2j}\right\}$ is the matrix of all incremental forces, and [K] is the master stiffness matrix. Equation (29) can be solved by iteration. Iteration is the same as incrementing, only the incremental loads are zero. The main virtue of this method is that the stiffness needs to be formulated and reduced only once.

Method 2: The stress and strain can be written as

$$\{\sigma\}_j = \{\sigma\}_{j-1} + \{\Delta\sigma\}_j \quad , \text{ and}$$
$$\{\varepsilon\}_j = \{\varepsilon\}_{j-1} + \{\Delta\varepsilon\}_j \quad , \tag{30}$$

where $\{\sigma\}_{j-1}$ and $\{\varepsilon\}_{j-1}$ are known from the last iteration. Substituting these equations into Eq. (27),

$$\{\Delta\sigma\}_j = [\bar{D}]_j \{\Delta\varepsilon\}_j - [\bar{G}]_j \{\sigma\}_{j-1} + [\bar{H}]_j \{\varepsilon\}_{j-1} \quad , \tag{31}$$

where

$$[\bar{D}]_j = ([I] + f(\zeta_j)\,[G])^{-1}\,([D] + f(\zeta_j)\,[H]) \quad ,$$

$$[\bar{G}]_j = ([I] + f(\zeta_j)\,[G])^{-1}\,f(\zeta_j)\,[G] \quad ,$$

$$[\bar{H}]_j = ([I] + f(\zeta_j)\,[G])^{-1}\,f(\zeta_j)\,[H] \quad , \text{ and}$$

$[I]$ is 6 x 6 identity matrix.

Substituting Eq. (31) into the virtual work Eq. (26),

$$\sum_{k=1}^{K} \Bigg(\{\Delta\bar{u}\}^T_{jk} \int_{V_k} [B]^T [\bar{D}]_j [B]\, dV - \int_{V_k} \{\sigma\}^T_{j-1,k} [\bar{G}]_j [B]\, dV$$

$$+ \int_{V_k} \{\varepsilon\}^T_{j-1,k} [\bar{H}]_j [B]\, dV - \int_{V_k} \{\Delta F\}^T_{jk} [N]\, dV \tag{32}$$

$$- \int_{S_k} \{\Delta T\}_{jk}^{T} [N] \, dS \Big) \, \delta\{\Delta \bar{u}\}_{jk} = 0 \quad .$$

This equation can be written as

$$\{F_{1j}(\zeta_j) + F_{2j}\} = [K(\zeta_j)] \, \{\Delta U\} \quad . \tag{33}$$

Notice with this method that both the force vector and stiffness matrix must be reformulated with each increment or iteration. Again, Eq. (33) can be solved for each increment by iteration. The virtue of this method is that it would probably be more stable. A method similar to this was used in Ref. 5 for creep analysis.

Because of its simplicity, Model 1 was coded into the NONSAP code for axisymmetric continuum elements. To check this Model, I analyzed a cylinder in tension. The stress-strain results from this analysis were quite different from those expected, so I decided to study the behavior of the endochronic viscoplasticity model with the uniaxial problem.

IV. UNIAXIAL PROBLEM

For the uniaxial problem shown in Fig. 1, the following assumptions apply.

(1) *Normal stress* $\sigma_{11} = F/A$, $\sigma_{22} = 0.$, *and* $\sigma_{33} = 0.$

(2) Shearing stresses $\sigma_{12} = 0.$, $\sigma_{23} = 0.$, and $\sigma_{31} = 0.$

(3) Shearing strains $\varepsilon_{12} = 0.$, $\varepsilon_{23} = 0.$, and $\varepsilon_{31} = 0.$

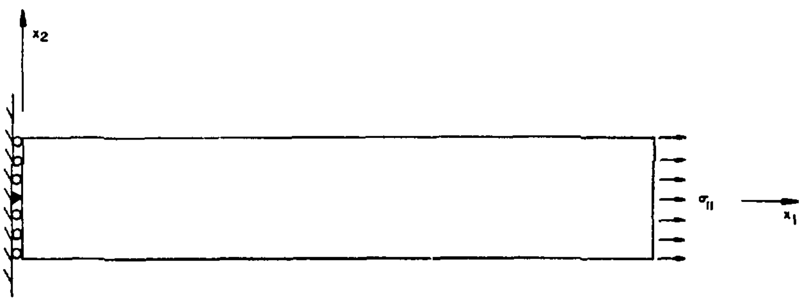

Fig. 1.
Uniaxial problem.

From these assumptions for the uniaxial problem and Eqs. (22), (23), and (24) in Sec. II, the endochronic viscoplasticity relation can be derived as

$$\Delta\sigma_{11} = E\,\Delta\varepsilon_{11} - 2(1+\nu)\alpha_1\left(\frac{\beta\,\Delta\zeta}{1+\beta\zeta}\right)\sigma_{11} + E\gamma_1\left(\frac{\beta\,\Delta\zeta}{1+\beta\zeta}\right)\varepsilon_{kk} \quad , \tag{34}$$

where

$$\Delta\varepsilon_{22} + \Delta\varepsilon_{33} = -2\left\{\nu\Delta\varepsilon_{11} + \frac{(1-2\nu)(1+\nu)}{E}\alpha_1\left(\frac{\beta\,\Delta\zeta}{1+\beta\zeta}\right)\sigma_{11} + (1+\nu)\,\gamma_1\left(\frac{\beta\,\Delta\zeta}{1+\beta\zeta}\right)\varepsilon_{kk}\right\} \quad ,$$

$$(\beta\,\Delta\zeta)^2 = K_1(\Delta\varepsilon_{kk})^2 + K_2(\Delta\varepsilon_{11}^2 + \Delta\varepsilon_{22}^2 + \Delta\varepsilon_{33}^2) \quad , \text{ and} \tag{35}$$

$$\Delta\sigma_{11} = 3k\,\Delta\varepsilon_{kk} + 9k\,\gamma_1\left(\frac{\beta\,\Delta\zeta}{1+\beta\zeta}\right)\varepsilon_{kk} \quad . \tag{36}$$

A computer code has been written to solve Eqs. (34) and (35) and is included in the report as the Appendix.

Consider the aluminum alloy material discussed in Ref. 6, page 173. A stress strain relation for this material is shown in Fig. 2.

Reference 7, page 16, states "that the yielding of a metal is, to a first approximation unaffected by a hydrostatic pressure or tension." Thus, let γ_1 be zero. Also, assume K_2 equal to zero, and then Eq. (35) can be written as

$$\beta\,\Delta\zeta = \frac{\sqrt{K_1}}{3k}\,\Delta\sigma_{11} \quad . \tag{37}$$

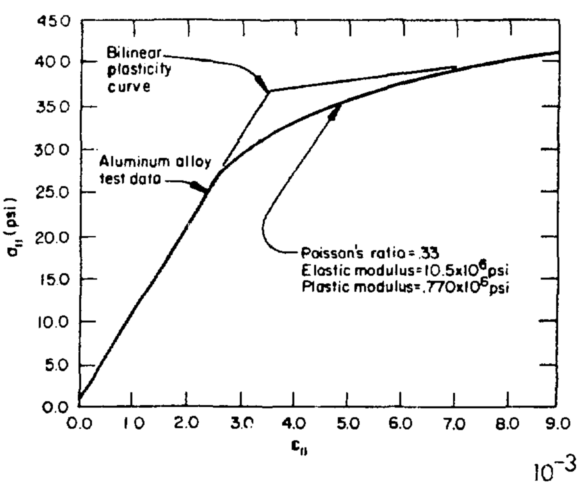

Fig. 2.
Stress (σ_{11}) strain (ε_{11}) relation for aluminum alloy uniaxial problem.

This equation can be integrated to give

$$\beta\zeta = \frac{\sqrt{K_1}}{3k} \sigma_{11} \quad . \tag{38}$$

Substituting Eqs. (37) and (38) into Eq. (34) gives

$$\Delta\sigma_{11} = E \, \Delta\varepsilon_{11} - 2(1 + \nu) \, \alpha_1 \, \frac{\left(\sqrt{K_1}/3K\right) \Delta\sigma_{11} \, \sigma_{11}}{1 + (\sqrt{K_1}/3K) \, \sigma_{11})} \quad . \tag{39}$$

For σ_{11} = 40 500 psi in Fig. 2,

$$\text{plastic modulus} = \frac{\Delta\sigma_{11}}{\Delta\varepsilon_{11}} \quad . \tag{40}$$

Also, for σ_{11} = 40 500 psi, and K_1 = 15 555,

$1 + \frac{\sqrt{k_1}}{3k} \sigma_{11} \doteq \frac{\sqrt{k_1}}{3k} \sigma_{11}$ and Eq. 39 reduces such that Eq. 40 becomes

$$\text{plastic modulus} = \frac{E}{1 + 2(1 + \nu) \, \alpha_1} \quad . \tag{41}$$

From the values in Fig. 2,

$$\alpha_1 = 4.74 \quad . \tag{42}$$

Using the following data for the code in the Appendix, I generated Fig. 3.

NINC = 30
NIT = 5
EMOD = 10.5 x 10^6 psi
BETA1 = 15555
ALPHA1 = 4.74
DELP = 4190 lbs

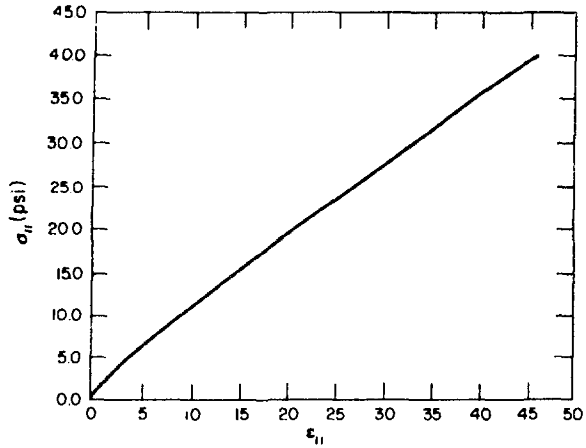

Fig. 3.
Endochronic model for aluminum alloy.

EL = 10 in.

AREA = 3.0 in.^{2}

ECVGE = .00001

ENU = .333

GAM1 = 0.0

BETA2 = 0.0

Figures 3 and 2 do not agree. An examination of the calculation revealed that $\Delta\epsilon_{kk}$ was constant and did not allow the model to react as expected. Note that if one example can be found that can not be solved with the endochronic model presented, then the model is not a general viscoplasticity model. The fact that Fig. 3 does not model the aluminum alloy in Fig. 2 does not imply that it is impossible. There are many other possibilities. Thus many values of α_1 and β_1 were used with little improvement in the results. Another approach used was to set $\beta_1 = 0$ and use many values for α_1 and β_2. These results were better; however, β_2 is needed for shear data, so this approach is not entirely satisfactory. Yet another approach that might be tried is to let K_1 and K_2 be functions of strain invariants in the code and see if better results can be calculated.

Another concern is that for the uniaxial problem, assumptions (1) and (2) are stress boundary conditions while assumption (3) is a limiting condition on the displacements. If assumption (3) were not used, would a more physical solution be forthcoming? Would such a solution overcome the difficulties seen with the present solution?

This uniaxial problem has not shown that the endochronic viscoplasticity is not a general model. However, the difficulty encountered in trying to use it for such a simple problem makes the model's generality questionable. If further investigation demonstrates that this model is not general, this will only prove that intrinsic time is not a material property.

V. RECOMMENDATIONS

My recommendations follow.

(1) Do the testing discussed in Sec. II. I believe that the triaxial test would show that instrinsic time is not a material property and that the endochronic viscoplasticity model is not a general viscoplasticity model. If this is the case, perhaps intrinsic time could be changed until it is a material property. Dr. B. J. Hsieh of ANL shows some very impressive results when he modifies the intrinsic time definition in Refs. 8 and 9. I think much more work is needed in this area of correlation of the endochronic model with testing.

(2) Develop theoretical endochronic codes for tests specimen, like the code presented in the Appendix. Also, develop a nonlinear least-squares code to be used to fit the test data with the theoretical model.

REFERENCES

1. K. C. Valanis, "A Theory of Viscoplasticity Without a Yield Surface, Part I, General Theory, Part II, Application to Mechanical Behavior of Metals," Archives of Mechanics 23, 517-551 (1971).

2. R. M. Christensen, "Theory of Viscoelasticity: An Introduction," (Academic Press, New York, 1971).

3. C. R. Wylie, Advanced Engineering Mathematics 4th Ed. (McGraw-Hill, New York, 1975).

4. K. J. Bathe, E. L. Wilson, and R. H. Iding, "NONSAP - A Structural Analysis Program for Static and Dynamic Response of Linear Systems," Dept. of Civil Eng., Univ. of California report USCESM 74-3 (February 1974).

5. C. A. Anderson, "Cavity Stability: Finite Element Analysis for Steady and Transient Creep," Los Alamos Scientific Laboratory report LA-5769 (February 1975).

6. F. R. Shanley, Strength of Materials (McGraw-Hill, New York, 1957).

7. R. Hill, The Mathematical Theory of Plasticity (Oxford at the Clarendon Press, 1950).

8. B. J. Hsieh, "On the Uniqueness and Stability of Endochronic Plasticity Theory," Argonne National Laboratory report ANL-CT-78-51 (September 1978).

9. B. J. Hsieh, "The Use of Endochronic Plasticity For Multi-Dimensional Small and Large Strain Problems," Argonne National Laboratory report ANL-CT-79-19 (March 1979).

APPENDIX

VARIABLES INPUT

NINC - Number of load increments

NIT - Maximum number of iterations allowed for convergence of endochronic viscoplasticity relation

EMOD - E (Young's Modulus)

BETA1 - $\beta_1 = (K_1)^{1/2}$

ALPHA1 - α_1

DELP - Δp (load increment)

EL - ℓ (length of bar in Fig. 1)

AREA - A (cross sectional area of bar in Fig. 1)

ECVGE - Criterion for convergence of endochronic viscoplastiticy relation

ENU - ν (Poisson's ratio)

GAM1 - γ_1

BETA2 - $\beta_2 = (K_2)^{1/2}$

PROGRAM ENDO

LASL Identification No.LP-2024.

```
      program endo (inp,out1,tape5=inp,tape6=out1,output)
      dimension s(500),u(500),fzeta(500),bzeta(500),e(500),e2(500)
   10 continue
      read (5,1000) ninc,nit,emod,beta1,alpha1,delp,el,area,ecvge
 1000 format (2i5,7f10.0)
      read (5,1010) enu,gam1,beta2
 1010 format (8f10.0)
      if (ninc.eq.0) call donepl
      if (ninc.eq.0) call exit
      write (6,2001) ninc,nit,emod,beta1,alpha1,delp,el,area,ecvge
 2001 format (1h1,*increment iteration e-modulus beta1 alpha1 delta-p l
     1area convergence criteria*/5x,2i10,7e12.5)
      write (6,2010) enu,gam1,beta2
 2010 format (1x* nu*e15.7,5x,*gamma1*e15.7,5x,*beta2*e15.7)
      funct=0.0
      ekk=0.0
      ekkq=0.0
      functe=0.0
      do 20 i=1,ninc
      s(i)=0.0
      u(i)=0.0
      e(i)=0.0
      e2(i)=0.0
      fzeta(i)=0.0
      bzeta(i)=0.0
   20 continue
      p=delp
      do 50 n=2,ninc
      write (6,2002) p
 2002 format (1h0*p-load*,e15.7)
      do 40 m=1,nit
      delu=el*(delp-p*funct)/(area*emod)
      delu=delu-el*functe*ekk/emod
      deleps=delu/el
      delep2=-enu*deleps+(1-2.*enu)*funct*p/(2.*emod*area)
      delep2=delep2-functe*ekk*(1.+enu)/emod
      ekk=e(n)+deleps+2.*(e2(n)+delep2)
      bdelz=(beta1*(deleps+2.*delep2))**2
      bdelz=sqrt(bdelz+beta2*beta2*(deleps*deleps+2.*delep2*delep2))
      bzet=bzeta(n-1)+bdelz
      funct=-2.*(1.+enu)*alpha1*bdelz/(1.+bzet)
      functe=emod*gam1*bdelz/(1.+bzet)
      convge=abs((ekk-ekkq)/ekk)
      write (6,2000) m,n,deleps,delep2,funct,convge,ekk,akkq,functe
 2000 format (1x*m n a e2 f cge ekk ekkq fe*2i2,7e11.4)
      ekkq=ekk
      if (convge.lt.ecvge) go to 44
   40 continue
   44 ekkq=0.0
   45 continue
      u(n)=u(n-1)+delu
      e(n)=e(n-1)+deleps
      e2(n)=e2(n-1)+delep2
      s(n)=p/area
      fzeta(n)=funct
      bzeta(n)=bzeta(n-1)+bdelz
      p=p+delp
      write (6,2003) m,n,u(n),e(n),e2(n),s(n),fzeta(n),bzeta(n)
 2003 format (1h0*iteration increment displacement strain strain2 stress
     1 f(zeta) beta.zeta*/10x,2i5,6e15.7)
   50 continue
      go to 10
      end
```