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FERMION MASSES AND NEUTRINO MIXING IN AN $SU(5)_{GUT} \otimes SU(8)_{ETC}$ MODEL

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Abstract

We extend the $SU(3) \otimes SU(2) \otimes U(1)$ model without scalars to $SU(5)_{GUT} \otimes SU(8)_{ETC}$. In our model, the mixing in the leptons is identical to that for the quarks, so that the Cabibbo angle determines the mixing of ν_e and ν_μ. The quark masses and mixing angles are studied for two and three generations of quarks.

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I. INTRODUCTION

There has lately been a great deal of experimental and theoretical interest in the phenomenon of neutrino mixing. Very sensitive experiments have been proposed.[1] It has been very difficult in Grand Unified Theories (GUTs) to obtain, for example, a large $\nu_e - \nu_\mu$ mixing.[2] We are interested in GUTs because of the possibility of connecting quark and lepton sectors.

The quark mass spectrum and mixing have been studied in extended technicolor models.[3-6] We extend this approach to GUTs in order to exploit the connection between the quark and lepton sectors. This allows us to make predictions for neutrino mixing different from the other GUT models that make use of Higgs scalars.

In the $SU(3) \otimes SU(2) \otimes U(1) \otimes SU(8)_{ETC}$ model we previously studied[4] we obtained reasonable quark masses and mixing. It is our aim to extend such a model to a GUT ⊗ ETC in order to make predictions for the lepton mixing angles. In SO(10) all particles and their conjugates are in the <u>16</u> representation, so that one cannot obtain mass differences between up- and down-quarks. We therefore consider an SU(5) GUT as the simplest generalization of the aforementioned model.[3,4] Quark mixing will result in a manner similar to that evidenced in the $SU(3) \otimes SU(2) \otimes U(1) \otimes SU(8)_{ETC}$ model.

The assignment of the representation of SU(8) differs in the low energy gauge group from that of our previously considered model. Because of this difference, the mass spectrum of the present model will differ from that of Refs. 3 and 4.

In Sec. II, we introduce our model. In Sec. III, we consider the implications for the mixing in the lepton sector. In Secs. IV and V, we study the quark mass spectrum and mixing in the two- and three-generation cases, respectively. In Sec. VI, we make some comments and discuss some difficulties with the present approach.

II. THE MODEL

In the previous paper,[4] we used the gauge group $SU_L(2) \otimes U_Y(1) \otimes SU_C(3) \otimes SU_{ETC}(8)$ with representations $(i = 1, \ldots, 5)$

$$\begin{aligned}
(\,2\ \tfrac{1}{6}\ 3\ 8\,)_L &= \begin{pmatrix} U_i & t & c & u \\ D_i & b & s & d \end{pmatrix}_L \,, \\
(\,1\ \tfrac{2}{3}\ 3\ 8\,)_R &= (\,U_i\ \ t\ \ c\ \ u\,)_R \,, \\
(\,1\ -\tfrac{1}{3}\ 3\ \bar{8}\,)_R &= (\,D_i\ \ b\ \ s\ \ d\,)_R \,,
\end{aligned} \tag{1}$$

We wish to consider here the model $SU_{GUT}(5) \otimes SU_{ETC}(8)$ with representations

$$\begin{aligned}
(\,10\ 8\,)_L &= \begin{pmatrix} U_i & t & c & u & U_i^c & t^c & c^c & u^c & E_i^c & \tau^c & \mu^c & e^c \\ D_i & b & s & d & & & & & & & & \end{pmatrix}_L \,, \\
(\,\bar{5}\ \bar{8}\,)_L &= (\,D_i^c\ \ b^c\ \ s^c\ \ d^c\ \ N_i\ \ \nu_\tau\ \ \nu_\mu\ \ \nu_e\ \ E_i\ \ \tau\ \ \mu\ \ e\,)_L \,, \\
(\,1\ \bar{8}\,)_L &= (\,N_i^c\ \ \nu_\tau^c\ \ \nu_\mu^c\ \ \nu_e^c\,)_L \,.
\end{aligned} \tag{2}$$

The decomposition of the group $SU_{GUT}(5) \otimes SU_{ETC}(8)$ down to $SU_L(2) \otimes U_Y(1) \otimes SU_C(3) \otimes SU_{ETC}(8)$ is

$$(\,10\ 8\,)_L \rightarrow \left\{ \begin{aligned}
(\,2\ \tfrac{1}{6}\ 3\ 8\,)_L &= \begin{pmatrix} U_i & t & c & u \\ D_i & b & s & d \end{pmatrix}_L \\
(\,1\ \tfrac{2}{3}\ 3\ \bar{8}\,)_R &= (\,U_i\ \ t\ \ c\ \ u\,)_R \,, \\
(\,1\ -1\ 1\ \bar{8}\,)_R &= (\,E_i\ \ \tau\ \ \mu\ \ e\,)_R
\end{aligned} \right. \tag{3}$$

and

$$(\,\bar{5}\ \bar{8}\,)_L \rightarrow \left\{ \begin{aligned}
(\,1\ -\tfrac{1}{3}\ 3\ 8\,)_R &= (\,D_i\ \ b\ \ s\ \ d\,)_R \\
(\,2\ -\tfrac{1}{2}\ 1\ \bar{8}\,)_L &= \begin{pmatrix} N_i & \nu_\tau & \nu_\mu & \nu_e \\ E_i & \tau & \mu & e \end{pmatrix}_L
\end{aligned} \right. \,, \tag{4}$$

$$(\,1\ \bar{8}\,)_L \rightarrow (\,1\ 0\ 1\ 8\,)_R = (\,N_i\ \ \nu_\tau\ \ \nu_\mu\ \ \nu_e\,)_R \,. \tag{5}$$

Here the upper and lower case letters refer to technifermions and ordinary fermions, respectively, the superscript c refers to charge conjugation, and it is understood that the model may be supplemented by additional fermions so that the model is anomaly free in $SU_{GUT}(5) \otimes SU_{ETC}(8)$ and asymptotically free. A comparison of Eq. (1) with Eqs. (3) and (4) reveals that the $\underline{8}$ of SU(8) has been conjugated in the right-handed multiplets. This has an effect on the up- and down-quark mass matrices, M^U and M^D, allowing them to differ. Had we used an SO(10) GUT, we would have been forced to have the $\underline{16} = \underline{10} + \underline{\bar{5}} + \underline{1}$ assigned to the $\underline{8}$ of SU(8). It would then have been impossible to assign the $\underline{8}$ to the $\underline{10}$ multiplet and the $\underline{\bar{8}}$ to the $\underline{\bar{5}}$. Thus, the up- and down-quark mass matrices could not have been different.

The gauge boson mass matrix which results from the interaction of the gauge bosons and the Higgs scalars takes the form[4]

$$\tfrac{1}{2}g^2[W^{a\dagger}(\zeta\times\zeta^T+A^2+v^2\,1)W^a+W^{a\dagger}AvW^{a*}+W^{aT}AvW^a] = (W^1_a\ W^2_b)\ M^2\begin{pmatrix}W^1_c\\ W^2_d\end{pmatrix} \qquad (6)$$

with the gauge bosons $W_i^{\,a}$ which mediate the interaction between ordinary and technifermions combined to form

$$W^1_{aj} = (W^a_j+W^{a\dagger}_j)/\sqrt{2}, \quad W^2_{aj} = (W^a_j-W^{a\dagger}_j)/i\sqrt{2},$$

where a is the generation index and $j = 1,\ldots,5$. The Higgs scalars involved in the interaction with the gauge bosons can be represented in the form

$$\varphi_{ij} = \begin{pmatrix} A & & & & & \\ & v & & & & \\ & & v & & & \\ & & & v & & \\ & & & & v & \\ & & & & & v \end{pmatrix}$$

and $\varphi^\alpha_j = (\zeta^\alpha\ 0\ 0\ 0\ 0\ 0)$, $\alpha = 1, 2, 3$. With these two representations, we can break $SU(8)_{ETC}$ down to $SO(5)_{TC}$. This could be regarded as a condensate obtained as in Refs. 3 and 4. In the three generation case, for the definitions

$$W^a = \begin{pmatrix} W^1 \\ W^2 \\ W^3 \end{pmatrix}, \quad A = \begin{pmatrix} a & d & f \\ d & b & e \\ f & e & c \end{pmatrix}, \quad \zeta^\alpha = \begin{pmatrix} x^\alpha \\ y^\alpha \\ z^\alpha \end{pmatrix}, \tag{7}$$

where the subscript j is suppressed in W^a, the above matrix M^2 of Eq.(6) is determined by

$$M^2 = \tfrac{1}{4}g^2 \times \begin{pmatrix} A^2+v^2\mathbb{1} +2Av+\zeta\times\zeta^T & 0 \\ 0 & A^2+v^2\mathbb{1} -2Av+\zeta\times\zeta^T \end{pmatrix} . \tag{8}$$

The sum over indices has been suppressed for the ζ term. The inverse of M^2 enters in the four-fermion interaction in the one-loop diagram connecting ordinary quarks, techniquarks, and gauge bosons.

The techniquarks Q form a condensate, $\langle\bar{Q}Q\rangle_0 = \frac{1}{3}(\sqrt{2}/G_F)^{3/2}\,(3/5)^{1/2}$; thus for $(4\pi)^{-1} = \frac{1}{6}(\sqrt{2}/G_F)^{3/2}\times(3/5)^{1/2})$,[7]

$$M^2 = \frac{g^2}{4\pi}\begin{pmatrix} (M^D + M^U)^{-1} & 0 \\ 0 & (M^D - M^U)^{-1} \end{pmatrix} \tag{9}$$

In Secs. III and IV, we shall identify M^2 in Eq.(9) with that from Eq.(8). From this identification, one can show that

$$|\det M^D| \geq |\det M^U| , \tag{10}$$

which must be satisfied by the solutions found there. This has been discussed in Ref. 5 for the case in which the representations chosen are different from ours of Eqs.(3) and (4); the $\underline{8}$ has been switched with $\bar{\underline{8}}$ in the right-handed components. For this reason the inequality in Eq.(10) is the opposite of that of Ref. 5. We shall assume that suitable representations of SU(8) are introduced to make the SU(8) anomaly-free as in Ref. 3.

The formalism of the model embodied in Eqs.(8) and (9) determines the masses and relates the quark mass matrices to the parameters in our model. It is convenient to define matrices

$$\mathcal{M}^{\pm} = [\,(M^D+M^U)^{-1} \pm (M^D-M^U)^{-1}] \;, \tag{11}$$

so that the equality of M^2 from Eqs.(8) and (9) reads

$$\mathcal{M}^{-} = 4\eta vA \;,\quad \mathcal{M}^{+} = 2\eta[A^2+v^2 1 + \zeta\times\zeta^T] \;. \tag{12}$$

The term $\zeta\times\zeta^T$ contains terms $(x)^2 = \sum_\alpha (x^\alpha)^2$, $(y)^2 = \sum_\alpha (y^\alpha)^2$, etc. These terms, since they must be positive definite, give conditions

$$V^2 - \mathcal{M}^{+}_{ii} V + \tfrac{1}{4} (\mathcal{M}^{-})^2_{ii} \leq 0 \;. \tag{13}$$

Here we take $V \equiv 2\eta v^2$ and index $i = 1,2,3$ corresponding respectively to x,y, and z. The requirement that x^2y^2, for example, be identical to $(xy)^2$ results in consistency conditions

$$[V^2 - \mathcal{M}^{+}_{ii} V + \tfrac{1}{4}(\mathcal{M}^{-})^2_{ii}][V^2 - \mathcal{M}^{+}_{jj} V + \tfrac{1}{4}(\mathcal{M}^{-})^2_{jj}] - [\mathcal{M}^{+}_{ij} V - \tfrac{1}{4}(\mathcal{M}^{-})^2_{ij}]^2 = 0 \;, \tag{14}$$

where $i \neq j = 1, 2, 3$. These equations (11) through (14) will be solved in Secs. IV and V for the quark masses for the cases of two and three generations, respectively.

We may always take one of the phenomenological quark mass matrices $M^{U,D}$ as diagonal. In the three-generation case, the other will be non-diagonal and expressed in terms of the Kobayashi-Maskawa[8] mixing matrix

$$U_{KM} = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3+s_2s_3 & c_1c_2s_3-s_2c_3 \\ -s_1s_2 & c_1s_2c_3-c_2s_3 & c_1s_2s_3+c_2c_3 \end{pmatrix} ,$$

where $c_k = \cos\theta_k$ and so on. We shall find it convenient to take

$$M^D \pm M^U = M^D_{diag} \pm U_{KM}\, M^U_{diag}\, U_{KM}^{\dagger}, \tag{15}$$

where $M^U_{diag} = \mathrm{diag}\,(m_u, m_c, m_t)$ and $M^D_{diag} = \mathrm{diag}\,(m_d, m_s, m_b)$.

III. NEUTRINO MIXING

In order to discuss the fermion sector, we shall write down the four-fermion interactions mediated by the gauge bosons W_i^a. The currents in our model are given by (a = 1,2,3)

$$\mathcal{J}_\mu^{ai} = J_\mu^{ai} + j_\mu^{ai} ,$$

where

$$J_\mu^{ai} = \overline{u_L^a}\, \gamma_\mu\, U_L^i - \overline{U_R^i}\, \gamma_\mu\, u_R^a + \overline{d^a}\, \gamma_\mu\, D^i , \tag{16}$$

and

$$j_\mu^{ai} = -\overline{E^i}\, \gamma_\mu\, e^a - \overline{N_L^i}\, \gamma_\mu\, \nu_L^a + \overline{\nu_R^a}\, \gamma_\mu\, N_R^i . \tag{17}$$

The fermion masses are obtained by the interaction mediated by W_i^a. There are two interaction terms. In the following equations, the first term is SU(5)-invariant and the second term is SO(5)-invariant. Only these two are invariant under $SO(5)_{TC}$ constructed from $\mathcal{J}_\mu^{ai}$ and $\mathcal{J}_\mu^{\dagger ai}$. The fact that there are no condensates between ordinary quarks and techniquarks means that the mass terms coming from $\mathcal{L}_{INT}$ are

$$\mathcal{L}^q = J_\mu^\dagger X_1 J^\mu + \tfrac{1}{2} (J_\mu X_2 J^\mu + \text{h.c.}) , \tag{18}$$

and

$$\mathcal{L}^\ell = j_\mu^\dagger X_1 j^\mu + \tfrac{1}{2} (j_\mu X_2 j^\mu + \text{h.c.}) . \tag{19}$$

The matrices X_1 and X_2 are essentially gauge boson propagators. The first (second) term in Eq.(18) yields the down-quark (up-quark) mass matrix $M^D (M^U)$ after a Fierz transformation

$$J_\mu^\dagger X_1 J^\mu = (\overline{D^i}\, \gamma_\mu\, d^a)\, X_1^{ab}\, (\bar{d}^b\, \gamma^\mu\, D^i) + \ldots$$

$$= - (\overline{D^i} D^i)(\bar{d}^b X_1^T\, d) + \ldots \quad .$$

Thus, since $\mathcal{L}^q = -\bar{d}M^D d - \bar{u}M^U u$, we find

$$M^D = \langle \bar{D}D \rangle X_1 , \tag{20}$$

where we have taken X_1 symmetric.[9] Similarly,

$$\tfrac{1}{2}(J_\mu X_2 J^\mu + h.c.) = \tfrac{1}{2}\left[-(\overline{u^a_L}\gamma_\mu U^i_L)(X_2^{ab} + X_2^{ba})(\overline{U^i_R}\gamma^\mu u^b_R) + h.c. + \ldots\right]$$

$$= (\overline{U^i_R}U^i_L)\,\bar{u}_L\,(X_2+X_2^T)\,u_R + h.c. + \ldots$$

$$= (\overline{U^i_R}U^i_L)\,\bar{u}_L\,(X_2+X_2^T)\,u_R + (\overline{U^i_L}U^i_R)\,\bar{u}_R\,(X_2^\dagger+X_2^*)\,u_L + \ldots .$$

Taking X_2 real and symmetric and using $\langle \bar{U}_L U_R \rangle = \langle \bar{U}_R U_L \rangle = \tfrac{1}{2}\langle \bar{U}U \rangle$, we find

$$M^U = -\langle \bar{U}U \rangle X_2 . \tag{21}$$

Proceeding in a similar fashion for the first term in Eq.(19), we obtain

$$M^\ell = \langle \bar{E}E \rangle X_1 \tag{22}$$

Then the second term in Eq.(19) is, after a Fierz transformation,

$$\tfrac{1}{2}(j_\mu X_2 j^\mu + h.c.) = 2(\bar{N}_L N_R)(\bar{\nu}_R X_2 \nu_L) + [\bar{N}_L (N_L)^c][\overline{(\nu_L)^c} X_2 \nu_L] + [\overline{(N_R)^c} N_R][\bar{\nu}_R X_2 (\nu_R)^c]$$

$$+ h.c. + \ldots .$$

We therefore find that

$$M_L = -\langle \bar{N}_L (N_L)^c \rangle X_2 = B_1 X_2 \tag{23}$$

$$M_R = -\langle \overline{(N_R)^c} N_R \rangle X_2 = B_2 X_2 \tag{24}$$

$$M = -\langle \bar{N}_L N_R \rangle X_2 = B_3 X_2 \tag{25}$$

Eq.(24) has already been considered by Mohapatra.[10]

We note that Eqs.(23), (24), and (25), which determine the neutrino mass matrix, can be written in the form of a 6 x 6 matrix

$$\mathcal{M}^{\nu} = \begin{pmatrix} M_L & m \\ m & M_R \end{pmatrix} = \begin{pmatrix} B_1 & B_3 \\ B_3 & B_2 \end{pmatrix} X_2 = BX_2 \tag{26}$$

When the matrix B is diagonalized[11] we obtain three left-handed and three right-handed Majorana-type neutrinos.[12] From Eqs. (20) and (22), we see that

$$M^{\ell} \propto M^{D} \quad .$$

From Eqs. (21) and (26), we see that

$$M^{\nu} \propto M^{U} \quad ,$$

where M^{ν} is the mass matrix of the three left-handed neutrinos. Thus, the mixing angles among the neutrinos are determined by the quark (Kobayashi-Maskawa) mixing matrix.[13] In particular, the mixing angle of ν_e and ν_{μ} is the Cabibbo angle.

IV. QUARK MASSES IN TWO GENERATIONS

We shall use the equations developed in Sec. III to present the formulas for the quark masses, parametrized by

$$M^U = \begin{pmatrix} \alpha & \gamma \\ \gamma & \beta \end{pmatrix}, \quad M^D = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}.$$

Therefore, equating expressions for M^2 in Eqs.(11) and (12), we write the relevant determinants $D_+ \equiv (\alpha+m_d)(\beta+m_s) - \gamma^2$ and $D_- \equiv \gamma^2 - (\alpha-m_d)(\beta-m_s)$. Define quantities $\mathcal{P} \equiv \frac{1}{2}(D_-^{-1}+D_+^{-1})$ and $\mathcal{M} \equiv \frac{1}{2}(D_-^{-1}-D_+^{-1})$. The condition $x^2 \geq 0$ in Eq.(13) implies that V is between the limits

$$V_\pm^x = \beta\mathcal{P}-m_s\mathcal{M} \pm \left\{(\beta^2-m_s^2)(\mathcal{P}^2-\mathcal{M}^2)-\gamma^2\mathcal{M}^2\right\}^{\frac{1}{2}}, \tag{27}$$

and $y^2 \geq 0$ implies that V is between the limits

$$V_\pm^y = \alpha\mathcal{P}-m_d\mathcal{M} \pm \left\{(\alpha^2-m_d^2)(\mathcal{P}^2-\mathcal{M}^2)-\gamma^2\mathcal{M}^2\right\}^{\frac{1}{2}}. \tag{28}$$

The condition $(xy)^2 = x^2y^2$ of Eq.(14) leads to the quartic equation

$$\begin{aligned} &V^4 - 2(\alpha\mathcal{P}-m_d\mathcal{M}+\beta\mathcal{P}-m_s\mathcal{M})V^3 \\ &+ \left\{4(\beta\mathcal{P}-m_s\mathcal{M})(\alpha\mathcal{P}-m_d\mathcal{M}) + (m_s\mathcal{P}-\beta\mathcal{M})^2 + (m_d\mathcal{P}-\alpha\mathcal{M})^2 - 4\gamma^2\mathcal{P}^2 + 2\gamma^2\mathcal{M}^2\right\} V^2 \\ &- 2\left\{(\beta\mathcal{P}-m_s\mathcal{M})(m_d\mathcal{P}-\alpha\mathcal{M})^2 + (\alpha\mathcal{P}-m_d\mathcal{M})(m_s\mathcal{P}-\beta\mathcal{M})^2 + \gamma^2\mathcal{M}^2(\beta\mathcal{P}-m_s\mathcal{M}+\alpha\mathcal{P}-m_d\mathcal{M}) \right. \\ &\left. + 2\gamma^2\mathcal{P}\mathcal{M}\left[(m_s\mathcal{P}-\beta\mathcal{M}) + (m_d\mathcal{P}-\alpha\mathcal{M})\right]\right\} V + \left\{(m_d\mathcal{P}-\alpha\mathcal{M})(m_s\mathcal{P}-\beta\mathcal{M}) - \gamma^2\mathcal{M}^2\right\}^2 = 0 \end{aligned} \tag{29}$$

We solve Eqs.(27) through (29) to yield m_u, m_d, m_s, and m_c. The mixing angle for the two generation case is θ_c ($\sin\theta_c \simeq 0.2$) and $\alpha = m_u\cos^2\theta_c + m_c\sin^2\theta_c$, $\beta = m_u\sin^2\theta_c + m_c\cos^2\theta_c$ and $\gamma = (m_u-m_c)\sin\theta_c\cos\theta_c$. We impose condition (10) on the masses in the solution.

There is no reasonable solution to Eqs.(27), (28) and (29). We find analytically, for example, that for $\theta_c = 0$ that $m_d \geq m_u$ and $m_s \geq m_c$ is required.

V. QUARK MASSES IN THREE GENERATIONS

The procedure in the three-generation case is similar to that of the two-generation case. However, the level of complication rises rapidly. There are three mixing angles, not just one. There are in addition three quartic equations for V which must be within the three bounds of Eq.(13) and must have a simultaneous root for V in all three quartic equations.

We searched for a solution over a wide range of masses in the three-generation case. One acceptable solution is $m_u = 3 \times 10^{-4}$ GeV, $m_d = -0.0125$ GeV, $m_s = -0.225$ GeV, $m_c = -1.5$ GeV, $m_b = -5$ GeV, $m_t = -20$ GeV, and $\theta_c \sim 0.2°$, $\theta_2 = 0$, and $\theta_3 = -48°$, which is obtained using the parameters in units of GeV

U = 2330	a = 31950	d = -1950
X = 562	b = 5.29	e = 2460
Y = 6.23	c = 2320	f = -1890
Z = 8.38		

Note that the solution quoted has very reasonable masses, but that θ_c is very small. The Cabibbo angle seems to depend very strongly on m_c and m_d; for a value of $m_c \sim -0.1$ GeV, $\sin\theta_c \sim 0.2$.

We may ask how a very small θ_c and very large $|\theta_3|$ can arise. If we consider the three-generation case in which only c- and t-quark masses are mixed, we can generate a solution in the two generation c, t sector, as in Sec. IV. This solution is only possible for a very large c, t mixing angle $|\theta_3| \sim 40°$. Otherwise the terms analogous to the arguments of the square-roots in Eqs.(27) and (28) are negative. This behavior is carried over into our full-blown solution in the three generation case.

Other calculations in three generation cases[4,5,14] to obtain masses also fail to be compelling, having difficulties in obtaining "expected" results for the masses.

VI. CONCLUSIONS

In models such as ours, in which ordinary and techniquarks are in the same multiplet, flavor-changing neutral currents are automatically generated.[3,5,14] No entirely satisfactory way is known to suppress them.[3,5] Also, we do not treat CP violation in this model, setting the relevant phases equal to zero.

There are other possible ETC gauge groups and several choices of assignment of representations to each fermion multiplet. There may have been a better solution if some other possibility had been chosen.[5,6,14]

We expect that information on $\nu_e - \nu_\mu$ mixing will soon be forthcoming, assuming that the neutrinos are massive and that there is a mass difference. In Higgs approaches, it is extremely difficult to get large $\nu_e - \nu_\mu$ mixing. In this approach without scalars, the mixing angle is the Cabibbo angle. The experimental results will provide some choices on the validity of the various models with and without Higgs scalars.

In the three-generation case, no satisfactory values of quark masses and mixing angles have been obtained. We would hope that improved values can eventually be found.

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