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Modal Study of Refractive Effects on X-ray Laser Coherence

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Abstract

The role of smoothly varying transverse gain and refraction profiles on x-ray laser intensity and coherence is analyzed by modally expanding the electric field within the paraxial approximation. Comparison with a square transverse profile reveals that smooth-edged profiles lead to: (1) a greatly reduced number of guided modes, (2) the continued cancellation of local intensity from a loosely guided mode by resonant free modes, (3) and the absence of extraneous (or anomalous) free mode resonances. These generic spectral properties should enable a considerable simplification in analyzing and optimizing the coherence properties of laboratory soft x-ray lasers.

1. Introduction

Current X-ray laser (XRL) designs rely on amplifying spontaneous emission in a high temperature plasma.[1] An important issue in the study of XRL's is the degree of transverse spatial coherence necessary for holographic applications.[2] Longitudinal coherence appears to be satisfactory, but transverse coherence remains problematic and requires further optimization study.

Recently, London *et al.*[3] have undertaken a study of transverse coherence based on a modal decomposition of the electric fields in an amplifying medium. With this ansatz for the laser fields, the paraxial wave equation is transformed into two equations which separately govern the longitudinal and transverse behavior. The longitudinal equation describes the usual longitudinal amplification from a distributed noise source, whereas the transverse equation is of the Schrödinger-type with complex "potential" arising from the gain or imaginary part of the atomic susceptibility. The analysis of this latter equation leads to a spectrum of eigenmodes which determines the possible transverse profiles of intensity and coherence.

The above mathematical characterization for the fields has formed the basis in the literature for the predicted phenomenon of "excess noise" in an amplifying medium.[4] In particular, the inherent non-self-adjoint property of a general amplifying medium presumably allows for the possibility that loosely guided or bound transverse eigenmodes may dominate the profiles at large transverse distances from the lasant medium. Such a prospect has serious implications for coherence and intensity studies since the predicted profiles will be unacceptably sensitive to the precise value of gain used.

Previous use of the modal approach to understand XRL phenemena has been restricted to the bound or discrete portion of the transverse eigenmode spectrum. For sufficiently large values of gain-length product this restricted analysis can obtain accurate transverse profiles of intensity and coherence. Unfortunately, most gain-length products observed in the present generation of amplified spontaneous emission (ASE) XRL experiments are not sufficiently large to justify use of this truncated approach in general.

More recently, Amendt *et al.*[5] have reexamined the modal approach by appending the continuum or free modes to the bound mode portion of the transverse spectrum for the particular example of transverse square gain and refraction strength profiles. The primary motivation for including the continuum is that by virtue of the non-orthogonality of the eigenmodes, sufficient cancellation from cross-terms in the expression for the modal intensity may occur and possibly eliminate to a large extent the "excess noise" phenomenon. It is found that for small and moderately large gain-length products the anomalously large intensities associated with one loosely bound transverse mode are significantly reduced by the inclusion of neighboring free eigenmodes. This feature has the two-fold effect of greatly reducing the level of "excess noise" and of removing the source of undue sensitivity of previous modal modelling to the exact value of the gain parameter adopted.

Amendt *et al.*[5] address some fundamental problems arising in a general ASE XRL environment, but they do not determine the degree of transverse coherence relevant for an ASE XRL experiment. In particular, the square gain and refraction strength profiles explored in that analysis were intended mostly for analytic ease and conceptual clarity. What remains to be shown is whether the effective modal intensity cancellation persists as effectively for rounded profiles which now allow for the beneficial effect of refractive defocussing. In this paper we continue our analysis of modal XRL coherence by considering some consequences of rounded gain and refraction strength profiles in a finite geometry.

2. Modal Analysis

Our starting point is the paraxial equation for the slowly varying wave electric field amplitude E:[3]

$$\left[\frac{1}{k}\nabla_\perp^2 - 2i\partial_z - h(x) + ig(x)\right]E(x,z) = -4\pi k\, P_{sp}(x,z)\,, \qquad (1)$$

where k is the free-space longitudinal (or parallel to z-axis) wavevector, $\nabla_\perp^2$ is the transverse Laplacian, $h=\omega_{pe}^2(x)/kc^2$ is the refraction strength, ω_{pe} is the electron plasma frequency, g(x) is the atomic gain of the medium, and P_{sp} is the random (in x and z), spontaneous atomic polarization. Upon writing $E(x,z) = \Sigma c_n(z)u_n(x)$ we find a transverse mode equation:

$$\left[\frac{1}{k}\partial^2_{xx} - F_e\left(\eta\hat{h}(x) - i\hat{g}(x)\right)\right]u_n(x) = -E_n u_n(x)\,, \tag{2}$$

and a longitudinal transfer equation:

$$\sum_n \left[u_n\,\partial_z c_n - \frac{i}{2}\,E_n c_n u_n\right] = -i\,P_{sp}\,, \tag{3}$$

where $F_e = kg_0a^2$ is an effective Fresnel number, a is the lasant half-width, E_n is the eigenvalue, x -> xa, z->zka^2, P_{sp}->$P_{sp}/2\pi(ka)^2$, $\eta=h_0/g_0$, and $h = h/h_0$ and $g=g/g_0$ are normalized transverse profiles. Since eq. (2) is non-self-adjoint, the eigenvalues are generally nonreal and the eigenfunctions are *biorthonormal*: $\int u_n u_m dx=\delta_{nm}$ ($\neq\int u_n u_m^* dx$). This feature specifically gives rise to the problem of "excess noise" where a "loosely" bound mode may dominate the field intensity $<|E|^2>/8\pi$ at large transverse distance (x>>a).[4] Amendt *et al*[5]. have previously shown that free modes ($\lim_{x\to\infty} u_n < \infty$) contained in the spectrum of eq. (2) tend to compensate for "excess noise" arising from marginally bound modes ($\lim_{x\to\infty} u_{n'} = 0$) for the particular example of a square gain and refraction strength profile. However, the case of a hard-edged profile is not a realistic feature of XRL's that are based on amplifying spontaneous emission.

3. Modal Study with Refraction

A useful method for considering smooth transverse profiles in the modal approach is to numerically relax the square profile into a hyperbolic secant squared profile as follows: $h(x), g(x) \to (1-\varepsilon)\,f(x,\varepsilon) + \varepsilon\,\mathrm{sech}^2(x)$, where ε->1 and

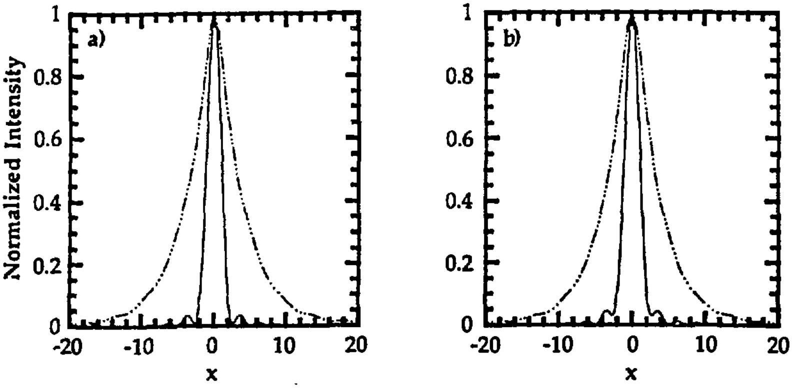
$$f(x,\varepsilon) = \begin{cases} 0 & |x| > 1+\varepsilon \\ \sin^2\left[\dfrac{\pi(|x|-1-\varepsilon)}{4\varepsilon}\right] & 1-\varepsilon < |x| < 1+\varepsilon \\ 1 & |x| < 1-\varepsilon \end{cases}\,, \tag{4}$$

and then solving (by shooting methods) the transverse eigenvalue equation (2) at each incremental step in ε using the eigenvalue for the previous ε as a guess. The sech^2(x) profile is a convenient choice because: 1) $h(0)$ equals unity as for the square profile, (2) the integrated area under h coincides with the square profile, and (3) the bound mode eigenvalues are analytically known for the case of an unbounded geometry:

$$E_n = -\left[\frac{1}{2}(1 + 4iF_e(1 + i\eta))^{1/2} - (n + \frac{1}{2})\right]^2 \qquad , n=0, 1, 2, \ldots . \tag{5}$$

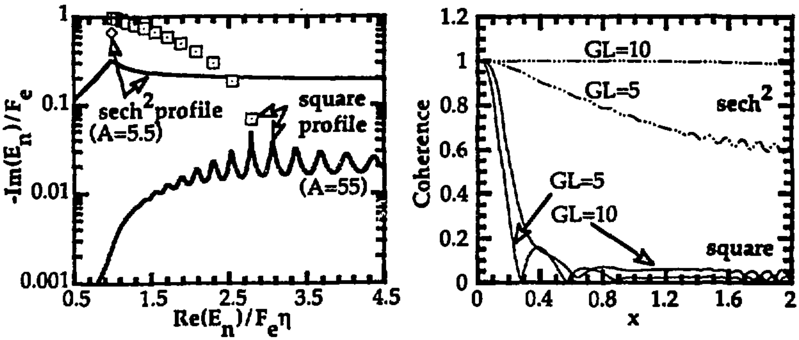
The general requirement for a bound mode is that Im (E_n) < 0, leading to only three bound modes for a Ni-like Se XRL,where typically F_e = 1500 and η = 59.[3] By contrast, the number of modes n_g in a hard-edged laser scales as:[3] $2F_e/(\pi\ln[F_e/(1+\eta)])$, giving nearly 300 bound modes for the Ni-like Se XRL. Clearly, the coherence length (or distance at which the normalized correlation function $<E(0,z)E^*(x,z)>\cdot 1/[<|E(0,z)|^2><|E(x,z)|^2>]^{1/2}$ is reduced to 0.85) is far greater for a smooth profile than for a square profile, if we consider only the bound mode contribution. The key question becomes whether the inclusion of free modes can affect this conclusion.

To consider this question, we first show how the free mode portion of the spectrum compensates for anomalous intensities arising from loosely bound modes as in the square profile case. The free modes are obtained by employing a reflecting boundary condition at x=± A, where A>>1. Figures (1a-b) display the compensated



Figs.(1a-b): Normalized compensated (solid line) and uncompensated (dotted-dash) intensity for the square profile (a) and the sech²(x) profile (b) for F_e=0.5, η=0, and geometric half-width A=25.

intensity for one loosely bound mode compared to the uncompensated or "bare" intensity. Note how the bound mode intensity dramatically cancels beyond x~±2 for the uncompensated case, while in the absence of resonant free modes a considerable surplus of energy resides outside the lasant medium ($|x|\geq 1$). This example illustrates the "excess noise" phenomenon for a smooth profile, but with a significant degree of reduction occurring due to cross-correlations between neighboring (in $Re(E_n)$) free modes and the one marginally bound mode.



Figs.(2,3): In fig.(2) (left) are shown the point and free mode (even parity) spectra for square and sech²(x) profiles with F_e=100 and η=10. Note the larger value of A (10x) used for the square profile. In fig.(3) (right) are shown coherence profiles for A=5.5, F_e=100, and η=10 with gain-length parameter GL=5, 10.

In fig.(2) we show the point (or bound mode) and continuum spectra for both square and sech²(x) profiles. Note the absence of continuum resonances beyond the one (even parity) bound mode for the $sech^2(x)$ profile; such resonances effectively act as loosely bound modes in the case of the square profile. This generic feature of the smooth profile is seen to simplify the analysis: we need only to include those free modes which are resonant with a given bound mode for a viable description of coherence and intensity in the moderate to large GL (>5) regime. In fig.(3) we compare coherence profiles for the square and $sech^2(x)$ gain profiles by considering only free modes in the neighborhood of bound modes (but excluding the higher continuum resonances for the square profile on the basis of gain discrimination). The improved coherence for the $sech^2(x)$ profile is due mainly to the fewer number of bound modes in the system which is attributed to refractive defocussing in a smoothly varying medium. This phenomenon actually consists of two parts: (1) the usual bending of rays away from the lasant medium when $\eta \neq 0$ and (2) the effectively reduced region of maximum gain giving rise to fewer high-gain transverse modes in general. Both effects contribute significantly in discarding many of the bound modes responsible for degraded transverse coherence.

4. Discussion

Realistic coherence modelling of current XRL experiments must include the role of refractive defocussing arising from non-trivial gain and refraction profiles. We have begun to study this phenomenon from a modal viewpoint with the aim of both clarifying the role of "excess noise" in such a system and optimizing the degree of coherence for eventual holographic applications. The generally large values of F_e and η found in current XRL experiments require a streamlined use of the modal approach as outlined above in order to compare with existing numerical wave propagation codes.[6] We have shown that the role of free modes in a smoothly varying media appear not to be as important as for the square profile, requiring only that the few bound modes be "dressed" or compensated by coincident free mode resonances.

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