

## EFFECT OF THE ASPECT RATIO ON THE STABILITY LIMITS OF TJ-II-LIKE STELLARATORS

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The four-field-period device TJ-II [1] has a major radius of 1.5 m and an average plasma radius of 0.10-0.25 m, with a typical plasma aspect ratio  $A_p$  of 10.

In the infinite aspect ratio, helically symmetric limit, the region of the stability to low- $n$  modes has been shown to extend to average betas of at least 40%, for a relatively highly indented plasma [2]. It is possible to approximate the helically symmetric limit from the actual TJ-II parameters increasing the number of toroidal periods  $N_T$  and choosing the major radius  $R_0$  such as to obtain a constant helical pitch  $h = N_T/R_0$ . In this way the aspect ratio per period is also fixed.

In this work we analyze a shear-less TJ-II configuration with a rotational transform per period of 0.36 and a vacuum magnetic well of 3.5%. By taking for  $N_T$  the values  $N_T = 3, 4, 5, 8, 10, 11, 12, 19$  and 100 a sequence of equilibria is generated. These equilibria are calculated with the fixed boundary version of the VMEC code [3]. The Mercier stability properties are then analyzed.

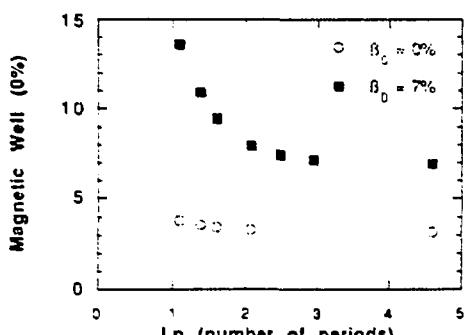


Fig. 1: Magnetic Well as a function of the function of the logarithm of the number of periods.

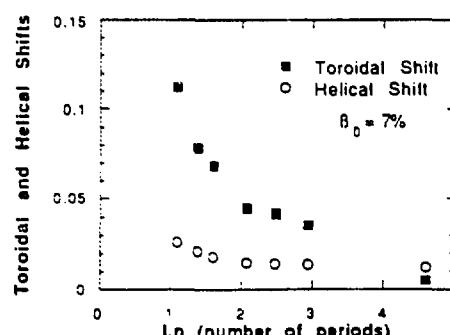


Fig. 2: Toroidal and helical shifts versus logarithm of the number of periods for  $B_0 = 7\%$ .

We compare first the equilibria. For finite beta calculations, we have considered a pressure profile linear in the toroidal flux. The rotational transform per period and the boundaries are the same. Since the aspect ratios are different, we expect different values of the

magnetic well and the toroidal and poloidal shifts. In Fig. 1 we show the differences on the magnetic well for the vacuum case and at  $\beta_0 = 7\%$ . We point out that these differences increase when  $\beta_0$  increases, and are higher for low values of  $N_T$ . This is related to the toroidal and poloidal shifts, that we plot in Fig. 2 at  $\beta_0 = 7\%$ . We see that increasing the aspect ratio lowers toroidal shift but has no effect on helical shift [4]. We see also that for  $N_T = 100$  the helical shift is higher than the toroidal one, as correspond to an almost helically symmetric configuration. It is also worth to point out that the differences are more important between the four-period and three-period cases than between the five-period and four-period cases.

We analyze now the Fourier coefficients  $R_{mn}$  and  $Z_{mn}$ , which parameterize the equilibrium flux surfaces in the VMEC code. Fig. 3 shows the  $R_{mn}$  term with  $m = 2$  and  $n = 2$ . Fig. 4 shows the  $R_{mn}$  and  $Z_{mn}$  terms with  $m = 2$  and  $n = 3$ . It is known that in the perfect helically symmetric case only the off-diagonal coefficients [ $m, m \pm 1$ ] are not zero. We see that these coefficients are also the more important in the non-symmetric cases, and their value is almost independent from the number of periods  $N_T$ . The diagonal coefficient reduces to a linear function of the normalized toroidal flux in the helically symmetric limit. That means that its contribution is the same for all the magnetic surfaces.

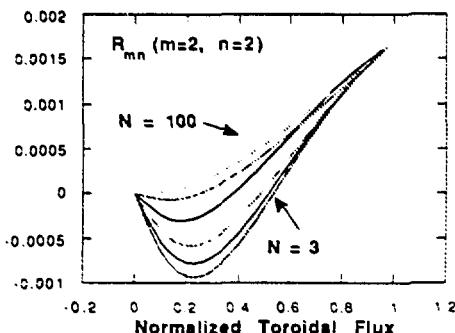


Fig. 3: Fourier coefficient  $R_{mn}$  for  $m = 2$  and  $n = 2$  as a function of the normalized toroidal flux for  $N_T = 3, 4, 5, 8, 12, 19$  and 100. The calculations was at  $\beta_0 = 7\%$ .

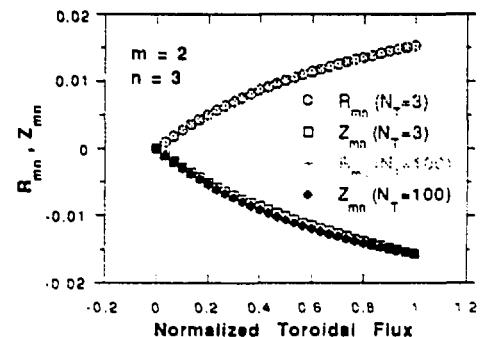


Fig. 4: Fourier coefficients  $R_{mn}$  and  $Z_{mn}$  for  $m = 2$  and  $n = 3$  as a function of the normalized toroidal flux for  $N_T = 3$  and 100. The calculations was at  $\beta_0 = 7\%$ .

The equilibrium quantities obtained were used to evaluate the Mercier stability criterion. It can be written as:

$$D_M = D_S + D_I + D_W + D_G > 0$$

where  $D_S$  is the contribution from the shear,  $D_I$  is that from net currents,  $D_W$  is that from the magnetic well and  $D_G$  is that from the geodesic curvature. In the configuration we are considering, with very low shear and zero current, the first two terms can be neglected. In configurations with a magnetic well, the term  $D_W$  is always positive. The term  $D_G$  is always negative. For these cases, we shall also use the following form of the Mercier criterion:

$$D_W / (-D_G) > 1.$$

The value of this quotient as a function of the logarithm of the number of periods is shown in Fig. 5 at  $\beta_0 = 7\%$  and an average radius  $r_0 = 0.913$  corresponding to a flux surface near the

boundary. We found that, in all cases we are considering, if  $D_M$  is positive for this radius it is also positive for  $r < r_0$ . In Fig. 6 we show separately the two terms  $D_W$  and  $D_G$  for  $N_T = 3, 4, 5, 8, 12$  and 19. The magnetic well term  $D_W$  grows faster than  $D_G$  when the number of periods increases, even though the depth of the magnetic well depth is shallower.

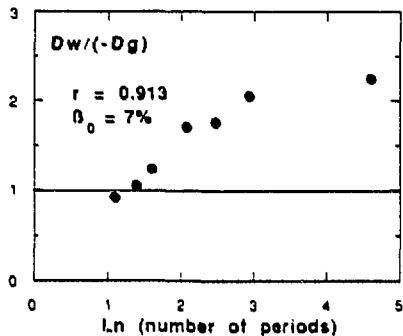


Fig. 5: Mercier criterion as a function of the logarithm of the number of periods at  $r_0 = 0.913$  and  $B_0 = 7\%$  for  $N_T = 3, 4, 5, 8, 12, 19$  and 100. The horizontal line separates the stable (upper) from the unstable zones.

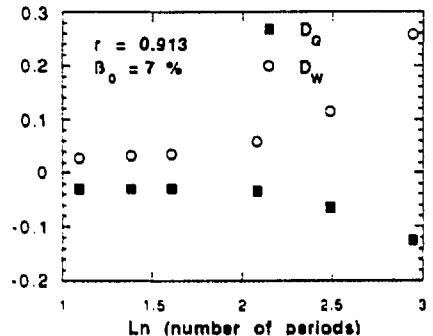


Fig. 6: Contributions to the Mercier criterion from the magnetic well,  $D_W$ , and geodesic curvature,  $D_G$ , for as a function of the logarithm of the number of periods for  $N_T = 3, 4, 5, 8, 12$  and 19.

It is interesting to see the evolution of the Mercier criterion  $D_M$  with respect to average beta. The value of  $D_M$  as a function of  $\langle\beta\rangle$  is shown in Fig. 7 for  $N_T = 3, 4$ , and 5, and for  $N_T = 8, 10$  and 12 in Fig. 8. In these figures,  $r_0 = 0.913$ . For  $N_T = 12$  no beta stability limit is reached, and we see a typical self stabilization behavior. The same result we obtain for higher  $N_T$ . We can say that for  $N_T > 11$  all the equilibria are Mercier stable.

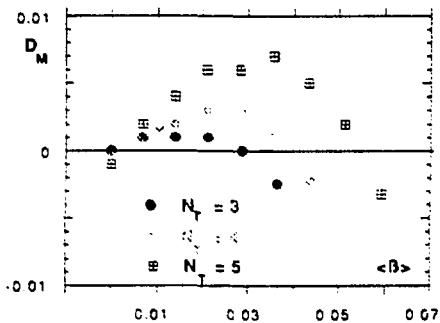


Fig. 7: Mercier criterion as a function of  $\langle\beta\rangle$  at  $r_0 = 0.913$  for  $N_T = 3, 4$ , and 5. The horizontal line separates the stable (upper) from the unstable zones.

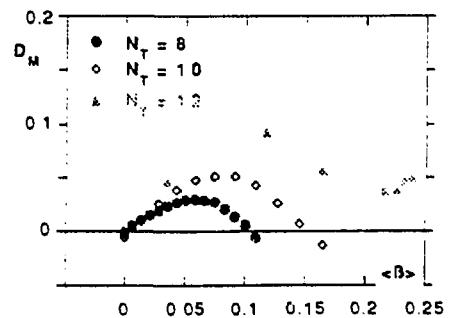


Fig. 8: Mercier criterion as a function of  $\langle\beta\rangle$  at  $r_0 = 0.913$  for  $N_T = 8, 10$  and 12. The horizontal line separates the stable (upper) from the unstable zones.

Finally we plot in Fig. 9 the average beta limit as a function of the number of periods. For low values of  $N_T$  the average beta limit increases linearly with respect to  $N_T$ . At higher values of  $N_T$  the average beta limit grows faster.

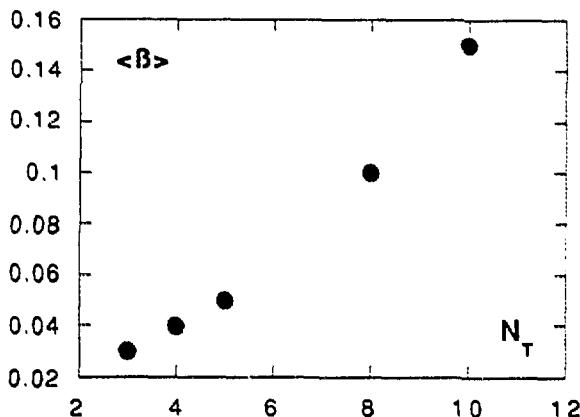


Fig. 9: Average beta stability limit as a function of the number of periods

In reference [5] a similar calculation was done for a different TJ-II configuration and varying  $N_T$  between 2 and 6. The configuration was symmetrized by suppressing small Fourier amplitudes of the boundary and by symmetrizing those which are non-vanishing in helical symmetry. In the present work no symmetrization was done. For low values of  $N_T$  both calculations yields the same results.

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