Fixed Field Alternating Gradient Recirculator for Heavy Ion Fusion*

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Abstract

A heavy ion fusion driver is considered in which a beam is passed repeatedly through a LIA (linear induction accelerator) by recirculating with two spiral sector FFAG (Fixed Field Alternating Gradient) 180 degree bends. The driver consists of three such rings: a 10-100 MeV low energy ring (LER), a 100-1000 MeV medium energy ring (MER), and a 1-10 GeV high energy ring (HER). Using a scaling field of 14 kG and taking the length of the straight sections to equal the path length in the bends, the circumference of the three rings would be 187, 590, and 1890 meters.

Four matching sections in each of the three rings provide the interface between the two straight sections accommodating the LIA and the FFAG bends. These matching sections consist of dipoles which provide a dispersion free match between the linear induction accelerator and the energy dependent equilibrium orbits of the FFAG ring.

The advantage in the use of the spiral sector FFAG over other recirculator concepts is that the fields are time invariant. This removes the problems associated with time dependent field penetration into the vacuum

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chamber and the large amount of energy which must be expended to change the magnetic field on the small time scale associated with the required pulse repetition frequency.

The disadvantage, as we show in this work is the relatively weak alternating gradient focusing. The FFAG will not accommodate the level of beam current possible in a separate function lattice of bending magnets and quadrupoles.

I. Introduction

The concept of an FFAG ring accelerator was developed in the 1950's at Midwestern Universities Research Association (MURA) as a possible alternative to the high energy proton synchrotron. Two types were studied, a radial sector and a spiral sector. The spiral sector FFAG is typically three times smaller in circumference than the radial sector FFAG, and for this reason we consider this design as a possible recirculator for heavy ion fusion.

In the FFAG system the fields of the quadrupoles and bending magnets are time invarient. Separate kicker magnets are used for beam injection and extraction from the rings. These are the only time varying magnetic fields used and are based on existing designs.

In prehistoric times theory predicted many nonlinear single particle resonances in an FFAG ring. We do not consider single particle resonances at all. A recirculator is not a synchrotron. It bears a resemblance to a race track microtron with the rf linac replaced by an induction linac. The energy gain per turn is a sizable fraction of the final energy of the particles, therefore particles experience different regions of the magnetic fields on successive passes and the concept of a single particle resonance becomes nebulous.

Another disadvantage foreseen was the limited acceptance in transverse phase space. We do not calculate the acceptance because investigation shows that the current carrying capability of the FFAG ring is discouragingly low for use as a recirculator for HIF.

We focus on FFAG rings for singly charged Bismuth 209 with rest mass of 194,651 MeV. This ion was selected because it is isotopically pure with mass around 200. The beam currents chosen in this work are based on delivering 5 MJ[1] on the fusion target at 10 GeV. This translates to a total charge of 500 μCoulombs. We assume that the beam is divided into four sub-beams. The beam current considered in the space charge calculations is one quarter this value, or 125 μCoulombs (2.1×10^{14} ions per beam). The low energy recirculator is shown schematically in Fig. 1.

II. Magnetic Fields and Particle Orbits

An FFAG ring is a scaling ring, which means that the orbit of a particle with energy E_2 is a photographic enlargement of the orbit at energy $E_1 < E_2$. The vertical and horizontal tunes are independent of energy. We will not discuss the magnetic field in detail. Much of the analytic theory of particle orbits can be found in Ref. 2 and in Ref. 3, "Selected Works of L. Jackson Laslett."

The median plane magnetic field $B_z(R,\theta,z)$ with $z = 0$ is given by

$$B_z(R,\theta,0) = B_0\left[\frac{R}{R_0}\right]^k\left[1 + f \sin\left\{N \tan\zeta \ln \frac{R}{R_0} - N\,\theta\right\}\right] \qquad (2.1)$$

The constants k, R_0, B_0, f, N, and z have the following meanings: k is the average field index, R_0 is the reference radius on which the magnetic field is B_0. N is the azimuthal periodicity around the ring, f is the field flutter

$(B_{max}-B_{min})/B_{max}$. The spiral angle, ζ, is measured from the radius vector to the locus of maximum magnetic field, "hill ridge." The value of B_z is shown as a function of R and θ in Fig. 2, which resembles a magic carpet in a strong wind. The average radius for a beam of energy E is found from the azimuthally independent field equation,

$$B_z(R) = B_0\left[\frac{R}{R_0}\right]^k \quad . \tag{2.2}$$

The average radius for a particle with momentum $p = eB\rho$ is given by

$$R = R_0\left[\frac{B\rho}{B_0R_0}\right]^{\frac{1}{k+1}} \tag{2.3}$$

The radial and vertical betatron oscillations are given approximately by

$$\nu_x^2 = 1 + k \quad , \tag{2.4}$$

$$\nu_y^2 = -k + (f \tan\zeta)^2 \quad . \tag{2.5}$$

The transverse oscillations with terms given by Eqs. (2.4) and (2.5) are oscillations about the forced orbit which is not a circle but an orbit that itself oscillates with one period every period of the ring.

In the following we shall employ the phase advance per cell for a single particle. That is, the phase advance in the absence of all coherent self-forces from the beam's space charge. We have

$$\sigma_{0x} = 2\pi\nu_x/N \ , \qquad \sigma_{0y} = 2\pi\nu_y/N \ . \tag{2.6}$$

III. Selecting the Ring Parameters

Each 180 degree bend must be matched on entrance and exit to the straight linear accelerator sections by means of four matching sections shown in Fig. 1. These sections have been designed in some detail, and consist of several strong focusing bending magnets. To lowest order these sections consist of a bend left and a bend right of equal bending angle. From consideration of these matching sections, and somewhat arbitrarily choosing a bending field of 4 kG at the inner (low energy) orbit and 14 kG at the outer (high energy) orbit, we choose the difference in these two radii to be about one or two meters. These constraints and use of Eq. (2.3) give the values of the field index k for the three rings shown in Table 1.

Table 1. FFAG Ring Parameters

Parameter	LER	MER	HER	Unit
Field Index k	20	30	82.5	
Tune σ_0	80	80	80	degrees
ν_x	4.58	5.568	9.138	
ν_y	3	4	8	
Periodicity N	20	25	42	
Spiral Angle ζ	87.34	87.889	88.817	degrees
$\tan\zeta$	21.54	27.13	48.415	
$2\pi R/N$	4.7	12	22	meters
$1/W = N\tan\zeta$	430	678	2034	
f/WN^2	0.269	0.2712	0.2883	
k/N^2	0.0500	0.0480	0.04677	

The flutter f is chosen to be 0.25 in all three rings.

Having chosen k for reasonable radial width, we now choose ζ and N. These choices must yield tunes well within the tune diagram shown in Fig. 3. We choose ζ to achieve $\nu_y \approx \nu_x - 1$ and N so that σ_{0x} is about 80 degrees. The expressions for ν_x and ν_y given in Eqs. (2.4) and (2.5) are valid provided that the two conditions

$$N^2 >> k + 1, \qquad f/WN^2 < 1,$$

are satisfied with $1/W = N \tan \zeta$. The coordinates in Fig. 3 are f/WN^2 and k/N^2. The values of σ_{0x} and σ_{0y} for the three rings are shown in the figure. The space charge will move the operating point to the left and down in the figure, but it must remain within the stability region. The complete parameter lists for the three rings is given in Table 1.

IV. Space Charge Limited Current

We employ only the beam's coherent electric self-field in this calculation, ignoring the coherent magnetic self-field. Following the usual cavalier treatment of "space charge," we consider the beam's charge to be uniform within the beam radius "a." Define the coordinate $x \equiv r - r_0$, where r_0 is the radius of the forced orbit. We have

$$E_x = \frac{Ix}{2\pi\varepsilon_0 a^2 v} \quad , \tag{4.1}$$

in which I is the instantaneous beam current. In the absence of space charge the single particle equation of motion is

$$\frac{d^2x}{d\theta^2} + (1 + k)\,x = 0 \quad . \tag{4.2}$$

The effect of space charge is to replace $1 + k$ in Eq. (4.2) by $1 + k - \delta k$. Using Eq. (4.1) we find the expression for δk. We introduce I_0 by the definition

$$I_0 = 4\pi\varepsilon_0 mc^3/e \quad . \tag{4.3}$$

For singly charged Bismuth 209, $I_0 = 6.5 \times 10^9$ A. In terms of this current, we have

$$\delta k = \frac{2I}{I_0\beta^3}\left(\frac{R}{a}\right)^2 \quad . \tag{4.4}$$

From the phase diagram in Fig. 3 we see that the maximum value of δk is bout .05 N^2. We designate the value of I necessary to obtain this value of δk as I_ℓ. We have

$$I_\ell = 1.625 \times 10^8 \beta^3 (a N/R)^2 \quad . \tag{4.5}$$

We really don't have much idea what the beam radius "a" will be in these devices, so we leave it in explicitly in the following numerical examples. Without space charge "a" can be calculated from the beam emittance and the ring parameters, but we are considering space charge effects to be large enough to invalidate that calculation. Noting that the values of $2\pi R/N$ are given for the rings in Table 1, we write Eq. (4.5) as

$$I_\ell/a^2 = 1.625 \times 10^8 \beta^3 \left(\frac{2\pi}{2\pi R/N}\right)^2 \quad . \tag{4.6}$$

We calculate I_ℓ/a^2 at injection and extraction for each of the 3 rings. The values are given in Table 2.

As mentioned above, the total charge in the beam $Q = 1.25 \times 10^{-4}$ Coulombs. The actual instantaneous current desired is Q/τ, where τ is the pulse duration of the beam. From pulsed power considerations we have some idea what values of τ are desired. These are given in Table 2 as well as values of Q/τ. The quantity a_{crit} in Table 2 is the value of "a" necessary to have I_{ℓ} equal to Q/τ. We see that a_{crit} is about 6 cm at extraction from the low energy and high energy rings. At extraction from the medium energy ring and at injection into all 3 rings, the value is 10 to 20 cm, which is larger than the pipe radius (7 or 8 cm) being contemplated for separate function recirculator rings.

This approximate calculation should yield values of the limiting current about a factor of 2 greater than the desired current, Q/τ, to justify further study of an FFAG recirculator.

Table 2. Space Charge Limited Current

	LER		MER		HER	
Parameter	INJ	EXT	INJ	EXT	INJ	EXT
β	0.01	0.032	0.032	0.1	0.1	0.31
$I_{\ell}/a^2 (kA/m^2)$	0.29	9.5	1.46	44.6	12.8	381
τ (μs)	40	4	4	0.33	0.33	0.1
Q/τ (A)	3.12	31.2	31.2	375	375	1,250
a_{crit} (cm)	10.4	5.7	14.6	9.2	17.1	5.73

References

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3. "Selected Works of L. Jackson Laslett" PUB-616, Lawrence Berkeley Laboratory, Berkeley, CA (1987).

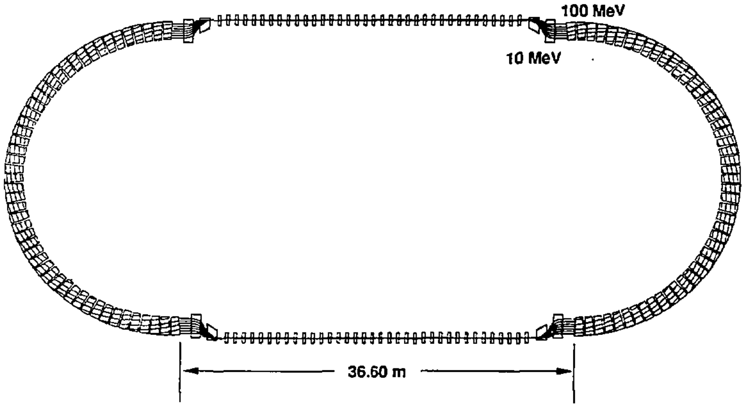

Figure 1

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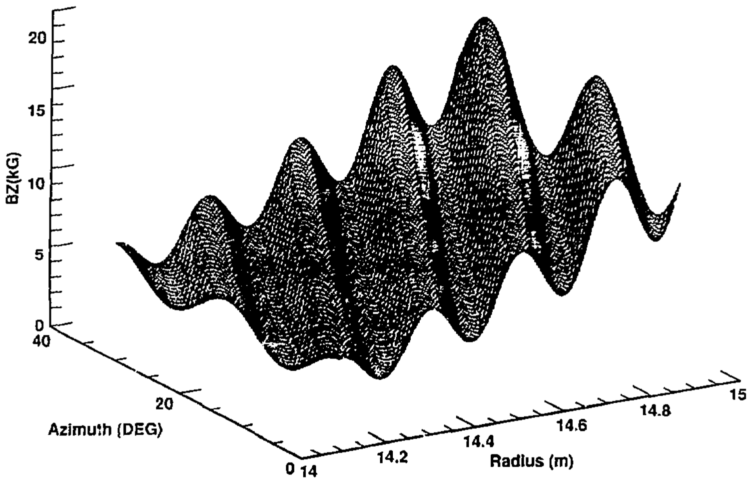

Figure 2

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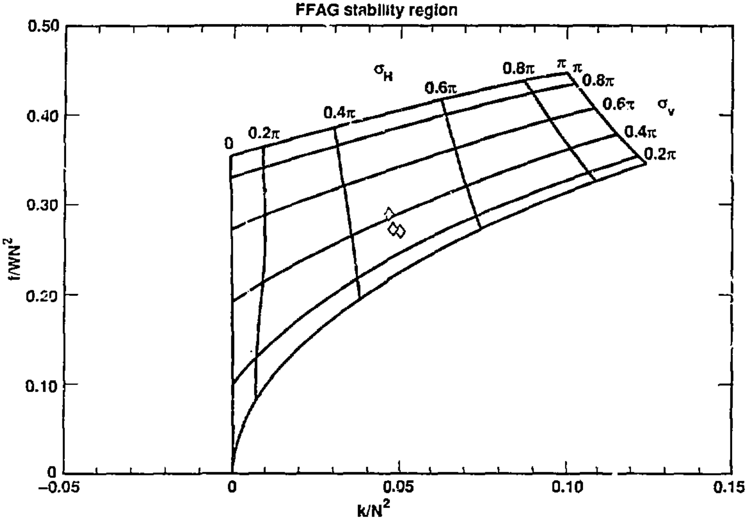
FFAG stability region
0.50
0.40
0.30
0.20
0.10
0
-0.05
0
0.05
0.10
0.15
k/N^2
f/WN^2
σ_H
0
0.2π
0.4π
0.6π
0.8π
π
π
0.8π
0.6π
σ_V
0.4π
0.2π

Figure 3