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NONLINEAR MAGNETO-  
HYDRODYNAMICS

Progress Report

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## PROGRESS REPORT

The proposal is structured as follows. The activity of the last two years is summarized in Section I. Abstracts of papers recently published and/or submitted are collected in Section II.

Activity during 1988-89

Our recent activity has been focussed on the following three subjects: (i) turbulent relaxation processes in driven, dissipative MHD fluids in a dc magnetic field and the relaxed steady states that result; (ii) variational principles, particularly those of the minimum-dissipation-rate variety, as an analytical predictor of the properties of the relaxed states; and (iii) the inclusion of variable mass density and compression in the evolution of MHD turbulence. These areas represent for us sensible abstractions of the problems involving the principal uncertainties for tokamaks, reversed-field pinches, and other current-carrying confinement devices, as well as being of great importance for solar-terrestrial MHD processes. We now outline briefly each of these areas.

(i) Turbulent relaxation processes

We have accumulated [1,2] extensive numerical evidence\* about the driven MHD problem of a periodic cylinder with rigid, conducting, poloidal boundaries. Computations have been done, over a period of five years, with both the full set of 3D MHD equations

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\*Bracketed numbers, here and elsewhere in this proposal, refer to the numbered abstracts in Section II. [Extensive bibliographies are part of each paper. Copies of the papers have been or are being transmitted to USDOE.]

and the Strauss approximation of "reduced" MHD. Highly non-equilibrium initial conditions, unrestricted by computationally-convenient symmetries, led to a turbulent phase sometimes close to a hundred Alfvén transit times long. At the end of this turbulent phase (which, by the way, had many features that are not shared by the homogeneous, isotropic MHD turbulence that had been investigated for several years previously) a relaxed state occurred. A summary and details of the features of the relaxed state, which differed only quantitatively, but not qualitatively, between the tokamak and RFP regimes have been presented in several places [1-8] and need not be reproduced here, except to remark on the two non-axisymmetric features that are the most striking: helical vortex pairs which in a poloidal plane show up as a double convection cell, and a helical contribution to the magnetic field and current channel. Both contributions are energetically only a very few per cent, compared to the axisymmetric parts of the magnetic field and current channels; but they are enough to alter the field line structure and to provide a candidate for an effective convective transport mechanism. Strikingly, they seem to persist [2] even in the presence of local temperature variation and sawteeth. That is, it is a helical, rather than an axisymmetric, magnetic channel that does the pulsating in Strauss-approximation sawteeth.

(ii) Variational Principles

Qualified but real success has been [5-8] recorded in calculating analytically the helically-distorted profiles observed computationally and discussed in the previous paragraph. Because

of the fact that the system is both electrically driven and resistive as well as viscous, all global ideal invariants are being supplied and dissipated at the same rate, in the steady state. This puts the problem outside the selective-decay [3] and dynamic-alignment [9] framework that have previously been the principal candidate for a dynamical basis of Taylor's "minimum-energy" state (e.g., *Revs. Mod. Phys.* 58, 741 (1986)), and the development of "Alfvénic" periods in the solar wind.

As a principle better adapted to driven, dissipative, steady-state systems, the principle of minimum rate of energy dissipation has proved to be quite useful [3, 5-8]. The principle has a good, if old, pedigree. It can be proved that the electric current density inside a non-uniform conductor will distribute itself in such a way as to minimize the ohmic dissipation, if the electrostatic potential is prescribed over the boundary. In hydrodynamics, the principle was never proved in full generality, but was shown (see, e.g., Lamb's Hydrodynamics, 6th edition) by Helmholtz, Korteweg and Lord Rayleigh to imply the elementary bounded hydrodynamic flows. What we have done [5, 6] has been to generalize the principle to MHD and explore the consequences of minimizing ohmic plus viscous dissipation for the boundary conditions and constraints that apply to uniform cylinders of electrically-driven, dissipative magnetofluid in a dc magnetic field with periodically identified ends. One of the surprises of the exploration has been the discovery [6] that finite velocity  $\vec{v}$  and helical deformation could lower the dissipation rate below the  $\vec{v} = 0$  axisymmetric minimum. It is also the case that the states

obtained of this kind can be explicitly represented, at least as a perturbation series, and look more than approximately like the states computationally obtained [6].

(iii) Density Fluctuations

Most of the MHD turbulence calculations that have been done have started from the assumption of incompressibility ( $\nabla \cdot \mathbf{y} = 0$ ), not so much because the necessary conditions for incompressible flow are so well satisfied in all cases as because incompressible fluid turbulence is much better understood than is compressible fluid turbulence, and most of what has been done in basic MHD turbulence has been modeled on what has been done for the Navier-Stokes case. We are continuing to explore [10,11] compressible MHD turbulence, not yet at the level of realistic boundary conditions but rather in 2D periodic boundary conditions. This is where we were with incompressible MHD turbulence twelve years ago. If the P.I. lives long enough, the intent is to give proper compressible treatments, numerical and analytical, parallel to those in [1-9]. At present, we are focussing on questions of density fluctuation spectra for homogeneous turbulence [10] and modifications to be expected for the dynamo effect introduced by compressibility and the presence of a non-zero mean dc magnetic field [11].

II. SUMMARY OF RECENT ACTIVITY,  
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The following abstracts represent work partially supported by the U.S. Department of Energy during 1988-89.

[1] Driven, Steady - State RFP Computations

J.P. Dahlburg, D. Montgomery, C.D. Doolen and L. Turner, J. of Plasma Physics, 40, 39-68 (1988).

A previously described, pseudo-spectral, three-dimensional MHD code is used to compute the dynamical behavior of a channel of magnetofluid carrying an axial current and magnetic flux. This situation contains the essential MHD behavior of the reversed-field pinch ("RFP"). An externally-imposed electric field is applied to an initially current-free magnetofluid, and drives currents which rise and eventually fluctuate about values corresponding to pinch ratios  $\Theta \sim 1.3, 2.2, \text{ and } 4.5$ . A period of violent turbulence leads to an approximately force-free core, surrounded by an active MHD boundary layer which is not force-free. A steady state is reached which can apparently be sustained indefinitely ( $\geq$  several hundred Alfvén transit times). The turbulence level and time variability in the steady state increases with increasing  $\Theta$ . The average toroidal magnetic field at the wall reverses for the  $\Theta \sim 2.2$  and  $4.5$  runs, but not for the  $\Theta \sim 1.3$  one. Negative toroidal current filaments are observed. The Lundquist numbers are of the order of a few hundred.

[2] Sawtooth oscillations about helical current channels

M.L. Theobald, D. Montgomery, G.D. Doolen, and J.P. Dahlburg, Physics of Fluids, B1, 766-773 (1989).

An existing pseudo-spectral code for solving the three-dimensional equations of "reduced" magnetohydrodynamics is extended by adding a temperature equation. Resistivities and thermal conductivities are given their (isotropic) Braginskii temperature dependences, and are advanced self-consistently. Realistic-looking sawtooth oscillations are observed at modest Lundquist and Reynolds numbers. However, the oscillations are excited upon, and relax back to, a helical (rather than an axisymmetric) current channel.

[3] MHD Turbulence: Relaxation Processes and Variational Principles

D. Montgomery and L. Phillips, Physica D (in press, 1989).

Turbulent relaxation processes seem to play a more prominent role in magnetohydrodynamics (MHD) than in hydrodynamics and exhibit a wider variety of behavior. In decaying turbulence, "relaxed" states can result from highly unequal decay rates of extensive, cascadable ideal invariants such as energy, magnetic helicity, cross helicity, etc. In externally-driven MHD systems, the relaxed states may result from other processes, less well understood. We are exploring a formulation based on a principle of the minimum rate of energy dissipation. This is a nineteenth

century principle from which seems to descend the more modern (and less well accepted) principle of "the minimum rate of entropy production." Consequences of the conjectured principle are described for two cases: (1) for the reversed-field pinch (RFP), an externally-applied electric field supplies magnetic helicity at a constant rate; (2) for the case of a constant rate of dissipation of cross-helicity, a relaxed state with aligned vorticity and current densities is predicted. The former problem models the steady-state operation of a familiar fusion confinement device. The latter models certain features of solar wind turbulence, regarded as a driven, steady-state system.

[4] Turbulent MHD Relaxation: Driven Steady States. D. Montgomery, in Turbulence and Nonlinear Dynamics in MHD Flows, ed. by M. Menguzzi, A. Pouquet, and P.L. Sulem, (Elsevier/North-Holland, Amsterdam, 1989) pp. 165-170. (Proceedings of a 1988 Cargese Summer School.)

It is suggested that the appropriate direction for magneto-hydrodynamic turbulence theory to move is away from the concerns of classical homogeneous turbulence theory. As an example of an application in the direction of increasing realism, a recent computational and analytical approach to turbulent relaxation in the reversed-field pinch is reviewed. The analytical calculation is based upon a minimum-energy-dissipation-rate variational principle, appropriate to driven systems. An application involving constant rate of supply and dissipation of cross helicity is also remarked upon.

[5] Minimum Dissipation Rates in Magnetohydrodynamics D. Montgomery and L. Phillips, Physical Review A, 38, 2953-2964 (1988).

Minimum dissipation rate states are explored for a current-carrying channel of magnetofluid, supported by a dc magnetic field and driven by an applied electric field. The minimization is carried out subject to the constraints of constant axial (toroidal) magnetic flux and constant time-averaged rate of supply of magnetic helicity. The solutions of the resulting Euler-Lagrange equations are sensitive to boundary conditions on the current density,  $\mathbf{j}$ . One set of boundary conditions on  $\mathbf{j}$  leads to the same consequences as Taylor's "minimum energy" theory. A different set leads to significantly different consequences, including departure from the "force-free" magnetic profile and a toroidal component of current density that does not reverse at the wall when the toroidal magnetic field reverses.

[6] Helical, Dissipative, Magnetohydrodynamic States with Flow D. Montgomery, L. Phillips and M.L. Theobald, accepted by Physical Review A (1989, to appear in August issue).

It is shown that for an axially periodic column of magnetofluid driven by an applied axial electric field, the total rate of energy dissipation (ohmic plus viscous) can be lowered by permitting a helical component with vortical flow in the solution. The principle of minimum energy dissipation rate suggests that this partially helical state will be preferred to the axisymmetric one

that exists for the same parameters. The result is consistent with the repeated appearance of such partially-helical states in several full three-dimensional numerical computations, and is not inconsistent with the data from some confinement experiments.

[7] Relaxed States in Driven, Dissipative Magneto-Hydrodynamics: Helical Distortions and Vortex Pairs

D. Montgomery. To appear in Trends in Theoretical Physics, Vol. 1, P.J. Ellis and Y.C. Tang, eds. (New York, Addison-Wesley, 1989). (Colloquium given at the University of Minnesota, May 18, 1989)

For more than thirty years discussion of the confined states of a magnetized, current-carrying plasma column has been dominated by a paradigm of an ideal axisymmetric MHD equilibrium with no steady-state flow velocity. Anything else that has happened has been interpreted as a consequence of "instabilities," regarded as perturbations of such equilibria. We question this way of viewing the problem and suggest that it has only been possible to sustain it this long because of the absence of good internal diagnostics for the relevant MHD variables. Electrically-driven, magnetically-supported, cylindrical magnetofluids seem naturally to relax into states which contain a pair of low-mode-number, counterrotating, helical vortices. The current channel and magnetic topology are the result of an axisymmetric current distribution plus a smaller helical component. The evidence for the configuration is numerical and analytical. The analytical argument is based on the assumption that the relaxed states of driven, dissipative MHD are those of the minimum rate of energy dissipation.

[8] Minimum Dissipation States and Vortical Flow in MHD

D. Montgomery, J.P. Dahlburg, L. Phillips, and M.L. Theobald. Submitted to IUTAM Symposium on Topological Fluid Mechanics, Cambridge (U.K.) August 13-18, 1989. (Lectures [7] and [8] discuss essentially the same material, but differ considerably in details.)

For more than thirty years, discussion of the confined states of a magnetized, current-carrying plasma column has been dominated by a paradigm of an ideal axisymmetric MHD equilibrium with no steady-state flow velocity. Anything else that has happened has been interpreted as a consequence of "instabilities," regarded as perturbations of such equilibria. We question this way of viewing the problem, and suggest that it has only been possible to sustain it this long because of the absence of good internal diagnostics for the relevant magnetohydrodynamic variables. Electrically driven, magnetically-supported, cylindrical magnetofluids seem naturally to relax into states which contain a pair of low-mode number, counterrotating, helical vortices. The current channel and magnetic topology are the result of an axisymmetric current distribution plus a small helical component. The evidence for the configuration is numerical and analytical. The poloidal flow is such as to connect the hot geometrical center of the plasma with the cooler perimeter. The flow thus becomes a candidate for a dominant heat loss mechanism. Ideal and slightly non-ideal configurations may be as different in MHD as they are in hydrodynamics, and the former may play an equally minor role in practice.

[9] The Evolution of Cross Helicity in Driven/Dissipative Two-Dimensional Magnetohydrodynamics

S. Ghosh, W.H. Matthaeus, and D. Montgomery, Physics of Fluids, 31, 2171-2184 (1988).

A numerical study of the evolution of cross helicity in driven/dissipative MHD is presented. The magnetofluid is incompressible, and a 2D periodic geometry is considered. Cross helicity, a measure of the correlation between fluctuations in the magnetic field and the velocity field, is injected by use of correlated Gaussian forcing over a finite bandwidth in wavenumber. Numerical experiments include driving and initially uncorrelated spectra with high correlated forcing and driving of correlated spectra with anti-correlated forcing. A recurring and persistent feature of the simulations is the appearance of oppositely signed cross helicity at small scales relative to large scales. A simple argument based on the Elsässer-variables and previously used in the context of decaying turbulence explains many of the observed features. The effect of a uniform external magnetic field is considered, and the relation to purely decaying 2D MHD turbulence is discussed.

[10] Turbulent MHD Density Fluctuations

J.V. Shebalin and D. Montgomery, J. of Plasma Physics, 39, 339-367 (1988).

A spectral-method numerical code is used to compute mass-density fluctuation spectra in turbulent magnetofluids. The computations are used to test and extend a recent analytical theory of density variations in slightly compressible magnetofluids. The computations are used to test and extend a recent analytical theory of density variations in slightly-compressible magnetofluids given by Montgomery, Brown, and Matthaeus, and used by them to infer inertial-range density-fluctuation spectra for the nearby interstellar medium and solar wind. A local equation of state is assumed, relating density to pressure. Constant, scalar resistivities and viscosities are used. In the limit of low Mach numbers and high mechanical-to-magnetic pressure ratios, the fit of the computations to the analytical theory is seen to be close.

[11] Magnetic Field Amplification in Mechanically Driven Compressible Magnetohydrodynamic Turbulence (revised)

J.V. Shebalin and D. Montgomery (submitted to AIAA Journal, 1989)

Magnetic field amplification in a homogeneous, dissipative, isentropic polytropic, two-dimensional, turbulent magneto-fluid is simulated numerically. The magneto-fluid is in a number of cases, mechanically forced so that energy input balances dissipation, thereby maintaining a steady state. In the presence of a spatially uniform (i.e., mean) magnetic field, a magneto-fluid whose initial turbulent magnetic energy is zero quickly arrives at a state of non-zero turbulent magnetic energy. If the mean magnetic field energy density is small, the turbulent magnetic field can achieve a local energy density more than four hundred times larger than the energy density associated with the mean magnetic field; if the mean magnetic field energy density is large, then equipartition between

the turbulent magnetic and kinetic energy is achieved at the smaller scales. Compared to the presence of a mean magnetic field, compressibility appears to have only a marginal effect in mediating the transfer of turbulent kinetic energy into magnetic energy.

[12] Introduction to the Theory of Fluid and Magnetofluid Turbulence  
D. Montgomery, article in Nagoya Lectures in Plasma Physics and Controlled Fusion, ed. by Y.H. Ichikawa and T. Kamimura, (pp. 207-301) Tokai University Press, Tokyo, Japan, (1989).

[No abstract. These lectures have previously been transmitted to DOE as Nagoya Report IPPJ-670 (1984), and represent a series of lectures given while the P.I. was visiting professor at the University of Nagoya. They are only now being published.]