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NUMERICAL AND PHYSICAL MODELLING OF BUBBLY FLOW PHENOMENA

Progress Report

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by

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1 Introduction

The objective of the proposed research is to develop a theoretical framework for analyzing various two-phase flows, with special emphasis on the flows of gas-liquid dispersions. The macroscopic behavior of these flows depends on the details of the microstructure of the dispersion, and these details, in turn, depend on the nature of the flow. Given the very diverse nature of the flows and their complex dependence on the microstructure of the dispersion, it is unlikely that a single set of equations, similar to the Navier-Stokes equations for homogeneous fluids, will apply to all the different situations. What is possible, however, is to develop general methodologies that can be used to examine specific situations and a general understanding about different kinds of macroscopic flows. The aim of the proposed research is to develop efficient numerical techniques for carrying out dynamic simulations of flows of dispersions and to apply them to a carefully selected problems whose solutions would reveal important qualitative as well as quantitative insights into the complex interdependence of the microstructure and macroscopic properties of the flows. These numerical techniques are to be supplemented with the techniques of ensemble averaging and statistical physics to obtain results that could be used in modelling more complicated flows through a set of relatively simple equations.

2 Work accomplished during July 1990-Jan. 1992

Two classes of macroscopic flows were analyzed in detail during the current funding period. The first is the oscillatory flows, as in the case of acoustic or pressure wave propagation through bubbly liquids, and the second is convective flows as in bubbles rising through a liquid. Two articles based on the study of oscillatory flows have appeared in print (Sangani 1991 and Sangani, Zhang, and Prosperetti 1991). One manuscript based on a convective flow study has been submitted for publication, and the other is in preparation. In addition, a study on simulations of oscillatory flows is nearly completed, and a manuscipt based on this will be prepared over the next few weeks. The PI and his collaborator, Professor Prosperetti at The Johns Hopkins University, has also contributed a chapter on the simulations of bubbly flows in an upcoming book edited by Dr. Roco at the National Science Foundation (Prosperetti and Sangani 1992). A summary of the important findings is given below.

2.1 Oscillatory flows

The primary motivation for studying these flows was its simplicity. Since the bubbles are simply executing a simple harmonic motion around their mean positions, the determination of the microstructure is rather trivial. In the limit of small-amplitude motions, the only aspect of the microstructure that remains to be determined is the shape of the bubbles, and, consequently, the Monte-Carlo simulations are adequate for such flows. The problem is also of considerable practical significance because a number of non-invasive techniques employ acoustic probes to gain insight into the flows of suspensions.

2.1.1 Pairwise interactions in dilute bubbly liquids

The first study (Sangani 1991) was a theoretical analysis of the acoustic wave propagation through dilute bubbly liquids, i.e., dispersions in which β, the volume fraction of gas bubbles, is small. The initial motivation for examining this problem was simply the need to familiarize with the ensemble averaging techniques for deriving equations that govern the behavior of the dispersion at the macroscale. It turned out, however, that in the process we discovered a number of important effects that arise due to interactions among bubbles. The earlier theories, correct to $O(\beta)$, examined only the interaction of a single bubble with the incident planar wave. We found that the correction to this is $O(\beta^{3/2})$ and this causes the acoustic damping of waves. Thus, the waves are damped at finite volume fractions even in the absence of nonadiabatic thermal effects, finite compressibility or the finite viscosity of the liquid. This effect alone turned out to be quite significant in explaining the discrepancy between the predicted values of attenuation based on the $O(\beta)$ theories and its measured values for frequencies comparable to or smaller than the resonance frequency of bubbles. For example, the attenuation of sound at $\beta = 0.01$ near the resonance frequency of bubbles of radii 0.27 cm. predicted from the earlier theory was about a factor of 5 lower than its measured value, whereas the new theory gives values that are in excellent agreement with the experiments. The next corrections, of

$O(\beta^2 \log \beta)$ and $O(\beta^2)$, were also determined. In particular, we discovered a number of secondary resonances that arise due to the interaction among pairs of bubbles. A detailed account of various physical effects that become important may be found in Section 1.1 of Sangani (1991).

2.1.2 Added mass and viscous effects in non-dilute bubbly liquids

In Sangani, Zhang, and Prosperetti (1991), we presented the results of numerical simulation of oscillatory flows for bubbly liquids obtained by solving

$$\nabla \cdot \hat{\mathbf{u}} = 0, \qquad i\omega\rho\hat{\mathbf{u}} = -\nabla \hat{P} + \mu\nabla^2\hat{\mathbf{u}}. \tag{1}$$

Here, we have assumed that the velocity and pressure are proportional to $e^{i\omega t}$, t being the time. $\hat{\mathbf{u}}$ and $\hat{P}$ are the amplitudes of velocity and pressure variations, ω is the frequency, and ρ and μ are, respectively, the density and viscosity of the liquid. An efficient numerical technique was developed for solving the above equations by rigorously accounting for the interactions among all the bubbles in the limit of small viscosity and frequency. In this limit, the radial oscillations of the bubbles are negligible and the effect of viscosity of the liquid is essentially confined to thin Stokes layer near the surface of each bubble. The leading order effect in the limit of vanishingly small viscosity is the added mass effect and the correction to this is the Basset force effect for bubbles contaminated with surface-active impurities and a simple viscous force effect for bubbles free of impurities. Thus, these calculations provide, in addition to the estimates of speed of sound and its attenuation in the low frequency limit, the coefficients of added mass, Basset force, and viscous drag. This turns out to be one of the simplest formulation one can consider for determining these coefficients. In particular, it clarified some of the issues regarding what the proper definition of the added mass coefficient is, and how to recocile the two different results for that coefficient that were derived earlier by other investigators (van Wijngaarden 1976, Biesheuvel and Spoelstra 1989) in the limit of small β. We carried out numerical analysis for both spatially periodic and random dispersions and for different velocity distributions of the bubbles (or particles) by varying their density from 0 to ∞. Interestingly, we found that the added mass and viscous drag coeffcients were both relatively insensitive to the details of the spatial and velocity distributions of the bubbles. Thus, we expect that these results may also be useful in other flows provided that the spatial distribution of the bubbles is nearly uniform. We also presented simple formulas for estimating these coefficients.

2.1.3 Attenuation of sound waves at large frequencies

As mentioned in Sec. 2.1.1, the agreement between the $O(\beta^2)$ theory of Sangani (1991) and the experimental data on the attenuation of sound waves as reported by Silberman (1957) is excellent at frequencies comparable to and below the resonance frequency of the bubbles. At higher frequencies, however, there is a considerable discrepancy between the theory and the experiments. For example, the attenuation predicted by the theory is about 50 to 100 percent higher than the experimental values at frequencies two to five times the resonance frequency, even for β as small as 0.01. This difference was somewhat puzzling, and therefore we undertook a separate study to evaluate attenuation via numerical simulations without invoking the assumptions regarding the magnitude of the volume fraction, the ratio of radius of the bubbles to the effective wavelength, or the compressibility of the liquid phase. The results of such analysis serve as an independent check on the theory and allow us to account for other effects, such as that of size and spatial distributions of the bubbles or the nonadiabatic thermal effects. Moreover, it was suggested in Sangani (1991) that the out-of-phase mode resonance effects between the pair of bubbles could be responsible for this large discrepancy. This mode occurs only at frequencies greater than the resonance and were not included in the theory as it contributes to the attenuation only at $O(\beta^{5/2})$, and the resulting analysis for determining it analytically was quite involved.

We developed a scheme for rigorously solving the multiple scattering problem via direct numerical simulations. In this, we combine the use of planar periodic singular solutions, developed earlier by Sangani and Behl (1989) for solving nonlocal problems in which the averaged quantities vary over the lengthscale comparable to the size of the dispersed phase, with a scheme of multiple scattering calculations developed earlier by Rayleigh (1898), Ewald (1916), and Twersky (1962). This leads to an eigenvalue problem for determining the effective wavenumber Γ and, hence, the attenuation of the acoustic waves. We found that this formulation, which directly determines Γ in the bulk of the dispersion, actually has multiple solutions, especially at frequencies above the resonance frequency, and this poses some numerical difficulties in detecting the smallest eignevalue, which is the one that is likely to be observed in experiments. We may note here that, at finite β, the pressure variations in the bubbles are approximately 180 degree out-of-phase with the pressure variations in the mixture for frequencies greater than the resonance frequency of the bubbles, and thus the bubbles behave as if they have a negative compressibility. As a consequence, the bubbly liquid behaves as an acoustically opaque medium with the intensity of sound simply decreasing exponentially with the distance into the bubbly medium. Thus, it is not aphysical to have multiple eigenvalues for this problem at higher frequencies. Because of the numerical difficulties involved in determining the smallest eigenvalue of the resulting equations, we subsequently used a different formulation in which we considered propagation of sound waves through a finite layer of bubbly liquids. The solution of this problem is given by

$$\hat{\varphi} = e^{-ikx_1} + \sum_{n=0}^{\infty} \sum_{m=0}^{n} \sum_{\alpha=1}^{N} A_{nm}^{\alpha} Y_{nm}(\partial x_1, \partial x_2, \partial x_3) \left\{ H(\mathbf{x} - \mathbf{x}^{\alpha}) + \frac{2\pi}{ik\tau} e^{-ik|x_1 - x_1^{\alpha}|} \right\} . \tag{2}$$

Here we have assumed that there is a periodicity in arrangement of the bubbles in the $x_2 - x_3$ plane, τ being the area of the unit cell. The incident wave is in the x_1 direction with the corresponding wavenumber in pure liquid equal to k, and $\hat{\varphi}$ is the amplitude of the velocity potential. $\mathbf{x}^{\alpha}$ is the center of a representative bubble α and H is the planar periodic singular solution of the Helmholtz equation. Y_{nm} is the differential operator related to the solid spherical harmonics, i.e., $Y_{nm}(x_1, x_2, x_3)$ is the solid harmonic of degree n and order m. There are $2n + 1$ independent harmonics for each n. N is the total number of bubbles used in the simulation and A_{nm}^{α} is the strength of a 2^n-multipole associated with the bubble α. It turns out that at larger frequencies and for $\beta = 0.01$, the above solution can be truncated to just monopoles, i.e., terms with $n = 0$ in (2), with a reasonable degree of accuracy. This allowed us to carry out simulations with as many as 100 bubbles without much difficulty. Once the monopoles are determined for the given frequency and size and spatial distributions of the bubbles, we plot the logarithm of the magnitude of the monopole versus its x_1 coordinate and determine the attenuation from the slope of the line drawn by best fitting the results with a straight line after discarding the results for the bubbles that were near the edge of the bubbly region. The scatter of the magnitude of the monopole from the straight line was quite small in all cases we examined, and hence, this method allowed us to estimate the attenuation to a high degree of accuracy.

Our numerical results are in very good agreement with the dilute theory of Sangani (1991) and, thus, substantially higher than the experimetal values. We also carried out calculations in which the bubbles' size distribution was 50 percent polydispersed and found that this actually results in even higher values of attenuation. Similarly, we carried out simulations in which the spatial distribution was not uniform, and this too resulted in higher values. Thus, none of these factors can contribute to the discrepancy between the theory and experiments. We now believe that the measurement of attenuation at higher frequencies by Silberman (1957) may not be very accurate.

This work is nearly completed and a manuscript will be submitted for publication shortly.

2.2 Convective flows

2.2.1 Dynamic simulations

In Sangani and Didwania (1991), we have presented the results of dynamic simulations of bubbles rising through a liquid. This situation is representative of the flows in which the mean of the relative motion between the two phases is not zero. The Reynolds number of the flow, based on the radius and the terminal rise velocity of the bubbles, is assumed to be large compared to unity. As shown by Moore (1963), the effect of viscosity in large Reynolds number flows past a single bubble is essentially confined to a small region near the surface of the bubbles. The velocity field determined from the potential or the irrotational flow approximation is accurate to the leading order, i.e., $O(Re^0)$, everywhere in the liquid. The correction to this is at most of $O(Re^{-1/2})$ and confined to a thin boundary layer of nondimensional thickness $O(Re^{-1/2})$ near the surface of the bubble. Unlike the flow past rigid bluff objects, for which a boundary layer separation leads to an $O(1)$ deviation in the velocity from the potential flow over the region that in width is comparable to the size of the object, the width of the wake behind the bubble is small, of $O(Re^{-1/4})$, and the velocity correction in the wake is also small. It is possible that a bubble rising in the non-dilute dispersion, that we were interested in studying, may leave behind it a nonvanishing vorticity which, in turn, may affect the motion of the bubbles following it. Since it is quite difficult to determine this vorticity distribution and its effect on the motion of the bubbles, we decided to carry out simulations in which we ignore this effect, leaving the examination of this effect to a future work. Thus, the simulations we have carried out solves for the potential flow around many bubbles and then estimates the viscous forces on the bubbles using two different methods. The first is based on computing the gradient of the total viscous dissipation with respect to the velocity of the individual bubbles – a method used by Biesheuvel and van Wijngaarden (1982) and Kok (1989) who determined the viscous drag on pair of bubbles, and the other is based on exact analysis of the viscous effects in small amplitude oscillatory flows by Sangani (1991) and Sangani, Zhang, and Prosperetti (1991). Both methods yield identical values for the drag on individual bubbles in non-dilute dispersions. The second method, however, turned out be computationally far more efficient, and hence all the dynamic simulations were subsequently carried out using that method.

To keep the analysis simple, we assumed further that the Weber number, which is the ratio of inertial to surface tension forces, is small compared to unity so that the bubbles may be assumed to remain spherical. We further assumed that the bubbles do not coalesce. The conditions of large Reynolds number and small weber number are satisfied by bubbles approximately 1 mm. in diameter. Careful experimental observations of the dynamics of pairs of bubbles approximately 1 mm. in size are reported by Kok (1989). He reported that, while bubbles in pure water generally coalesce, an addition of a small amount of surface-active impurity prevented bubbles from coalescing. He further noted that, in the latter situation, the bubbles bounced away from each other rather instantaneously. He also compared the observed trajectories of the pairs of bubbles with the corresponding trajectories evaluated using the potential flow approximation and the viscous forces estimated from the dissipation method mentioned earlier and found an excellent agreement between the two. Finally, he also carried out experiments in which the concentration of the surface-active impurities was high, and found that a wake of finite size forms behind the bubbles and that the potential flow approximation broke down for such high concentrations. Thus, the calculations we carried out are likely to apply to systems in which a slight amount of surface-active impurities is present.

In accordance with the above experimental observations, we assume that whenever any two bubbles collide, they bounce back almost instantaneously. At large Reynolds numbers, the viscous effects are negligible and, therefore, we assume that the momentum and the kinetic energy of the entire dispersion is conserved during the collision. We show that the normal component of the relative motion of the colliding pair of bubbles reverses its direction upon collision, just as in the case of two elastic spheres colliding in vacuum; but unlike the latter case, we find that the velocity of all the bubbles in the dispersion also undergo a change in their velocities during the collision. Since the bubbles are essentially massless, they collide in such a manner so as to conserve the total momentum and kinetic energy of the liquid, and this is possible only if the other bubbles adjust their speeds so as to accomodate the changes in the liquid velocity.

We developed a numerical method for simulations of the motion of bubbles under the conditions described above. Considerable attention was devoted to making the computer code for solving the interactions among the bubbles as efficient as possible. With careful programming and the use of vectorization, we were able to reduce the computational time by two orders of magnitude over a period of one year. As a result, the CPU time on the supercomputer at the Cornell Theory Center (IBM 3090) for solving for the interaction among 30 bubbles placed randomly within a unit cell is now less than 2s. A typical dynamic simulation was carried out for 5 to 10 thousand time steps.

Our dynamic simulations showed that the random state of bubbly liquids under the aforementioned conditions is unstable and that the bubbles form large aggregates by arranging themselves in the plane transverse to the direction of the mean relative motion. These aggregates form even when the size distribution of the bubbles is nonuniform. The instability results primarily from the nature of inertial interaction among pairs of bubbles which cause them to attract toward each other when they aligned in the plane perpendicular to their velocities and to repel when aligned in the same direction as their velocities. Interestingly though, we find that the presence of viscous forces further facilitates in the formation of planar aggregates. In fact, if the initial velocity distribution of the bubbles is sufficiently nonuniform (50 percent variance), we find that there is no significant evidence of aggregate formation if the viscous forces are absent.

We found that the averaged properties of the dispersions are profoundly affected by the formation of these aggregates, particularly at smaller values of β. For example, the added mass coefficient for an initially uniform and random spatial distribution of the bubbles is approximately 1.3 for $\beta = 0.1$, in agreement with what we found in the study of oscillatory flows described in Sec. 2.1. After a non-dimensional time of about 100 units, when the aggregate formation was nearly complete, we found that the added mass coefficient had increased to about 4. Similarly, high values were also observed for the viscous drag coefficients.

2.2.2 Dispersed phase stress tensor in bubbly liquids

In this study we considered the problem of deriving a set of averaged equations for the flows of bubbly liquids. The novel feature of this work is the derivation of the expression for the stress tensor to be used in the force balance equation for the dispesed phase. This equation reads

$$\frac{\partial \mathbf{I}}{\partial t} + \mathbf{U}^G \cdot \nabla \mathbf{I} = -\frac{1}{\beta} \nabla \cdot \beta \tau + 12\pi \mu a C_d (\mathbf{U} - \mathbf{U}^G) - \frac{4\pi a^3}{3} \rho \mathbf{g}, \tag{3}$$

where a is the radius of the bubbles, C_d the drag coefficient, $\mathbf{g}$ the gravitational acceleration, $\mathbf{U}$ the ensemble-averaged velocity of the gas-liquid mixture, $\mathbf{U}^G$ the ensemble-averaged velocity of the gas phase, and $\mathbf{I}$ the average impulse defined by

$$\mathbf{I} = < -\rho \int \varphi \mathbf{n} dS >, \tag{4}$$

the integral being carried over the surface of a representative bubble in the dispersion and $\mathbf{n}$ the unit outward normal on such a surface. The angular brackets denote the operation of averaging. Finally, τ is the stress tensor.

The goal of the analysis is to determine the force acting on a representative bubble which is in the midst of a dispersion when there are spatial and temporal variations in the volume fraction and the velocities of the two phases. The term on the left-hand side of (3) is the usual added mass force and can be expressed in terms of the added mass coefficient C_a by means of

$$\mathbf{I} = \rho \frac{4\pi a^3}{3} \left[\frac{1}{2} C_a (\mathbf{U}^G - \mathbf{U}) - \mathbf{U} \right]. \tag{5}$$

The second and third terms on the right-hand side of (3) are the usual viscous and buoyancy forces. The existence of a term corresponding to the dispersed phase stress tensor, as represented by the first term on the right-hand side of (3) has been postulated for many years and is believed to play an important role in stabilizing the void fraction waves. A term similar to this has been used in the fludization literature for over thirty years, and recently there has been a number of studies which discuss the origin of such a force in dispersions. For example, in a recent study of void fraction waves in fluidized beds, Batchelor (1988) noted that such a term would arise from the transport of momentum by the random translational motion of the particles (or bubbles), the collision of the particles, and the fluid dynamic stresses. However, he did not specify the means of evaluating this stress tensor from a detailed study of particle interactions. For rapid granular flows, Jenkins and Richman (1985) were able to derive an expression for the dispersed phase stress tensor which included the translational and collisional contributions. For dilute bubbly liquids, van Wijngaarden and Kapteyn (1990) have obtained an expression for the stress tensor which includes only the translational contribution. Finally, Biesheuvel and Gorissen (1990) developed a kinetic theory for the bubbly liquids and obtained an expression for the stress tensor assuming that the interaction among the bubbles can be decomposed into a sum of pair potentials. They, however, did not specify exactly how such a pair decomposition can be accomplished from the details of the flow.

Determination of the exact expression for this stress tensor turned out to a quite difficult problem, and a significant portion of our efforts during the past year was spent in solving it. We have now derived the expression for the stress tensor in terms of multipoles associated with each bubble. These quantities can be evaluated directly from the dynamic simulations, and thus it is possible for the first time to determine the stress tensor directly for different flow situations. This work will be submitted for publication in the near future.

3 Future work

The oscillatory flows are now reasonably well understood and, hence, the research will focus mostly on the convective flows. The approach will be to combine the theoretical ideas from statistical mechnics and dynamics of granular media to dynamic simulations of flows of bubbly liquids. At the same time, attention will be given to improving further the efficiency of the computer code for simulations and to modifying it so that it can allow us to gain insights into the flows of bubbly liquids that are more complicated.

The results of our dynamic simulations described in Sec.2.2.1 showed that the random state of bubbly liquids is unstable and that large planar aggregates form as a result. An obvious question that arises is how realistic are the conditions and approximations that were made in these simulations? This is particularly important since such aggregates are not reported by any experimental investigations on dispersions. Kok (1989) *did* observe that pairs of bubbles align themselves in the plane perpendicular to the gravity, but that is the only study that we are aware of which reports this tendency. Matuszkiewicz, Flamand and Boure' (1987) have reported an extensive data on the void fraction waves. They found stable void fraction waves for β less than about 0.25, and unstable ones, or transition from a bubbly to a slug flow, for higher volume fractions. Howev··, they report that the flow was turbulent (superficial velocity of the flows were greater than 1 m/s). Most of the assumptions we have made appear quite reasonable, but there are two effects that warrent further study. The first is the effect of nonzero vorticity distribution, and the second is the effect of turbulence.

In non-dilute dispersions at finite Reynolds number, the flow may not be regarded as irrotational since the vorticity distribution may be significant even though the mean vorticity may be zero. One possibility for modelling this effect is to allow for random forces acting on each bubble during simulation. This is similar to Brownian dynamic simulations of collidal suspensions, except that it is unclear how to relate the magnitude of the random forces to the vorticity generated at the surface of the bubbles. At any rate, before doing any simulations of this kind, it appears that first understanding in detail some simple calculations regarding the vorticity distribution for flow past a single bubble is essential. Therefore, we are currently studying the rigorous analysis of Kang and Leal (1988) on the viscous effects at large Reynolds number flows past a single bubble. These investigators have obtained a formal solution for the vorticity and the viscous corrections to the velocity and pressure. They showed that although, the actual expression for the viscous correction to the pressure is very complicated and involves a detailed knowledge of the vorticity distribution for its determination, it is not necessary to evaluate this distribution if one is interested only in evaluating the drag on a single bubble. This is because of symmetry in the flow around a single bubble. In fact, the drag evaluated using the dissipation method described in Sec. 2.2 is exact to $O(Re^{-1})$ in this case. We have extended this solution to the case of many bubbles and found that it is now necessary to evaluate the vorticity distribution. Thus, it is not clear how, if at all, this method will reconcile with the dissipation method that we used in the simulation of the motion of many interacting bubbles. The formal solution of Kang and Leal (1988) involves decomposing the vorticity into poloidal fields. We have been able to integrate this vorticity equation to obtain a set of ordinary differential equations in time for the integrals of these poloidal decomposition over the boundary layer regions of each flow. At present, we are examining these equations for the special case of two bubbles to better understand the relation between this exact analysis of the viscous effects to the dissipation method. It is hoped that with, a better understanding of the nature of vorticity distribution, it may become possible to extend it to simulations involving many bubbles.

We shall also examine the effect of liquid turbulence. At this point, it seems that a rigorous approach is not possible. However, since fairly extensive data are available on the bubbly flows in this regime (see, for example, Matuszkiewicz, Flamand and Boure' 1987), it may be worthwhile to test some simple ideas through simulations. Thus, for example, we may carry out simulations in the absence of viscous and gravitational forces. For this situation, the kinetic energy and momentum are invariant throughout the simulations and depend on the initial velocity distribution of the bubbles. Corresponding to different velocity distributions, it is possible to estimate various average properties of the dispersion, and in particular, it is easy to evaluate the Reynolds stress in the liquid. Thus, one possibility is to relate various properties to the Reynolds stress and then to solve the averaged equations for the stability of the void fraction waves for which experimental data are available. The stability analysis for void fraction waves should be carried out by including the energy equation for the dispersed phase in addition to the usual continuity and momentum equations. Such an energy equation has been derived by us, and all the quantities that appear in it can be determined directly from dynamic simulations. We may also need to add an additional term in the energy equation corresponding to turbulent energy production so that at steady state, with no gradients in β or velocity, the rate of viscous energy dissipation must equal the turbulent energy production.

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4 List of publications

The publications based on the work described here are listed below.

In print

Sangani A. S. 1991 Pairwise interactions theory for determining the linear acoustic properties of dilute bubbly liquids. *J. Fluid Mech.* **232**, 221-284.

Sangani A. S., Zhang D. Z., and Prosperetti A. 1991 The added mass, Basset, and viscous drag coefficients in non-dilute bubbly liquids undergoing small-amplitude oscillatory motion. *Phys. Fluids A*, **3**, 2955-2970.

To appear

Prosperetti A. and Sangani A. S. (1992) Numerical simulation of the motion of particles at large Reynolds numbers. Chap. 28 in *Particulate Two-Phase Flow*, ed. M. C. Roco, Butterworths, Boston, MA.

Under review

Sangani A. S. and Didwania A. K. Dynamic simulations of flows of bubbly liquids at large Reynolds numbers. (submitted Dec. 1991) *J. Fluid Mech.*

In preparation

Sangani A. S. and Didwania A. K. Dispersed phase stress tensor in flows of bubbly liquids at large Reynolds numbers.

5 Presentations at National Meetings and Seminars

Seminars

Acoustic properties of bubbly liquids, SUNY at Buffalo, NY (1991).

Dynamics of Bubbly liquids, City College at the City University of New York, NY (1991).

Symposium

Dynamic simulations of flows of bubbly liquids, International Union of Theoretical and Applied Mechanics (IUTAM) symposium on fluidized beds, Stanford University, CA (1991).

Meetings

Sangani A. S. Acoustic properties of bubbly liquids, Ann. Meeting of Am. Inst. Chem. Eng., Chicago, IL (1990).

Zhang D. Z., Prosperetti A. and Sangani A. S. The added mass effects in bubbly liquids, Ann. Meeting of Am. Phys. Soc., Cornell University, NY (1990).

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