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Measurement Accuracy of Macroscopic Quantum Circuits with RF-Biased Josephson Junction Arrays

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Abstract The dc voltage output from an hysteretic Josephson junction which is locked to an ac frequency source differs from the ideal Josephson relation if the junction drives a current about a closed superconducting circuit. The difference in voltage ΔV from two hysteretic Josephson junctions driven in series opposition is proportional to the difference in their driving frequencies $\Delta\omega$ if the junctions are each biased to the nth voltage step. It is shown here, however, that ΔV is systematically smaller than the voltage difference ΔV_0 predicted by the ideal Josephson relation $\Delta V_0 = (n\hbar/2e)\Delta\omega$. If the loop inductance approaches zero, the smallest detectable voltage difference ΔV_0 between two junctions is limited by the intrinsic Josephson inductance. For arrays of more than one junction, however, ΔV_0 remains proportional to the loop inductance.

1. Introduction

Microlithographic series-arrays of up to 18992 rf-biased Josephson junctions have been developed [F. L. Lloyd et al. 1987] for metrology [R. L. Steiner and B. F. Field 1989]. The use of these series-arrays in macroscopic quantum circuits (MQC) has recently been proposed [L. Z. Wang and R. V. Duncan] as a new method for ultra-accurate interferometric readout for such applications as gravity wave detection and Sagnac-effect gyroscopes. Here we calculate the measurement accuracy of these readout circuits based on the interactions of the two rf-biased Josephson devices within the MQC.

Consider two rf-biased Josephson devices which are connected in series-opposition with superconductive wire, forming the MQC with a total self-inductance L. The supercurrent I in the MQC builds up over time t due to the difference in electric potential ΔV between the two rf-biased devices as:

$$I = \frac{1}{L}\int_0^t \Delta V\, dt \qquad (1.1)$$

The supercurrent is then read by a SQUID which is magnetically coupled to the MQC. This method of ultra-precise potentiometry was developed by Clarke [1968], and more recently used by Jain et al. [1987] to obtain measurements with a precision of $\Delta V / V \sim 3\times 10^{-19}$ using single rf-biased junctions operating at about 0.3 mV. Kautz and Lloyd [1987] have used this technique to compare the voltage output of two series-arrays, each containing 2,076 Josephson junctions and biased to $V \sim 1$ volt, to a precision of $\Delta V / V \sim 2\times 10^{-17}$. In these measurements both Josephson devices were biased to the same step (n) by the same rf source to check for deviations from $I = 0$. Other experiments [T. D. Bracken and W. O. Hamilton 1972] have been performed which have used two different rf sources of known detuning $(\Delta\omega)$ to bias the junctions, and thus to generate an increasing current consistent with equation (1) and the prediction that

$$\Delta V = \Delta V_0 = n\hbar\Delta\omega / 2e, \qquad (1.2)$$

where $\hbar$ is Planck's constant divided by 2π, and e is the charge of the electron.

Referring to equation (1), it is clear that an arbitrarily precise measurement of ΔV may be made by either decreasing L, increasing the length of the measurement interval, or both. The value of the self-inductance L of the MQC need not be limited by the SQUID's input inductance [Jain et al. 1987]. We show below, however, that the measured ΔV of two phase-locked hysteretic junctions is not simply equal to $\Delta V_0 = n\hbar\Delta\omega / 2e$. Rather, there is a correction to the voltage difference which becomes large as L goes to zero. This correction has been calculated within the framework of the Stewart-McCumber model [W. C. Stewart 1968, D. E. McCumber 1968] in the absence of external noise. This analysis, following Kautz [1981] applies only to hysteretic junctions operating near zero-current bias.

2. Analysis and Results

To illustrate the correction to the ideal Josephson relation, we consider for simplicity a comparison of two identical hysteretic Josephson junctions. Suppose that each junction is phase-locked to the same voltage step and that they are placed in series opposition in a closed superconducting loop with inductance L. For zero current biases, the phase differences across junction #1 and junction #2, ϕ_1 and ϕ_2 respectively, obey the following coupled equations of motion:

$$\ddot{\phi}_1 + \gamma\dot{\phi}_1 + \omega_0^2(\phi_1 + \phi_2) + A\sin(\phi_1) = B_1\cos(\Omega_1 t + \delta_1) \qquad (2.1)$$

$$\ddot{\phi}_2 + \gamma\dot{\phi}_2 + \omega_0^2(\phi_2 + \phi_1) + A\sin(\phi_2) = B_2\cos(\Omega_2 t + \delta_2) \qquad (2.2)$$

Equations (2.1) and (2.2) have been written in dimensionless form so that time is scaled by the period of the rf drive. Parameters are defined as follows: $\gamma = \frac{1}{RC\omega}$, $\omega_0^2 = \frac{1}{LC\omega^2}$, $A = \frac{2e}{\hbar C\omega^2} I_0$, $B_{1(2)} = \frac{2e}{\hbar C\omega^2} I_{1(2)}$, and $\Omega_{1(2)} = \frac{\omega_{1(2)}}{\omega}$, where R is the junction shunt resistance, I_0 is the junction critical current, C is the junction capacitance, ω is the average of the two driving frequencies, and the driving amplitudes for junctions #1 and #2 are I_1 and I_2 respectively. We have taken I_1 and I_2 to be equivalent, and we have chosen the phases δ_1 and δ_2 to be zero and π respectively. Typical junction parameters are $R = 10\Omega$, $C = 20pF$, $I_0 = 200\mu A$, $\omega/2\pi = 96\ GHz$, and $I_1 = 20mA$ [C. A. Hamilton et al. 1985]. A typical measurement-loop inductance L is 2 μH [R. L. Kautz and F. L. Lloyd 1987]. On substituting these values, the dimensionless parameters in (2.1) and (2.2) are $\gamma = 10^{-2}$, $\omega_0^2 = 10^{-7}$, $A = 10^{-1}$, $B = 10$, and $\Omega_{1(2)} \cong 1$.

In the mechanical analogy [Lounasmaa 1974] in which the phase difference across a junction is identified with the rotation angle of a physical pendulum, equations (2.1) and (2.2) describe a system consisting of two pendula whose rotations are coupled by a torsion bar with stiffness ω_0^2. When phase locked, the pendula undergo n complete rotations per drive cycle. The tiny difference in drive frequencies causes one pendulum to rotate slightly faster than the other. This creates an increasing angular separation between the pendula while storing energy in the torsion bar. We note that for non-zero ω_0^2 the potential energy of each pendulum is no longer purely sinusoidal. The periodic gravitational torque becomes "tilted" by the torque due to the torsional spring. In such a case, the phases may only be *effectively* locked, for collapse from the rotating cycle will occur when either the potential energy stored in the torsional spring is of the order of the rotational kinetic energy of the pendula, or when the restoring torque of the torsional spring exceeds the maximum gravitational torque. Nevertheless, two Josephson junctions will remain effectively locked for a time which is much longer than typical measurement times if the voltage difference between them is sufficiently small. The question we ask is whether in the meantime there is any change in the junction voltages because the system is no longer periodic. To study the effect of the inductive coupling, we assume solutions to (2.1) and (2.2) of the form

$$\phi_1(t) = \phi_1^0(t) + x_1(t) \qquad (2.3)$$

$$\phi_2(t) = \phi_2^0(t) + x_2(t) \qquad (2.4)$$

where $\phi^0_{1(2)}(t)$ is the steady-state phase-locked solution for an isolated junction, and $x_{1(2)}(t)$ is the deviation caused by the coupling ω_0^2. We substitute (2.3) and (2.4) in (2.1) and (2.2), and solve approximately for $x_{1(2)}(t)$ in the regime in which A/Ω^2 is small. On the average, $x_{1(2)}(t)$ increases (or decreases) linearly in time, such that the measured voltage difference ΔV is always suppressed from the ideal voltage difference ΔV_0 of two isolated junctions by an amount $\delta(\Delta V_0)$. We find

$$\delta(\Delta V_0) = -\Delta V_0 \frac{2\omega_0^2}{2\omega_0^2 + \sqrt{A^2 J_n^{\,2}(B) - \gamma^2 n^2}} \qquad (2.5)$$

where $J_n(x)$ is the ordinary Bessel function of the first kind of order n. The same analysis may also be applied to a circuit containing two opposing multiple-junction array voltage standards. The calculation of the deviation from the ideal voltage difference is straightforward, even if the individual junctions within the array are non-identical and biased to different voltage steps. It is most illustrative, however, to consider a simplification in which (i) the two arrays are identical, (ii) there are N identical junctions in each array, and (iii) each junction is biased to the nth voltage step (in the same direction). In this case, the analogue of (2.5) is

$$\delta(\Delta V_0) = -\Delta V_0 \frac{2\omega_0^2}{2\omega_0^2 N + \sqrt{A^2 J_n^{\,2}(B) - \gamma^2 n^2}} \qquad (2.6)$$

In (2.6), the voltage difference $\Delta V_0 = Nn(\hbar/2e)(\Omega_1 - \Omega_2)$. We see that as the loop inductance approaches zero the percentage of deviation from the ideal voltage difference is no longer as large as it was in the case of only two junctions. While the maximum deviation in the voltage is still equal to the voltage difference between two junctions, this is only $1/N$ times the total voltage difference between the two arrays. The N junctions share the burden of building the magnetic field associated with the current loop. Each junction is affected less, and there is therefore a proportionally smaller voltage deviation. The voltage deviation expressed in (2.5) and (2.6) represents a departure from ideal Josephson behavior as expressed by (1.2). It is as though the fundamental constant $\hbar/2e$ has become effectively smaller. If the only purpose of the array comparison is to detect the presence of a voltage difference, then a small voltage suppression may be of little importance. However, the voltage deviation represents a potentially severe problem if array intercomparisons are to be used as accurate measures of very small frequency differences.

That the actual voltage difference will be suppressed from the ideal voltage difference in a junction intercomparison implies also that the average current in the MQC will be less than ideal. In fact, for two junctions driven by slightly different frequencies, the buildup of dc supercurrent in the measurement loop is given by

$$I = \frac{1}{L}\Delta V\, t = \frac{\Delta V_0}{L}\left[\frac{\sqrt{A^2 J_n^{\,2}(B) - \gamma^2 n^2}}{2\omega_0^2 + \sqrt{A^2 J_n^{\,2}(B) - \gamma^2 n^2}}\right] t \qquad (2.7)$$

We now consider the limits of expression (2.7) with respect to the relative magnitudes of the loop inductance L and the Josephson inductance $L_J = \hbar/2eI_0$. For our junctions $L_J \sim 1pH$. When the inductance is large, so that $L_J << L$ ($\omega_0^2 << A$), we recover expression (1.1); the dc current induced in the measuring loop is inversely proportional to the loop inductance L. On the other hand, when the inductance is such that $L << L_J$ ($A << \omega_0^2$), the voltage difference ΔV is proportional to the loop inductance L, and in the limit in which L approaches zero, the current is independent of L:

$$I = \frac{e}{\hbar} I_0 \Delta V_0 \, t \times \sqrt{J_n^2(B) - \left(\frac{\hbar\omega}{2e}\right)^2 \frac{n^2}{I_0^2 R^2}} \tag{2.8}$$

We observe in (2.8) that the dc current is limited by an effective inductance L_i ;

$$L_i = L_J \times \left(\sqrt{J_n^2(B) - \left(\frac{\hbar\omega}{2e}\right)^2 \frac{n^2}{I_0^2 R^2}} \right)^{-1} \tag{2.9}$$

where L_J is the Josephson inductance $\hbar / 2eI_0 \sim 1pH$.

In summary, we have quantified the effects of a closed current loop on the operation of hysteretic Josephson junctions at zero current bias. We have analyzed a particular situation in which two junctions are placed in series opposition and driven by slightly different frequencies. We have shown that the difference in their voltages is smaller than what is predicted by the ideal Josephson relation for isolated junctions. The deviation from ideal behavior is important in estimating the rate at which current will increase to a detectable level in the MQC. In the absence of noise, we find that the rate at which the loop-current increases for two single hysteretic junctions is limited by the intrinsic Josephson inductance as $L \rightarrow 0$. Further analysis of multiple junction arrays shows that this rate is limited by the inductance in the measurement loop [Dunlap and Duncan].

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