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THREE-BODY FORCES MANDATED BY POINCARÉ INVARIANCE

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INTRODUCTION

The three-body potential V_{123} in the three-nucleon Hamiltonian

$$H = H_0 + V_{12} + V_{23} + V_{31} + V_{123} \tag{1}$$

is inherently ambiguous since all observables (energies, cross sections, decay times etc.) are invariant under simultaneous unitary transformations of the Hamiltonian H and the wave function ψ,

$$\overline{H} = UHU^{+} \quad ; \quad \overline{\psi} = U\psi \ . \tag{2}$$

This ambiguity is greatly reduced by a number of essential (usually tacit) assumptions: (1) The three-body potentials V_{ijk} in the many-body Hamiltonian

$$H = H_0 + {}^1/_2 \sum_{ij} V_{ij} + {}^1/_3 \sum_{ijk} V_{ijk} + \ldots \tag{3}$$

are the same for all nuclei. (2) The interactions are dominated by the two-body potentials, and the importance of n-body forces decreases rapidly with increasing n. (3) Two-body (n-body) operators in the charge and current densities are determined by the supressed subnuclear degrees of freedom (mesons and/or quarks).

While meson field theory provides guidance to important features of the two- and three-body forces nonrelativistic nuclear potentials are essentially phenomemnological. The presence of three-body forces is required by the well-known failure of two-body potentials alone to account quantitatively for the binding of few-body nuclei and of nuclear matter.

There are obvious reasons to inquire into the importance of relativistic invariance for the dynamics of the three-nucleon system. Electromagnetic probes of the the short-range character of the wave functions involve large momenta for which a nonrelativistic theory prima facie suspect. Even in ground-state of three-body nuclei at rest the velocities of nucleons are sufficiently large that one may expect relativistic effects on the binding energy comparable to the effects of the three-body forces.

The states of any quantum system undergo unitary transformations under rotations, boosts and translations in space and time. Nonrelativistic systems transform according to the inhomogeneous Galilei group. Only the time translations depend on the dynamics, all other transformations are kinematic. Relativistic systems must transform according to the inhomogeneous Lorentz group (Poincare group). In that case the group structure demands that transformations other than the time evolutions depend on the dynamics. The only the representation of a kinematic subgroup remains independent of the interactions. The choice of this kinematic subgroup is to somne extent arbitrary and leads to different "forms of dynamics" which are unitarily equivalent as far as the observable consequeces are concerned. In the familiar "instant form" the kinematic subgroup (translations and rotations) leaves the hyperplanes t=const. invariant, and the Lorentz boosts are dynamical transformations. In the "front-form" dynamics the kinematic subgroup leaves the light front $\tau \equiv x^0+x^3=0$ invariant, and the rotations about any transverse axis are dynamical transformations. The front form is particularly convenient because the kinematic subgroup includes the Lorentz transformations which are important in the calculation of electromagnetic form factors and inelastic structure functions.

The Lagrangian dynamics of relativistic field theories formally satisfies all these requirements in both the instant-form and the the front-form Fock representations, but truncation of the Fock space to a finite number of particles destroys the relativistic invariance. Moreover any practical use of Lagrangian field theories has perurbative features which cannot be justified for strong nuclear interactions.

It is therefore of interest to examine Poincaré invariant models for the three-nucleon system which have the same heuristic relation to field theories as the nonrelativistic nuclear models. The construction of Poincare invariant dynamical models consisting of a finite number of particles is based on the following observations. The generators of the infinitesimal dynamical transformations can be obtained as functions of the kinematic generators, the invariant mass operator of the interacting system and additional operators which *may* be obtained from the noninteracting system. These additional operators are the components of the Newton-Wigner position operator in the instant form, and the transverse components of the spin in the front form.

RELATIVISTIC DYNAMICS

The generators of infinitesimal Poincare transformations are the four momentum $\{P^0,\vec{P}\}$, the angular momentum $\vec{J}$, and the Lorentz boosts $\vec{K}$. The generators of the front-form kinematic subgroup are $P^+ \equiv P^0+P^3$, the transverse component of the momentum $\vec{P}_T$, the longitudinal components of the angular momentum J_3 and the boosts K_3, and

$$\vec{E} \equiv \vec{K}_T + \vec{n}\times\vec{J}_T \qquad ; \quad \vec{n} \equiv \{0,0,1\}. \tag{4}$$

The dynamic generators are $P^- \equiv P^0-P^3$ and the transverse components of the angular momentum $\vec{J}_T$. The mass operator M is, of course, related to the four-momentum by

$$M^2 = P^+P^- - \vec{P}_T^2 . \tag{5}$$

A bound state must be an eigenfunction of the spin operator $\vec{I}$ as well as the mass operator M. The spin is related to the Pauli-Lubanski vector $\{W^0,\vec{W}\} \equiv \{\vec{P}\cdot\vec{J},P^0\vec{J}+\vec{P}\times\vec{K}\}$ by

$$I_3 := W^+/P^+ \qquad ; \quad M\vec{I}_T := \vec{W}_T - \vec{P}_T \tag{6}$$

Conversely, if the mass M and the transverse spin $\vec{I}_T$ are known then the dynamic generators P^- and $\vec{J}_T$ determined by

$$P^- = [M^2 + \vec{P}_T^2]/P^+ \tag{7}$$

and

$$\vec{J}_T = \frac{M}{P^+}\vec{I}_T + \frac{P^+ - P^-}{2P^+}\overline{n}\times\overline{E} + \frac{\vec{P}_T}{P^+} I_3 + \frac{\vec{n}\times\vec{P}_T}{P^+} K_3 \quad . \tag{8}$$

Projection of a meson field theory onto the two- or three-nucleon sector of the Fock space produces and expression for M^2 which is invariant under the kinematic subgroup, but the projections of the spin components do not satisfy the correct commutation relations and do not commute with M^2 . Poincare invariant dynamical model can be constructed by assuring that M^2 commute with the total spin of the free particles.

TWO-NUCLEON SYSTEMS

States $|\psi\rangle$ of a single nucleon are represented by square integrable functions $\psi(\mathbf{p},\mu)$, where $\mu=\pm{}^1/_2$ is the longitudinal component of the spin and $\mathbf{p} \equiv \{p^+,\vec{p}_T\}$. States $|\psi\rangle$ of a two-nucleon system are represented by square integrable functions $\psi(\mathbf{p}_1,\mu_1,\mathbf{p}_2,\mu_2)$. All the kinematic generators are additive in the two nucleons. Appropriate internal variable are

$$\eta := p_1{}^+/P^+ \qquad \text{and} \qquad \vec{k}_T := \vec{p}_{1T} - \eta\vec{P}_T , \tag{9}$$

where $\mathbf{P} := \mathbf{p}_1+\mathbf{p}_2$. The mass operator M is given by

$$M^2 = \frac{m^2 + \vec{k}_T{}^2}{\eta(1-\eta)} + 4mV_{12} = M_0{}^2 + 4mV_{12} . \tag{10}$$

V_{12} is the nucleon-nucleon potential. Lorentz invariance requires that V_{12} commute with $\mathbf{P}$ and be independent of $\mathbf{P}$. Further the dynamics so formulated is Poincare invariant if and only if M^2 commute with the spin. This can be assured in the following manner. *Define* the longitudinal component of the internal momentum $\vec{k}$ as a function of η and $\vec{k}_T$ by

$$\vec{k}\cdot\vec{n} := \frac{1}{2}\left\{ M_0\eta - \frac{m^2 + \vec{k}_T^{\,2}}{M_0\eta} \right\} \tag{11}$$

The spin of the noninteracting two-nucleon system can then be expressed in the form

$$\vec{I} = i\nabla_k \times \vec{k} + \mathbf{R}(\eta, \vec{k}_T, m, M_0)\vec{s}_1 + \mathbf{R}(1-\eta, -\vec{k}_T, m, M_0)\vec{s}_2 \quad , \tag{12}$$

where $\mathbf{R}$ denotes a Melosh rotation,

$$\mathbf{R}(\eta, \vec{k}_T, m, M_0) := \frac{m + M_0\eta - i\vec{\sigma}\cdot(\vec{n}\times\vec{k}_T)}{\sqrt{(m+M_0\eta)^2 + \vec{k}_T^{\,2}}} \quad . \tag{13}$$

Expressed as a function of the vector $\vec{k}$ the mass operator M is given by

$$M^2 = 4(k^2 + m^2 + mV_{12}) \tag{14}$$

where V_{12} must commute with the spin (12). Thus the dynamical equations for the internal coordinates have the same form as in the nonrelativistic case. The relations of $\vec{k}$ and $\mathbf{P}$ to the individual nucleon momenta p_1 and p_2 differ, of course, from the nonrelativistic relations. This difference becomes manifest in in three-nucleon systems as well as in form factors of the deuteron. For slow nucleons we have the nonrelativistic approximation

$$\vec{p}_1+\vec{p}_2 \approx \{P^+-2m,\vec{P}_T\} \quad ; \quad {}^1/_2(\vec{p}_1-\vec{p}_2) \approx \{m(2\eta-1),\vec{k}_T\} \tag{15}$$

The two-body dynamics formulated here implies an obvious deacription of two nucleons in the presence of an noninteracting spectator. The transition to a fully interacting three-nucleon syatem involves new problems which I will adress in the next Section.

THREE NUCLEON SYSTEMS

As for nonrelativistic systems the convenient choice of internal vaiables distinguishes one of the three particles. Let

$$\mathbf{P} := \mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3 \quad , \tag{16}$$

$$\eta := p_1^+/(p_1^+ + p_2^+) \quad ; \quad \xi := p_3^+/P^+ \quad , \tag{17}$$

$$\vec{k}_T := \vec{p}_{1T} - \eta(\vec{P}_T - \vec{p}_{3T}) \quad ; \quad \vec{q}_T := \vec{p}_{3T} - \xi\vec{P}_T \quad . \tag{18}$$

The mass operator M_{12} of the interacting 12 subsystem is given by Eqs. (10) or (14). All the Poincare generators are additive in two-body cluster and the spectator. The mass and spin operators are unambiguously defined as functions of those generators. However, the spinoperator so defined depends on the interaction V_{12} and the operator $M_{12,3}^2$,

$$M_{12,3}^2 = \frac{M_{12}^2 + \vec{q}_T^{\,2}}{1-\xi} + \frac{m^2 + \vec{q}_T^{\,2}}{\xi} = M_0^2 + \frac{4mV_{12}}{1-\xi} \; . \tag{19}$$

does not commute with the spin $\vec{I}_0$ of the noninteracting three-nucleon system, which commute with M_0^2,

$$M_0^2 = \frac{m^2 + \vec{k}_T^2}{\eta(1-\eta)(1-\xi)} + \frac{m^2}{\xi} + \frac{\vec{q}_T^2}{\xi(1-\xi)} \quad . \tag{20}$$

The noninteracting spin operator $\vec{I}_0$ is

$$\vec{I}_0 = i\nabla_q \times \vec{q} + R(1-\xi, -\vec{q}_T, M_{012}, M_0)\vec{I}_{12} + R(\xi, \vec{q}_T, m, M_0)\vec{s}_3 \quad , \tag{21}$$

where the longitudinal component of the vector $\vec{q}$ is defined by

$$\vec{q}\cdot\vec{n} := \frac{1}{2}\left\{ M_0\xi - \frac{m^2 + \vec{q}_T^2}{M_0\xi} \right\} . \tag{22}$$

The interaction-dependent spin operator $\vec{I}_{12,3}$ that commutes with $M_{12,3}$ can be obtained from (21) and (22) by replacing M_0 and M_{012} by $M_{12,3}$ and M_{12} respectively.

The Hamiltonian $H_{12,3} \equiv P^-_{12,3}$,

$$P^-_{12,3} = \sum_i p^-_i + 4mV_{12} / (p^+_1 + p^+_2) \tag{23}$$

has all the required invariance properties, but the addition of two or three two-body interactions destroys the invariance unless an appropriate three-body interaction is added. The expected result for the fully interacting three-nucleon system is

$$P^- = \sum_i p^-_i + \sum_{i<j} 4mV_{ij} /(p^+_i + p^+_j) + 6mV_{123}/P^+ \qquad (22)$$

and

$$M^2 = M_{12,3}{}^2 + M_{31,2}{}^2 + M_{23,1}{}^2 - 2M_0{}^2 + 6mV_{123} \qquad (23)$$

The task at hand is to show that a three-body potential V_{123} which establishes the invariance of the three-body dynamics exists, can be constructed explicitly and is small. To accomplish this we first construct an operator $\overline{M}_{12,3}$ which commutes with $\overline{I}_0$ and describes the same two-body dynamics as $M_{12,3}$, as well as the unitary transformation $B_{12,3}$ which relates them.

The operator V_{12} in Eq. (19) has the matrix representation

$$(k',\ell',S|\hat{V}_{12}|S,\ell,k) \times (\vec{q}_T',\xi',\mu_{12}',\mu_3'|1|\mu_3,\mu_{12},\xi,\vec{q}_T)\,; \qquad (24)$$

The operator $\overline{V}_{12}$ designed to commute with $\overline{I}_0$ can be defined by the matrix representation

$$(k',\ell',S|\hat{V}_{12}|S,\ell,k) \times (q',L',I',\mu'|1|\mu,I,L,q)\,. \qquad (25)$$

Manifestly the the dynamics of the two body subsystem is completely specified by the operator $\hat{V}_{12}$ which is the same in (24) and (25). The subsystem mass operators

$$M_{12}{}^2 = M_{012}{}^2 + 4mV_{12} \quad \text{and} \quad \overline{M}_{12}{}^2 = M_{012}{}^2 + 4m\overline{V}_{12} \qquad (26)$$

yield the same two-body bound-state energies and the same scattering observables. The same is true for $M_{12,3}$ defined by Eq.(19) and $\overline{M}_{12,3}$

$$\overline{M}_{12,3} = \sqrt{(\overline{M}_{12}{}^2 + q^2)} + \sqrt{(m^2+q^2)} \; . \qquad (27)$$

They commute respectively with $\vec{I}_{12,3}$ and $\vec{I}_0$. Therefore there exists a unitary transformation $B_{12,3}$ which transforms $\vec{I}_{12,3}$ into $\vec{I}_0$ and $\overline{M}_{12,3}$ into $M_{12,3}$. It follows that

$$B_{12,3}\,\xi\,B^+{}_{12,3} - \xi = [\vec{n}\cdot\vec{q} + \sqrt{(m^2+q^2)}]\,[M_{12,3}{}^{-1} - M_0{}^{-1}] \qquad (28)$$

and

$$B^+{}_{12,3}\,\vec{n}\cdot\vec{q}\,B_{12,3} - \vec{n}\cdot\vec{q} =$$

$$\frac{1}{2}\left\{(M_{12,3} - M_0)\xi - \frac{m^2+\vec{q}_T{}^2}{\xi}\,[M_{12,3}{}^{-1} - M_0{}^{-1}]\right\} . \qquad (29)$$

Eqs. (28) and (29) provide the basis for an approximation. The effect of $B_{12,3}$ is small, i.e. $B_{12,3} \approx 1 + i\beta_{12,3}$ with $\beta_{12,3}$ of the order $\|V_{12}M_0{}^{-1}\|$.

Since $\overline{M}_{12,3}$ commutes with the spin $\vec{I}_0$ the mass operator

$$\overline{M}^2 = \overline{M}_{12,3}{}^2 + \overline{M}_{31,2}{}^2 + \overline{M}_{23,1}{}^2 - 2M_0{}^2 + 6m\overline{V}_{123} \qquad (30)$$

is invariant for any three-body interaction $\overline{V}_{123}$ that commutes with the spin $\vec{I}_0$. The choice of $\overline{V}_{123}$ is subject to the same arbitrariness and restrictions as the nonrelativistic three-body potential. A three-nucleon dynamics based $\overline{M}$ and $\vec{I}_0$ satisfies all the invariance requirements, but the dynamic generators do not become additive if one of the particles is at a large distance. This macrocausality condition calls for a unitary transformation B that transforms (30) into (23), and $\vec{I}_0$ into $\vec{I}$.

$$B^{+}\overline{M}B = M \quad ; \quad B^{+}\vec{I}_0 B = \vec{I} \tag{31}$$

An appropriately defined product of $B_{12,3}$, $B_{23,1}$ and $B_{31,2}$ will serve that purpose. For the three-nucleon system the approximate form

$$B \approx 1 + i(\beta_{12,3} + \beta_{23,1} + \beta_{31,2}) \tag{31}$$

should be adequate.

The three-nucleon duynamics so constructed satisfies all the Poincare invariance requirements and has the correct cluster separabilty properties. The transformation (31) introduces three-body interactions in (23) even if $\overline{V}_{123}$ vanishes. The effects of the three-nucleon forces required by Poincare invariance can be expected to be relatively small.

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