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CONF-880220--7

FLAW IDENTIFICATION FROM FORCED VIBRATION TESTING

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SAND--87-1360C

DE88 002641

ABSTRACT

A continuous system approach to the vibration characteristics of a symmetrically slotted beam has been developed. The beam was modelled as a Bernoulli-Euler beam with a reasonable stress distribution associated with the slots. The equation of motion was developed and resonant frequencies obtained from Rayleigh's Quotient. An experimental study of an aluminum beam with symmetrical slots was performed. The resonant frequency and apparent mass (a measure of damping) were measured. Reasonably good correlation in natural frequencies between theory and experiment was obtained. The reduction in natural frequency was a more sensitive indicator of damage than was the increase in damping.

INTRODUCTION

A simple, quick and reliable method for determining the integrity of a structure has long been a goal for engineers. The development of such a test would allow easy periodic inspection of structures and devices with applications ranging from production quality control to evaluation of in-service and stockpiled items. One family of methods is the use of forced vibration as a diagnostic of structural integrity.

The identification of a flaw in a complicated structure is a very difficult task. In small components, methods such as ultrasonic and radiographic inspection may be used to locate flaws. However, these methods are not readily applicable to larger structures or assemblies of components. A method is needed which would identify a fairly small defect in a structure from

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a reasonably easy and cost effective test. The forced vibration testing of a structure is one candidate method.

Previous work in this field [1,2,3,4,5] has centered on the change in natural frequency of a structure due to reduced stiffness resulting from material removal as the best indicator of damage. Recent work [6] indicates that for certain types of damage the change in damping is more sensitive than is the change in natural frequency. Thus, it seems that a test method which gives a measure of natural frequency and of system damping would provide a better opportunity to identify the existence of damage.

In the current work, a forced vibration test method is used which measures the resonant frequency and the damping of the test system. The tests were performed on an aluminum beam with symmetric slots. The driving point force and acceleration were measured and analyzed to give the resonant frequency and system damping. The equation of motion for the system was obtained from a continuous system approach using Bernoulli-Euler theory modified to account for the slots.

THEORETICAL DEVELOPMENT

A uniform beam containing symmetric slots was modelled using Bernoulli-Euler beam theory modified to account for the presence of the slots. A similar development was done by [5] using a variational approach. However, some insight into the implicit assumptions of the derivation may be obtained from the following development.

Consider the beam shown in Fig. 1. The beam depth is 2d and is slotted top and bottom at a distance x_c either way from its center. The beam is of length 2L. Each slot, shown in Fig. 2, has a depth a such that

$$2d = 2a + 2h \qquad (1)$$

The local beam depth coordinate is ξ.

For an unslotted beam, Bernoulli-Euler beam theory gives the equation of motion

$$EI \frac{\partial^2 y}{\partial x^2} + \rho \frac{\partial^2 y}{\partial t^2} = 0 \qquad (2)$$

where

E = the modulus of elasticity
I = the area moment of inertia
ρ = the mass per unit length

Equation (2) assumes that E and I are uniform along the beam. In addition, Eq. (2) assumes that the elastic flexure formula and associated stress distribution from classical elastic beam theory are valid. In the case of a slotted beam, this is not the case and a modification to standard beam theory must be considered.

The existence of the slots may be considered in terms of its effect on the stress field. It is assumed that the effect of the slot is a maximum at the slot location and that it dies away at locations removed from the slot. Thus, the longitudinal stress in the beam may be written as

$$\sigma(x,\xi) = [-\xi + f(x,\xi)]\ S(x,t) \qquad (3)$$

where

$\sigma(x,\xi)$ = the longitudinal normal stress
$S(x,t)$ = the stress function
$f(x,\xi)$ = the slot function

We further assume that the stress distribution in the y direction at the slot is linear. This is known to be incorrect, but should give a reasonable and tractable development. The stress distribution must drop to zero immediately upon entering the slot and is assumed to return to nominal unslotted values in an exponential manner. One slot function which meets these criteria is

$$f(x,\xi) = [\xi - m\xi H(h - |\xi|)] \exp(-\alpha |x - x_c|/d) \qquad (4)$$

where

$H(h - |\xi|)$ = the unit step function;
α = a decay parameter to be selected.

If we require that the moment carried by the slotted section be the same as that carried by an unslotted section, the constant m may be determined from equilibrium by

$$\int_A (-\xi\ S(x,t))\xi\ dA =$$

$$\int_{A_r} (-\xi + f(x,\xi))\ S(x,t)\xi\ dA \qquad (5)$$

which reduces to

$$m = (I/I_r) \qquad (6)$$

where

A_r = the cross section at the slot;

I_r = the area moment of inertia at the slot.

Thus, the longitudinal normal stress in the beam is given by

$$\sigma(x,\xi) = \{-\xi + [\xi - m\xi H(h - |\xi|)]$$

$$\exp(-\alpha|x - x_c|/d)\}\ S(x,t) \qquad (7)$$

Since the stress function must be correct for the unslotted case, the stress function is given by

$$S(x,t) = M(x,t)/I \qquad (8)$$

where

M(x,t) = the bending moment.

In order to write the equation of motion for the slotted beam, one must first consider the moment equation for the beam in terms of the elastic curve. Assuming that shear effects are negligible and that the beam is still elastic, the radius of curvature, r, may be written as

$$\frac{1}{r} = \frac{d^2y}{dx^2} = \frac{\sigma(\xi)}{-E\xi} = \frac{M[-\xi + f(x,\xi)]}{-EI\xi} \qquad (9)$$

From which the moment equation may be written as

$$M(x,t) = EIQ(x) \frac{\partial^2 y(x,t)}{\partial x^2} \quad (10)$$

where

$$Q(x) = [1 + (m-1) \exp(-\alpha|x - x_c| /d)]^{-1} \quad (11)$$

The moment equation may be differentiated twice with respect to x and the equation of motion for the slotted beam may now be written as

$$EIQy^{iv} + EIQ'y''' + EIQ''y'' + \rho\ddot{y} = 0 \quad (12)$$

where
primes indicate derivatives with respect to x;
dots indicate time derivatives.

Equation (12) along with appropriate boundary conditions describes the motion of the slotted beam.

EXAMPLE

To test this development and to investigate the sensitivity of damping and natural frequency to slotting, a specific example was selected. A free-free beam of length 2L with symmetric slots located a distance x_c from the beam's center (see Fig. 1) was considered. Assuming that the slots are far removed from both the center of the beam and from the free ends and assuming symmetry, the appropriate boundary conditions are

$$\begin{aligned} &y'(0,t) = 0 \\ &EIy''(L,t) = 0 \\ &EIy'''(L,t) = 0 \\ &2EIy'''(0,t) + m_b\ddot{y}(0,t) = F(t) \end{aligned} \quad (13)$$

where
m_b = the base mass from the transducers.

A closed form solution to Eq. (12) was not obvious; therefore, an approximate solution for the natural frequencies using Rayleigh's quotient was obtained. The terms which appear in Rayleigh's quotient are potential and kinetic energy terms. For the example, the potential energy term is

$$V(t) = \int_0^L EIQ\left[\frac{\partial^2 y}{\partial x^2}\right]^2 dx \qquad (14)$$

And the kinetic energy term is

$$T(t) = \int_0^L \rho\left[\frac{\partial y}{\partial t}\right]^2 dx \qquad (15)$$

Which, in the neighborhood of a mode, reduces to

$$T(t) = \int_0^L \omega^2 \rho y^2 dx \qquad (16)$$

And Rayleigh's quotient becomes

$$\omega^2 = \frac{\int_0^L EIQ\left[\frac{\partial^2 y}{\partial x^2}\right]^2 dx}{\int_0^L \rho y^2\, dx} \qquad (17)$$

Thus, Rayleigh's quotient gives an approximation to the natural frequency, ω.

The quality of the approximation is strongly dependent on the assumed mode shape function, $y(x)$. This function must be selected since a closed form solution to the equation of motion was not obtained. The mode shape function selected here was obtained by assuming that the moment equation (Eq. 10) could be rearranged to give the second derivative of the mode shape; and by assuming that

$$\frac{M}{EI} = \frac{\partial^2 y}{\partial x^2} \text{ (unslotted)} \qquad (18)$$

so that

$$\frac{\partial^2 y}{\partial x^2} \text{ (slotted)} = \frac{\partial^2 y}{\partial x^2} \text{ (unslotted)} \; \frac{1}{Q} \qquad (19)$$

The unslotted problem with these boundary conditions (Eq. 13) has been solved by [7]. Thus, the use of the unslotted mode shapes is advantageous. The second derivative of the mode shape for the slotted case is assumed to be

$$\frac{d^2 y}{dx^2} = \lambda^2(-A \cos \lambda x - B \sin \lambda x + C \cosh \lambda x + D \sinh \lambda x) \frac{1}{Q(x)} \qquad (20)$$

where

λ = the beam parameter;

A, B, C, & D = the constant calculated from [7].

The value of λ is obtained from the unslotted eigenvalue equation shown by [7] to be

$$\frac{m_b}{2\rho L} (1 + \cos \lambda L \cosh \lambda L) + \frac{1}{\lambda L} (\cos \lambda L \sinh \lambda L + \sin \lambda L \cosh \lambda L) = 0 \qquad (21)$$

The mode shape function was obtained by twice integrating Eq. (20). This mode shape was then substituted into Eq. (17) to obtain an estimate of the natural frequency. The parameter, α, was selected to give good correlation with experimental data.

TEST DESCRIPTION

The beam was slotted symmetrically top and bottom at locations 3" each direction from the center of the beam. The slot depth was increased from zero to 0.625" in 0.125" steps. The slot width was 0.10" so that the slots remained fully open during testing.

The tests were performed using sinusoidal excitation from an electrodynamic shaker at the first flexural resonance of the beam system. The force and acceleration at the driving point, the center of the beam, were measured using piezoelectric transducers. The driving point acceleration was maintained at 10 g's peak, and the force was monitored to identify resonance.

The measurement of force and acceleration at the driving point is sufficient to obtain a measure of the system damping. It has been shown [6,7] that the damping at resonance for a free-free beam is given by

$$\eta = (F/a)D \qquad (22)$$

where

- η = the loss factor, a measure of damping;
- F = the driving force magnitude;
- a = the driving point acceleration magnitude;
- D = a function of mode number and geometry.

Thus, damping is proportional to the ratio of force to acceleration, the apparent mass, at the driving point; and this test method allows for the investigation of the change in damping and change in natural frequency as a function of slot depth. In addition, a comparison of theoretical development with test results may be performed.

DISCUSSION OF RESULTS

The results of the tests were in the form of natural frequencies and a relative damping measure, the apparent mass. Only the natural frequencies could be compared with the theoretical development so that data will be considered first.

The parameter α of Eq. (4) was a free parameter which was adjusted to obtain a good fit of the natural frequency data. The physical significance of α is that it is the decay parameter in the longitudinal normal stress. The effect of α on the mode shape is shown in Figs. 3 and 4. As α increases, the beam becomes stiffer giving a mode shape closer to the unslotted mode shape.

The effect of α on the natural frequency was also significant as might be expected. The natural frequency is shown as a function of slot depth in Figs. 5 and 6. In each case, the ratio of natural frequency to unslotted natural frequency is shown. The correlation between theory and experiment is acceptable in each case but is significantly better in Fig. 6. This results from allowing α to increase with increasing slot depth. Thus, the assumed mode shape function is probably not the best candidate. A more accurate description of the stress field would provide an improved shape function which would not require adjustment of parameters with slot depth. However, until such a mode shape function is derived, the current shape function gives very acceptable results.

The natural frequency is seen from Figs. 5 and 6 to roll off smoothly with increasing slot depth. The reduction is natural frequency being nearly 30% when the slot depth reaches 62.5% of the beam depth.

The change in damping (apparent mass) with slot depth is shown in Fig. 7. The damping remains nearly constant from the unslotted state to a slot depth of half of the beam depth, with variations in this region due to measurement noise. On increasing the slot depth to 62.5% of the beam depth, the damping increased significantly. Thus, the damping measurement does not seem to be sensitive to this type of damage unless the damage is quite severe.

CONCLUSIONS

1. The theoretical development leads to a reasonable equation of motion, and good comparisons with test data may be obtained if α is allowed to vary with slot depth.

2. The change in natural frequency is a more sensitive indicator of slots in a beam in bending than is the change in damping.

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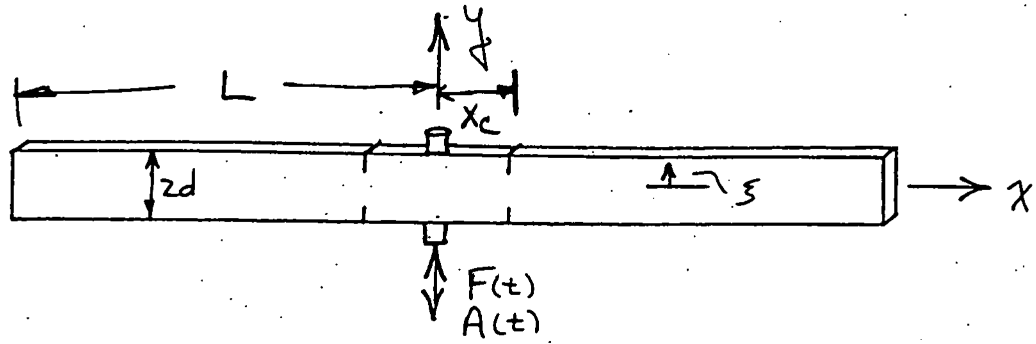

Figure 1. Slotted beam specimen.

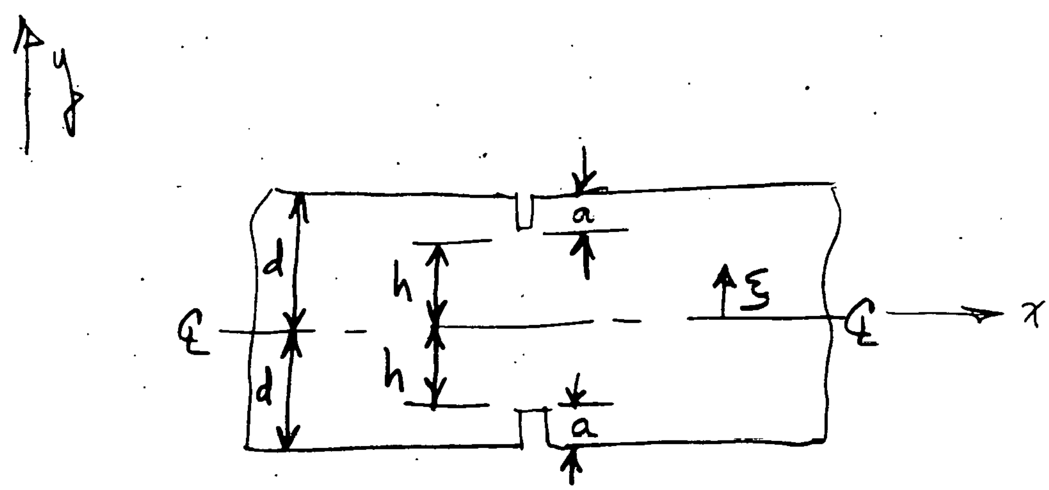

Figure 2. Slot geometry

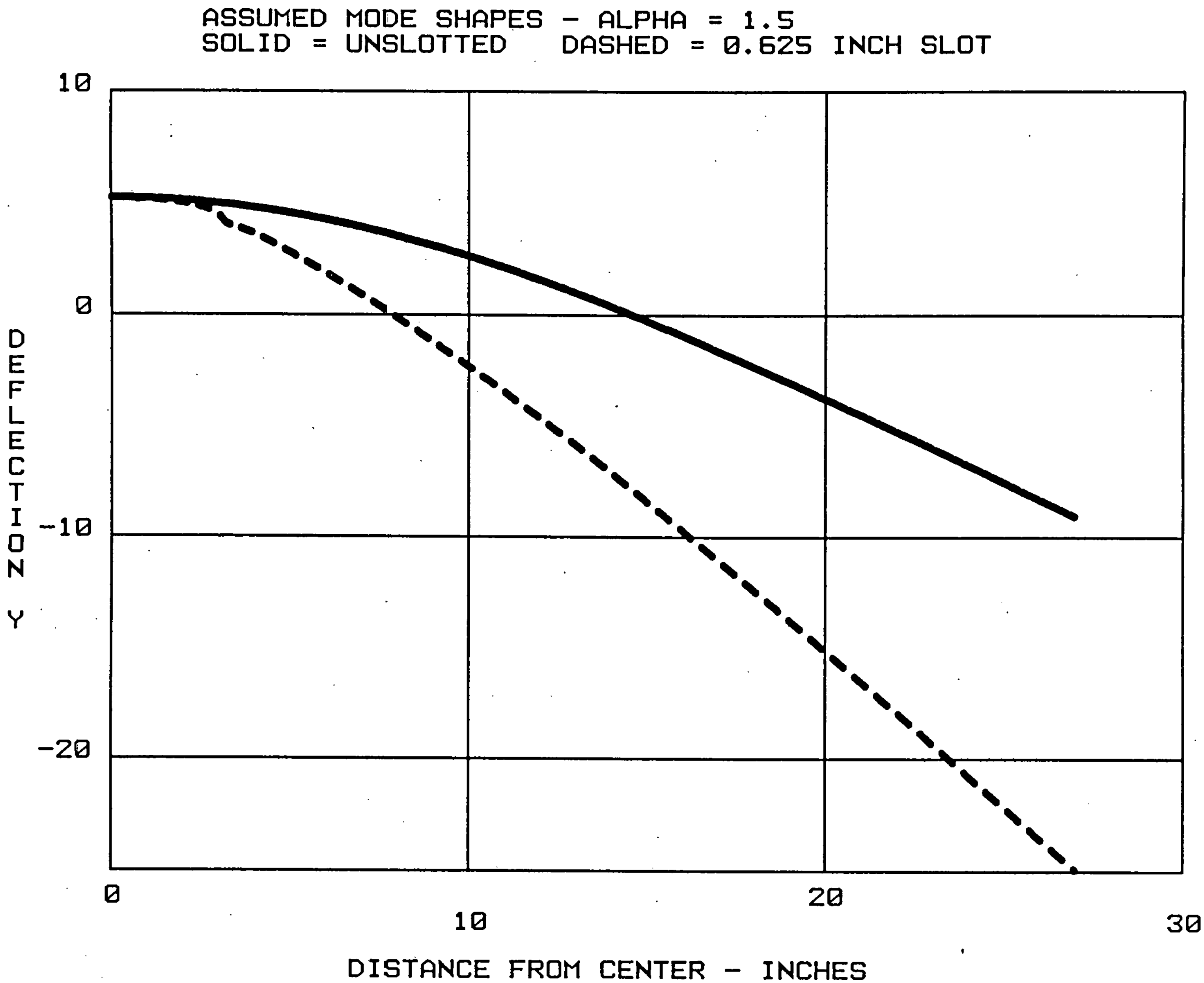

Fig. 3 Mode Shape for $\alpha = 1.5$

$\alpha = 2.0$

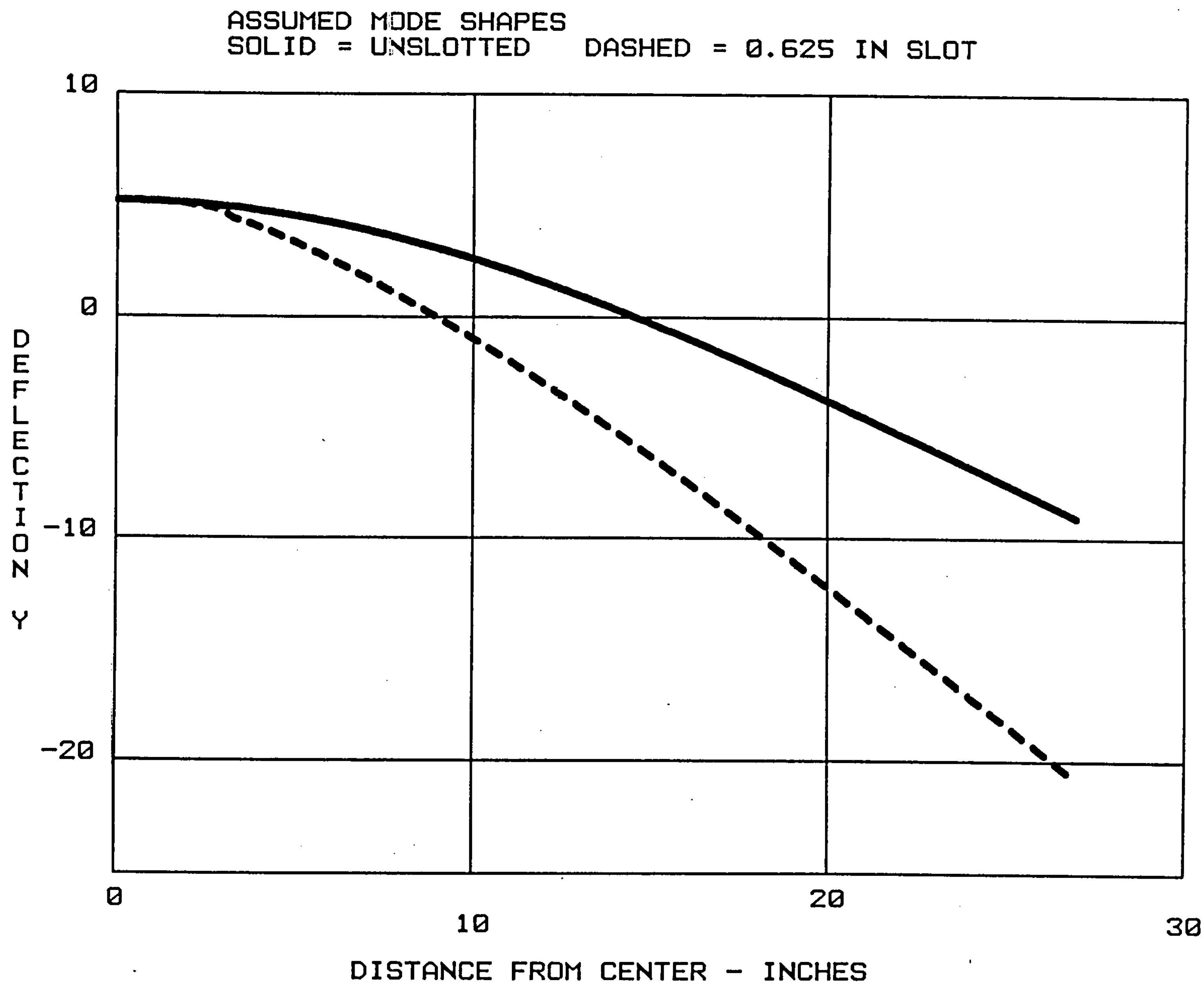

Fig. 4. Mode Shape for $\alpha = 2.0$

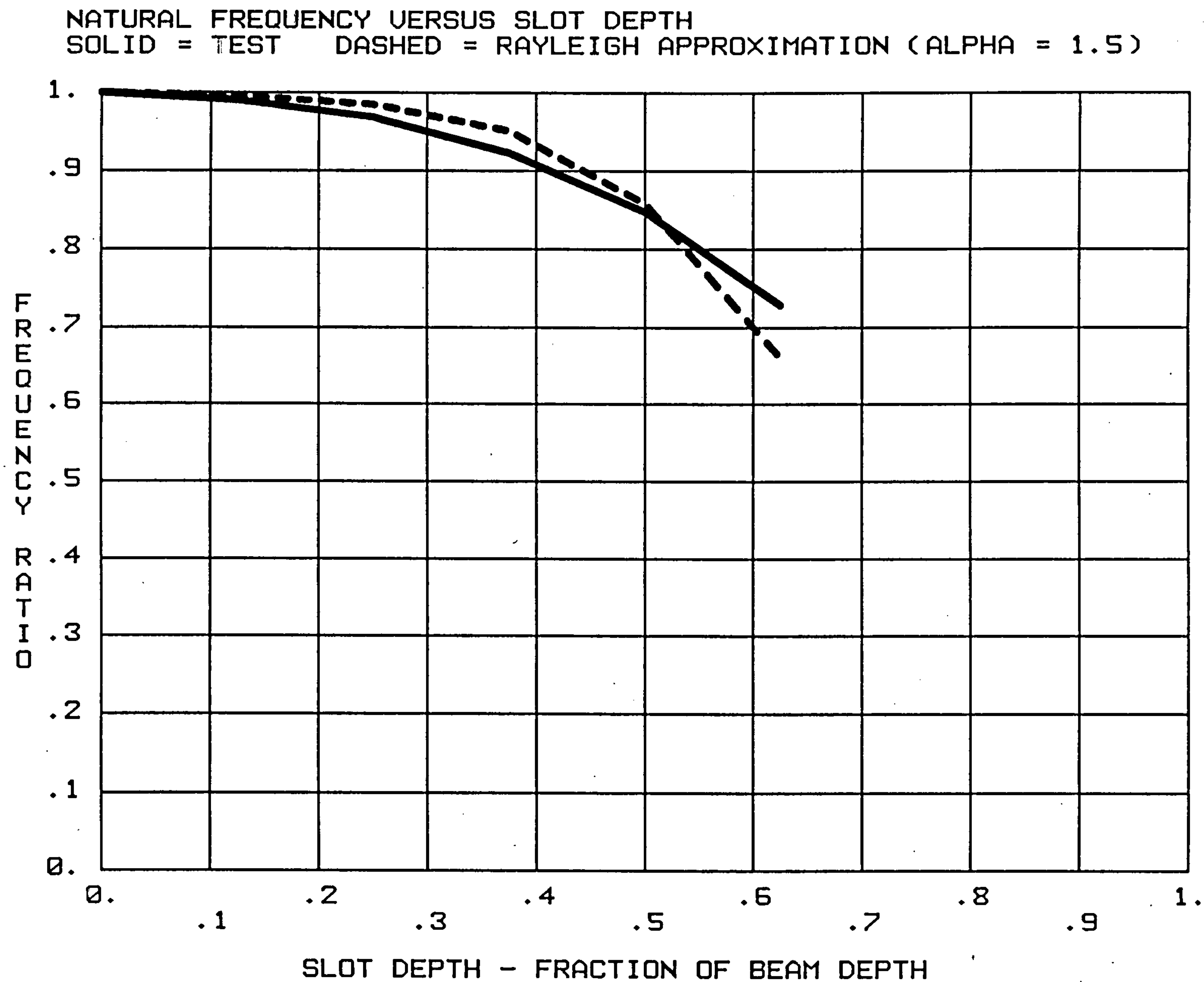

Fig. 5 Natural frequency versus slot depth, $\alpha = 1.5$

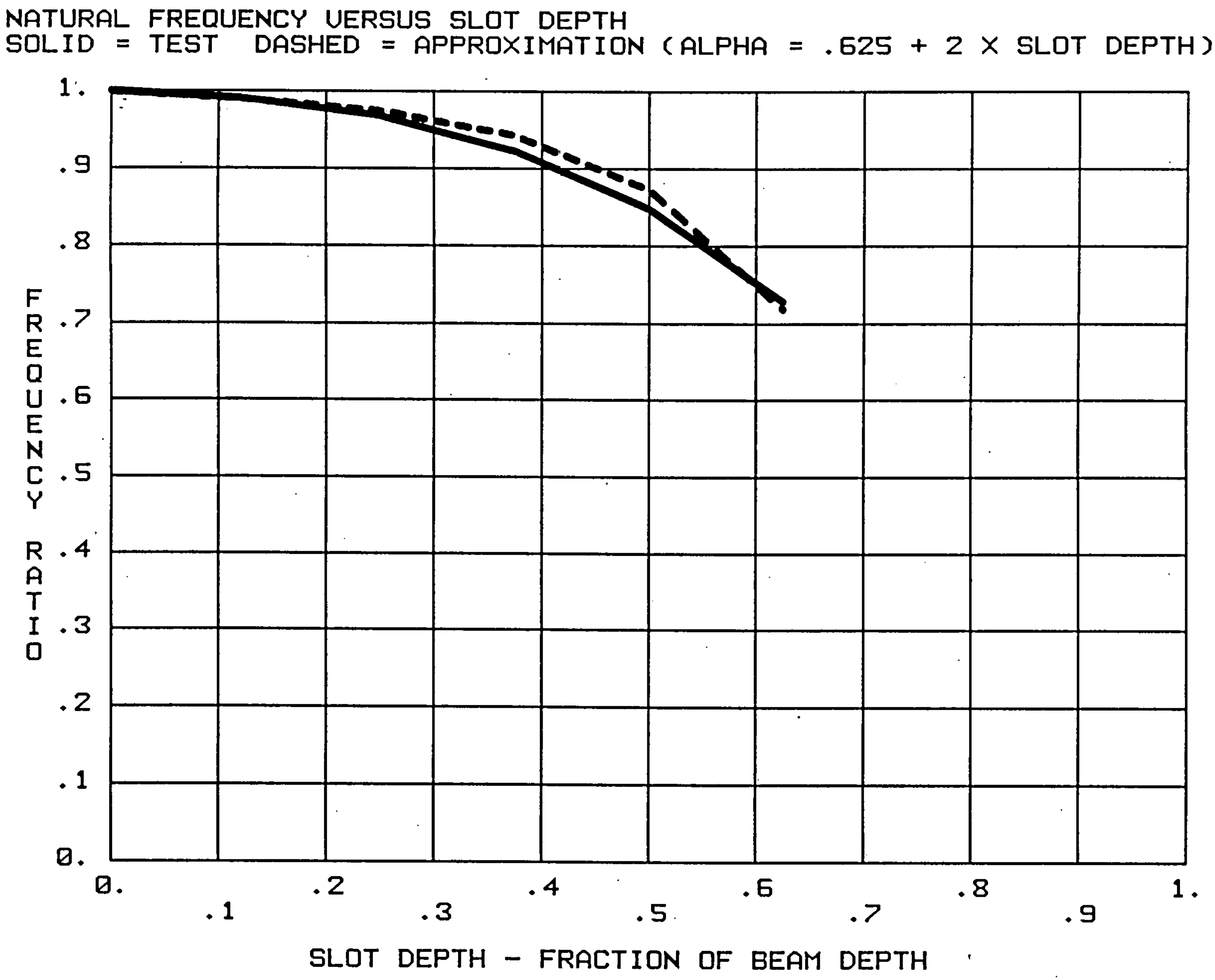

Fig. 6. Natural frequency versus slot depth, α linear with depth.

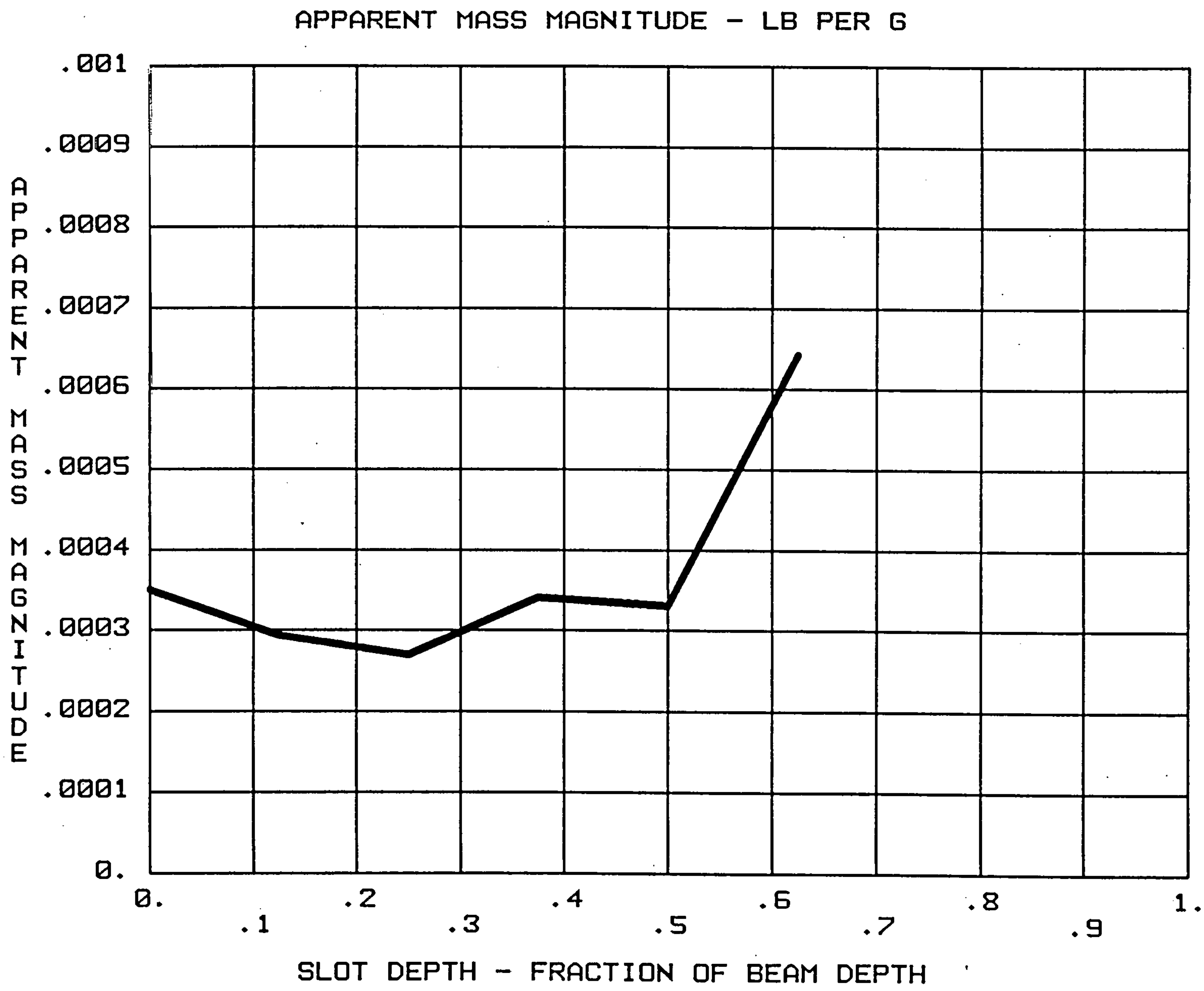

Fig. 7. Damping (apparent mass) versus slot depth.