LEGIBILITY NOTICE

A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.

LA-UR -87-3582 CONF-880220--2

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36

TITLE: EXPERIMENTAL IDENTIFICATION OF NONLINEAR STRUCTURAL MODELS

LA-UR--87-3582

DE88 001795

AUTHOR(S): N. F. Hunter, WX-11
T. L. Paez, SNL

SUBMITTED TO: 6th Int'l Modal Analyis Conf.,
Orlando, FL
Feb. 1, 1988

Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

EXPERIMENTAL IDENTIFICATION OF NONLINEAR STRUCTURAL MODELS

Thomas L. Paez

Experimental Mechanics Department
Sandia National Laboratories
Albuquerque, New Mexico

Norman F. Hunter

Group WX 11
Los Alamos National Laboratories
Los Alamos, New Mexico

A fundamental objective of structural dynamic engineering is to understand the behavior of physical structures subjected to field environments. In practical situations, many structures execute nonlinear motions, and under the influence of extreme excitations structures often show strongly nonlinear responses. Recent investigations have shown that the Volterra model provides a means for system characterization that holds great potential for the description of nonlinear structural response. Simulations have shown that when the Volterra model of a structure is available, it can provide a reasonable match to numerically simulated nonlinear response. The present investigation considers the identification of the functions used in the Volterra model for physical structures tested in the laboratory. It is shown that the frequency domain form of the Volterra functions can be estimated directly using measured excitation and response data. A coherence-like measure of the observed structural response and the nonlinear model is developed, and it is shown that this quantity can be used to evaluate the accuracy of the nonlinear structural model. Finally, the characteristics of structures most suitably modelled by the Volterra series are discussed.

EXPERIMENTAL IDENTIFICATION OF NONLINEAR STRUCTURAL MODELS

Norman F. Hunter
Analysis and Testing Group
Los Alamos National Laboratories
Los Alamos, New Mexico

Thomas L. Paez
Experimental Mechanics Department
Sandia National Laboratories
Albuquerque, New Mexico

Introduction

Though all real mechanical systems excited by field inputs display some degree of nonlinear response, the majority of all mechanical engineering dynamic analyses performed, assume that the structure under consideration is linear. Further, the algorithms used to control laboratory shock and vibration tests assume structural linearity. In many situations the linearity assumption is satisfactory. However, when increased analytical or test precision is required, or when a mechanical system displays substantially nonlinear behavior, it may be advantageous to use a nonlinear model for the system.

Structural nonlinearity can be treated in several different ways. In the analytical framework, sets of equations that model specific nonlinearities in mechanical systems are often developed and approximate solutions established in closed form. (See References 1 and 2.) When the equations of motion cannot be solved in closed form then numerical solution techniques are sometimes developed. (See, for example, Reference 3.) Another general approach to the solution of nonlinear problems is to model a structure using a general mathematical model whose features may be made to match the characteristics of a mechanical system.

Examples of general mathematical models for nonlinear systems are the Volterra, Wiener, and harmonic generating transfer function (HGTF) models, and the nonlinear autoregressive-moving average (ARMA), and the threshold nonlinear ARMA models. These general approaches to system modelling are powerful and useful because they can be used to model entire classes of nonlinear system behavior. (See References 4 through 8.)

When the functions required to characterize a mechanical system, in the framework of a general mathematical model, are known, then the behavior of the system can be described (and perhaps controlled) in the general framework of the mathematical model. This is advantageous when the specific mathematical form of a system nonlinearity is unknown, yet it is required to predict or control the system response.

This paper studies the Volterra and HGTF models for nonlinear structural systems. An efficient method for estimating the frequency domain functions of the models is developed, and the use of this technique is demonstrated for a structure with a cubic nonlinearity. A method for using ordinary coherence to establish the accuracy of a nonlinear model is also developed. Finally, a discussion of the Volterra model and its practical use in analysis and testing is presented.

Identification of Volterra Models

The Volterra model for a nonlinerar system is expressed by the equation

$$
\begin{aligned}
(1) \quad z(t) = & \int_0^t h^{(1)}(t-\tau)\, x(\tau)\, d\tau \\
& + \int_0^t d\tau_1 \int_0^t d\tau_2 \; h^{(2)}(t-\tau_1, t-\tau_2)\, x(\tau_1)\, x(\tau_2) \\
& + \int_0^t d\tau_1 \int_0^t d\tau_2 \int_0^t d\tau_3 \; h^{(3)}(t-\tau_1, t-\tau_2, t-\tau_3)\, x(\tau_1)\, x(\tau_2)\, x(\tau_3) \\
& + \cdots \\
& t \geq 0
\end{aligned}
$$

where z(t) is the structural response at a point, x(t) is the excitation, and h(j)(t1,...,tj) is the jth order, j-variate kernel of the Volterra model. The function h(1)(t) is the structure impulse response function, and the higher order Volterra kernels can be thought of as higher order impulse response functions.

The Fourier transform of a time function reveals the source of its power in the frequency domain. Fourier transformation of both sides of

(1) yields (See Reference 5.)

$$(2) \quad Z(f) = H^{(1)}(f)\, X(f)$$
$$+ \int_{-\infty}^{\infty} H^{(2)}(\alpha, f-\alpha)\, X(\alpha)\, X(f-\alpha)\, d\alpha$$
$$+ \int_{-\infty}^{\infty} d\alpha_1 \int_{-\infty}^{\infty} d\alpha_2\, H^{(3)}(\alpha_1, \alpha_2, f-\alpha_1-\alpha_2)\, X(\alpha_1)\, X(\alpha_2)\, X(f-\alpha_1-\alpha_2)$$
$$+ \cdots$$
$$-\infty < f < \infty$$

where Z(f) and X(f) are the Fourier transforms of z(t) and x(t), respectively, and H(j)(f1,...,fn) is the j-fold Fourier transform of h(j)(t1,...,tj). H(1)(f) is the frequency response function of the structure, and the higher order terms, H(j)(f1,...,fj), can be thought of as higher order frequency response functions. The H(j)(f1,...,fj) will be referred to as the Volterra frequency functions, and they are the coefficients in an expression that characterizes the Fourier transform of the response as a power series in the Fourier transform of the excitation. Because the Fourier transform is unique, (2) can be used to compute the response as well as (1).

Several approaches to the identification of the Volterra kernels have been established for both the analytical and the experimental cases. (See References 5 and 6.) The Volterra frequency functions can also be estimated using an experimental approach. To develop a method for the identification of the Volterra frequency functions we consider the discrete form of (2). It is

$$(3) \quad Z_k = H^{(1)}_{k;k}\, X_k$$
$$+ \sum_{k_1} H^{(2)}_{k_1, k-k_1; k}\, X_{k_1}\, X_{k-k_1}$$
$$+ \sum_{k_1} \sum_{k_2} H^{(3)}_{k_1, k_2, k-k_1-k_2; k}\, X_{k_1}\, X_{k_2}\, X_{k-k_1-k_2}$$
$$+ \cdots$$
$$k = 0, \ldots, n-1$$

In this expression it is assumed that the Fourier transforms are discrete Fourier transforms (DFT) of discrete time functions like x_j, $j=0,\ldots,n-1$, defined at times, $t_j=j\Delta t$, $j=0,\ldots,n-1$. The DFTs are defined at frequencies $f_k=k/n\Delta t$, $k=0,\ldots,n-1$. Z_k, X_k, and $H^{(j)}_{k1,\ldots,kj}$, correspond to the functions $Z(f)$, $X(f)$, and $H^{(j)}(f_1,\ldots,f_j)$, respectively. The Volterra frequency functions, $H^{(j)}$, have been written with a group of subscripts followed by a final subscript separated by a semicolon. The first group indicates the incices of the frequencies where the excitation originates; the final subscript indicates the frequency where the response is considered. Because the sum of the first group equals the final subscript, this notation is redundant. However, it emphasizes the source of the input power in relation to the response, and it simplifies the generalization of the model to be done later.

To identify the Volterra frequency functions we excite the structure under consideration with a stationary, normal random process. The Volterra frequency functions can be identified using a sequence of computations. First, we identify the functions $H^{(1)}_k$. To do this, multiply both sides of (3) by X_m^* and take the expected value of the resulting expression. The result is

$$(4) \quad E[Z_m X_m^*] = H^{(1)}_{m;m} \, E[|X_m|^2], \quad m=0,\ldots,n-1$$

The orthogonality characteristic of the components of the DFT of a stationary random process was used to obtain this result. The orthogonality characteristic is described in Reference 7, and, in summary states the following facts. When X_k, $k=0,\ldots,n-1$, are the components of the DFT of a stationary random process: (1) The expected value of the product of any odd number of components or their complex conjugates is zero. (2) The expected value of the product of any pair of terms is zero except when their subscripts are equal and one term is the conjugate of the other. (3) The expected value of the product of any quadruplet of terms is zero except when pairs of terms have equal subscripts and one term in each pair is the conjugate of the other, etc.

The first order Volterra frequency function can be obtained from (4). It is

$$(5) \quad H^{(1)}_{k;k} = \frac{E[Z_k X_k^*]}{E[|X_k|^2]}, \quad k=0,\ldots,n-1$$

The higher order functions can be obtained in a similar manner. To get the second order Volterra frequency functions we multiply both sides of (3) by $X^*_{m1}X^*_{m2}$ and take the expected value. The result is

$$(6) \quad E\left[Z_m X^*_{m_1} X^*_{m-m_1}\right] = H^{(2)}_{m_1, m-m_1; m} \; E\left[|X_{m_1}|^2 |X_{m-m_1}|^2\right]$$

$$m_1, m = 0, \ldots, n-1$$

where, again, the orhtogonality property of the elements in the DFT of a stationary random process has been used. The second order Volterra frequency function is

$$(7) \quad H^{(2)}_{k_1, k-k_1; k} = \frac{E\left[Z_k X^*_{k_1} X^*_{k-k_1}\right]}{E\left[|X_{k_1}|^2 |X_{k-k_1}|^2\right]}$$

$$k_1, k = 0, \ldots, n-1$$

The third order Volterra frequency function can be obtained using the same approach. It is

$$(8) \quad H^{(3)}_{k_1, k_2, k-k_1-k_2; k} = \frac{E\left[Z_k X^*_{k_1} X^*_{k_2} X^*_{k-k_1-k_2}\right]}{E\left[|X_{k_1}|^2 |X_{k_2}|^2 |X_{k-k_1-k_2}|^2\right]}$$

$$k_1, k_2, k = 0, \ldots, n-1$$

Equations (5), (7), and (8) are used as the basis for estimating the Volterra frequency functions. In an experimental framework (either analytical or laboratory), the following sequence of operations can be used to establish estimates.

1. Measure excitation and response signals and discretize them.
2. Divide the signals into blocks, and DFT the signals. (The analyst may window the signals, first, if desired. He may use the FFT algorithm to perform the DFT.)
3. Using the DFT components in each block, form the products that appear in the numerators and denominators of (5), (7), and (8)
4. Average the products over all blocks to obtain the expected value estimates.

5. Form the appropriate ratios of expected value estimates to establish estimates of Volterra frequency functions.

A computer program that executes these operations has been written, and some results are presented later in this paper.

A generalization of the Volterra model, given in (3), is possible, and easily described in terms of the Volterra frequency functions defined above. It is known as the harmonic generating transfer function model, and it was introduced in Reference 7. The model is

$$
\begin{aligned}
(9) \qquad Z_k = & \sum_{k_1} H^{(1)}_{k_1;k} X_{k_1} \\
& + \sum_{k_1}\sum_{k_2} H^{(2)}_{k_1,k_2;k} X_{k_1} X_{k_2} \\
& + \sum_{k_1}\sum_{k_2}\sum_{k_3} H^{(3)}_{k_1,k_2,k_3;k} X_{k_1} X_{k_2} X_{k_3} \\
& + \cdots \\
& k = 0, \ldots, n-1
\end{aligned}
$$

This model adds a degree of complexity and a degree of freedom to the Volterra model by using an additional sum in each term on the right hand side. For example, in a second order term, the response at frequency index k has the potential of being produced by inputs at any pair of frequencies k1 and k2, not simply the frequencies k1 and k-k1, as in the Volterra model. This introduces the potential for modelling subharmonic power generation. The frequency functions of the HGTF model can be identified using formulas like (5), (7) and (8), except that arbitrary combinations of indices can be used before the semicolon, and these are not necessarily related to the response index following the semicolon.

Spectral Density

The spectral density of the response of an HGTF system can be established using (9). We multiply each side of (9) by its own complex conjugate, take the expected value, then normalize the result by multiplying each side by $\Delta t/n$. The result is

(10)

$$
\begin{aligned}
(S_{zz})_k = & \sum_{k_1} |F^{(1)}_{k_1;k}|^2 (S_{xx})_{k_1} \\
& + 2\left\{ \sum_{k_1} \sum_{k_2} |F^{(2)}_{k_1,k_2;k}|^2 (S_{xx})_{k_1} (S_{xx})_{k_2} \right\} \\
& + 6\Big\{ \sum_{k_1} |F^{(3)}_{k_1,k_1,k_1;k}|^2 (S_{xx})^3_{k_1} \\
& \quad + \sum_{k_1 \neq} \sum_{k_2} |F^{(3)}_{k_1;k_1;k_2;k}|^2 (S_{xx})^2_{k_1} (S_{xx})_{k_2} \\
& \quad + \sum_{k_1 \neq} \sum_{k_2 \neq} \sum_{k_3} |F^{(3)}_{k_1,k_2,k_3;k}|^2 (S_{xx})_{k_1} (S_{xx})_{k_2} (S_{xx})_{k_3} \\
& + \cdots \qquad k = 0, \ldots, n-1
\end{aligned}
$$

where (SZZ)k and (SXX)k are the spectral densities of the response and the excitation, respectively, at frequency fk. The functions F(j) are normalized forms of the HGTF frequency functions that eliminate dependence of the estimates on the analysis time period. Note in (7) and (8) that the power of the terms in the numerator does not match the power of the terms in the denominator. The functions F(j) are defined as

$$
(11) \qquad F^{(j)}_{k_1,\ldots,k_j;k} = T^{(j-1)/2} \; H^{(j)}_{k_1,\ldots,k_j;k}
$$

where $T = n\Delta t$

and the occurrence of these functions in the computations leading to (10) confirms the time independence of the response spectral density.

The computer program (mentioned above) executes this computation of (1) by estimating the spectral density of the measured excitation, then using this result in (10) to obtain (SZZ)k. Results obtained from the computer code are presented later in the paper.

Coherence Test for Nonlinear Models

Two of the reasons for forming nonlinear models of mechanical systems (like the Volterra model) are (1) to assist us in understanding system behavior, and (2) to permit us to predict responses to arbitrary excitations. To assure ourselves that a nonlinear model correctly reflects the behavior of a system, it is necessary to perform tests on the model that confirm its validity. One such test, based on random excitation and response, might be described as follows: (1) Measure the excitation and response of a structure. (2) Use the measurements to identify the system parameters. (3) Estimate the spectral density of the measured response using standard statistical techniques. (4) Compute the spectral density of the response using the model and the measured excitation. (5) Compare the response spectral densities from (3) and (4), above. The quality of the model for predicting the amplitude character of the response is reflected by the closeness of the match described in (5). (Only the amplitude predicting ability of the model is tested because phase information is ignored by the spectral density.)

It is also desirable to test the phase predicting ability of a model, and an ordinary coherence test that does this, in the random vibration framework, is described below. Let $\{z(m)(t)\}$ denote the random process that is the source of the measured response, and let $\{z(c)(t)\}$ denote the random process that is the source of the computed response. Let $(S(m))k$ denote the spectral density of $\{z(m)(t)\}$, and let $(S(c))k$ denote the spectral density of $\{z(c)(t)\}$. Let $(S(mc))k$ denote the cross-spectral density between $\{z(m)(t)\}$ and $\{z(c)(t)\}$. Then the ordinary coherence between the two random processes is defined

$$(12) \quad (\gamma^2)_k = \frac{|(S^{(mc)})_k|^2}{(S^{(m)})_k (S^{(c)})_k}, \quad k = 0, \ldots, n-1$$

If the model exactly predicts the measured response, then the two random processes are identical, and the numerator in (12) is simply the square of the response spectral density. Further, the two spectral densities in the denominator are identical, and each equals the response spectral density. Therefore, the coherence is unity. When the random process $\{z(c)(t)\}$ does not yield, exactly, the random process $\{z(c)(t)\}$, then the coherence (12) is not one. The value of the coherence reflects the quality of the model in predicting the measured response. In general, the coherence is a strict measure of the similarity in two random processes.

The estimation of the coherence can be implemented in a manner similar to the test procedure described above. To estimate the coherence we repeat the first four steps listed in the test sequence. Next, we use the measured and computed responses to estimate the cross-spectral density between these two random processes. (This requires the use of a standard statistical procedure.) Finally, we use the estimated cross-spectral density and the estimated autospectral densities to form the ratio in (12).

The computer program developed in this study can perform the coherence computation, and the results of an example are provided in the Discussion section.

Application of Estimation Techniques to a Classical Nonlinear System

The methods described in this paper for estimating parameters for the Volterra and HGTF models and their associated spectral densities have been applied to several nonlinear systems. Typical results are illustrated by application of the techniques to a classical single degree of freedom system with a cubic stiffness characteristic.

Description of the Nonlinear System

Since base excited systems are commonly encountered in vibration testing and in some modal applications, a base excited single degree of freedom oscillator (with one rigid body mode) was chosen for this example. A diagram of this system is shown in Figure 1. The equations of motion which describe this system are as follows:

$$(13)\quad z'' + 2\zeta\omega_n(z'-x') + \omega_n^2((z-x) + \beta(z-x)^3) = 0$$

where $\zeta = 0.05$, $\omega_n = 2\pi 100$, x = base displacement, z = mass displacement, $\beta = 20\times10^6$.

The system is excited by a base acceleration x''. The cubic term is exercised to a degree where it is significant, but not dominant. To achieve this condition the system is excited by a random noise input of approximately 25 g's RMS over a bandwidth of about 700 Hz. The spectral density of the system input is shown in Figure 2. Both the input and response spectral densities for the system are based on 8192 data points which are averaged in 32 blocks with 256 points/block. The effective sampling rate for the data is 2048 samples/second and the Δf is 8 Hz. The spectral density of the system output is shown in Figure 3. A resonant peak is evident at approximately 130 Hz and a second peak occurs at 390 Hz. The 130 Hz peak represents the fundamental response of the system. This resonance has been shifted upward from the 100 Hz which would be expected for a linear system by the stiffening effect of the cubic term. The second peak at 390 Hz is produced by third harmonic contributions from the 130 Hz resonance.

Spectral Density Estimates From the Zero, First

Order HOTF, and Cubic Volterra Models

Zero Order Model Results

A linear system is completely represented by its zero order transfer function. This transfer function is computed from (5) and the output spectral density is then estimated using (10). The system response and the spectral density computed from the zero order model are compared in Figure 4. The coherence between the system and model outputs is shown in Figure 5. The spectral density estimate from the zero order model provides a good estimate of the system response from low frequencies up to frequencies in the vicinity of 200 Hertz. Above 200 Hertz, and expecially in the 300 to 450 Hertz region, the zero order spectral density estimate is poor. This is as expected, as the zero order model does not incorporate any means of accounting for the harmonic generation that occurs in a cubic system.

First Order Model Results

The next step in modeling the cubic system is an estimation of the first order HOTF terms. These terms are computed from (5) and the associated spectral densities are then calculated using (10). The results of the spectral density compuation based on this raw H(1) calculation are shown in Figure 6. In contrast to the zero order model, which either closely approximates or underestimates the response spectral density of the system, the first order model overestimates the spectral density everywhere except in the 300 to 450 Hz region. In the process of estimating the magnitude and phase of the first order transfer function terms in (5) an average over 32 blocks is computed. The expected value of this average is zero if the magnitude of the HOTF term being computed is in fact insignificant. The variance of the computation, based on the 32 averages, is nonzero. In general for each HOTF estimate a nonzero spectral density will result even when the true magnitude of the HOTF term is in fact insignificant. Overestimation of the spectral density occurs due to a summation of the spectral densities computed from numerous low level H(1) terms. To offset this problem a confidence test is used to determine the significance of the H(1) terms. Application of this test effectively eliminates many of the H1 terms. At a confidence level of 50 percent virtually all of the H(1) terms are eliminated and the result for the first order model is identical to that for the zero

--- Raw First Order Model Results
— cubic system output

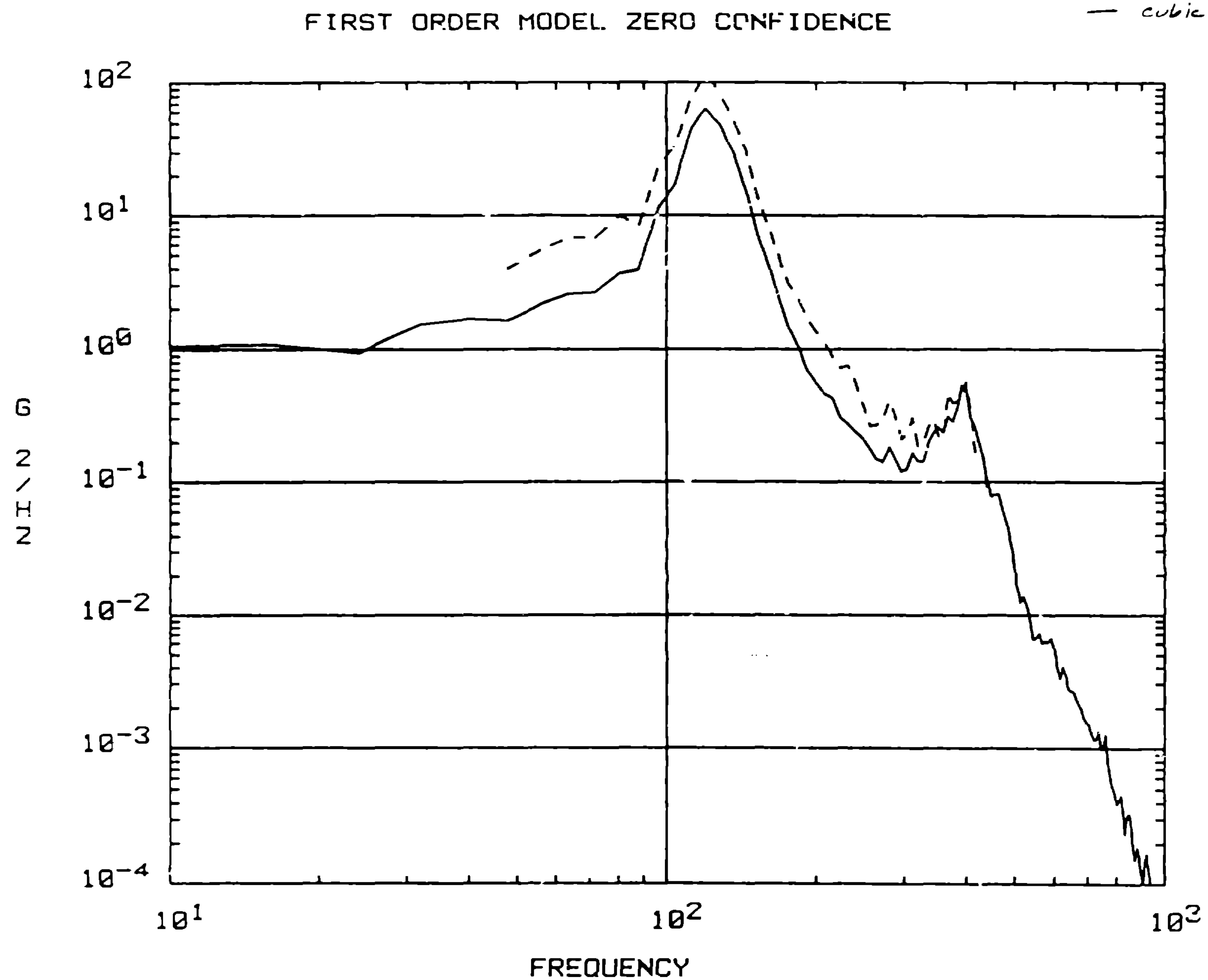
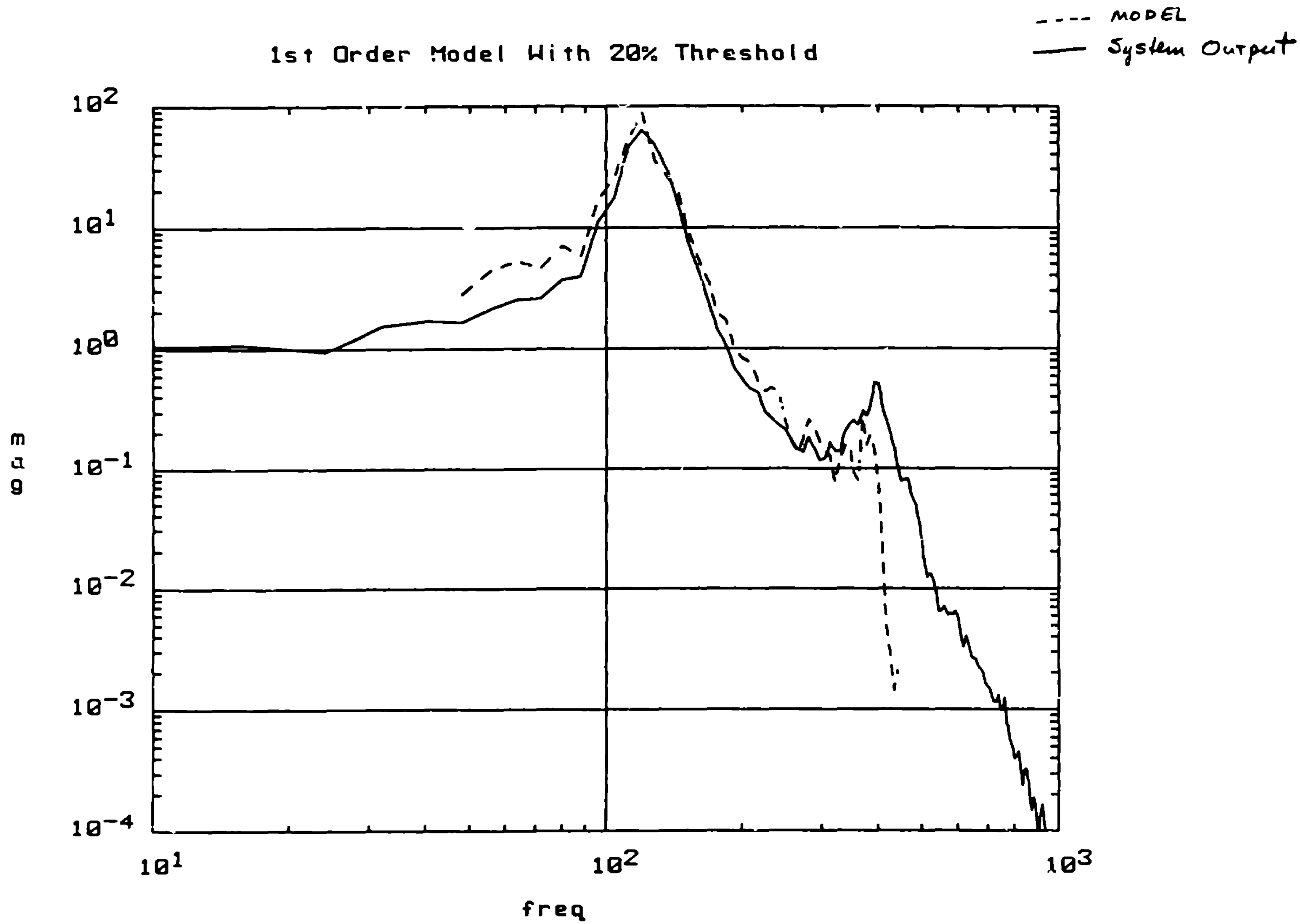


FIGURE 6

First Order Model Results at a 20% Confidence Level

Figure 7



~~Figure 11~~

Confidence

~~First order Model Results at a 20% Level~~

order model. Using only first order terms, a good fit to the response spectral density occurs at a confidence level of 20 percent. These results are shown in Figure 7. Addition of enough first order terms to fit the third harmonic response results in an overestimate of the spectrum in the 50 to 200 Hz region. This should be expected since the significance of the first order terms in a cubic system should be minimal. In fact, the fit which occurs to the third harmonic peak using linear terms is puzzling. In the author's opinions, the significance of the first order terms in this sort of analysis should be limited.

Third Order Volterra Model Results

Volterra transfer function terms were computed using (5), (7) and (9). The response spectral density estimate was then computed from the transfer function using (10). The results are shown in Figure 8 where no terms have been dropped (zero confidence level). Choosing confidence levels from 10 percent to 50 percent gradually reduces the level of the model spectral density until at 50 percent confidence there are no cubic terms included and the response degenerates to the zero order model. Inherent weaknesses in the confidence test are believed to be the reason for the low confidence levels at which many cubic terms are rejected. Since relatively few Volterra terms are included in the response spectral density estimate for the third order model, noise terms are generally less significant than in the case of the first order model. Coherence was computed for the cubic case and it is quite low. Some possible reasons for this low coherence are discussed in the following section.

Phase and Coherence Measurements

To clarify the reasons for the low coherence observed with the cubic model discussed above, a phase measurement program was implemented. Since the spectral estimates from the third order Volterra model are accurate, the low coherence is attributed to problems in the phase estimates. The computed phase for the transfer function was compared to the phase of the system output for each of the data blocks for both the linear and nonlinear models. The mean and variance of the difference between the computed phase for the transfer function and the phase of each data block was summarized for various frequencies.

Recall that resonance for the linear system occured at 100 Hertz. For the nonlinear system the resonance is more difficult to define accurately but lies in the 120 to 140 Hertz range. Phase for the linear system is as would be expected. Below resonance (32 Hz.) the system input and output are essentially in phase and the block to block phase variance is very low. At resonance the linear system shows a mean phase difference between input and output of about -90 degrees (4.90 radians) and above resonance the input and output are close to pi radians out of phase. The greatest variance in the phase measurements for the linear system is .53 radians at 100 Hertz. For the cubic system the phase measurements generally show a much greater variance (.36 to 1.28 radians) even for the zero order transfer function. Phase differences for higher order models show such large variances (about 2 radians) that assigning a clear meaning to the phase results is difficult. The large variance of these results may be explained by considering the nonlinear nature of the resonance in the cubic system. In this system the resonant frequency varies with input level. Consequently each block of data will cause the system to exhibit a different resonant frequency. Since the phase of a resonant system changes rapidly in the vicinity of resonance it is clear that much more variance in the input-output phase difference would be expected for the cubic system. This variance is further magnified when higher order transfer functions are considered. In the cubic model the response phase would be expected to vary approximately 3 times as widely as the input phase. This is indeed observed as the variance of the input phase in the third order model varies from .5 to 1 radian and the variance of the output phase varies from 2 to 3 radians. Such wide variations in phase response inevitably lead to poor coherence estimates.

Conclusions and Discussion

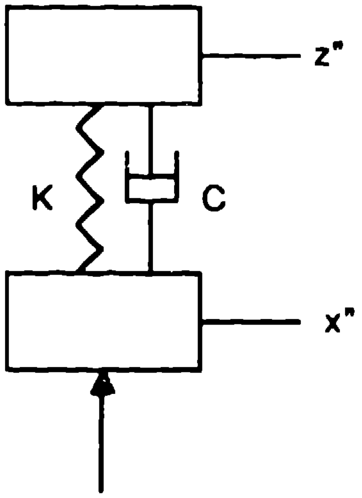
It is clear from the above results that the methods outlined in this paper for computing higher order transfer functions and spectral densities can yield acceptable results when applied to a system with a cubic stiffness nonlinearity. Spectral estimates obtained from the third order Volterra terms match the system response in the region of the third harmonic reasonably well. The significance of the first order terms is clearly open to question due to the additive effects of noise in the spectral estimates. Prediction of spectral responses using the zero order transfer function is good in the region of the fundamental response but poor in the region of the third harmonic response. Coherence derived from all estimates, zero through third order, is poor in the region of the third harmonic response. Since coherence is strongly phase sensitive, and since the magnitude results are fairly accurate, the low coherence is attributed to poor estimate of the phase of the response in the 300 to 450 Hertz region.

Nonlinear systems present many problems in analysis when compared to linear systems. Variations in "resonant frequency" and harmonic generation are present even in a relatively simple system with a cubic stiffness characteristic. For systems exhibiting significant nonlinearity the concept of the transfer function must be extended to include higher order transfer functions. These functions regulate the transfer of energy from one frequency to another. These transfer functions may be defined in various forms, including the Volterra and Harmonic Generating forms noted here. A method of estimating these higher order transfer functions has been discussed in this paper, and the spectral density estimates resulting from using these transfer functions compared to the actual system response. Inherent problems in the phase estimation techniques used here lead to low coherence between the system output and the model responses. Considerable work needs to be done in further defining the optimal form of higher order transfer functions for various types of nonlinear systems.

References

1. Stoker, J., Nonlinear Vibrations in Mechanical and Electrical Systems, Interscience Publishers, New York, 1950.
2. Minorsky, N., Introduction to Nonlinear Mechanics, J. W. Edwards, Ann Arbor, Michigan, 1947.
3. Clough, R., Penzien, J., Dynamics of Structures, McGraw-Hill, New York, 1975.
4. Bedrosian, E., Rice, S., "The Output Properties of Volterra Systems (Nonlinear Systems with Memory) Driven by Harmonic and Gaussian Inputs," Proceedings of the IEEE, V. 59, No. 12, December, 1971.
5. Schetzen, M., The Volterra and Wiener Theories of Nonlinear Systems, Wylie-Interscience, New York, 1980.

6. Marmarelis, P., Marmarelis, V., Analysis of Physiological Systems, The White Noise Approach, Plenum Press, New York, 1978.
7. Paez, T., "Nonlinear Structural System Modelling," Proceedings of the Structural Dynamics and Materials Conference, AIAA, 1987.
8. Lai, H., Hsieh, S., "Threshold Modelling of Nonlinear Dynamic Systems," Modal Testing and Analysis, DE Vol. 3, Asme, 1987.



Base Excited Single Degree of Freedom System

FIGURE 1

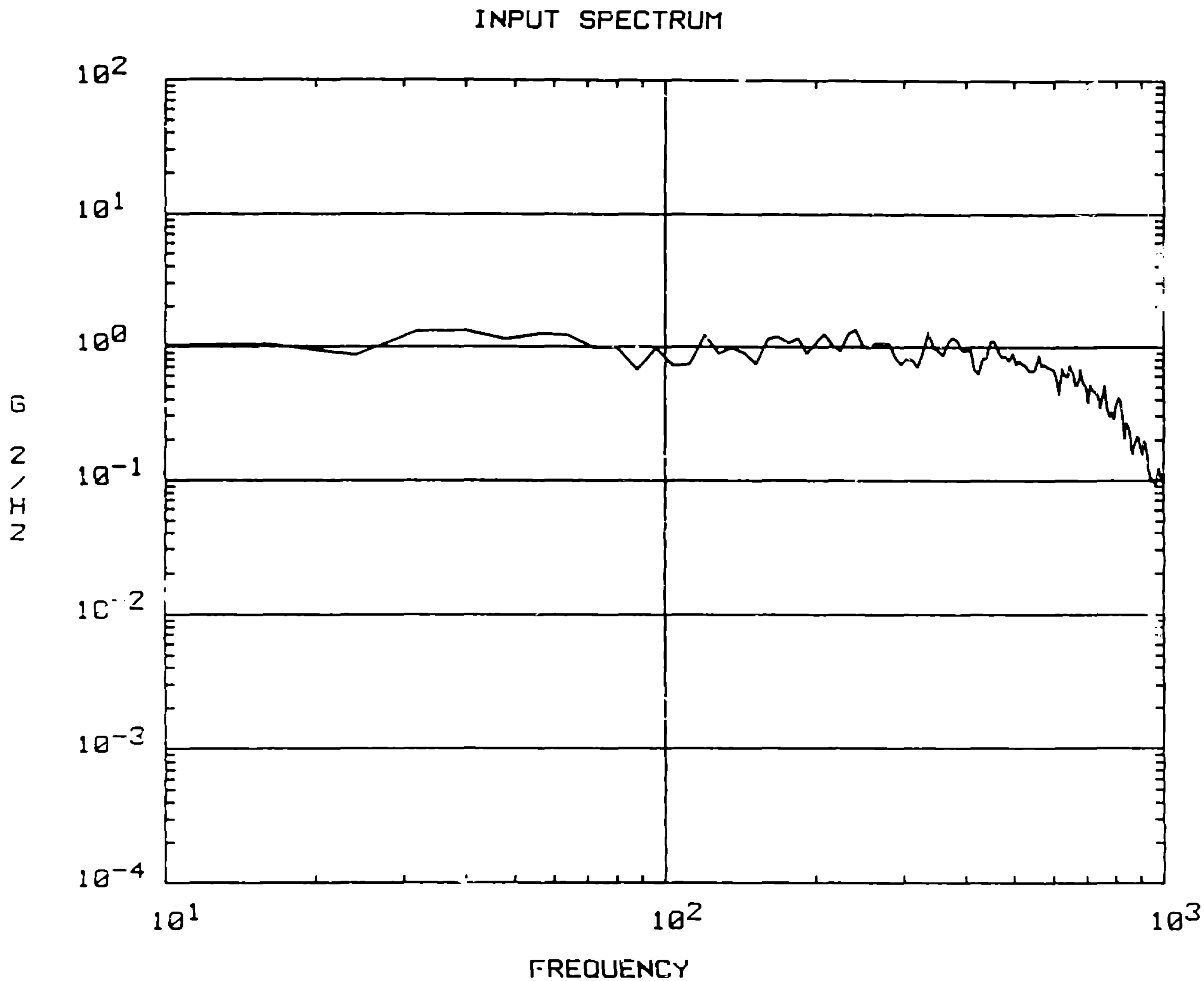


FIGURE 2

INPUT SPECTRAL DENSITY FOR THE CUBIC SYSTEM

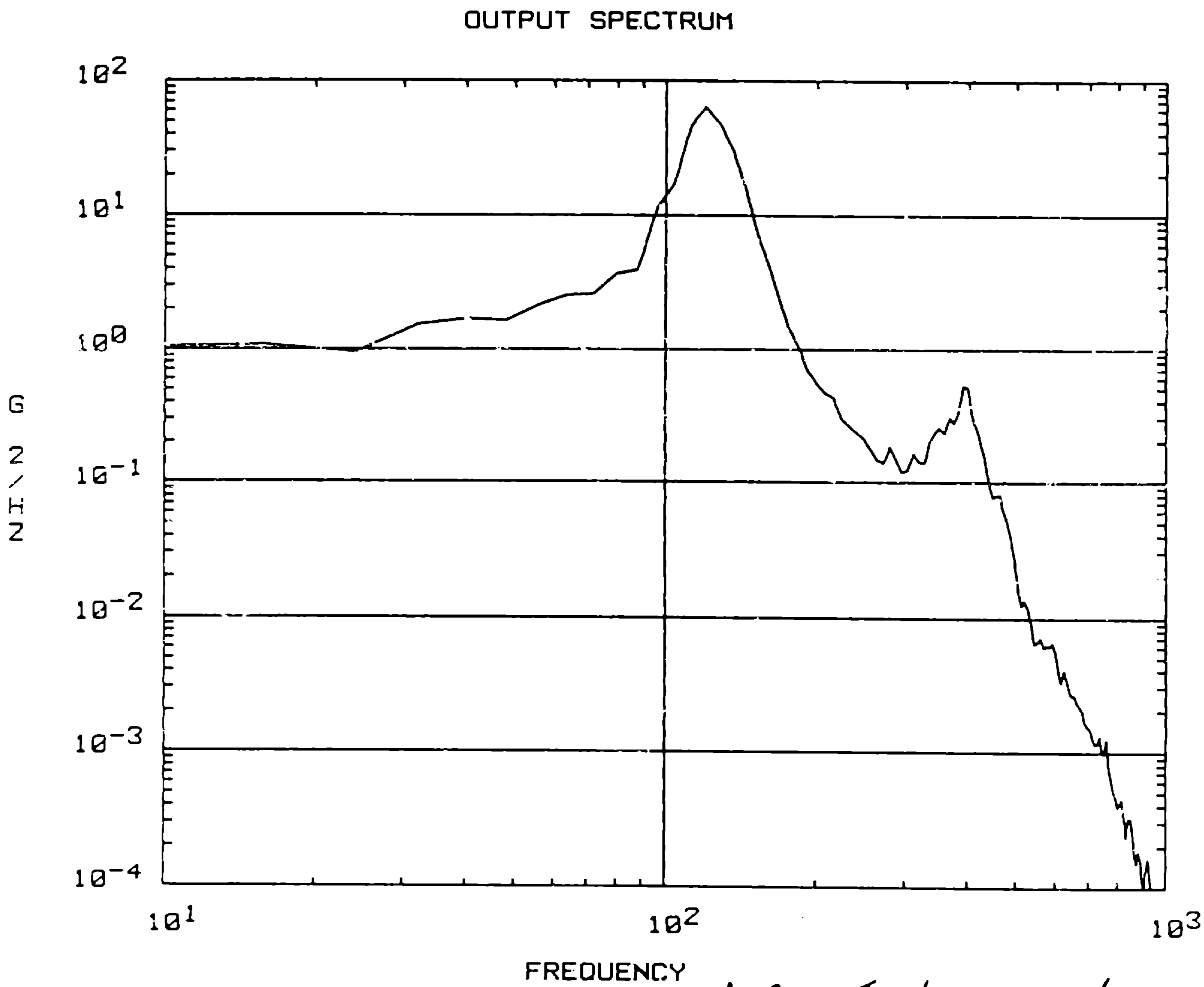
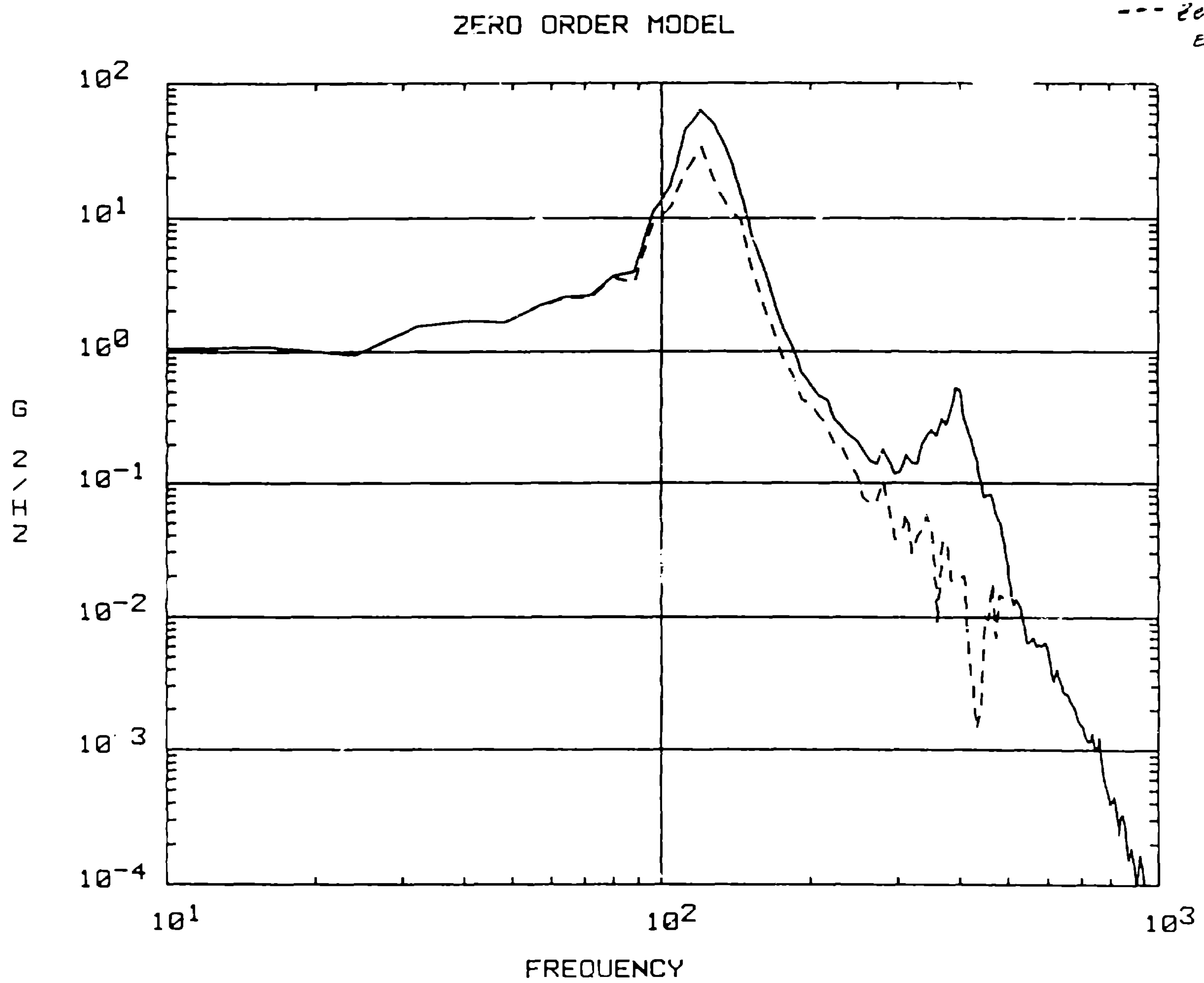


FIGURE 3 SPECTRAL DENSITY OF the Cubic System Output

— cubic system output
--- Zero order model Estimate



Comparison of the Cubic System Output and the Spectral Density
FIGURE [redacted]6 Estimated From the Zero Order Model

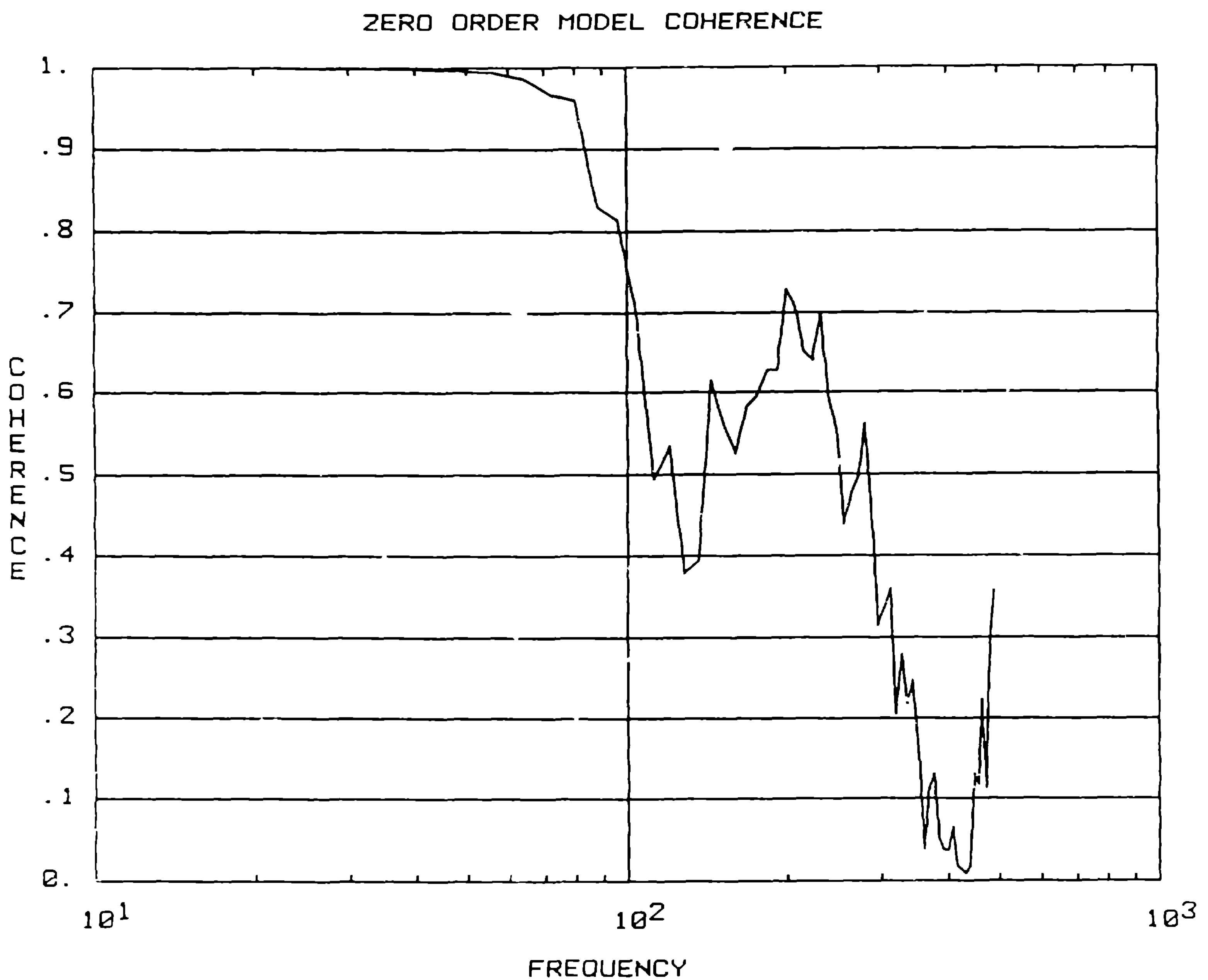


FIGURE 5
Zero Order Model Coherence

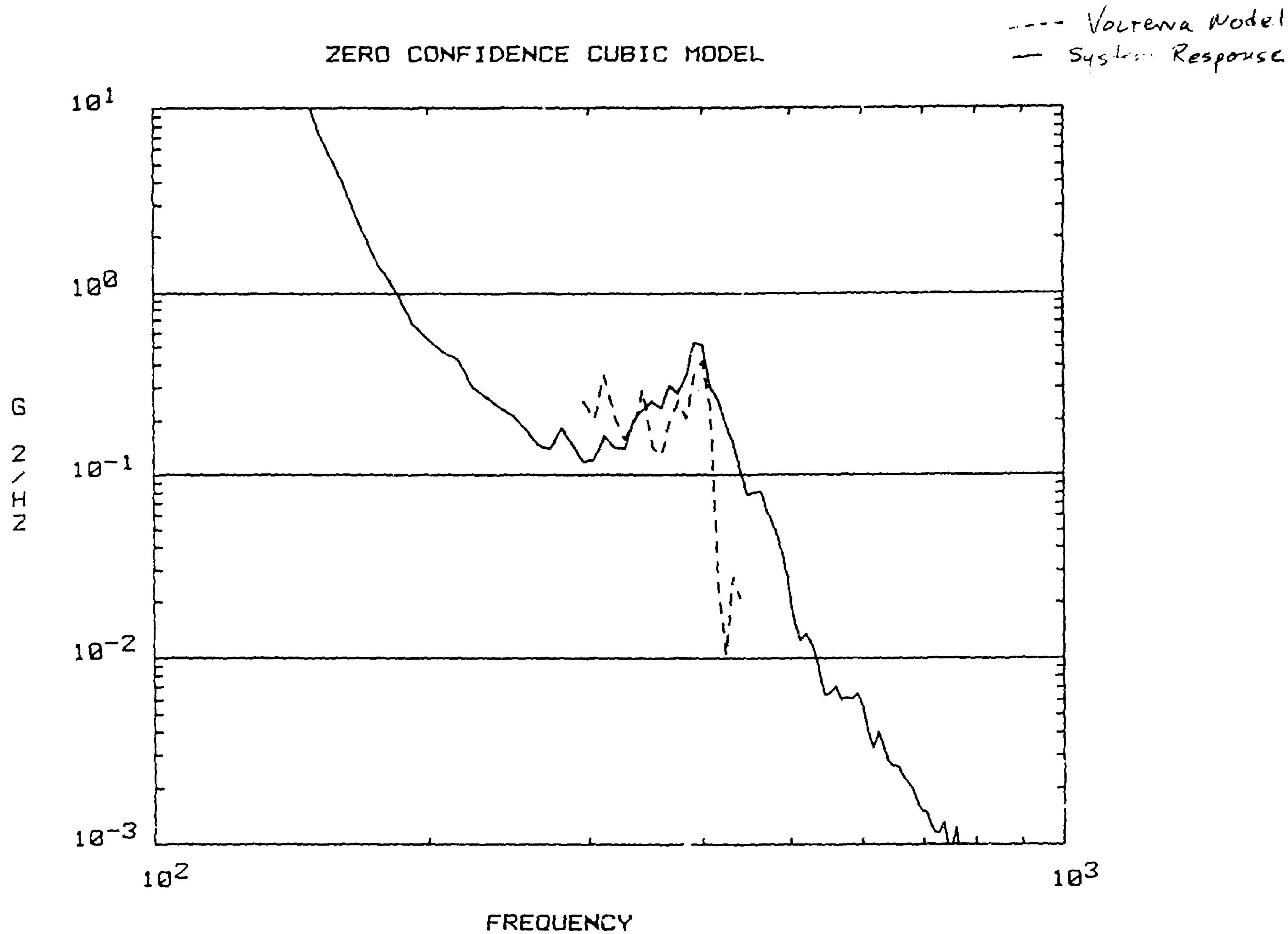


Figure
~~Volterra model results at a zero confidence level~~