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Mixing Angles in SU(2)$_L$ x U(1) Gauge Model

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MASTER

Abstract

Exact expressions for the mixing parameters are obtained in terms of mass ratios in the standard Weinberg-Salam model with permutation symmetry S_3 for six quarks. The CP violating phase is ignored and there are no arbitrary parameters except for the quark masses. In the lowest order, the angles defined by Kobayashi-Maskawa are $\sin\theta_1 = \sin\theta_c = (m_d/m_d + m_s)^{\frac{1}{2}}$, $\sin\theta_2 = -\sin\theta_3 = -m_s^2/m_b^2$ and $m_t m_s \geq m_c m_b = 7.2\ \text{GeV}^2$ or $m_t \geq 24$ GeV for $m_s = 0.3$ GeV.

DISCLAIMER

Recently, several authors have computed the mixing angles in gauge models for six quarks in terms of quark mass ratios.[1-7] The $SU(2)_L \times SU(2)_R \times U(1)$ gauge models are usually supplemented by discrete symmetries and left-right symmetry[2-5] whereas the standard Weinberg-Salam $SU(2)_L \times U(1)$ gauge models[8] are supplemented by permutation symmetry[6] and sometimes also by an additional discrete symmetry.[7] Our purpose is to supplement the $SU(2)_L \times U(1)$ gauge model with permutation symmetry S_3, a simple extension that enables one to compute all three mixing angles. The CP violating phase is ignored.

In the present model the quark mass matrices are generated by a quark Higgs-Yukawa interaction,

$$\mathcal{L} = \sum_{i,j,k=1,2,3} [g^k_{ij}(\bar{Q}_{iL}\phi_k n_{jR}) + h^k_{ij}(\bar{Q}_{iL}\tilde{\phi}_k p_{jR})], \tag{1}$$

where Q_{iL} represent the quark doublets $Q_{1L} = (u_o,d_o)_L$, $Q_{2L} = (c_o,s_o)_L$, $Q_{3L} = (t_o,b_o)_L$, and n_{jR} and p_{iR} represent the quark singlets $n_{jR} = (d_o,s_o,b_o)_R$ and $p_{iR} = (u_o,c_o,t_o)_R$. The coupling constants g^k_{ij} and h^k_{ij} are taken to be real. Three Higgs fields (ϕ_1,ϕ_2,ϕ_3) with vacuum expectation values (v_1,v_2,v_3) are required to implement S_3 symmetry, and $\tilde{\phi}_k = i\sigma_2\phi^+_k$.

Let each combination $\bar{Q}_{iL}n_{jR}$ couple to only one Higgs field and the Higgs field that couples to $\bar{Q}_{1L}n_{1R}$ is defined to be ϕ_1. The application of S_3 symmetry that we use is the permutations $12 \to 21$, $13 \to 31$, and $23 \to 32$. This leads to only three types of down quark mass matrices m^o_2 (and also similarly for the up quark mass matrix, m^o_1) after spontaneous symmetry breakdown, depending on whether the term $\bar{Q}_{1L}n_{2R}$ is coupled to ϕ_1, ϕ_2, or ϕ_3. For the case ϕ_2, we do not obtain a non-trivial unitary transformation to diagonalize the mass matrix.[9] The quark mass terms takes the form

$$\sum_{i=1,2} \bar{\psi}^o_{iL} m^o_i \psi^o_{iR} + \text{h.c.},$$

where $\psi^o_1 = (u_o,c_o,t_o)$, $\psi^o_2 = (d_o,s_o,b_o)$,

$$m_2^o = \begin{pmatrix} fv_1 & gv_3 & gv_2 \\ gv_3 & fv_2 & gv_1 \\ gv_2 & gv_1 & fv_3 \end{pmatrix} , \qquad m_1^o = \begin{pmatrix} kv_1 & jv_1 & jv_1 \\ jv_2 & kv_2 & jv_2 \\ jv_3 & jv_3 & kv_3 \end{pmatrix} . \qquad (2)$$

The alternative assignment of the mass matrices, namely, taking the down quark mass matrix to be m_1^o and the up quark mass matrix to be m_2^o does not lead to interesting physical results. We adjust the parameters in the Higgs potential so that $v_1 = 0$ and divide all matrix elements as well as the quark masses by v_3 and obtain

$$m_2^o = \begin{pmatrix} 0 & g & gr \\ g & fr & 0 \\ gr & 0 & f \end{pmatrix} , \qquad m_1^o = \begin{pmatrix} 0 & 0 & 0 \\ jr & kr & jr \\ j & j & k \end{pmatrix} , \qquad (3)$$

where $r = v_2/v_3$. From m_1^o of Eq. (2), we find that $m_u = 0$ is the necessary and sufficient condition for $v_1 = 0$, provided $r \neq 0$ and $m_c > 0$.

The characteristic equation for m_2 with eigenvalues $(-m_d, m_s, m_b)$ yields

$$\begin{aligned} -m_d + m_s + m_b &= f(1+r), \\ -m_d m_s + m_s m_b - m_d m_b &= f^2 r - g^2(1+r^2), \\ m_d m_s m_b &= g^2 f(1+r^3), \end{aligned} \qquad (4)$$

and for m_1^o with eigenvalues $(0, m_c, m_t)$ yields

$$m_t(1+q) = k(1+r), \quad m_t^2 q = r(k^2 - j^2), \quad q = m_c/m_t . \qquad (5)$$

When f and g are eliminated from (4), we obtain to order m_b^{-5}

$$r = \varepsilon_1(1 + \varepsilon_1\varepsilon_2 + \varepsilon_1^2\varepsilon_2), \qquad (6)$$

where $\epsilon_1 = (m_s - m_d)/m_b$ and $\epsilon_2 = m_s m_d / m_b^2$.

The mass matrices (3) are diagonalized by the following unitary transformations $\psi_2^o = U_2\psi_2$, $\psi_1^o = U_1\psi_1$

$$U_2 = \begin{pmatrix} n_1 & n_2 & n_3 \\ -n_1\dfrac{g}{m_d + fr} & n_2\dfrac{g}{m_s - fr} & n_3\dfrac{g}{m_b - fr} \\ -n_1\dfrac{gr}{m_d + f} & n_2\dfrac{gr}{m_s - f} & n_3\dfrac{gr}{m_b - f} \end{pmatrix}, \quad U_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -N_2(m_c - k) & N_3(m_t - k) \\ 0 & -N_2 j & N_3 j \end{pmatrix}, \tag{7}$$

where

$$n_1 = [1 + (g/m_d + fr)^2 + (gr/m_d + f)^2]^{-\frac{1}{2}},$$

$$n_2 = [1 + (g/m_s - fr)^2 + (gr/m_s - f)^2]^{-\frac{1}{2}},$$

$$n_3 = [1 + (g/m_b - fr)^2 + (gr/m_b - f)^2]^{-\frac{1}{2}},$$

$$N_2 = [(m_c - k)^2 + j^2]^{-\frac{1}{2}}, \quad N_3 = [(m_t - k)^2 + j^2]^{-\frac{1}{2}}. \tag{8}$$

Segre, Weldon and Weyers[6] constructed a similar unitary matrix[10] for U_1 of Eq.(7) and the transformation U_2 corresponds to that of Refs. 3 and 6. When $j = 0$, U_1 is the unit matrix in which case the mixing angles are determined by the down quark mass matrix only. We wish to incorporate the effects of the up quark mass matrix so that the form of U_1 of Eq.(7) is used. When $r = q$ is put into the results obtained below, the case when U_1 is a unit matrix is recovered.

The charged current coupled to physical quarks are $J_{\mu L} = \bar{\psi}_{1L}^o \gamma_\mu \psi_{2L}^o = \bar{\psi}_{1L} \Gamma \gamma_\mu \psi_{2L}$. From Eq.(7), we obtain

$$\Gamma = U_1^{+}U_2 =$$

$$\begin{pmatrix} n_1 & n_2 & n_3 \\ N_2 n_1 g\left(\frac{m_c - k}{m_d + fr} + \frac{rj}{m_d + f}\right) & -N_2 n_2 g\left(\frac{m_c - k}{m_s - fr} + \frac{rj}{m_s - f}\right) & -N_2 n_3 g\left(\frac{m_c - k}{m_b - fr} + \frac{rj}{m_b - f}\right) \\ -N_3 n_1 g\left(\frac{m_t - k}{m_d + fr} + \frac{rj}{m_d + f}\right) & N_3 n_2 g\left(\frac{m_t - k}{m_s - fr} + \frac{rj}{m_s - f}\right) & N_3 n_3 g\left(\frac{m_t - k}{m_b - fr} + \frac{rj}{m_b - f}\right) \end{pmatrix} . \quad (9)$$

This Γ can be identified with the Kobayashi-Maskawa[11] form

$$\Gamma = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & -s_2 s_3 + c_1 c_2 c_3 & c_1 c_2 s_3 + s_2 c_3 \\ s_1 s_2 & -c_2 s_3 - c_1 s_2 c_3 & -c_1 s_2 s_3 + c_2 c_3 \end{pmatrix}, \quad (10)$$

where $c_1 = \cos\theta_1$, $s_1 = \sin\theta_1$ etc., θ_1, θ_2, and θ_3 are the weak mixing angles. To the order, m_b^{-2}, we find the following matrix elements Γ_{ij} with the aid of Eqs.(4)-(10),

$$\Gamma_{11} = c_1, \quad \Gamma_{12} = s_1, \quad \Gamma_{13} = s_1 s_3,$$

$$\Gamma_{21} = -s_1[r/2r - q]^{\frac{1}{2}}, \quad \Gamma_{22} = c_1[r/2r - q]^{\frac{1}{2}}, \quad \Gamma_{23} = -[(r - q)/(2r - q)]^{\frac{1}{2}},$$

$$\Gamma_{31} = -s_1\{[r(r - q)]^{\frac{1}{2}} + (m_s r/m_b)\}, \quad \Gamma_{32} = c_1\{[r(r - q)]^{\frac{1}{2}} - (m_d r/m_b)\}, \quad \Gamma_{33} = 1, \quad (11)$$

where

$$c_1 = (m_s/m_s + m_d)^{\frac{1}{2}}, \quad s_3 = c_1(m_s/m_b)^2,$$

and from $\Gamma_{31} = s_1 s_2$, $s_2 = -c_1^2 (m_s/m_b)^2$ [for $r = q$]. From the reality condition of Γ_{ij}, one finds, $r \geq q$ which leads to

$$m_t m_s \geq m_b m_c = 7.2 (\mathrm{GeV})^2, \tag{12}$$

or $m_t \geq 24$ GeV for $m_s = 0.3$ GeV.

The choice of U_1 is valid in the limit $j/K = \epsilon \approx 0$. When $j/k = \epsilon$ is substituted in Eq.(5), one finds $q = r(1 - \epsilon^2)$ which suggests that $q \approx r$ so that $m_t m_s \approx m_b m_c$ must hold. This $q = r(1 - \epsilon^2)$ is substituted into $\Gamma_{31} = s_1 s_2$ of Eq.(11), then, for $\epsilon = 0$ $(q = r)$ $\theta_2 = 0.2°$ and θ_2 increases with increasing ϵ, for example, $\theta_2 = 1.2°$ for $\epsilon = 0.3$. The mixing angles θ_2 and θ_3 are predicted to be at most a degree, in this application of S_3 symmetry.

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9. The mass matrix that we obtain in this case (with $v_1 = 0$) is the hermitian conjugate of m_1^o in (3). The three eigenvectors are:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad [j^2r^2(m-k+j)^2/m^2 + (m-k)^2 + j^2r^2]^{-\frac{1}{2}} \begin{pmatrix} jr(m-k+j)/m \\ m-k \\ jr \end{pmatrix}$$

with $m = m_c$ or m_t. Putting $j/k = \epsilon$, these become,

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad (1+\epsilon^2+\epsilon^2r^2)^{-\frac{1}{2}} \begin{pmatrix} \epsilon \\ 1 \\ -\epsilon r \end{pmatrix} \text{ and } \quad (1+2\epsilon^2)^{-\frac{1}{2}} \begin{pmatrix} \epsilon \\ \epsilon \\ 1 \end{pmatrix}.$$

We see that these three eigenvectors become orthogonal only for $\epsilon = 0$.

10. The eigenvector of m_1^o of (7) with eigenvalue $m_u = 0$ is $(j+k,-j,-j)/((j+k)^2+2j^2)^{\frac{1}{2}}$ which becomes approximately (1,0,0) when $k >> j$.

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