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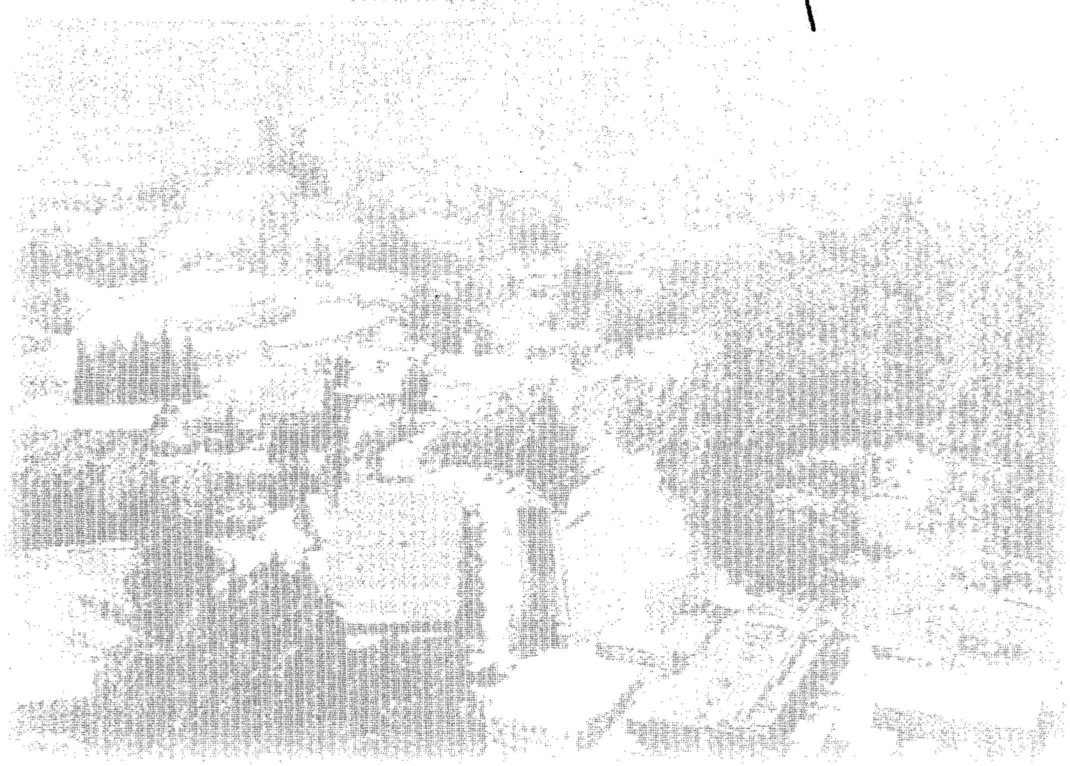

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Abstract

Slug tests have been used for over 30 years as a means of evaluating hydraulic parameters of aquifers. The interpretation of transient water level data from these tests has almost exclusively been based on fitting the data to analytical solutions or on using semi-analytical methods. Because these methods are constrained by simplifying assumptions, it is useful to investigate the conditions under which these assumptions are reasonable so that the interpretation of field data can be carried out with increased confidence. To this end, we investigate the transient flow of water in an unconfined aquifer during a slug test, using a numerical model that solves the generalized Richards' equation. The model accounts for saturated-unsaturated flow, time-dependent seepage face in the well, various combinations of blank casings and well screens, and injection or withdrawal tests. Parametric studies were conducted using a fully penetrating well in a 10 meter thick, homogeneous, isotropic aquifer with an initial hydrostatic condition in order to provide insights into such issues as (1) the difference in response between injection and bail-out tests, (2) the significance of flow through the transient seepage face, and (3) the role of the unsaturated zone. An examination of the flow anatomy suggests that flow in the unsaturated zone is significant and important, although the response of the water level in the well may not be very sensitive to the unsaturated zone processes. A second part of the present study investigated the reasonableness of widely used techniques of interpretation, namely the methods of Cooper et al. (1967), Boast and Kirkham (1971) and Bouwer and Rice (1976). For the limited set of parametric variations considered in this work, the results show that estimated hydraulic conductivities may vary by a factor as much as 2 or more from the true value.

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Introduction

Because of the ease and rapidity with which it can be conducted, the slug test is widely used by both practicing engineers and earth scientists to estimate the hydraulic parameters of aquifers. Many analytical and semi-analytical solutions have been proposed to interpret transient water level data representing the decay of the slug (e.g. Hvorslev, 1951; Cooper *et al.*, 1967; Boast and Kirkham, 1971; Bouwer and Rice, 1976; Nguyen and Pinder, 1984; Karasaki *et al.*, 1988). Mathematically, a very well posed statement of the slug test problem is that of Cooper *et al.* (1967) who considered a horizontal, homogeneous, confined aquifer of uniform thickness. For this problem, the Cooper *et al.* (1967) solution enables the estimation of hydraulic conductivity, K, as well as specific storage, S_s. However, many shallow aquifers are unconfined. In an unconfined aquifer, the upper boundary is the water table, across which the aquifer communicates with the atmosphere, via the vadose zone. A slug test in such an unconfined aquifer leads to a problem that is mathematically far more complicated than that pertaining to the confined aquifer. Current practice for interpreting slug test data from unconfined aquifers ignores flow in the unsaturated zone (Bouwer and Rice, 1976; Hyder and Butler, 1995) in order to simplify the solution process.

Very few papers in the literature deal specifically with slug tests on unconfined aquifers. Two papers that specifically address unconfined aquifers are those of Boast and Kirkham (1971) and Bouwer and Rice (1976). The Boast-Kirkham method is generally used by agronomists and agricultural engineers while the Bouwer-Rice method is better known among groundwater hydrologists. Both methods extend the idea of a variable head permeameter to the field set up. A basic consequence is that although the slug test involves a transient process, the interpretation neglects the specific storage (hydraulic capacitance) parameter. Thus, both methods provide estimates only of hydraulic conductivity. Both methods involve expressing hydraulic conductivity as a product of a shape factor and a rate of change of water level with time. Boast and Kirkham (1971) and Bouwer and Rice (1976) provide tables of values of the shape factor for various configurations to be used in the respective formulae. In both cases, the mathematical analysis is restricted to the saturated flow domain. The water table is treated as a constant potential boundary and no consideration is given to the formation of a seepage face at the well.

The aforesaid methods, developed before the availability of powerful digital computers, resorted to the use of reasonable mathematical approximations (Boast-Kirkham) or empirical approximations (Bouwer-Rice) to solve a problem that was otherwise too difficult to solve. Although these methods were assumed to give hydraulic conductivity estimates of acceptable accuracy, the assumptions inherent in these idealizations have so far not been tested independently. Considering the wide usage of these methods, it is of practical benefit to check the credibility of these methods with an alternate, independent approach so that future interpretations using these methods can be moderated by due judgment.

Accordingly, in the present work we use a numerical model as an independent tool of analysis to study the slug test process in an unconfined aquifer. The model is used to solve a generalized form of the Richards' equation (Narasimhan and Witherspoon, 1976; Narasimhan et al., 1978) which accounts for saturated-unsaturated flow in deformable porous media.

For the present study, the model was modified slightly to handle the time-dependent growth and decay of a seepage face at the well, accompanied by effects of well-bore storage. Fluxes into the well and drainage from the unsaturated zone were recorded as needed. The investigation consisted of two main parts. In the first, the nature of the dynamic flow domain in the vicinity of a well subjected to a slug test was studied to gain insights into the importance of the seepage face, the role of the unsaturated zone and the difference in aquifer response to a slug-withdrawal in comparison to a slug-injection. These insights are based on parametric studies on a single unconfined aquifer of finite thickness. The second part of the study was devoted to testing the accuracy of the hydraulic conductivity estimates from the Boast-Kirkham and Bouwer-Rice methods.

Nature of Flow Regime

A typical unconfined aquifer, initially under hydrostatic conditions, with a single well is shown in Figure 1. Depending on the disposition of the well casing and screen in the well, slug tests can be done in four different ways (Figure 2). If a blank casing exists from land surface to below the water table (Figures 2A and 2B), no seepage face can develop at the well; nor will the well communicate directly with the unsaturated zone. If the well is screened above and below the water table and a slug is injected (Figure 2C), water will move from the well into the unsaturated zone as well as the saturated zone. When the well is fully screened in the aquifer and a slug of water is withdrawn (Figure 2D), a transient seepage face will form in the well. Sometimes slug tests are conducted by simply dropping a rod or closed pipe into the well to raise the water level above the static water level. If the rod so dropped remains completely immersed in the water, the effect is essentially the same as injecting a similar slug of water. However, if the rod or closed pipe is quite long and projects about the water level, then the cross sectional area of the rod or pipe has to be subtracted from the cross sectional area of the casing in calculating the hydraulic capacitance of the well.

In the most general case of a slug test involving a seepage face (Figure 2D), the evolving flow pattern around the well can be described as follows. Assume, for convenience, that a slug of water is removed from the well and that the well is screened below the initial water table. In the beginning, a seepage face will form and grow with time. Water will enter the well through the well screen below the water level in the well as well as through the seepage face above. With time, the length of the seepage face will gradually decrease. Simultaneously, the water level in the well will rise. As time progresses, the water level in the well will catch up with the cone of depression and the seepage face vanishes. At infinite time, the water level in the well will go back to the initial static level.

Upon initiation of the slug, a transient front of potentiometric disturbance will migrate radially outward and upward into the aquifer and into the unsaturated zone. The extent of this front is the domain of influence of the slug test. As the system approaches the hydrostatic state that existed initially, the front will collapse back to the well. Thus the domain of influence will initially increase with time, attain a maximum size, and collapse back to zero. As a result, the "radius of influence" is a function of time; it starts with zero and ends with zero.

During a slug test (injection or bail-out), the quantity of water exchanged between the well and the aquifer will, at early times, be derived from the aquifer's compressible storage. But, after a long period of time, a mass of water equal to the slug of water will be exchanged between the unsaturated zone and the saturated zone. Thus the unsaturated zone plays a fundamental role in the overall flow dynamics accompanying a slug test in an unconfined aquifer.

Theoretical Basis

The governing equation idealizes the isothermal, saturated-unsaturated flow process accompanied by porous medium deformation and follows a generalized form of Richards' equation (Narasimhan and Witherspoon, 1977). In considering deformation of porous media in the unsaturated zone, it is recognized that effective stress decreases rapidly with moisture suction. Therefore, to handle the hydraulic capacitance of the aquifer material, we choose not to use specific storage, S_s. Rather, we use compressibility of the porous medium in conjunction with Bishop's χ-parameter (Narasimhan and Witherspoon, 1977) to account for the weak coupling between effective stress (σ') and moisture suction.

Given this overall physical basis, the equation of mass conservation for an elemental volume, n, in the flow system can be expressed in discrete form as follows,

$$(1) \qquad \sum_m U_{n,m}(\bar{\phi}_m - \bar{\phi}_n) + \sum_b U_{n,b}(\phi_b - \bar{\phi}_n) + \rho_w G_n = M_{c,n}\frac{\Delta\psi_n}{\Delta t} \quad ,$$

in which $U_{n,m}$ is the hydraulic conductance [M/LT] of the interface between volume element n and its neighbor m; $\bar{\phi}_m$ and $\bar{\phi}_n$ are time-averaged values (over a small time interval) of potentiometric head [L]; $U_{n,b}$ is the hydraulic conductance between element n and a boundary element b; ϕ_b is the prescribed potentiometric head at the boundary; ρ_w is mass density [M/L^3] of water; G_n is the volumetric rate of fluid generation [L^3/T]; $M_{c,n}$ is the hydraulic capacitance [M/L] of element n; ψ_n is the average change in gauge pressure-head [L] over volume element n and Δt is change in time[T]. In an axisymmetric system, a volume element is a cylindrical annulus. For two such adjoining cylindrical elements, (1) leads to,

(2) $$U_{n,m} = \frac{2\pi K H}{\ln\left(\frac{r_m}{r_n}\right)}, \quad r_m > r_n$$

in the radial direction, where K is hydraulic conductivity [L/T] which is a function of pressure head in the unsaturated zone, H is the thickness [L] of the cylindrical surface between elements n and m, and r_m and r_n are the radial distances [L] to the nodal points of elements n and m. In the vertical direction, considering the fact that the base area of a cylindrical element does not change with elevation, hydraulic conductance is defined using conventional finite differences as follows,

(3) $$U_{n,m} = \frac{\pi K (r_{out}^2 - r_{in}^2)}{|z_m - z_n|},$$

where, r_{out} and r_{in} are respectively the radial distances to the outer and inner surfaces of the annulus from the well axis, z_m and z_n are the vertical elevations of the mid planes of elements m and n. In (1), $M_{c,n}$ is the hydraulic capacitance of element n which denotes the change in fluid mass stored in element n accompanying a unit change in pressure head and can be expressed as,

(4) $$M_{c,n} = V_{s,n}\rho_w\left[\gamma_w S_w e\beta + \gamma_w S_w \chi' a_v + e\frac{dS_w}{d\psi}\right],$$

in which, V_s is
the volume of solids [L^3] contained in n; γ_w is unit weight [M/LT^2] of water; S_w water

saturation; e is void ratio; β is compressibility [LT^2/M] of water; $\chi' = \chi + d\chi/d\psi$, where χ is Bishop's parameter (Narasimhan and Witherspoon, 1977); $a_v = -(de/d\sigma')$ is the coefficient of compressibility (Lambe and Whitman, 1969).

The governing equations presented above constitute a special case of multi-phase flow in which the air phase is considered to be always at atmospheric pressure. Moreover, both for reasons of mathematical necessity and of data deficiency, this study ignores effects of hysteresis in relation to soil moisture characteristic as well as effective hydraulic conductivity under unsaturated conditions.

For the slug test problem it is assumed that the land surface as well as the horizontal surface bounding the bottom of the aquifer are impermeable to water. It is also assumed that the aquifer extends to infinity in the radial direction. The well itself is a volume element with a hydraulic capacitance of $M_{c,w} = \pi r_w^2$ where r_w is well radius. The only other boundary condition to address is the well screen above the water level in the well. Two cases are relevant here. The first pertains to the well screen below the initial water table and above the well water level during a bail-out (Figure 2D) while the second relates to the well screen above the initial water table and below the water level in the well during an injection test (Figure 2C).

We now discuss the special conditions that arise in applying equations (1) - (4) to the slug test problem. Consider first the bail-out test in Figure 2D. Point C denotes the time dependent intersection of the water level in the well with the well screen. Point D denotes the time dependent intersection of the water table with the well screen Point E denotes the intersection of the initial water table with the well screen. Along the inside of the well screen below point C the potentiometric head is ϕ_w, corresponding to the water level in the well. During a bail-out test, the potentiometric head in the well is less than the potentiometric head in the aquifer and water will flow into the well below point C. Between points C and D, the potentiometric head at the inside of the well screen is equal to the elevation because gauge pressure head at this boundary is zero($\psi_b = 0$). The corresponding potentiometric head in the aquifer is greater than elevation because gauge pressure head is positive. Thus, line CD is a time-dependent seepage face across which water flows into the well. Between D and E no water can flow into the well because water in the aquifer is held in the pores by moisture suction and hence, DE is a time dependent impermeable boundary.

Now consider the injection scenario shown in Figure 2C. Adjacent to point B, the formation is partially saturated and the potentiometric head in the aquifer at B is less than the elevation at point B (z_b). On the other hand there is a column of water existing above the point so that the potentiometric head inside the well is greater than z_b. Therefore water moves from the well into the formation at point B. However, at point A, the gauge pressure head is less than zero whereas the gauge pressure head on the inside of the screen in zero. Consequently, water is held by capillary forces in the aquifer and no water flows across the screen at A. Nevertheless, it must be noted that early during the test when the water level was above point A, some water did move to the well across A. Thus, in both cases (Figures 2C and 2D), the boundary conditions have geometrical dispositions that change with time.

The discretized equations governing transient flow of water given in Eq. (1) - (4) have been incorporated into the computer program TRUST (Narasimhan *et al.*, 1978) that is based on an integral-finite difference philosophy (Narasimhan and Witherspoon, 1976), assuring mass conservation as a necessary condition. Consequently, the computational output not only provides time dependent changes in potential for each cell, but also enables the evaluation of redistribution of water mass in the flow domain as a function of time. The model considers both saturated and unsaturated radial flow in a cylindrical system. To satisfy the specific needs of the present work, the program was modified to handle the time dependent boundary conditions relating to an evolving seepage face at the well (Figures 2D) and the flow of water directly into the unsaturated zone (Figure 2C).

Simulation Studies

Cases Studied

Parametric studies serve as useful means to gain insights into significant processes associated with slug tests. These processes include the fluxes of water into the well, the origination point of water that enters the well, the zone of influence, and mechanisms of storage. Shallow unconfined aquifers are of wide-ranging interest in the fields of civil engineering, groundwater hydrology and environmental engineering. To carry out parametric studies or sensitivity analysis to cover field conditions of interest in all these fields is neither realistic nor necessary. Our purpose in this study is merely to generate some useful insights on the basis of a hypothetical system whose dimensions and attributes are assumed to be reasonable. Accordingly, five series of numerical experiments, summarized in Table 1, were performed.

The numerical experiments presented below considered a 10-meter thick aquifer (H_{total} = 10 m, Figure 1) with a 0.08-meter radius well. The saturated portion of the horizontal aquifer was 6.0 meters thick The porous medium (a medium-grained sand) was assigned a void ratio of 0.429 (porosity = 0.3) at a reference effective stress of 1×10^5 Pa, a coefficient of compressibility (a_v) of 1×10^{-7} m^2/N, and a saturated permeability of 4.42×10^{-13} m^2. Under fully saturated conditions, the designated a_v leads to a specific storage (S_s) of approximately 1×10^{-3} m^{-1}. The assigned permeability yields a saturated hydraulic conductivity of 4.32×10^{-6} m/sec. The interdependence of pressure head, saturation, and permeability are shown in Figures 3A and 3B. The coupling between gauge pressure-head and effective stress in the unsaturated zone is known to be weak in coarse-grained soils. However, data pertaining to this coupling are hard to come by. In the absence of such data, we have arbitrarily specified the effective stress versus pressure-head relation in such a way that $\chi = 0$ for $\psi < -0.5$ m and $\chi = 1$ for $\psi \geq -0.1$ m. Effective stress was allowed to vary linearly in the range -0.5 m $< \psi \leq -0.1$ m.

We restricted our study to an isotropic material partly for convenience and partly for philosophical reasons. Slug test observations are invariably restricted to the well itself and do not involve observation wells. Mathematically it is not possible to interpret single-well test data for anisotropy; data from two or more observation well data are needed to enable interpretation of anisotropy. Secondly, in the presence of vertical infiltration from the unsaturated zone above, anisotropy (should it exist) may also be masked by pseudo anisotropic effects. Therefore, care needs to be exercised in studying the role of anisotropy. The role of anisotropy probably merits an independent investigation.

Results of Parametric Studies

Case 1: Model Verification: Confined Aquifer

The slug injection and bail-out tests that we consider in Case 1 were designed to conform to assumptions employed by Cooper *et al.* (1967). Injections of 1-, 2- and 3-meter slugs of water and bail-outs of 1-, 2-, 3-, 4- and 5-meter slugs of water were numerically simulated. The confined aquifer was 6.0 meters thick, the well was fully penetrating. To faciltate easy comparison with the table of values presented by Cooper et al. (1967), a_v was set equal to 1.698×10^{-7} Pa^{-1} so that the storage coefficient S works out to 1×10^{-2}. The initial potential in the well was set sufficiently high so that even during bailout tests the well screen remained fully submerged.

The Cooper *et al.* (1967) analytical solution enables one to calculate the normalized head in the well, H/H_o as a function of time for various values of the storage coefficient S (= S_sH). In Figure 4, plots of H/H_0 generated numerically are shown compared with the analytical solution of Cooper et al. (1967). In the confined aquifer case, injection and bail-out tests are essentially the same, except for the change in sign. Therefore, all the eight results fall on the same curve and cannot be distinguished. As can be seen in the figure, the numerical results agree very well with the analytical solution. These results show that the numerical model replicates the analytic solution with acceptable accuracy.

Case 2: Slug Injection in an Unconfined Aquifer

The slug injection simulations considered in Case 2 test the response in an unconfined aquifer in which a seepage face does not form. Injections of 1-, 2- and 3- meter slugs of water (Figure 2A) were numerically simulated. The well was screened for 6.0 meters in each case. As in Case 1, a_v was set to 1.698×10^{-7} Pa^{-1} so that $S = 1 \times 10^{-2}$. Whereas, in case 1, the upper boundary of the aquifer was impermeable, the upper boundary of the aquifer in case 2 is the water table, which communicates with the unsaturated zone above.

The simulated responses for the injection scenarios are presented in Figure 5 along with Cooper *et al.* solution for a confined aquifer with the same geometry, hydraulic conductivity, and specific storage values as those used in the Case 1 simulations. The closeness of these solution with those of Case 1 (Figure 4) shows that the decay of the slug in the well is not very sensitive to the conditions on the upper boundary of the aquifer.

This finding is of practical interest. Under conditions of injection in an unconfined aquifer or even under conditions when the seepage face may be very small, the Cooper et al. method can be reasonably used to estimate hydraulic parameters. One need not summarily reject the use of the Cooper et al. method for unconfined aquifers. In addition to being mathematically rigorous, this method also provides estimates of storage coefficient.

Case 3: Richards' Equation Perspective

The simulations of Case 3 were designed to throw light on the general consequences of implementing Richards' equation as a whole. The computer program was modified to allow a seepage face to form during the course of both an injection and a bail-out slug test. Injections of 1, 2 and 3 meters of water (Figure 2C) and bail-outs of 1, 2, 3, 4 and 5 meters of water (Figure 2D) were numerically simulated. The simulated responses for five of the Case 3 scenarios are presented in Figure 6. Also presented, for comparison and reference, is the Cooper *et al.* solution for a confined aquifer with the same geometry, hydraulic conductivity, and approximately the same storage coefficient values as those used in Case 1 simulations.

At the outset, it is easy to see that the curves for all the cases are mutually distinct. This indicates that the response of a well to a slug test in an unconfined aquifer depends on (a) whether the slug is injected or withdrawn and (b) the size of the slug itself. This finding is at variance with the basis of conventional methods of analyses such as those of Cooper *et al.*, Boast and Kirkham, or Bouwer and Rice. Implicit in these methods is the assumption that injection and withdrawal tests should give symmetric results. Therefore, estimates of parameters generated with these traditional methods should be expected to lead to different

ets of parameters for different slug tests conducted on the same unconfined aquifer. Note also that the Cooper *et al.* solution, shown by dotted symbols in Figure 5, cuts across the simulated curves and does not match with any one of them.

To provide an idea of the role of the seepage face, the fluxes across the seepage face for the eight scenarios of Case 3 are presented in Table 2. As can be seen, depending upon the height of the slug as compared with the saturated thickness of the aquifer, more than a quarter of the flux interchanged between the well and the aquifer may pass through the seepage face under some conditions. The total flux shown pertains to the end time defined to be the time when the water level in the well returns to equilibrium conditions.

From a process point of view, the results pertaining to Case 3 provide a comprehensive perception of the movement and storage of water in the vicinity of the well and the response of water level in the well during slug test in an unconfined aquifer. This comprehensive understanding is of considerable value in exercising judgement about the usefulness of other mathematical methods which are based on many restrictive assumptions pertaining to the key processes. The three injection scenarios show that the well does communicate directly with the unsaturated zone when conditions permit and that the degree communication increases with the size of the slug. So also, during slug withdrawal tests the role of the seepage face could be quite significant. Thus, the flow regime around a well in an unconfined aquifer is strongly three-dimensional during a slug test and the role of the unsaturated zone may not be negligible.

It is commonly assumed that the unsaturated zone can be ignored in the interpretation of data from unconfined aquifers slug tests (Bouwer and Rice, 1976; Hyder and Butler, 1995). If indeed flow in the unsaturated zone is unimportant, then the thickness of the unsaturated zone should have no effect on slug decay. Case 4 involved several simulations in an unconfined aquifer having a thin (0.5-m thick) unsaturated zone.

Case 4: Aquifer with a Thin Unsaturated Zone

In order to evaluate the role of the unsaturated zone during a slug test in an unconfined aquifer, it is necessary to set some criteria for the purpose. During a slug test, the system starts with an initial hydrostatic condition and after a transient period returns to that hydrostatic condition. Under these conditions, the change in storage within the aquifer is initially accommodated by the elastic properties of the aquifer. However, with time, the elastic change in storage in the aquifer is compensated by transfer of water between the unsaturated zone and the saturated zone. In other words, subject to a time lag, a mass of water, almost equal to the mass of the slug is ultimately accounted for by change in storage in the unsaturated zone. Thus, one criterion to evaluate the role of the unsaturated zone is to understand where the change in water storage ultimately occurs within the aquifer system.

Simulations pertaining to an aquifer with a thin unsaturated zone show that the fluxes into the well, tabulated in Table 3, differ from the results shown in Table 2 by less than one percent. This comparison indicates that the thickness of the unsaturated zone does not significantly influence the fluxes to the well, which are dominated at early and intermediate times by water derived from the saturated zone. However, further examination of the anatomy of the flow regime showed that significantly more water is transferred from the unsaturated zone to the saturated zone from the 4-m thick unsaturated zone (Case 3) than the 0.5-m thick unsaturated zone. Figure 7 shows the time dependent variation of cumulative vertical flux crossing the horizontal plane of the initial water table for the 4-meter slug injection case. This flux represents the amount of water dynamically transferred from the saturated zone to the unsaturated zone. In the case of the thick unsaturated zone, the mass of water that ultimately leaves the well is almost equal to the vertical flux of water from the saturated zone to the unsaturated zone, subject to a time lag. Thus, almost all the slug is accounted for by change of storage in the unsaturated zone. However, in the case of the thin unsaturated zone, a significant portion (about 30%) of the slug is accounted for by change in storage in the saturated zone. Note also in Figure 7 that after about an hour into the test, the differences in the contribution of water from the unsaturated zone are clearly discernible between the two cases. Although the rate at which the slug decays in the well is insensitive to the thickness of the unsaturated zone, it is apparent that the attributes of the unsaturated zone cannot be ignored in a broader context. For example, the role of the unsaturated zone could be quite important if one were interested in contaminant transport within the aquifer.

Case 5: Slug Tests in the Presence of Well Skin

A zone of altered hydraulic conductivity (well skin) can significantly influence the decay of the slug and in turn affect estimates of aquifer hydraulic conductivity (Faust and Mercer, 1984; Sageev, 1986). Several hypotheses exist regarding the proper treatment of slug test data when a well skin is present. At early times when the radius of influence is small, the pressure transient in the well will reflect the large resistance offered by the low-permeability skin material. At later times, as the pressure perturbance encompasses increasingly larger volumes of the aquifer, the total resistance will be far larger than the resistance offered by the skin zone. Hence, the late time response of the well will reflect the transmissivity of the aquifer. If a significant region around the well-bore is damaged during well drilling, the slug test would reflect the transmissivity of the silty material rather than the aquifer transmissivity (Moench and Hsieh, 1985).

A well skin factor, S_{skin}, is defined in the petroleum engineering literature as (Earlougher, 1977),

(5)
$$s_{skin} = \left(\frac{K_{aquifer}}{K_{skin}} - 1 \right) \ln \frac{r_{skin}}{r_w} \quad .$$

A positive well skin factor implies a degradation of permeability close to the well screen while a negative well skin factor implies enhanced permeability (e.g. a gravel pack).

In view of the significance of the well skin when interpreting slug testdata, Cases 5A and 5B examine slug tests for both low and high permeability well skins with a thickness of 0.0763 meter $r_{skin} = 0.156$ m). The damaged skin zone was assigned a permeability two orders of magnitude less than that of the aquifer. For the gravel pack scenario the material was assigned a permeability two orders of magnitude greater than that of the aquifer. The eight scenarios of Case 3 were repeated with these materials in place. The skin factor worked out to about 143 for Case 5A and -0.71 for Case 5B.

Consider first the damaged well skin. Figure 8 shows mutually distinct curves that are shifted to later times than the comparable curves in Figure 6. This is to be expected because a low permeability skin inhibits the movement of water from the aquifer into the well. To understand the role of the seepage face in the presence of a well skin, the fluxes across the seepage face for the eight scenarios are presented in Table 4. As can be seen, the seepage fluxes tend to be somewhat smaller in the presence of a skin, especially under slug injection.

For the gravel pack scenario, Figure 9 shows that response curves shift to earlier times than the comparable curves in Figure 6. Depending on the height of the slug relative to the hydrostatic conditions of the aquifer, Table 5 shows that a greater percentage of flow occurs through the seepage face for both bail-out and injection tests compared to Table 2. Here , the fluxes across the seepage face are enhanced, especially under slug injection.

Comparison with Bouwer-Rice and Boast-Kirkham Methods

The Bouwer-Rice and Boast-Kirkham methods provide simple means for the estimation of hydraulic conductivity from the temporal response in the well caused by a slug test in an unconfined aquifer. We analyzed our simulated results using the Bouwer-Rice and the Boast-Kirkham methods so as to compare the estimates so obtained with the values used in the simulations. Cooper et al. and Bouwer-Rice estimates for hydraulic conductivities were obtained through the computer program AQTESOLVE® (Duffield, 1996). A best-fit approximation of the data was used for all Boast-Kirkham solutions. In all cases the value of hydraulic conductivity used in the simulations was 4.32×10^{-6} m/sec.

The simulated data for the confined aquifer test of Case 1 yielded hydraulic conductivity estimates of 4.75×10^{-6} m/sec (Cooper *et al.*) and 4.48×10^{-6} m/sec (Bouwer-Rice). Note that the Bouwer-Rice method (1976) specifically pertains to an unconfined aquifer. Yet, the estimate obtained with this method for a confined aquifer is quite good. This agreement corroborates our earlier finding under Case 2 that the decay of the slug in the well is insensitive to the boundary conditions on the upper surface of the aquifer. The simulated results for Case 2 in an unconfined aquifer yielded a hydraulic conductivity estimate of 4.37

x 10^{-6} m/sec using the Bouwer-Rice method, which is nearly identical to the actual hydraulic conductivity.

The simulated results for Cases 3, 4, 5A, and 5B were analyzed using the Bouwer-Rice method and the Boast-Kirkham methods. The estimates for hydraulic conductivities so obtained are shown in Table 6. Looking at the injection and bail-out scenarios of Case 3, it is seen that the Bouwer-Rice method yields estimates varying from 65% to 185% of the "true" value of 4.32 x 10^{-6} m/sec. The Boast-Krikham method yields estimates for the 5 bail-out tests which vary from about 132% to about 166%. From this limited study it is reasonable to expect that estimates of unconfined aquifer hydraulic conductivity using these methods can deviate from "actual" value by a factor of about two.

As discussed earlier, the decay of head within the well is not very sensitive to the thickness of the unsaturated zone. Consequently, we see in Table 6 that the estimates for Case 4 are almost identical to the estimates for Case 3.

The results presented for Case 5A and Case 5B show that the estimates of hydraulic conductivity by the Bouwer-Rice method and the Boast-Kirkham method can be significantly influenced by the permeability of the material in the immediate vicinity of the well. In the presence of near-well heterogeneities, the estimates could be off by a factor of 4 to 10. In other words, the two methods provide estimates of materials close to the well bore.

Discussion

In the conceptualization of the process in the vicinity of a well subjected to a slug test in an an unconfined aquifer, the common practice is to restrict attention to flow in the saturated zone and treat the upper boundary of the aquifer as a constant potential boundary. The simulation results presented in the foregoing pages show that the flow pattern around a well, under these conditions, involves the saturated zone as well as the unsaturated zone, with the flow dynamics largely influenced by the saturated zone at early times and the unsaturated zone at late times.

The positioning of the well screen and its length in an unconfined aquifer play a very important role in a slug test. The attributes of the well screen determine whether a seepage face will form inside the well during a bail-out test or whether the well may directly communicate with the unsaturated zone following slug injection. Because of the possibility of formation of a seepage face and the possibility of direct communication between the well and the unsaturated zone, the flow geometry under conditions of bail-out and injection are significantly different. Moreover, the well response is also influenced by the size of the slug itself. Although one may readily recognize the relevance of these physical processes qualitatively, their quantitative treatment is mathematically quite cumbersome. At the present time these processes cannot be adequately handled by analytical solutions. Nevertheless, numerical models offer a means by which these systems can be quantitatively understood.

From a practical point of view, earth scientists and engineers have for decades had an important need to hydraulically characterize these systems, even if only approximately. The methods of Bouwer-Rice and of Boast-Kirkham came into existence to satisfy this practical need at a time when the personal computer revolution had not yet occurred. Yet, as we have seen, these methods give estimates of hydraulic conductivity within a factor of 2 or more. Experienced earth scientists and engineers who have a sense for the complexity and inaccessibility of the earth's subsurface often feel satisfied with estimates that are accurate within an order of magnitude. Such being the case, one may conclude that the Bouwer-Rice method and the Boast-Kirkham methods "work" quite well.

The reason why the Bouwer-Rice method and the Boast-Kirkham method seem to "work" despite the complexity of the actual flow process is that in a radial flow system, the change of the water level in the well is rather insensitive to the complexities of the flow dynamics within the aquifer. Thus, although methods such as those of Bouwer-Rice or Boast-Kirkham have served us admirably as inexpensive practical tools, it is useful for us to remember that they do not adequately account for relevant processes. Neither of these methods pertain to a well-defined problem from a mathematical point of view.

It is quite common to think about the effects of anisotropy in the context of slug tests. In an elegant paper during the 1960's, Papadopulos (1965) showed that two or more observation wells are needed to interpret data from pumping tests in terms of anisotropy. Analogously, the role of anisotropy cannot be reasonably interpreted unless the slug methodology is extended to include observation wells. However, slug tests involving interference between wells are not commonly known in the literature. Should sufficient motivation arise in the future, it should be of interest to investigate the role of anisotropy during slug tests in unconfined aquifers, giving due consideration to the vertical movement of water from the unsaturated zone to the saturated zone.

With the availability of sophisticated pressure measuring devices and automatic data loggers, we are now in a position to collect data of considerable time resolution not only from the well within which the slug is introduced but also in neighboring observation wells and piezometers. It stands to reason that these data are potentially capable of helping us understand the hydraulic structure of the aquifer far better detail than hitherto possible. However, interpretation of these observations will necessarily entail a conceptualization that is more refined in process content than the traditional simplifications that we have relied upon. Fortunately, numerical models that can solve the Richards equation rapidly on a desk-top personal computer can help us move forward. As we greatly extend our ability to gather more and more field data in space and in time, we must have the ability to interpret the data with a minimum set of assumptions. It is quite limiting to have a sophisticated data set from the field but not have an equally sophisticated interpretive tool with which analyze the data. We have the methodology and the technology to economically match the sophistication of field data with the sophistication of interpretation. Numerical codes (such as TRUST which was used in this study) can solve the Richards equation as applied to slug tests in an unconfined aquifer in a matter of seconds to minutes on a lap-top computer. We need to invest the time to make these tools help us interpret complex field experiments so that we can hydraulically characterize unconfined aquifers in greater and greater detail.

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Table 1. Slug tests: Cases studied

Case No.	Aquifer Thickness (m)	Unsaturated Zone Thickness (m)	Screened Interval (m)	Seepage Face Allowed	Type of Test	Slug Sizes (m)	Remarks
1	6.0	0.0	0.0 to 6.0	No	Injection Bailout	1, 2, 3 1, 2, 3, 4, 5	Simulations in a confined aquifer to compare against Cooper *et al.* type curve solutions.
2	10.0	4.0	0.0 to 6.0	No	Injection	1, 2, 3	Simulations in an unconfined aquifer with not direct communication between the well and the unsaturated zone.
3	10.0	4.0	0.0 to 10.0	Yes	Injection Bailout	1, 2, 3 1, 2, 3, 4, 5	Simulations in an unconfined aquifer with a transient seepage face.
4	6.5	0.5	0.0 to 7.0	Yes	Bail out	1, 2, 3, 4, 5	Simulations in an unconfined aquifer with a thin unsaturated zone to quantify vertical fluxes.
5A	10.0	4.0	0.0 to 10.0	Yes	Injection Bailout	1, 2, 3 1, 2, 3, 4, 5	Evaluation of damaged (low permeability) well skin effects in an unconfined aquifer.
5B	10.0	4.0	0.0 to 10.0	Yes	Injection Bailout	1, 2, 3 1, 2, 3, 4, 5	Evaluation of gravel pack (high permeability) well skin effects in an unconfined aquifer.

Table 2. Total Fluxes with Seepage Face Present

Slug size	Seepage face flux (kg)	Saturated flux (kg)	Total flux (kg)	Seepage face flux as % of Total flux
5m Bail-out	26.45	74.41	100.86	26.23%
4m Bail-out	15.19	65.60	80.79	18.81%
3m Bail-out	7.43	53.22	60.64	12.25%
2m Bail-out	2.89	37.55	40.44	7.16%
1m Bail-out	0.55	19.79	20.34	2.71%
1m Injection	-0.41	-19.70	-20.11	2.02%
2m Injection	-2.62	-37.59	-40.21	6.52%
3m Injection	-6.12	-54.20	-60.32	10.14%

Table 3. Total Fluxes: Aquifer with Thin Unsaturated Zone

Slug size	Seepage face flux (kg)	Saturated flux (kg)	Total flux (kg)	Percent seepage face flux
5m Bail-out	26.82	73.91	100.74	26.63%
4m Bail-out	15.24	65.46	80.70	18.89%
3m Bail-out	7.52	53.02	60.55	12.43%
2m Bail-out	2.88	37.50	40.39	7.14%
1m Bail-out	0.55	19.76	20.30	2.70%

Table 4. Total Fluxes for a Lower Permeability Well Skin

Slug size	Seepage face flux (kg)	Saturated flux (kg)	Total flux (kg)	Seepage face flux % of Total flux
5m Bail-out	25.75	75.13	100.88	25.53%
4m Bail-out	14.43	66.31	80.75	17.88%
3m Bail-out	6.78	53.83	60.61	11.18%
2m Bail-out	2.54	37.91	40.45	6.28%
1m Bail-out	0.34	19.93	20.28	1.69%
1m Injection	-0.12	-19.99	-20.11	0.59%
2m Injection	-1.03	-39.18	-40.21	2.56%
3m Injection	-1.82	-58.50	-60.32	3.01%

Table 5. Total Fluxes for a Higher Permeability Well Skin

Slug size	Seepage face flux (kg)	Saturated flux (kg)	Total flux (kg)	Seepage face flux % of Total flux
5m Bail-out	26.52	74.57	101.09	26.24%
4m Bail-out	15.48	65.42	80.90	19.13%
3m Bail-out	7.95	52.76	60.70	13.09%
2m Bail-out	3.22	37.27	40.49	7.96%
1m Bail-out	0.69	19.61	20.30	3.39%
1m Injection	-1.10	-19.00	-20.11	5.48%
2m Injection	-4.97	-35.35	-40.21	12.32%
3m Injection	-11.54	-48.78	-60.32	19.13%

Table 6. Estimated Hydraulic Conductivity using Bouwer-Rice and Boast-Kirkham methods.

	Case 3		Case 4		Case 5A Damaged		Case 5B Gravel Pack	
Slug size (m)	Bouwer-Rice K (m/s)	Kirkham-Boast K (m/s)	Bouwer-Rice K (m/s)	Kirkham-Boast K (m/s)	Bouwer-Rice K (m/s)	Kirkham-Boast K (m/s)	Bouwer-Rice K (m/s)	Kirkham-Boast K (m/s)
-5.0	2.82E-6	5.72E-6	2.83E-6	5.75E-6	1.53E-7	2.38E-7	3.89E-6	8.17E-6
-4.0	3.06E-6	6.02E-6	3.08E-6	6.00E-6	1.66E-7	2.43E-7	4.28E-6	8.68E-6
-3.0	3.32E-6	6.27E-6	3.33E-6	6.26E-6	1.77E-7	2.48E-7	4.71E-6	9.24E-6
-2.0	3.60E-6	6.51E-6	3.59E-6	6.51E-6	1.89E-7	2.54E-7	5.16E-6	9.87E-6
-1.0	3.88E-6	6.77E-6	3.88E-6	6.77E-6	2.00E-7	2.59E-7	5.76E-6	1.07E-5
1.0	5.44E-6				2.44E-7		7.90E-6	
2.0	6.87E-6				2.69E-7		1.01E-5	
3.0	8.05E-6				3.02E-7		1.32E-5	

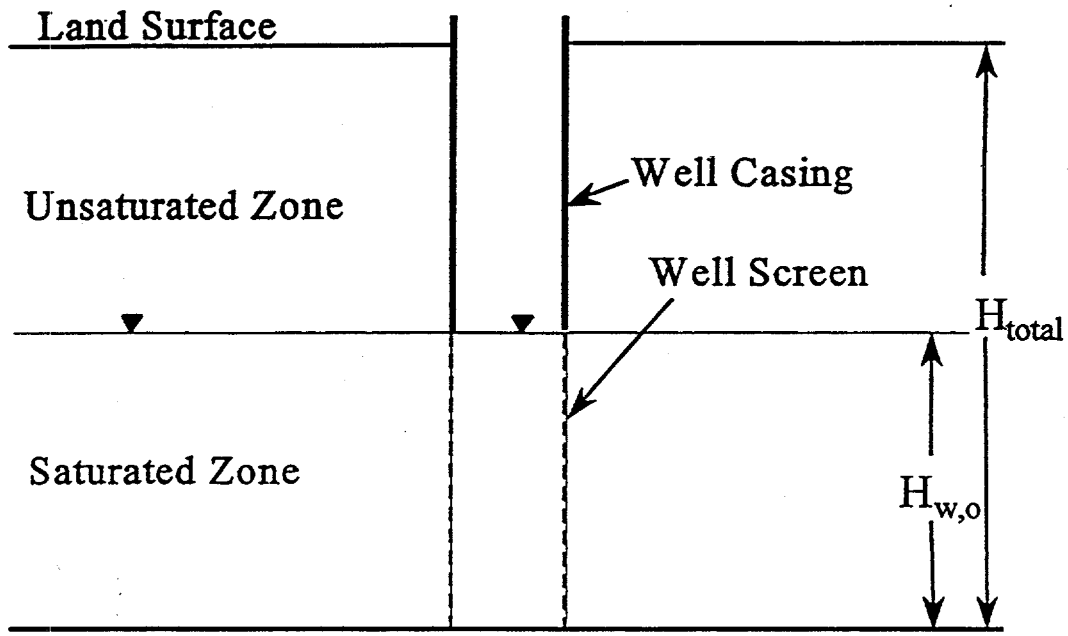

Figure 1. Idealization of a well fully piercing an unconfined aquifer

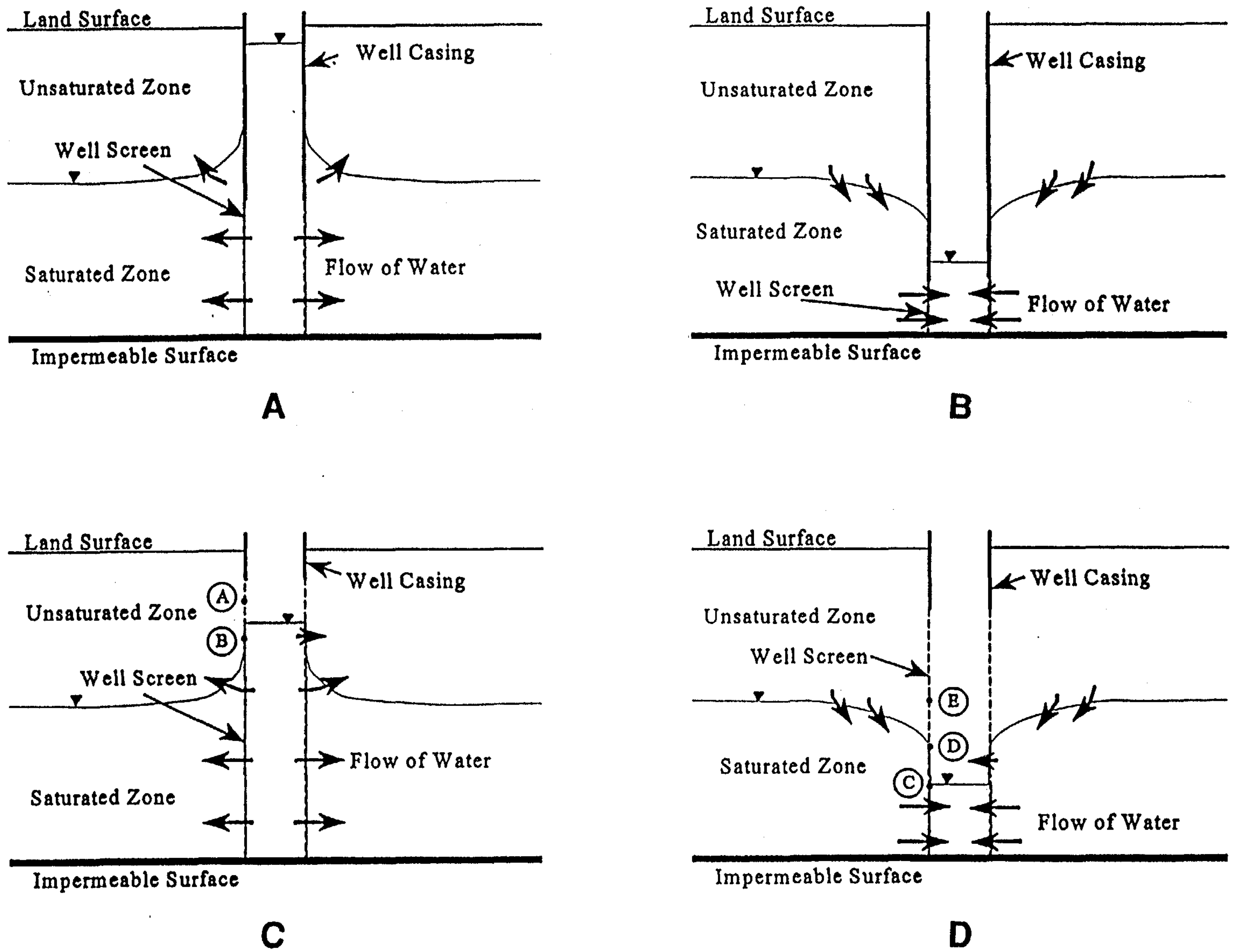

Figure 2. Four possible configurations for slug test in an unconfined aquifer, (A) Slug injection, no seepage face nor direct communication with unsaturated zone; (B) Slug bail-out, no seepage face; (C) Slug injection, direct communication with unsaturated zone; (D) Slug bailout, dynamic seepage face

Figure 3(A)

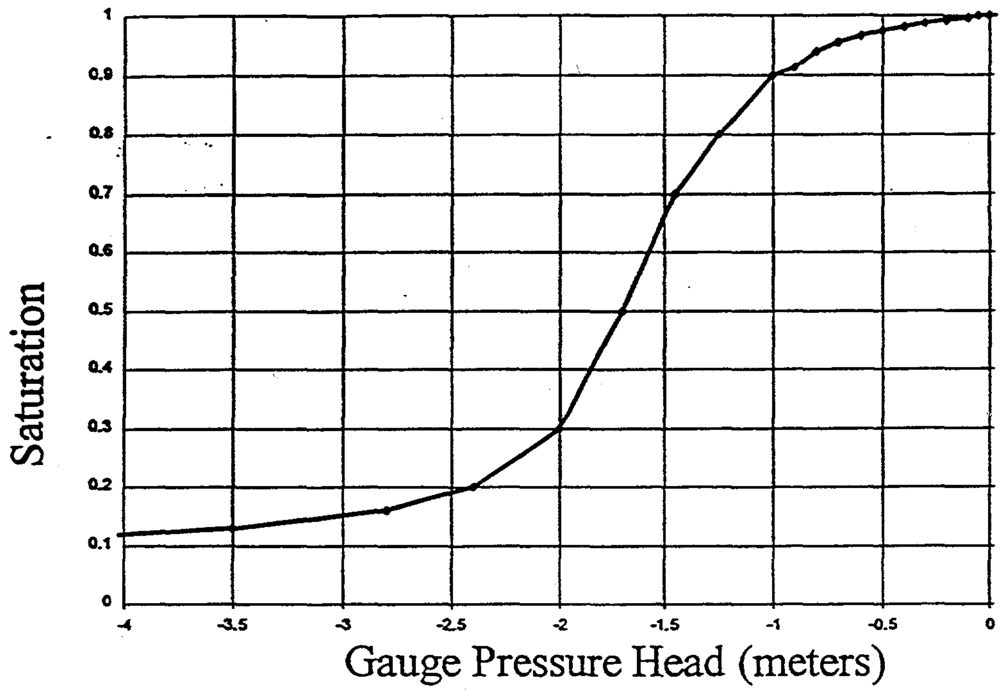

Figure 3(B)

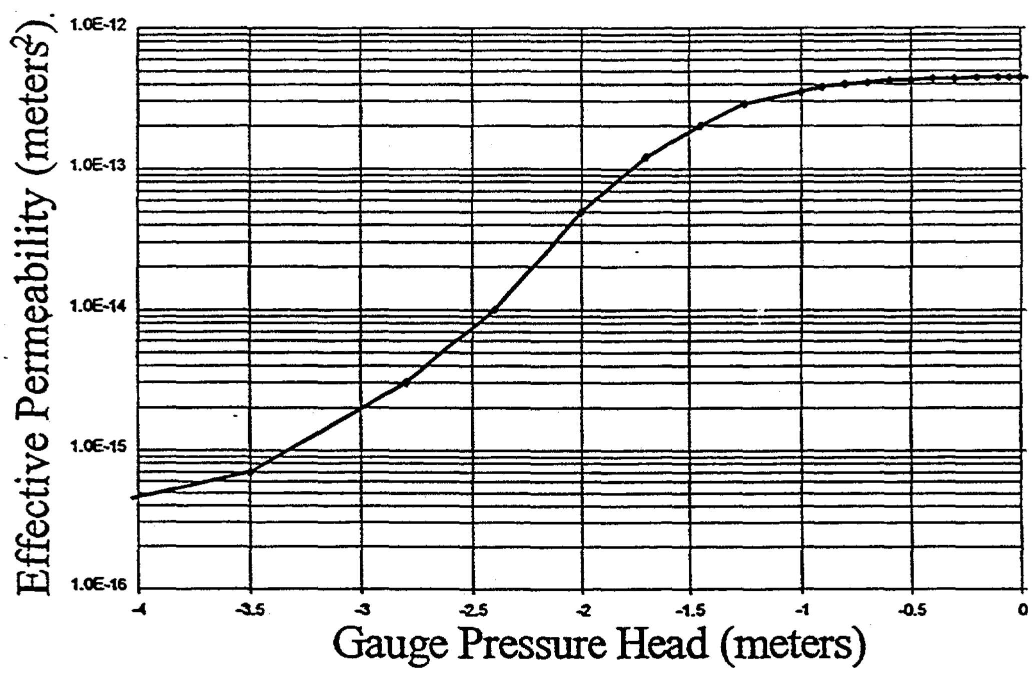

Figure 3. Unsaturated soil properties used in the simulations, (A) Saturation versus gauge pressure head; (B) Effective permeability versus gauge pressure head

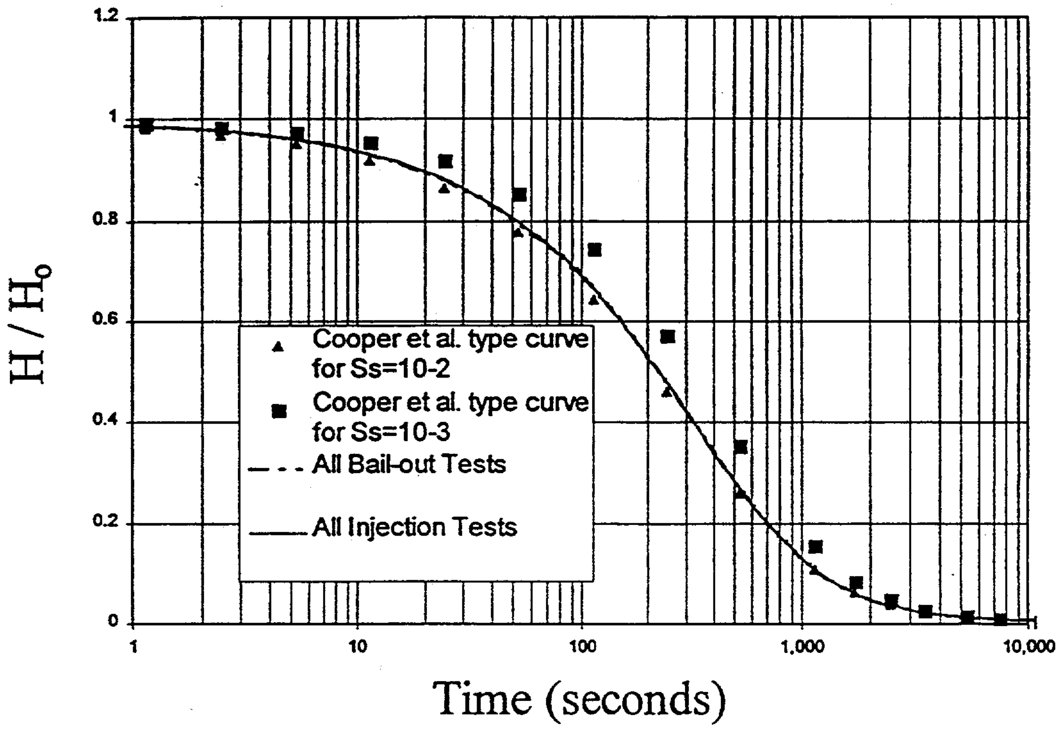

Figure 4. Slug test in a confined aquifer (Case 1), comparison of numerical analytical solutions

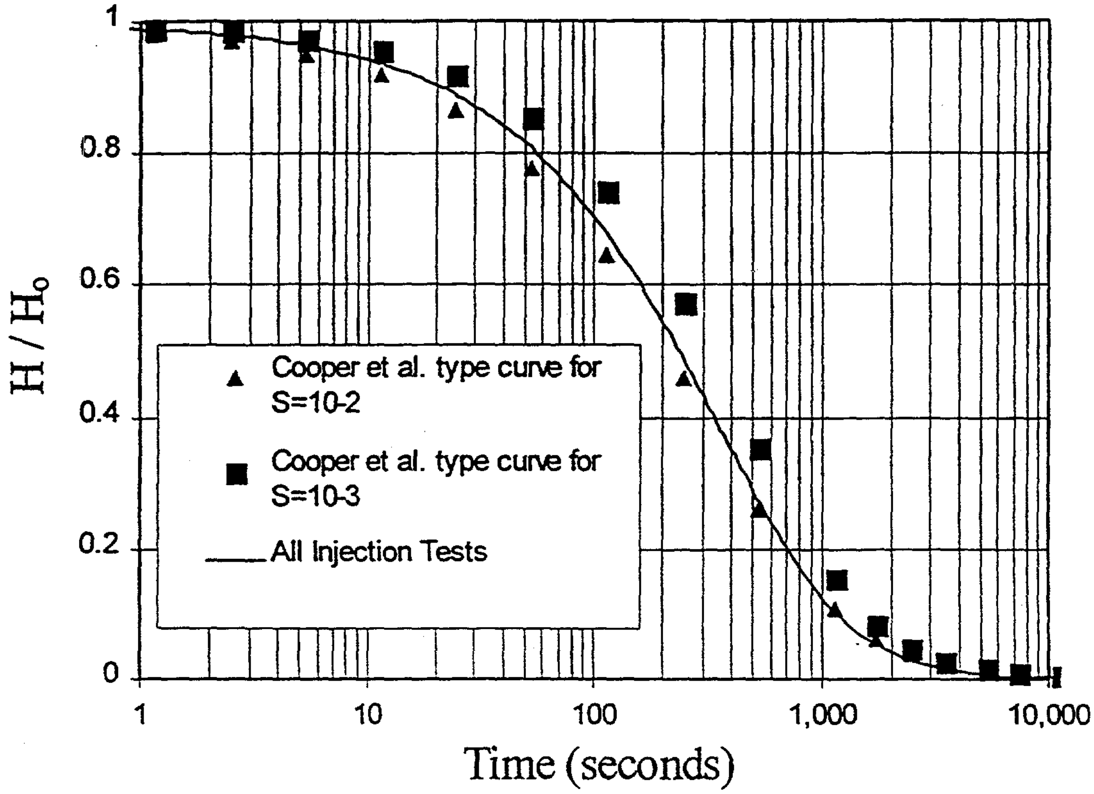

Figure 5. Slug test in an unconfined aquifer (Case 2), comparison of numerical and analytical solutions

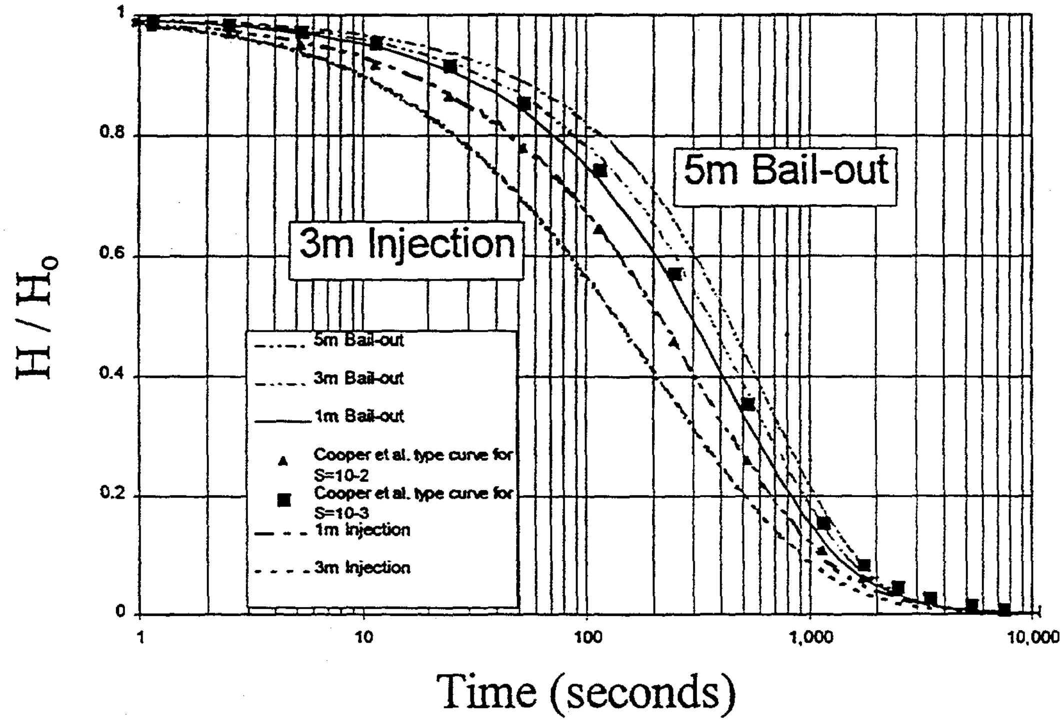

Figure 6. Slug test in an unconfined aquifer with seepage and direct communication with unsaturated zone (Case 3)

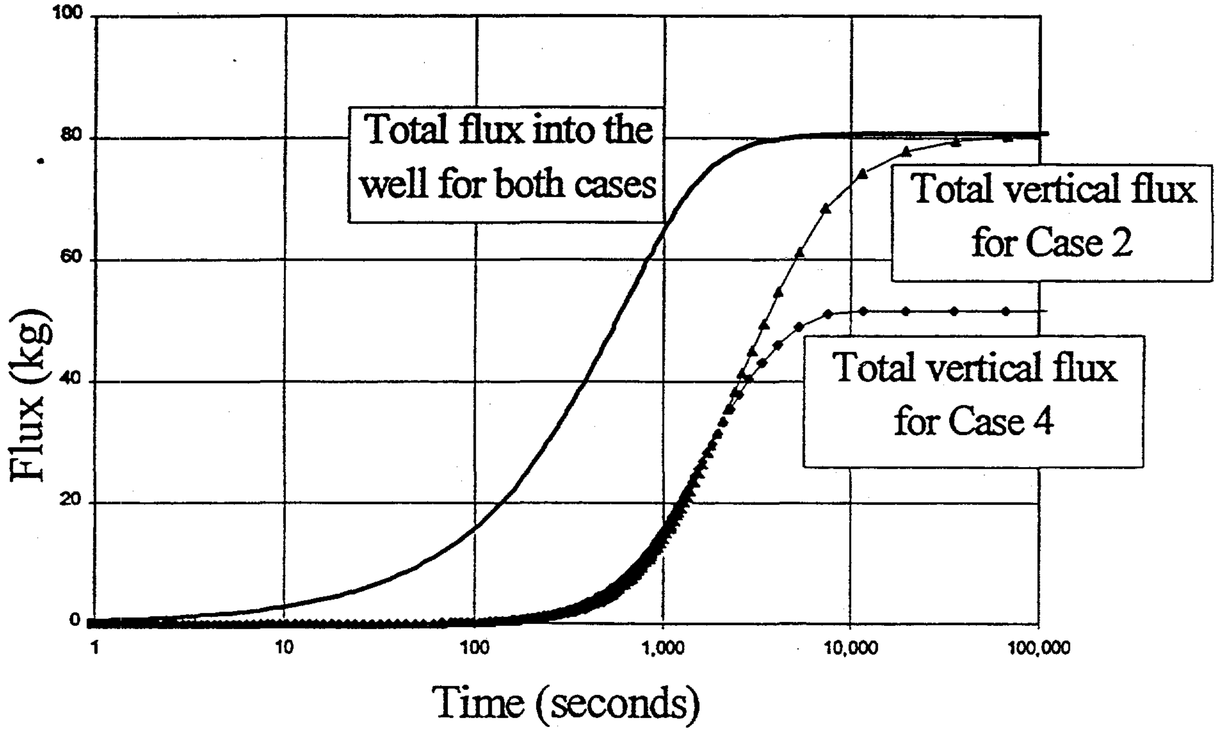

Figure 7. Comparison of cumulative flux from the unsaturated zone between thick and thin unsaturated zones

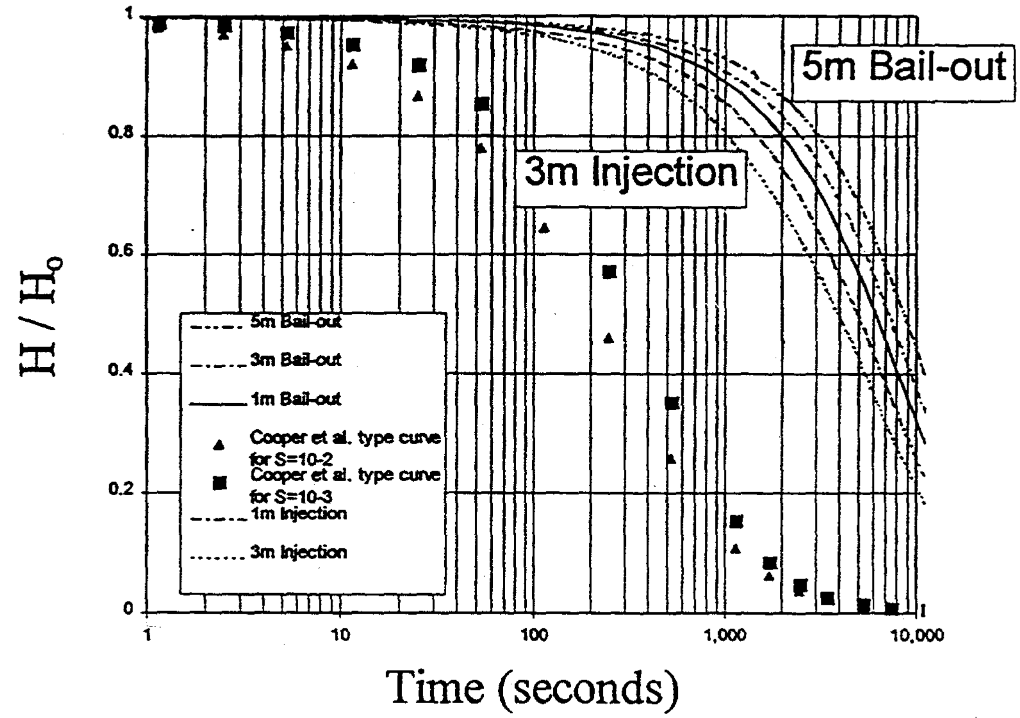

Figure 8. Slug test in an unconfined aquifer; well with damaged skin

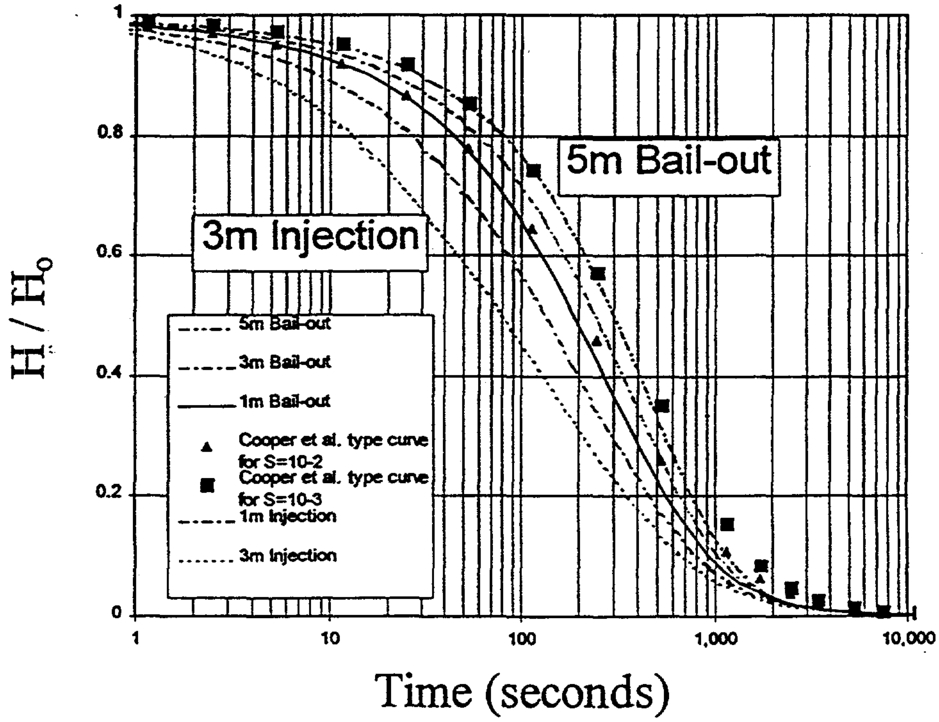

Figure 9: Slug test in an unconfined aquifer; well with a gravel pack