Mathematical Modeling of the Human Knee Joint*

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ABSTRACT

A model was developed to determine the forces exerted by several flexor and extensor muscles of the human knee under static conditions. The model was used to evaluate muscle tension and joint load for two static postures: the standing position and the position of the knee in an automobile simulation. The following muscles were studied: the gastrocnemius, biceps femoris, semitendinosus, semimembranosus, and the set of quadricep muscles. The rules of rigid body mechanics were employed to formulate force equilibrium equations for two rigid bodies: Rigid body 1 included the pelvis, femur and patella; Rigid body 2 included the tibia, fibula, and the collective bones of the foot. Planar motion at the knee joint was assumed. The selected muscles were modeled by their functional lines of action in space, which connected their points of origin to their points of insertion. Assumptions based on previous muscle tension data were used to resolve the indeterminacy.

INTRODUCTION

Musculoskeletal models provide the means for estimating individual muscle forces, stresses, and joint loads. Clinical applications of such models include modeling prosthetics to determine the change in joint load, and observing the effects of changes in muscle strength, size, and point of attachment, as well as loss of muscle functionality. Documented in this report is an illustrative description of the modeling technique used to evaluate selected flexor and extensor muscles of the knee in various static postures.

METHOD

Using the rules of rigid body mechanics, a static model of the knee joint was developed. We modeled the knee in two postures: (1) the standing position (full extension at the knee, 0 degrees of flexion) and (2) the knee in the position taken when a subject is seated in an automobile with the foot pressed against the brake pedal (35 degrees of flexion at the knee).

The individual skeletal components included in the model are the pelvis, femur, patella, tibia, fibula, and the collective bones of the foot. The following assumptions were made:

(1) Motion of the fibula with respect to the tibia is negligible.

(2) Rotation about the hip and ankle joint is not considered for the purpose of this model. Rotation occurs only at the knee joint.

(3) The patella was assumed to be fixed with respect to the femur. The position of the patella was determined as a function of the knee flexion angle.

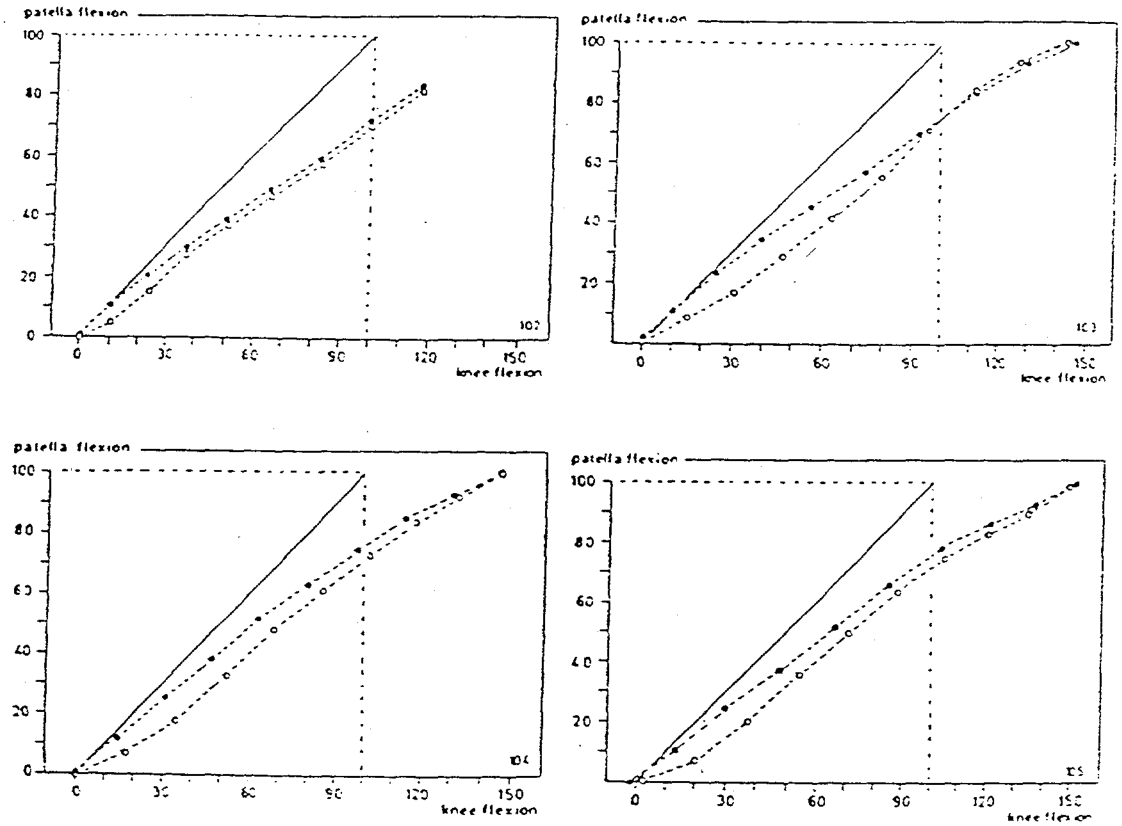


figure 1:

Patellar flexion as a function of knee flexion. (A. Van Kampen (1990) The Three Dimensional Tracking Pattern of the Human Patella)

(4) The knee is assumed to be a hinge joint, with motion occuring only in the sagittal plane.

Taking the above assumptions into account, two rigid bodies were considered. Rigid body 1 collectively includes the geometries of the pelvis, femur, and patella; Rigid body two includes the geometries of the tibia, fibula, and the collection of foot bones. Rigid body 1 will herein be referred to as the femur; Rigid body 2 will herein be referred to as the tibia.

With over 30 flexor and extensor muscles crossing the knee joint, selective muscle modeling is necessary to keep the problem of redundancy to a minimum. Data taken from the literature was used to select those muscles whose force contribution seemed most significant. The following muscles were selected: the gastrocnemius, rectus femoris, vastus intermedius, vastus lateralis, vastus medialis, semitendinosus, semimembranosus, and biceps femoris muscles.

For each of the muscles selected, single points representing the origin and insertion sites of each of the muscles were determined. Ambiguity was involved in assigning single points to muscles that attached to bone over a wide region. In these cases, we assumed that the midpoint of the region was the attachment point.

We modeled the muscles by their functional lines of action in space, defined by the line that connects the point of origin to the corresponding point of insertion. Muscles with more than one distinct component, or those with complex tendon paths, were modeled using multiple lines. In modeling the gastrocnemius muscle, which is comprised of two heads, we represented each head with a single line of action originating from the corresponding femoral condyle and inserting into the calcaneus bone of the foot (see figure 9).

Muscle	figure no.	force reference no.
Semitendinosus	2	1
Semimembranosus	3	2
Biceps Femoris	4	3
Rectus Femoris	5	4
Vastus Lateralis	6	5
Vastus Medialis	7	6
Vastus Intermedius	8	7
Gastrocnemius (medial)	9	8
Gastrocnemius (lateral)	9	9

table 1:

Muscle lines of action

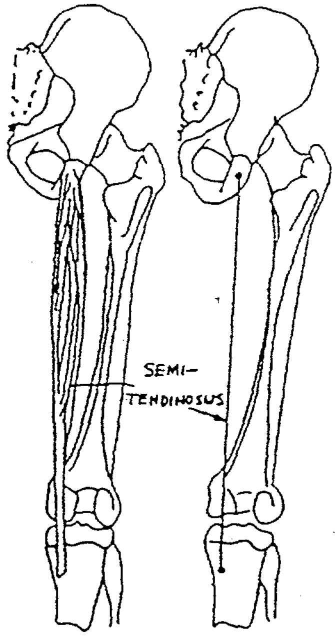


figure 2:

Line of Action for Semitendinosus*

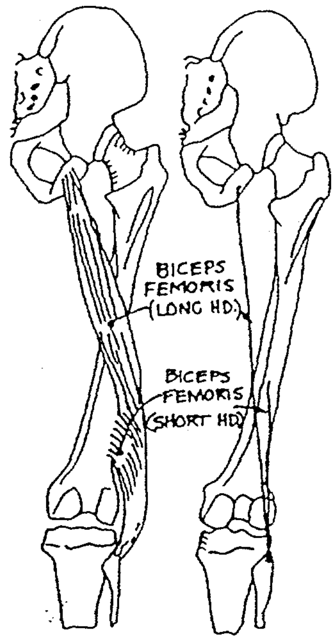


figure 4:

Line of Action for Biceps Femoris*

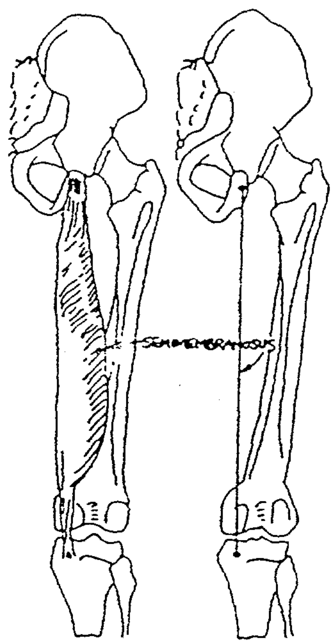


figure 3:

Line of Action for Semimembranosus*

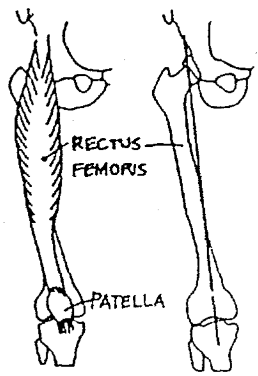


figure 5:

Line of Action for Rectus Femoris*

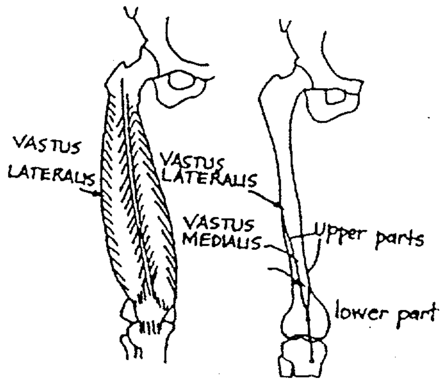


figure 6:

Line of Action for Vastus Lateralis*

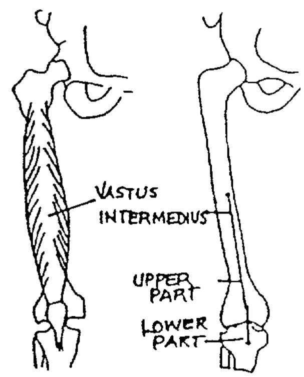


figure 8:

Line of Action for Vastus Intermedius*

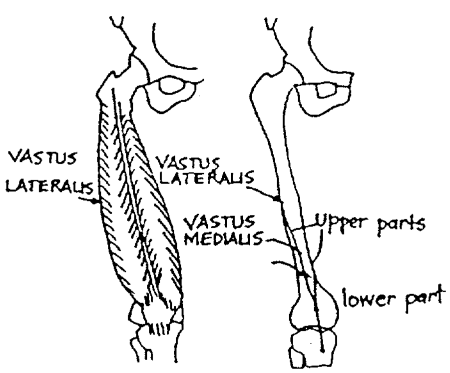


figure 7:

Line of Action for Vastus Medialis*

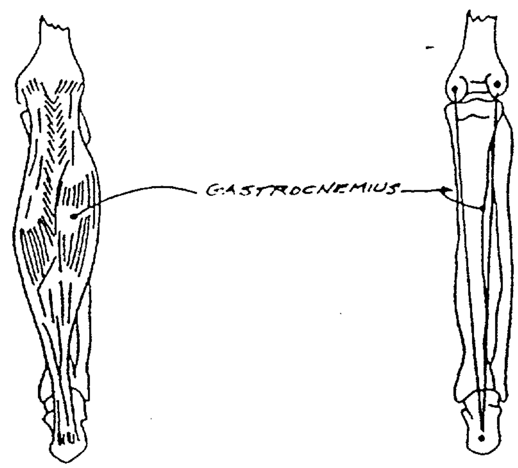


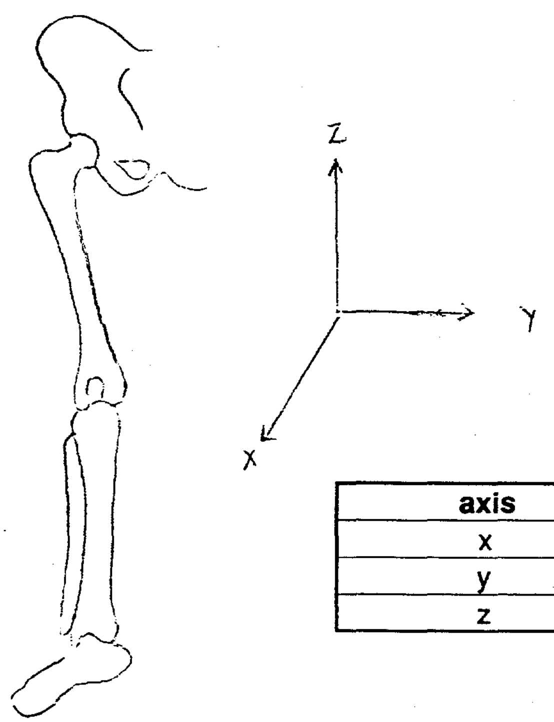
figure 9:

Lines of Action for Gastrocnemius*

* A. Seireg and R.J. Arkivar (1973) A Mathematical Model for Evaluations of forces in Lower Extremities of the Musculo-Skeletal System.

For each position modeled, the magnitude, direction, and point of application of an applied load was specified. In modeling the standing position, the magnitude of the applied load was specified as 344 N, acting down on the femoral head. This force magnitude value corresponds to the force produced by one-half of the body weight of a 70kg subject. In modeling the automobile situation, a force of 200 N acting perpendicular to the bottom of the calcaneus bone of the foot was applied, simulating the force of the brake pedal on the bottom of the foot.

Free body diagrams included the force vectors of the muscle tensions, the joint and ground reaction forces, the weights of the rigid bodies, and the applied load. Using the free body diagrams, we formulated force and moment equilibrium equations. A set of force component equations for each segment was developed, as well as a moment equation describing rotation about the knee joint. The following table and diagram indicates the reference axes used in formulating the force component equations:



axis	positive direction	negative direction
x	anterior	posterior
y	medial	lateral
z	upward	downward

figure 10:
Reference Axes

Our equation set consisted of six force component equations and one moment equation with a total of 10 unknowns (9 forces from muscle lines of action and one joint reaction force). In order to resolve the redundancy problem, the following assumptions were made:

(1) The forces produced by the medial and lateral heads of the gastrocnemius are equal in magnitude.

(2) The forces produced by the quadricep muscles (rectus femoris, vastus lateralis, vastus intermedius, vastus medialis) are equal in magnitude.

RESULTS

Free-body diagrams for both the femur and the tibia were constructed for both the standing and the automobile simultation positions.

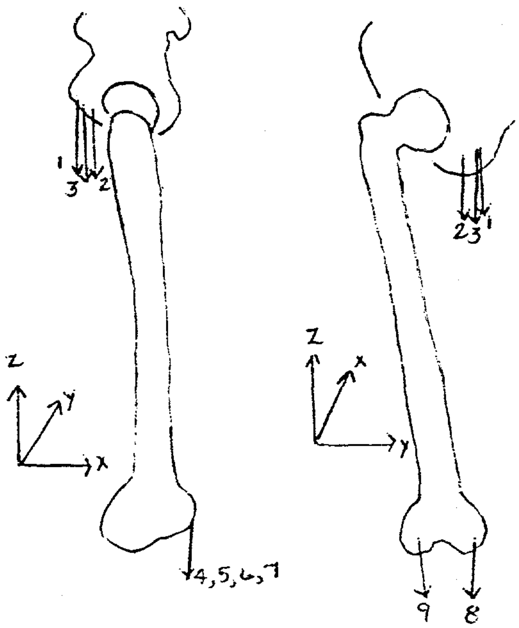


figure 11:

Free-body diagram for the femur, standing position

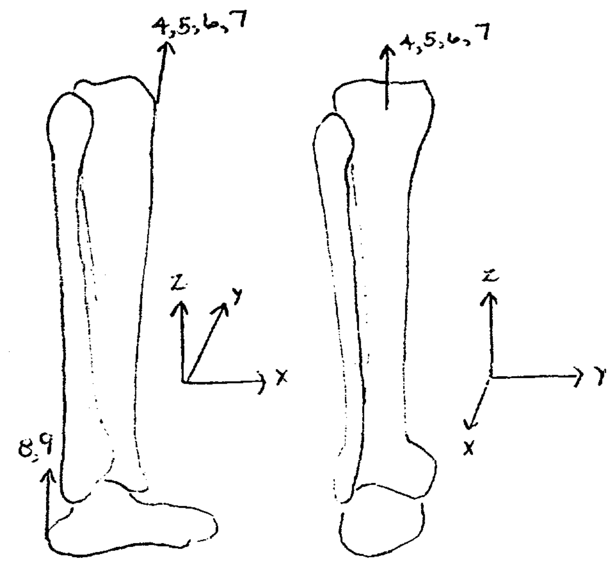


figure 12:

Free-body diagram for the tibia, standing position

Force equilibrium equations and a moment equation were formulated to describe the forces at each segment and rotation about the knee joint, respectively. The following equations were formulated for the standing position.

Force equilibrium equations for femur:

$$\Sigma F_X = F_{X1} + F_{X2} + F_{X4} + F_{X5} + F_{X6} + F_{X7} - (F_{X3} + F_{X7} + F_{X8} + F_{X9}) + R_X = 0$$

$$\Sigma F_Y = F_{Y4} + F_{Y7} + F_{Y9} + (F_{Y1} + F_{Y2} + F_{Y3} + F_{Y5} + F_{Y6} + F_{Y8} + R_Y = 0$$

$$\Sigma F_Z = -(F_{Z1} + F_{Z2} + F_{Z3} + F_{Z4} + F_{Z5} + F_{Z6} + F_{Z7} + F_{Z8} + F_{Z9}) + F_A + R_Z - W_F = 0$$

Force equilibrium equations for tibia:

$$\Sigma F_X = F_{X3} + F_{X7} + F_{X8} + F_{X9} - (F_{X1} + F_{X2} + F_{X4} + F_{X5} + F_{X6} + F_{X7}) - R_X = 0$$

$$\Sigma F_Y = F_{Y1} + F_{Y2} + F_{Y3} + F_{Y5} + F_{Y6} + F_{Y8} - (F_{Y4} - F_{Y7} - F_{Y9}) - R_Y = 0$$

$$\Sigma F_Z = F_{Z1} + F_{Z2} + F_{Z3} + F_{Z4} + F_{Z5} + F_{Z6} + F_{Z7} + F_{Z8} + F_{Z9} - R_Z - W_Z = 0$$

Moment equation describing rotation about the knee joint:

$$\Sigma M = F_1d_1 + F_2d_2 + F_3d_3 + F_4d_4 + F_5d_5 + F_6d_6 + F_7d_7 + F_8d_8 + F_9d_9 + W_F + F_A = 0$$

CONCLUSION

Applications. Determining muscle tensions and joint reaction forces in various postures is an immediate application of the model. Using the method outlined, several static situations (e.g. leaning, squatting, rising from a chair) can be modeled. The results of the standing and automobile situations will be used as boundary conditions for a finite element model of the human knee joint.

The concepts and equations developed for this model may be used as input for a computational model of the human knee joint. Specifically, a computational model can be used to solve for muscle forces using the optimization technique. One can use the model to analyze the effects of changes in muscle attachment points.

Suggestions for Improvement. Several changes can be made to improve the accuracy of the model.

- The data used to formulate the results presented in this paper exhibited considerable variability. Muscle length, moment arm length, and cross sectional area values vary between subjects. For the purpose of this model, data was obtained from several sources in the literature.

- The attachment of muscle to bone may occur over a variable region, making the task of assigning an origin and/or insertion point an ambiguous one. Lines of action should closely reflect the true physiological function of the muscle. Therefore, choosing a point of origin and a point of insertion should involve careful consideration of the muscles' kinetic characteristics.

- The assumptions made when reducing the number of unknown variables may not accurately reflect true physiological behavior of the muscles. EMG analyses and verification of the proposed assumptions should be performed before calculating muscle forces.

- Straight-line modeling does not account for the curvature of tendon paths. To take into account those muscle which wrap around bones, second and third order equations should be employed. With straight line modeling, the muscle's true tendon path is not represented.

- Eight muscles were included in this model. However, over 30 different muscles cross the knee joint. Including a greater number of muscles in the model would increase the accuracy of the results.

- The center of gravity of the segments defined in this model were estimated. The information included in the literature only accounts for the weights and centers of single bones.

- Several ligaments cross the knee joint and exhibit passive tension. Including the tension contribution of these ligament in the model would increase the accuracy of the results.

- The moment arm values of muscles vary according to the orientation of the bones they cross. In modeling different postures, one must take into account the change in moment arm length as a function of joint angle.

REFERENCES

V.C. Mow, W.C. Hayes (1991) *Basic Orthopedic Biomechanics*. Raven Press, Ltd., New York.

C.W. Spoor and J.L. Van Leeuwen (1992) Knee Muscle Moment Arms from MRI and from Tendon Travel. *J. Biomechanics* **25(2)**, pp. 201-206.

W. Herzog and L.J. Read (1993) Lines of Action and Moment Arms of the Major Force-Carrying Structures Crossing the Human Knee Joint. *J. Anat.* **182**, pp. 213-230.

A. Seireg and R.J. Arvikar (1973) A Mathematical Model for Evaluations of Forces in Lower Extremities of the Musculo-Skeletal System. *J. Biomechanics*, **6**, pp. 313-326.

A. Van Kampen, R. Huiskes (1990) The Three-Dimensional Tracking Pattern of the Human Patella. J. Orthopaedic Research, **8**, pp. 372-382.