CONF-870821--1

BROOKHAVEN NATIONAL LABORATORY

September 1987

BNL--40379

DE88 002118

WEAK NEUTRAL CURRENTS AND FUTURE Z MASS MEASUREMENTS - (ANTICIPATING SLC)*

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To appear in *Proceedings of the 1987 SLAC Summer Institute Topical Conference, August 1987*

* Based on work done in collaboration with U. Amaldi, A. Böhm, L.S. Durkin, P. Langacker, A. Mann, A. Sirlin, and H.H. Williams

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WEAK NEUTRAL CURRENTS AND FUTURE Z MASS MEASUREMENTS*
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Outline

1. INTRODUCTION

Several years ago, a collaboration[1] was formed for the purpose of collecting and analyzing all neutral current data. Our idea was to carefully scrutinize the experimental and theoretical uncertainties in those results and to consistently include effects of electroweak radiative corrections. That undertaking involved examining a great many diverse measurements which included (in approximate order of precision): deep-inelastic $\nu_\mu N$ scattering, $W^\pm$ and Z masses, eD scattering asymmetry, atomic parity violation, νe scattering, νp scattering, e^+e^- annihilation, μC scattering etc. The goals of our work were:

1. To test the standard $SU(2)_L \times U(1)$ model at the tree and quantum loop level.
2. Provide a precise determination of $\sin^2\theta_W$ which could, for example, be used to rule out or at least constrain various grand unified theories (GUTS).
3. Look for hints of "new physics."

Based on work done in collaboration with U. Amaldi, A. Böhm, L.S. Durkin, P. Langacker, A. Mann, A. Sirlin, and H.H. Williams

In this talk, I will outline some of the main results of that collective effort and discuss various implications. The topics I have chosen to elaborate on are radiative corrections, top quark mass bounds, grand unified theories (GUTS), and extra Z' boson constraints. In addition, P. Langacker, A. Sirlin and I[2] recently examined the implications that a precise Z boson mass determination at SLC would have when combined with existing neutral current data. We found that such a measurement could tightly constrain the top quark mass or imply new physics beyond the standard model. Given the special interest much of this audience has in m_Z, I will also describe the results of that analysis.

2. RESULTS OF A GLOBAL NEUTRAL CURRENT ANALYSIS

Before stating some of the main results of our recent[1] global analysis of existing neutral current data and $W^\pm$ and Z masses, I will briefly describe the basic assumptions that went into it. We assumed 3 generations of fermions and one Higgs doublet with an underlying $SU(3)_C \times SU(2)_L \times U(1)$ gauge symmetry constituted the standard model. Within that framework, radiative corrections to all relevant neutral current processes as well as $W^\pm$ and Z mass formulas were accounted for. Those corrections depend on the couplings and masses in the model. Two of those parameters, m_t (the top quark mass) and m_H (the Higgs scalar mass) are presently undetermined. Whereas, the radiative corrections are not very sensitive to m_H variations, they are sensitive to m_t if it is $\gtrsim$90 GeV. Therefore, in some parts of our analysis, we allowed m_H to vary from 10 GeV to 1 TeV and merely required $m_t \lesssim 100$ GeV while in other parts $m_H \simeq 100$ GeV and $m_t = 45$ GeV were assumed for definiteness. Because of the sensitivity to large m_t, we were also able to place bounds on m_t, as we shall see. Some of the principal results of the global analysis of all existing neutral current data and $W^\pm$ and Z masses were:

1. There is at present no evidence for any deviation from the standard model.
2. For $m_t \lesssim 100$ GeV and $m_H \lesssim 1$ TeV, we found the world average $\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2 = 0.230 \pm 0.0048$. For $m_t \simeq 45$ GeV and $m_H \simeq 100$ GeV, the uncertainties were lowered slightly to $\sin^2\theta_W = 0.230 \pm 0.0044$.
3. Allowing $\rho \equiv m_W^2/m_Z^2\cos^2\theta_W$ as well as $\sin^2\theta_W$ to vary, we obtained from a two parameter fit to all data $\sin^2\theta_W = 0.229 \pm 0.0064$, $\rho = 0.998 \pm 0.0086$.

4. For $m_t \simeq 45$ GeV and $m_H = 100$ GeV, radiative corrections are confirmed at about the 3σ level, primarily in the comparison of deep-inelastic $\nu_\mu N$ scattering with m_W and m_Z.
5. Consistency of all data at the quantum loop level requires $m_t \lesssim 200$ GeV for $m_H \lesssim 1$ TeV. If $m_H \lesssim 100$ GeV, the tighter constraint $m_t \lesssim 180$ GeV is obtained. Those constraints also apply to a 4th generation mass difference $(m_{t'} - m_{b'})$.
6. Lower limits on extra Z' boson masses were obtained for a variety of popular GUT models. The bounds ranged from 120 GeV to 300 GeV depending on their specific couplings to fermions.

For a detailed discussion, the reader should consult Ref. 1.

The results described above have many implications. In the remainder of this talk I will elaborate on a few topics that we considered.

3. RADIATIVE CORRECTIONS AND $\sin^2\theta_W$

The weak mixing angle θ_W plays a central role in the $SU(2)_L \times U(1)$ model. Writing out the electroweak neutral current interaction Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm int} = & -eA^\mu(x)\sum_f Q_f \bar{f}\gamma_\mu f - \frac{2e}{\sin 2\theta_W} Z^\mu(x) \\ & \times \sum_f \left(T_{3f}\bar{f}_L\gamma_\mu f_L - \sin^2\theta_W Q_f \bar{f}\gamma_\mu f\right) \\ e = & g_2 \sin\theta_W \end{aligned} \tag{3.1}$$

with T_{3f} = weak isospin and Q_f =electric charge, we see that θ_W occurs both at the fermion weak neutral current level and in the normalization of the $SU(2)_L$ coupling g_2 relative to the electric charge e. In addition, for the simplest Higgs doublet scenario, it enters the W-Z mass relationship via

$$m_W = m_Z \cos\theta_W \tag{3.2}$$

So, one tests the standard model and the underlying concept of electroweak unification by measuring $\sin^2\theta_W$ in as many different ways as possible. A deviation in the value obtained from one experiment as compared with another would signal new physics.

Of course, radiative corrections must be accounted for in any precise determination of $\sin^2\theta_W$, so that they will not be confused with new physics. In some cases, electroweak radiative corrections can be quite large. For example, consider the lowest order natural relationship

$$\sin^2\theta_W^0 = \left(e^0/g_2^0\right)^2 = 1 - \left(m_W^0/m_Z^0\right)^2 \tag{3.3}$$

where 0 indicates bare (unrenormalized) parameters. In terms of physical measureable quantities, that relationship is modified by *finite* $O(\alpha)$ loop corrections. The size of those corrections depends on the definitions employed; but for typical definitions they can be quite large. Defining the renormalized weak mixing angle by

$$\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2 \tag{3.4}$$

where m_W and m_Z are physical masses and defining the renormalized charges e and g_2 via

$$\alpha = e^2/4\pi = 1/137.036 \tag{3.5a}$$

$$G_\mu = \frac{g_2^2}{4\sqrt{2}m_W^2} = 1.16636 \times 10^{-5}\text{GeV}^{-2} \tag{3.5b}$$

leads to[3]

$$\begin{aligned} m_W = m_Z\cos\theta_W &= \left(\frac{\pi\alpha}{\sqrt{2}G_\mu}\right)^{\frac{1}{2}} \frac{1}{\sin\theta_W\left(1-\Delta r\right)^{\frac{1}{2}}} \\ &= \frac{37.281\text{GeV}}{\sin\theta_W\left(1-\Delta r\right)^{\frac{1}{2}}} \end{aligned} \tag{3.6}$$

where[1]

$$\Delta r = 0.0713 \pm 0.0013 \tag{3.7}$$

for $m_t \simeq 45$ GeV and $m_{\text{Higgs}} \simeq 100$ GeV. The radiative corrections in $\Delta r \simeq O(\alpha)$ are quite large primarily due to vacuum polarization renormalization of e relative to g_2. It is, however, in my opinion a mistake to call that effect a QED correction. Fermion loops enter in the photon propagator as well as the $W^\pm$ and Z propagators. So, the relative correction is calculable only because of electroweak unification.

When either m_W or m_Z determinations are used to obtain $\sin^2\theta_W$ via Eq. (3.6), Δr causes a sizeable $\simeq 7\%$ shift in the value found. Similarly, neutral current scattering cross sections and interference measurements must be corrected for $O(\alpha)$ quantum loops in extracting $\sin^2\theta_W$. The effects of such electroweak radiative corrections are illustrated in Table 1 where several determinations of $\sin^2\theta_W$ are summarized. (For a more complete discussion see

Table 3.1: Values of $\sin^2\theta_W = 1 - m_W^2/m_Z^2$ before and after electroweak radiative corrections (R.C.) are included. The values $m_t \simeq 45$GeV and $m_H \simeq 100$GeV were employed in the radiative correction.

Experiment	$\sin^2\theta_W^{unc.}$	R. C.	$\sin^2\theta_W$
Atomic P.V.	0.201 ± 0.018 ± 0.014	+0.008	0.209 ± 0.018 ± 0.014
eD Asymmetry	0.226 ± 0.015 ± 0.013	-0.005	0.221 ± 0.015 ± 0.013
$\overset{(-)}{\nu}_\mu e$	0.221 ± 0.019	+ 0.002	0.223 ± 0.019
$\overset{(-)}{\nu}_\mu P$	0.208 ± 0.033	+ 0.002	0.210 ± 0.033
$\nu_\mu N$ deep-inel.	0.242 ± 0.003 ± 0.005	-0.009	0.233 ± 0.003 ± 0.005
$m_W = 80.9 \pm 1.4$ (UA1 - UA2)	0.212 ± 0.008	+0.017	0.229 ± 0.008
$m_Z = 91.9 \pm 1.8$ (UA1 - UA2)	0.208 ± 0.011	+0.022	0.230 ± 0.011
World Average			0.230 ± 0.0044

Ref. 1) At present, deep-inelastic ν_μ scattering provides the best determination of $\sin^2\theta_W$. In fact, much of the uncertainty in that extraction is theoretical in the sense that a model is employed to correct for charm threshold effects. It, therefore, appears that those measurements have been pushed about as far as possible.

Comparing the $\sin^2\theta_W^{unc.}$ and $\sin^2\theta_W$ columns in Table 1, it is clear that the uncorrected value obtained from deep-inelastic $\nu_\mu - N$ scattering differs from the m_W and m_Z values, but the corrected $\sin^2\theta_W$ are in good agreement. So, the standard model has been tested at the level of its $O(\alpha)$ radiative corrections (at about the 3σ level) if m_t is actually near 45 GeV. (See section 4).

The present world average of all data

$$\sin^2\theta_W = 0.230 \pm 0.0044 \tag{3.8}$$

now carries a rather small uncertainty. It can therefore be used to rule out or constrain GUTS (see section 5).

Experimentalists should continue to strive to measure $\sin^2\theta_W$ as precisely as possible. Fortunately, m_Z will be measured to better than ± 50 MeV at SLC and LEP and that will determine $\sin^2\theta_W$ to $\pm$ 0.00025 via Eq. (3.6) (after m_t and m_H are pinpointed). Given such high precision, what role can other experiments play? To test the standard model or look for hints of new physics, one *must* compare distinct measurements. So, high precision measurements of $\sin^2\theta_W$ should be undertaken in as many processes as possible. Indeed, in section 7, I will illustrate what one can learn by combining a precision measurement of m_Z with existing neutral current data.

4. TOP QUARK MASS

Radiative corrections to $\sin^2\theta_W$ can be quite sensitive to the value of m_t, if it is large. In fact, they grow like $\alpha m_t^2/m_W^2$ in most processes.[3,4] Only $\sin^2\theta_W$ determined from deep-inelastic $\nu_\mu N$ scattering (due to a subtle cancellation) is insensitive to variations in m_t. Therefore, the present good agreement between $\sin^2\theta_W$ obtained from $\nu_\mu N$ data and other experiments such as m_W and m_Z measurements gives us confidence that m_t is probably not greater than $\approx 45 \sim 100$ GeV. In fact, as stated in section 2, by varying m_t in the radiative corrections, we found (for $m_H \leq 1$ TeV)

$$m_t \lesssim 200\text{GeV} \qquad (90\%\ \text{CL}) \tag{4.1}$$

(That bound also applies to a fourth generation mass difference[5] $|m_{t'} - m_{b'}|$.)

The effect of changing m_t on our global fits is illustrated in Table 4.1. There I have given the values of $\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2$ and $\sin^2\theta_W\,(m_W)_{\overline{\text{MS}}}$ defined by $\overline{\text{MS}}$ (modified minimal subtraction). Note that those two distinct definitions of the weak mixing angle differ by terms of $O\left(\alpha m_t^2/m_W^2\right)$; hence, their difference grows as m_t increases. Whereas, $\sin^2\theta_W$ extracted from deep-inelastic $\nu_\mu N$ data is quite insensitive to m_t, that is not so for $\sin^2\theta_W\,(m_W)_{\overline{\text{MS}}}$. In contradistinction, $\sin^2\theta_W$ extracted from either m_W or m_Z via Eq. (3.6) is sensitive to the value of m_t through Δr, while $\sin^2\theta_W\,(m_W)_{\overline{\text{MS}}}$ extracted from m_W or m_Z (particularly m_W) is much less dependent on m_t. (Of course, a precise determination of both m_W and m_Z would determine $\sin^2\theta_W$ independent of m_t or any radiative corrections.) We, therefore, have a situation in which at present, neither definition's value can be precisely given without some assumption regarding m_t. In section

Table 4.1: World average values for the weak mixing angle as a function of m_t (keeping $m_H = 100$ GeV).

m_t (GeV)	$\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2$	$\sin^2\theta_W\,(m_W)_{\overline{\rm MS}}$
25	0.229 ± 0.0044	0.227 ± 0.0044
45	0.230	0.228
60	0.230	0.228
100	0.227	0.229
200	0.222	0.233
400	0.209	0.248

7, we will turn the argument around and show how a precise determination of m_Z can in the short term constrain m_t.

5. GAUGE COUPLINGS AND GUTS

The standard $SU(3)_C \times SU(2)_L \times U(1)$ model of strong and electroweak interactions contains 18 independent couplings and masses. Grand Unified Theories[6,7] (GUTS) correlate the three gauge couplings g_3, g_2 and g_1 by embedding the standard model in a compact simple group such as $SU(5)$, $SO(10)$, E_6 etc. Indeed, the high degree of symmetry naturally renders the bare couplings equal, explains charge-color quantization and promotes $\sin^2\theta_W^0$ from an infinite counterterm parameter to a rational number (generally 3/8). Unfortunately, GUTS have so far provided little new insight regarding the 15 mass and quark mixing parameters. Therefore, although GUTS represent a significant theoretical advancement, they cannot be the final word.

In this section, I will update the gauge coupling values. Those quantities are extremely important because they provide much of the basis for our belief in GUTS and a severe constraint on model building. In the case of the QCD coupling, the situation has not changed significantly during the last few years. Upsilon decays and high energy jet data are consistent with

$$\Lambda^{(4)}_{\overline{\rm MS}} \simeq 150^{+150}_{-75}\,{\rm MeV} \tag{5.1}$$

(The errors are quite conservative.) Assuming $m_t \simeq 45\text{GeV}$ and using $m_W = 80.7\text{GeV}$ (which corresponds to $\sin^2\theta_W = 0.23$), that range leads to[8]

$$\alpha_3(m_W) = 0.107^{+0.013}_{-0.008} \tag{5.2}$$

The analogous electroweak parameters, also defined by $\overline{\text{MS}}$ (modified minimal subtraction) have the short-distance values[1,9]

$$\alpha^{-1}(m_W) = 127.8 \pm 0.3 \tag{5.3}$$

$$\sin^2\theta_W(m_W) = 0.228 \pm 0.0044 \tag{5.4}$$

where the value of $\sin^2\theta_W(m_W)$ in Eq. (5.4) follows from the result $\sin^2\theta_W = 0.230 \pm 0.0044$ (for $m_t = 45$ GeV) found by the global analysis described above. As discussed in section 4, the $\overline{\text{MS}}$ definition and $\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2$ differ by order α radiative corrections which for $m_t = 45$ GeV imply[1] $\sin^2\theta_W(m_W) = 0.9907\sin^2\theta_W$. For other values of m_t, that relationship is modified as are the central values of both $\sin^2\theta_W(m_W)$ and $\sin^2\theta_W$ extracted from experiment (i.e. the radiative corrections to each experiment also depend on m_t). (See section 4.) It should be noted that the world average for the weak mixing angle in Eq. (5.4) has increased from the old $\sin^2\theta_W(m_W) = 0.219 \pm 0.006$ value primarily because of more precise deep-inelastic ν_μ scattering data and refinements in the $W^\pm$ and Z mass determinations. Also, as indicated in table 5.1, if m_t is > 45 GeV, $\sin^2\theta_W(m_W)$ increases even more. That higher value has very important implications for GUTS, as we shall see.

Employing the relationships

$$\alpha_1(m_W) = 5\alpha(m_W)/3\cos^2\theta_W(m_W) \tag{5.5a}$$

$$\alpha_2(m_W) = \alpha(m_W)/\sin^2\theta_W(m_W) \tag{5.5b}$$

leads to the gauge coupling values

$$\alpha_1(m_W) = 0.0169 \pm 0.0001 \tag{5.6a}$$

$$\alpha_2(m_W) = 0.0344 \pm 0.0007 \tag{5.6b}$$

Previously, the central value of $\alpha_2(m_W)$ was 0.036. Assuming that there are no other new thresholds between the standard model's mass scale of m_W and the grand unification scale of m_X, one can evolve the gauge couplings to higher energies using (for 3 generations)[7]

$$\mu\frac{\partial}{\partial\mu}\alpha_i(\mu) = b_i\alpha_i^2 + \ldots, i = 1,2,3 \tag{5.7a}$$

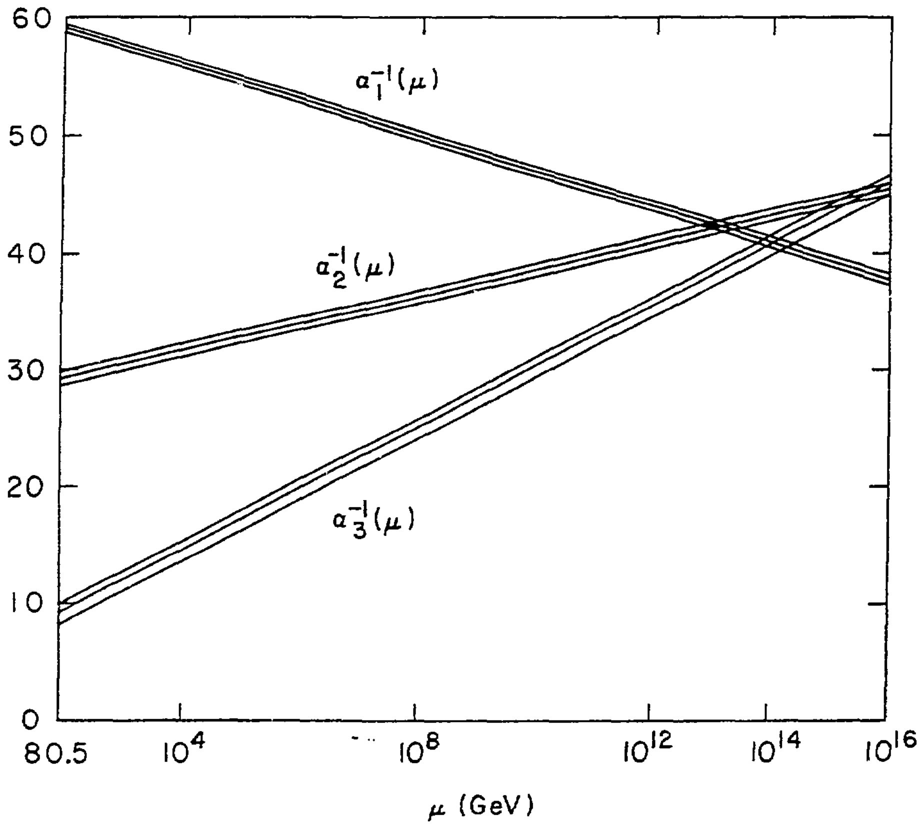

Fig. 5.1: Evolution of the $\alpha_i^{-1}(\mu)$, $i = 1, 2, 3$ couplings assuming no new physics beyond the standard model.

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{1}{2\pi} \begin{pmatrix} -41/10 \\ 19/6 \\ 7 \end{pmatrix} \tag{5.7b}$$

and the values of $\alpha_i(m_W)$ given above. If the three couplings meet at a single point, that would be clear evidence for grand unification. When $\alpha_2(m_W)$ was 0.036, they tended to meet near $\mu \sim 2 \times 10^{14}$GeV. That meeting was taken as strong confirmation of GUTS and perhaps an indication of no new physics thresholds at low or intermediate mass scales. Using the new value for $\alpha_2(m_W)$ in Eq. (5.6b), one finds that is no longer the case (see fig. 5.1).

The couplings $\alpha_1(\mu)$ and $\alpha_3(\mu)$ continue to meet near 1.5×10^{14}GeV; however, $\alpha_2(\mu)$ now crosses $\alpha_1(\mu)$ at 1.5×10^{13}GeV and meets $\alpha_3(\mu)$ near 1.0×10^{16}GeV. Is grand unification ruled out? No, this development merely implies that new physics thresholds between m_W and m_X must change the evolution of the couplings such that they meet at a single value. In my opinion, the near equality of the couplings at high energies that we find using Eq. (5.7) should still be taken as a strong indication of grand unification. At issue is: What new physics rectifies the evolution and at what energy will it be manifested?

The above remarks are nicely illustrated by the SU(5) Georgi-Glashow model.[6] In the so-called minimal version, one assumes the existence of a great desert between m_W and m_X, the unification mass scale. That simplistic assumption had an appealing consequence, it led to rather definite testable predictions. (The predictions hold in many GUTS with great deserts.) Indeed, using $\alpha^{-1}(m_W) \simeq 127.8 \pm 0.3$ and $\Lambda^{(4)}_{\overline{MS}} = 150^{+150}_{-75}$MeV, one predicts

$$m_X = \left(2.0^{+2.1}_{-1.0}\right) \times 10^{14}\text{GeV} \tag{5.8}$$

$$\sin^2\theta_W(m_W) = 0.214^{+0.003}_{-0.004} \tag{5.9}$$

Unfortunately, both of these predictions are now ruled out by experiment. The IMB proton decay bound[10]

$$1/\Gamma\left(P \to e^+\pi^-\right) \geq 3.1 \times 10^{32}\text{yr} \tag{5.10}$$

require $m_X \gtrsim 7 \times 10^{14}$GeV, while the $\sin^2\theta_W(m_W)$ prediction conflicts with the world average in Eq. (5.4). (It gets worse if m_t is > 45 GeV.) The latter disagreement is, of course, just another quantitative way of describing the apparent lack of unification of gauge couplings in fig. 5.1 when current $\alpha_i(m_W)$ values are employed. These failures of the minimal SU(5) model do not rule out SU(5) as a viable grand unification group. They do indicate that new physics appendages in the form of additional scalars or fermions must be introduced[11] to render $m_X \geq 10^{15}$GeV and increase the prediction for $\sin^2\theta_W(m_W)$. Another possibility is that a bigger GUT such as SO(10) or E_6 with intermediate stages of symmetry breaking must be employed. I will now describe how low energy supersymmetry[12] may do the trick for SU(5) or any other GUT.

The basic idea of supersymmetry is that each known boson (fermion) has a fermion (boson) partner. In those scenarios, the $\alpha_i(\mu)$ evolution in equations (5.7) change when we pass the supersymmetry thresholds. In leading order, one finds for three generations of fermions and N_H light Higgs doublets[12,13]

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{1}{2\pi}\begin{pmatrix} -6-\frac{3}{10}N_H \\ -\frac{1}{2}N_H \\ 3 \end{pmatrix} \tag{5.11}$$

Taking $N_H = 2$ (the minimal value) and using the $\alpha_i(m_W)$ values Eqs. (5.2) and (5.5) in section 5 as input, we can solve for $m_{\rm SUSY}$ and m_X. One finds in leading order[9]

$$\ln(m_X/m_W) \simeq \frac{\pi}{2}\left\{\frac{1}{\alpha_2(m_W)} - \frac{1}{\alpha_3(m_W)}\right\} \tag{5.12}$$

independent of $m_{\rm SUSY}$. Using the values of $\alpha_2(m_W)$ in Eq. (5.6b) and $\alpha_3(m_W)$ in Eq. (5.2) then gives the range of predictions

$$m_X \simeq 2\times 10^{14} \sim 2\times 10^{16}\text{GeV} \tag{5.13}$$

The lower mass range corresponds to very large $m_{\rm SUSY}$ while the higher values require $m_{\rm SUSY}$ to be nearer m_W. In SUSY GUTS, one expects the gauge boson mediated decay rate to be

$$\begin{aligned} 1/\Gamma\left(p\to e^+\pi^0\right) &\simeq 1.3\times 10^{29\pm0.7} \\ &\times\left(\frac{m_X}{2\times 10^{14}\text{GeV}}\right)^4 \text{yr} \qquad (\text{SUSY}) \end{aligned} \tag{5.14}$$

The IMB bound in Eq. (5.10) then rules out the $m_X \lesssim 10^{15}$GeV region in Eq. (5.13) but leaves open the possibility of $m_{\rm SUSY} \lesssim 10^6$GeV as the "new physics" we are looking for. SUSY GUTS also predict[1]

$$\sin^2\theta_W(m_W) = 0.237^{+0.003}_{-0.004} - \frac{4}{15}\frac{\alpha}{\pi}\ln(m_{\rm SUSY}/m_W) \tag{5.15}$$

which is in good accord with experiment (see table 4.1), particularly if m_t, $\Lambda^{(4)}_{\overline{\rm MS}}$ or $m_{\rm SUSY}$ is on the high side. This example illustrates how a new physics threshold (supersymmetry in this case) can bring GUTS into agreement with low energy phenomenology. It also demonstrates the complementarity between proton decay and high energy experiments. If $m_{SUSY} \lesssim 10$TeV, it is likely to be discovered at the SSC, and in this example, the proton decay rate is too slow to observe. On the other hand, if m_{SUSY} is beyond 10 TeV, the value of m_X is lower and the detection of proton decay is more likely. Of course, the use of a single supersymmetry mass scale is rather simplistic. Nevertheless, this example points out the importance of pushing the search for proton decay as far as possible.

6. EXTRA Z' BOSONS

Additional neutral gauge bosons (generically called Z' bosons) arise naturally in GUTS larger than SU(5).[14] The SO(10) model has one such additional boson which is often denoted by Z_χ while E_6 has Z_χ as well as a second flavor diagonal neutral boson Z_ψ. (At Brookhaven we call it the Z_J.) Their couplings are specified up to renormalization effects which are calculable if the entire particle spectrum is known.

The interaction Lagrangian for Z_χ and Z_ψ is given by

$$\mathcal{L}_{\text{int}} = -\sqrt{3/8}g_2 \tan\theta_W \sum_{i=\chi,\psi} \sqrt{\lambda_i} Z_i^\mu J_\mu^i \tag{6.1a}$$

$$J_\mu^i = \sum_f \left(Q_{f_R}^i \bar{f}_R \gamma_\mu f_R + Q_{f_L}^i \bar{f}_L \gamma_\mu f_L \right) \tag{6.1b}$$

where $\sqrt{\lambda_i}$ is a renormalization parameter that is generally ≈ 1. The charges in Eq. (6.1b) are completely specified (for each generation)

$$Q_{e_L}^\chi = Q_{\nu_L}^\chi = -Q_{d_R}^\chi = 3Q_{u_R}^\chi = 3Q_{e_R}^\chi = -3Q_{u_L}^\chi \;\; - 3Q_{d_L}^\chi = 1 \tag{6.2a}$$

$$Q_{\nu_L}^\psi = Q_{e_L}^\psi = Q_{u_L}^\psi = Q_{d_L}^\psi = -Q_{e_R}^\psi = -Q_{u_R}^\psi = -Q_{d_R}^\psi = \sqrt{5/27}. \tag{6.2b}$$

One expects Z_χ and Z_ψ to mix with one another (and probably mix somewhat with the ordinary Z). Ignoring potential small mixing[1] with the usual Z, the mass eigenstates can be denoted by $Z(\beta)$ and $Z'(\beta)$

$$Z(\beta) = Z_\psi \sin\beta + Z_\chi \cos\beta \tag{6.3a}$$

$$Z'(\beta) = Z_\psi \cos\beta - Z_\chi \sin\beta, \tag{6.3b}$$

with $m_{Z(\beta)} \le m_{Z'(\beta)}$.

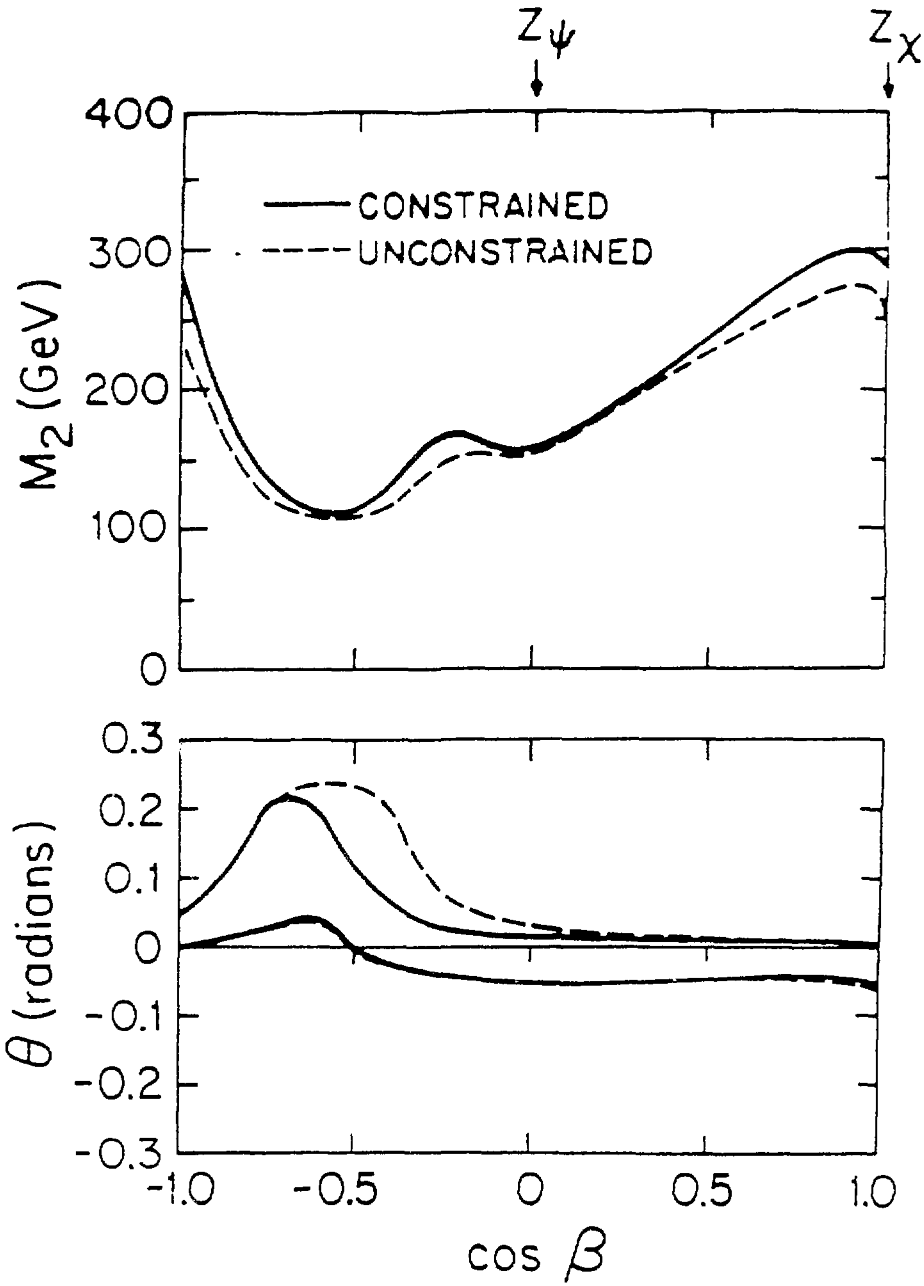

Fig. 6.1: Lower limits on the mass (M_2) of an E_6 boson $Z(\beta) = Z_\Psi \sin\beta + Z_\chi \cos\beta$. The dashed line corresponds to an unconstrained Higgs mechanism, i.e. $\rho \neq 1$. Also illustrated is the allowed range of mixing, θ, between $Z(\beta)$ and the standard model Z boson.

Bounds were placed on $m_{Z(\beta)}$ by the neutral current analysis in Ref. 1. They ranged between about 120 GeV for $\cos\beta \simeq -0.6$ to 300 GeV for $\beta \simeq 0$ (i.e. Z_χ). In fig. 6.1, experimental bounds are given for for $Z(\beta)$ mass (called M_2) allowing for possible mixing with the ordinary Z. (θ is the $Z - Z(\beta)$ mixing angle.) Note that the constraint is not very good for $Z(\beta)$ near the superstring inspired E_6 model $\cos\beta \simeq -0.6$. In fact, the data (in particular DESY e^+e^- annihilation results) slightly favors a $Z(\beta)$ near $\cos\beta \simeq -0.6$ which mixes with the ordinary Z. Of course, if such a scenario is correct, it will be easy to sort out at SLC. It is very important to push the bounds in fig. 6.1 into the TeV region or better yet find a Z'. To that end, the SSC will have a discovery potential for finding a Z' that should extend to 5-10 TeV.

What if a Z' is discovered? Such a discovery combined with a measurement of its couplings would almost certainly pinpoint the underlying GUT symmetry group. The absolute couplings, which could be obtained by comparing its production and decays with the standard Z, would then give us $\sqrt{\lambda_i}$ in Eq. (6.1a) and thus provide further important information about coupling evolutions and new thresholds. I should note that in the E_6 scenario, the mixing angle β should be relatively easy to determine since the branching ratios

$$\frac{\Gamma\left(Z(\beta) \to f\bar{f}\right)}{\Gamma\left(Z(\beta) \to \text{ all}\right)} = \frac{\left(Q^\beta_{f_L}\right)^2 + \left(Q^\beta_{f_R}\right)^2}{\sum_f \left(Q^\beta_{f_L}\right)^2 + \left(Q^\beta_{f_R}\right)^2} \tag{6.4}$$

$$Q^\beta_f \equiv Q^\chi_f \cos\beta + Q^\chi_f \sin\beta$$

depend only on β.

In the time between now and SSC physics, it will be interesting to see if hints of a Z' boson of any kind emerge from low energy phenomenology. In that regard, atomic parity violation and νe scattering experiments may reach high enough precision to probe for such particles up to $\simeq$ 800GeV during the intervening years. If evidence for a Z' is found, a super SLC capable of sitting on that resonance will be very desirable.

7. Implications of Precise Z Mass Measurements

What is the value of m_t? At present, one has the bounds

$$m_t \geq 26\text{GeV} \qquad \left(e^+e^- \text{ data}\right)^{15,16} \tag{7.1}$$

$$m_t \gtrsim 44\text{GeV} \qquad (UA1)^{17} \tag{7.2}$$

and the upper bound in Eq. (4.1). Furthermore, the recent $B_d^0 - \bar{B}_d^0$ oscillation signal observed by the ARGUS[18] collaboration seems to imply $m_t > 50$ GeV and some have suggested that it must be considerably larger.[19] That is to be compared with recent analyses[20] of $\sigma\left(p\bar{p} \to W \to e\nu\right)/\sigma\left(p\bar{p} \to Z \to e^+e^-\right)$ which favor $m_t \lesssim 65$ GeV. Unfortunately, neither argument is compelling. The top quark's mass could still be anywhere from 44 to 200 GeV. Experimental determination of m_t may, therefore, be several years away.

Given the m_t quandry, Paul Langacker, Alberto Sirlin and I[2] recently considered the following scenario. It is quite likely that the value of m_Z will be precisely determined to within ± 100 MeV at SLC before the top quark is discovered, particularly if m_t is large. That accuracy is to be compared with the present average[21]

$$m_Z = 91.9 \pm 1.8 \text{ GeV} \qquad (UA1 \ \& \ UA2) \tag{7.3}$$

or with somewhat better results of our global fit to all data[2] (assuming 10 GeV $\leq m_H \leq 1$ TeV and allowing m_t to vary)

$$m_Z = 91.8 \pm 0.9 \text{ GeV} \qquad \text{(all existing data)}\,. \tag{7.4}$$

Such a measurement will clearly represent a significant advancement; but we cannot use it alone to precisely determine $\sin^2\theta_W$ via Eq. (3.6) until m_t is known. (It will better determine $\sin^2\theta_W\,(m_W)_{\overline{\text{MS}}}$.) That point is illustrated in table 7.1 where Δr values are given as a function of m_t.

A value of $\sin^2\theta_W$ can be derived from Eq. (3.6)

$$\sin^2\theta_W = \frac{1}{2}\left[1 - \left(1 - \frac{1}{1-\Delta r}\left(\frac{74.562\text{Gev}}{m_Z}\right)^2\right)^{1/2}\right] \tag{7.5}$$

only if both Δr and m_Z are known.

One can, however, combine a precise SLC measurement of m_Z with existing neutral current data as well as UA1 and UA2 results for m_W and m_Z. Such a fit is illustrated in figure 7.1. Note that for $m_Z \geq 93.3$ GeV, no experimentally allowed value of m_t is consistent with neutral current data. Therefore, a high value of m_Z could well signal the presence of new physics beyond the standard model. If, on the other hand, m_Z turns out to be on the low side $\lesssim 90.5$ GeV, a large m_t or fourth generation would be preferred. For m_Z values in between, a bound on m_t is implied.

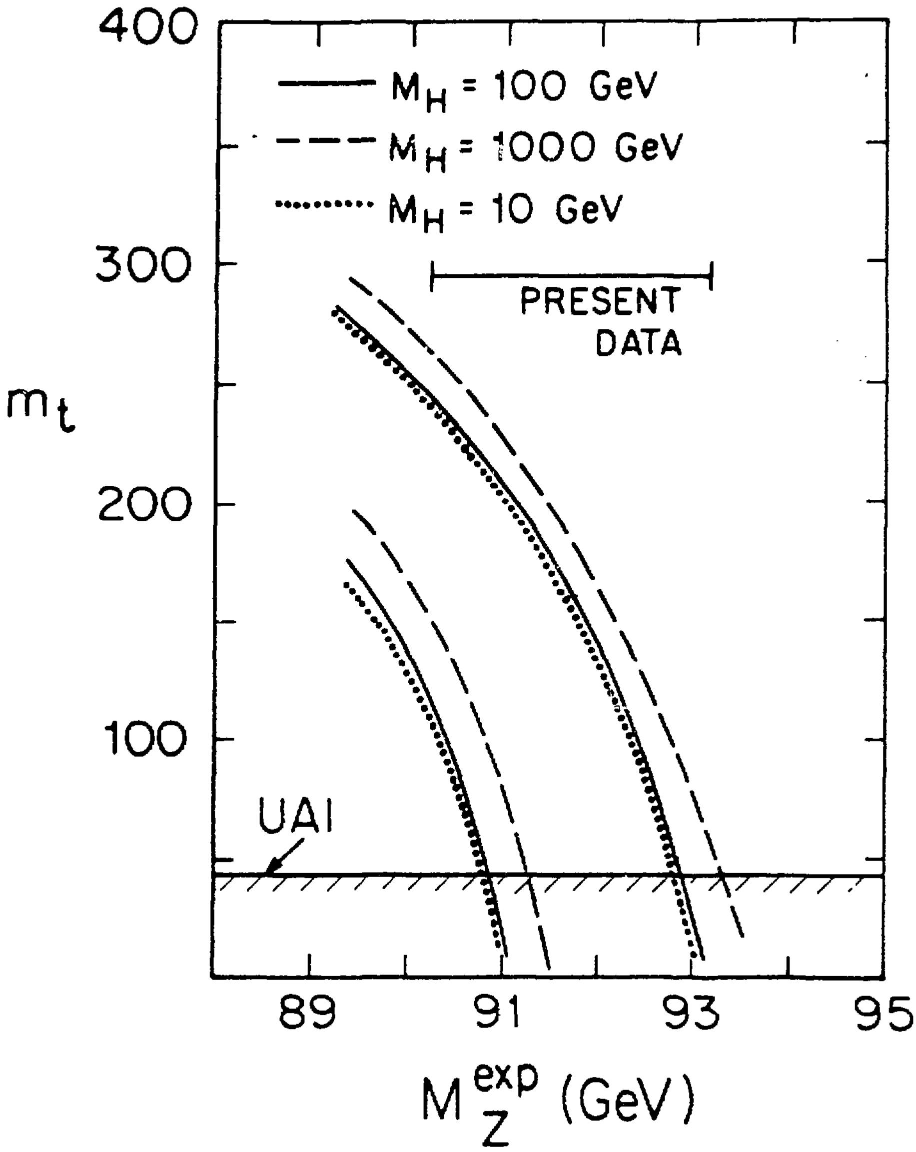

Fig. 7.1: 90% C.L. range allowed for m_t by combining existing data with a measurement $m_Z = m_Z^{\mathrm{exp}} \pm 100$ MeV, shown as a function of m_Z^{exp} for three values of the Higgs mass.

Table 7.2: Predicted Δr values for $\sin^2\theta_W \simeq 0.23$ and $m_H =$ 100 GeV. For $m_H = 1$ TeV, 0.0090 should be added while for $m_H = 10$ GeV, 0.0045 should be subtracted.

m_t (GeV)	Δr
45	0.0713 ± 0.0013
90	0.0606
120	0.0512
150	0.0412
180	0.0300
210	0.0173
240	0.0030

The constraints in fig. 7.1 are dominated by m_Z and deep-inelastic $\nu_\mu N$ scattering. Whereas $\sin^2\theta_W = 0.233 \pm 0.003 \pm 0.005$ obtained from the latter is very insensitive to m_t, $\sin^2\theta_W$ obtained from Eq. (7.5) implicitly depends on m_t via Δr. One could therefore obtain constraints on m_t (which are tighter than fig. 7.1) merely by comparing those two measurements. That should illustrate the utility of carrying out high precision measurements of $\sin^2\theta_W$ in as many distinct ways as possible. Comparison of very different types of measurements can provide distinct probes of new and old physics.

8. CONCLUSION

The $SU(2)_L \times U(1)$ model is in very good shape, even at the quantum loop level. Indeed, $\sin^2\theta_W$ has been determined with a precision of about 2%, if $m_t \lesssim 100$ GeV. That determination represents a world average of many diverse experiments which span Q^2 from 0 to m_z^2. It sets a standard that individual experiments should strive to attain or surpass. In that regard, we can expect high precision measurements of $\sin^2\theta_W$ via m_W, $\overset{(-)}{\nu}_\mu e$ scattering, atomic parity violation and polarization asymmetries in the future. Of course, anticipated measurements of m_Z at SLC and LEP have the potential to determine $\sin^2\theta_W$ to ± 0.0001, but only if we assume Δr in Eq. (7.5) is known. Otherwise, it will still play an important role in its comparison with other experiments.

The comparison of m_Z and deep-inelastic $\nu_\mu N$ scattering already probes the standard model at the loop level. That sensitivity provides the bounds on m_t

that were discussed. After m_t is known, we can use such comparisons to look for hints of new physics. Already, our global fits provide bounds on extra Z' bosons which are quite constraining. If one looks at where those bounds come from, it appears that atomic parity violation, $\overset{(-)}{\nu}_\mu e$ scattering and e^+e^- annihilation are particular good probes of Z' boson effects. Indeed, forthcoming experiments in those areas have the potential of searching up to masses $\simeq$ 800 GeV in the E_6 inspired models. Evidence for a Z' boson would of course provide a terrific window to the physics of grand unification and/or superstrings.

The coming years should be exciting times for high energy physics. SLC, TEVATRON and LEP have tremendous discovery potential. Of course, the advent of SSC will open up a completely new energy domain 1-10 TeV which may be filled with surprises. Rather than finding a desert, I think that physics beyond the Z will be richer than even most optimists have anticipated and will further challenge our creative imagination.

ACKNOWLEDGMENT

Work supported by the Department of Energy under Contract No. DE-AC02-76-CH00016.

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