



Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Physics, Computer Science & Mathematics Division

Presented at the 1981 Banff Summer Institute on Particles and Fields, Calgary, Alberta, Canada, August 16-28, 1981; and to be published in the Proceedings

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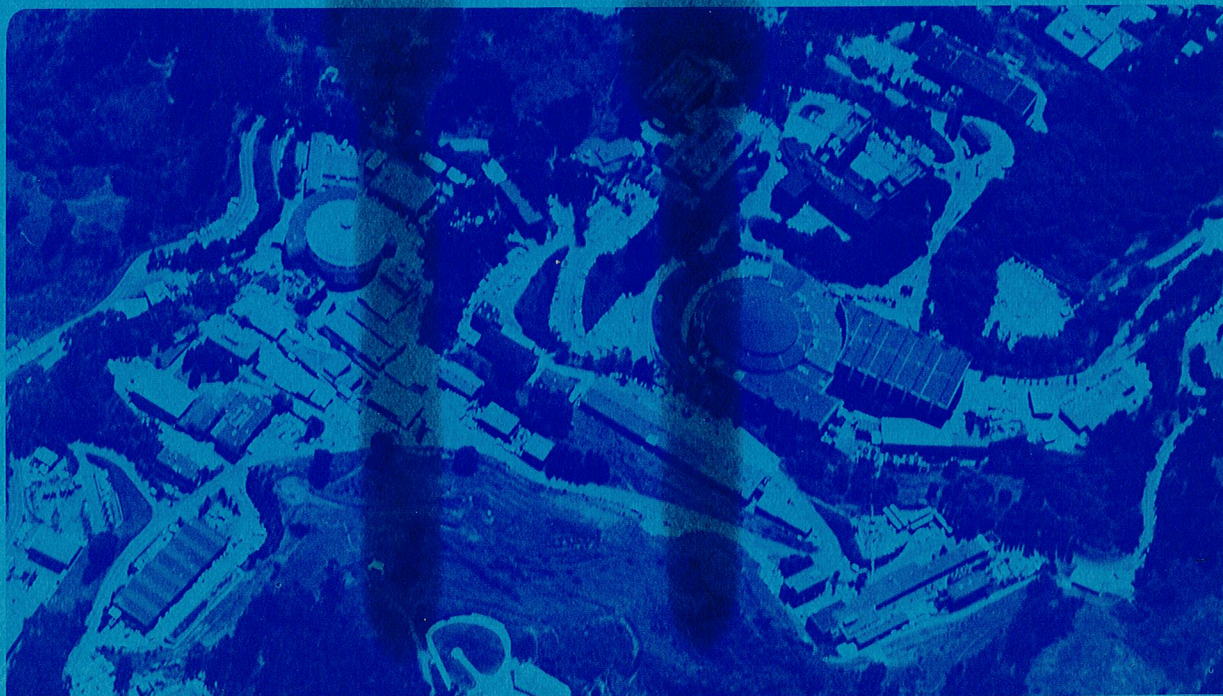
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SMALL $\bar{\theta}$ AND AN INVISIBLE AXION AS AUTOMATICCONSEQUENCES OF GRAND UNIFICATION[†]

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ABSTRACT

I explore the possibility of solving the strong CP problem in a grand unified theory in which a global Peccei-Quinn symmetry is an automatic consequence of the gauge structure. The low energy effective theory differs from that of the standard model only by the presence of an extremely light, weakly interacting axion.

^{*} This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

[†] To appear in Proceedings of 1981 Banff Summer Institute.

The QCD Lagrangian has two possible sources of CP violation.

The \tilde{FF} term, and the quark mass matrix:

$$\mathcal{L}_{\text{QCD}} = \dots - \frac{\theta g^2}{32\pi^2} \tilde{FF} + \dots - \bar{q}Mq .$$

Although the former is a total divergence, it may not be removed from the action because of instanton effects [1]. Under an axial phase rotation of the quark fields, $U(1)_A$, both the argument of the determinant of the quark mass matrix (ϕ_M) and θ change, such that only the combination $\bar{\theta} \equiv \theta - \phi_M$ is invariant and hence observable. The experimental limit on the electric dipole moment of the neutron [2] implies that $\bar{\theta} \lesssim 10^{-9}$. Why is the parameter $\bar{\theta}$ of the theory so small? This is the strong CP problem.

Various solutions have been suggested, but none is completely satisfactory. CP invariance may not be imposed on the Lagrangian of the standard model, because CP violation has been observed in kaon decays. Imposing CP invariance only on the dimension four couplings is a possibility, but usually leads to unappealing models. Finally, θ may be made unobservable by arranging for the Lagrangian to possess a global "symmetry" which is conserved up to a color anomaly. This is the Peccei-Quinn (PQ) mechanism [3], and the additional phase rotation is referred to as $U(1)_{\text{PQ}}$.

In this talk I will describe four models which incorporate the PQ mechanism. The simplest model has two $SU(2)$ doublets of scalar particles, but predicts a light (~ 100 keV) pseudo-Goldstone boson (PGB), the axion [4], which has yet to be experimentally confirmed. The second model removes such an axion, at the expense of introducing

a new fine tuning problem. This is resolved by embedding the model in a grand unified theory (GUT). Finally, a GUT which seeks to explain the origin of $U(1)_{PQ}$ is given. Although much of what follows involves the first three models, I have contributed only to the fourth.

The first model contains two Higgs doublets (ϕ_1, ϕ_2) and, for simplicity, a single generation of quarks. By demanding invariance (up to the anomaly) under a $U(1)_{PQ}$ phase symmetry with relative quantum numbers

	ϕ_1	ϕ_2	u_R	d_R	All other fields
$U(1)_{PQ}$:	1	-1	-1	-1	0

the terms of interest in the Lagrangian are

$$\mathcal{L}_1 = \dots - \left(\frac{m_u}{\lambda_1}\right) e^{\frac{i\theta}{2}} \bar{u}_L u_R \phi_1^0 - \left(\frac{m_d}{\lambda_2}\right) e^{\frac{i\theta}{2}} \bar{d}_L d_R \phi_2^{0*} + \text{h.c.} + V(\phi_1 \phi_2) \quad (1)$$

where ϕ has been rotated into the Yukawas by a $U(1)_A$ transformation, $\frac{m_u}{\lambda_1}$ and $\frac{m_d}{\lambda_2}$ are chosen real by choice of Higgs field phases, and the potential $V(\phi_1 \phi_2)$ is invariant under independent phase rotations of ϕ_1 and ϕ_2 . Since $U(1)_{PQ}$ is anomalous, θ is indeed unobservable in this model. Hence, on formation of CP conserving vacuum expectation values (vevs)

$$\langle \phi_i \rangle = \lambda_i e^{i\omega_i} \quad i = 1, 2$$

$$\langle \bar{u}u \rangle = v e^{i\theta_q}$$

$$\langle \bar{d}d \rangle = v e^{-i\theta_q},$$

with λ_i and v real, it is not surprising that the vacuum energy

$$V = \langle 0|H|0 \rangle = 2m_u v \cos\left(\frac{\theta}{2} + \theta_q + \omega_1\right) + 2m_d v \cos\left(\frac{\theta}{2} - \theta_q + \omega_2\right) \quad (2)$$

is minimized for $\theta + \omega_1 - \omega_2 = \bar{\theta} = 0$.

The $\langle \phi_i \rangle$ clearly break $U(1)_{PQ}$ and it appears that spontaneous symmetry breaking has led to a massless Goldstone boson. This is fallacious because Goldstone bosons couple to anomaly free currents. There are three independent neutral anomaly free currents broken by the above vevs: $J_3(SU(2)_L)$, the neutral broken weak gauge current, $J_3(SU(2)_{L+R})$, the neutral broken chiral $SU(2)$ current, and J'_{PQ} which is an anomaly free combination

$$J'_{PQ} = J(U(1)_{PQ}) + \alpha J_3(SU(2)_{L+R}) + \beta J(U(1)_A). \quad (3)$$

(The argument of each current denotes the symmetry with which it is associated.) The Yukawas explicitly break chiral symmetry, and the Π^0 which is coupled mainly to $J_3(SU(2)_{L+R})$ picks up a mass m_π . By virtue of Eq. (3) the PGB coupled to J'_{PQ} then picks up a mass αm_π . The properties of the axion, including the value of α , may be determined rather easily from the PGB mass matrix, In a basis where

the modes are given by variations in the vev phase angles $\delta\omega_1$, $\delta\omega_2$ and $\delta\theta_q$ the mass matrix is obtained simply by evaluating double derivatives of $V(\text{Eq. (2)})$ at its minimum:

$$M_{ab}^2 = \frac{1}{f_a f_b} \frac{\partial^2 V}{\partial \omega_a \partial \omega_b} \Big|_v = v \begin{pmatrix} \frac{m_u}{\lambda_1} & 0 & \frac{2m_u}{\lambda_1 f_\pi} \\ 0 & \frac{m_d}{\lambda_2} & \frac{2m_d}{\lambda_2 f_\pi} \\ \frac{2m_u}{\lambda_1 f_\pi} & \frac{2m_d}{\lambda_2 f_\pi} & \frac{4(m_u + m_d)}{f_\pi^2} \end{pmatrix}$$

where f_π is the pion decay constant. The eigenvectors are:

$$Z_L = \begin{pmatrix} \frac{\lambda_1}{\lambda} \\ \frac{\lambda_2}{\lambda} \\ \frac{f_\pi}{\lambda} \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \text{which has zero mass and is eaten}$$

to become the longitudinal Z ($\lambda = \sqrt{\lambda_1^2 + \lambda_2^2}$),

$$\pi^0 = \begin{pmatrix} \frac{m_u}{\lambda_1} \\ \frac{m_d}{\lambda_2} \\ \frac{2(m_u + m_d)}{f_\pi} \end{pmatrix} \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{with } m_\pi^2 = \frac{4(m_u + m_d)v}{f_\pi^2}, \text{ and}$$

$$A^0 \approx \begin{pmatrix} \frac{\lambda_2}{\lambda} \\ -\frac{\lambda_1}{\lambda} \\ \frac{f_\pi}{\lambda} \end{pmatrix} \sim \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad \text{with } m_A \sim \left(\frac{f_\pi}{\lambda}\right) m_\pi \quad (\text{I have set } \frac{\lambda_1}{\lambda_2} = 1$$

to get this value). The parameter α of Eq. (3) is $\frac{f_\pi}{\lambda} \sim 10^{-3}$, and the axion can be considered to have a small $\left(\frac{f_\pi}{\lambda}\right)$ admixture of Π^0 in it. The axion properties [4,5] may be summarized by:

$$\text{mass} \quad \left(\frac{f_\pi}{\lambda}\right) m_\pi$$

$$\text{coupling to matter} \quad \left(\frac{f_\pi}{\lambda}\right) (\pi^0 \text{ coupling to matter}), \text{ and}$$

$$\text{pseudoscalar fermion coupling} \quad \left(\frac{m_f}{\lambda}\right) \bar{f} \gamma_5 f A^0.$$

I now take the viewpoint that this conventional Weinberg-Wilczek axion has not been observed. Its absence in nature is far from certain [6]. The key observation [7,8], which allows the PQ mechanism to be retained while removing this axion, is that λ appears in the axion properties not because it is the scale at which weak interactions are broken, but because it is the scale which breaks $U(1)_{PQ}$. If $U(1)_{PQ}$ were broken at scale $V \gg \lambda$, λ would be replaced by V in the axion properties. The axion would be invisible.

This is precisely the situation in the following model [8] where a complex $SU(3) \times SU(2) \times U(1)$ singlet field S is added to the previous theory. Demanding invariance under

	ϕ_1	ϕ_2	u_R	d_R	S	Others
$U(1)_{PQ}$	1	-1	-1	-1	-1	0

the terms of interest in the Lagrangian are

$$\mathcal{L}_2 = \mathcal{L}_1 + \dots + g\phi_2^+ \phi_1 S^2 + V(S), \quad (4)$$

with g chosen real, and $V(S)$ invariant under independent phase rotations of S and chosen such that $\langle S \rangle = v e^{i\omega_0}$. The $\phi_2^+ \phi_1 S^2$ term is vital, it ensures that S takes part in $U(1)_{PQ}$ rather than a phase rotation all on its own. It contributes a large term to $V = \langle 0|H|0 \rangle$, but this fixes ω_0 , and does not spoil the alignment of $\bar{\theta}$ to zero. The PGB mass matrix is easily obtained as before. The matrix is 4×4 , although the fourth eigenvector is not a PGB as there is no additional global symmetry in this model. The Z_L and Π^0 are essentially as before. However, the axion is now mainly S , with mass and couplings $\sim \left(\frac{f}{v}\right)$.

What is this new scale V ? Without severely tuning parameters of the Higgs potential, V could be made one or two orders of magnitude larger than λ . However, constraints from the radiation of light axions in stellar physics [9, 10] suggest that if V is to be this large, then it must be larger than 10^9 GeV. Such a large scale requires an accurate fine tuning in this model, and suggests that it should be embedded in a GUT. The GUT must have an anomalous $U(1)_{PQ}$ which does not survive the symmetry breaking at the superheavy scale M . The resulting axion essentially decouples from the low

energy effective theory [11] which mimics the standard model except that $\bar{\theta}$ is zero in tree approximation.

A simple model which demonstrates how this comes about [10] is an $SU(5)$ theory with tens (T) and five bars (\bar{F}) of left handed fermions, two fundamentals of Higgs (H_1) and an adjoint of complex scalars (Σ). By demanding invariance under

	T	\bar{F}	H_1	H_2	Σ
$U(1)_{PQ}$	1	1	-2	2	2

θ is made unobservable and the Lagrangian takes the form

$$\begin{aligned} \mathcal{L}_3 = & \dots + g_1 TTH_1 + g_2 \bar{F}H_2 + \text{h.c.} \\ & + gH_2^+ \Sigma^2 H_1 + hH_2^+ H_1 \text{Tr}(\Sigma^2) + \text{h.c.} + V(H_1 H_2 \Sigma) \end{aligned} \quad (5)$$

where $V(H_1 H_2 \Sigma)$ is invariant under arbitrary phase rotations of H_1 , H_2 and Σ . The hypercharge component of Σ now plays the role of S and gets a superheavy vev breaking $U(1)_{PQ}$. As with all GUTs, this model does have a fine tuning problem. However, in this respect it is no worse than the minimal GUT; there is a single finely tuned cancellation which keeps the true Higgs doublet light. It may seem surprising that the low energy effective theory has only a single Higgs doublet; how is $\bar{\theta}$ to be rotated away? Because the phases of the two Yukawa couplings of the effective theory are now related, $\bar{\theta} = 0$ is still the minimum of the vacuum energy, even though the same Higgs doublet couples to both up and down quarks.

The Lagrangians $\mathcal{L}_{1,2,3}$ were all constructed by demanding invariance under a phase rotation $U(1)_{PQ}$. Two objections to this may be raised. Firstly, global symmetries seem to be additional unnecessary features of any gauge theory. Secondly, $U(1)_{PQ}$ is anomalous and imposing it does not even increase the symmetry of the theory. These objections demand a reason for imposing the $U(1)_{PQ}$ constraint. In Reference 12, Howard Georgi, Mark Wise and I proposed that situations exist in which even though the Lagrangian has a $U(1)_{PQ}$ invariance, there is no $U(1)_{PQ}$ constraint (i.e. the $U(1)_{PQ}$ phase rotation does not exclude any term from appearing in the Lagrangian). This is an example of an automatic symmetry which comes about as follows. A gauge theory is described by a gauge group and particle representations. When all possible renormalizable gauge invariant terms of the Lagrangian are written down, any global symmetries which remain we call automatic. In the standard model both baryon number (B) and lepton number (L) are automatic. Another example is B-L in minimal SU(5).

This example is worth investigating further because it is an automatic symmetry of a type which would not do for $U(1)_{PQ}$. Minimal SU(5) has the following automatic global symmetry:

$$\begin{array}{cccc}
 & T & F & H & \Sigma \\
 X: & 1 & 3 & -2 & 0 .
 \end{array}$$

This escapes being spontaneously broken by any vev by combining with the gauge hypercharge generator and becoming B-L. Further, X is anomaly free, which can be understood by embedding SU(5) in O(10) where B-L is a gauge generator. For an automatic symmetry to do the

job of $U(1)_{PQ}$, it must be spontaneously broken at superheavy scales, and must have an anomaly. It would never be possible to gauge such a current in a higher unified gauge theory; $U(1)_{PQ}$ is to be understood simply as an automatic symmetry.

These ideas can be illustrated by a straightforward extension of the GUT gauge group to SU(N) [12, 13]:

<u>L.H. field of model 3</u>		<u>L.H. field of model 4</u>	<u>Reason</u>
(a = 1 ... 5)		($\alpha = 1 \dots N$)	
T^{ab}	\rightarrow	$\psi^{\alpha\beta}$	} Retains $10 + \bar{5}$ family structure.
\bar{F}_a	\rightarrow	$(N-4)\chi_\alpha$	
Σ_b^a	\rightarrow	Σ_β^α	
H_1^a	\rightarrow	$\xi_{\alpha\beta\gamma\delta}$	$\xi\psi\psi$ gives U masses
H_2^a	\rightarrow	$(N-4)H_\alpha$	$\psi\chi H$ gives D masses.

Where is the $SU(3) \times SU(2) \times U(1)$ singlet scalar, the analogue of S of model 2? SU(N) gauge singlets could be added, or Σ_β^α could be taken complex, but such complications are preferably avoided. For $N \geq 9$ ξ has such singlets when its indices all lie in non-SU(5) directions. The Lagrangian

$$\mathcal{L}_4 = \dots + f_U \psi\psi\xi + f_D \psi\chi H + V(\xi, H, \Sigma)$$

has only two Yukawa couplings which limit possible automatic symmetries to linear combinations of P_1 and P_2

	ψ	χ	H	ξ
P_1 :	1	-1	0	-2
P_2 :	0	1	-1	0.

Further restrictions result from N ality breaking terms of the Higgs potential. For $N = 6, 7, 8$ and 12 these prove disastrous; for example $\epsilon_8 \xi^2$ for $N = 8$ (ϵ is the eight indexed totally antisymmetric tensor). For $N \geq 14$ there are no further restrictions. In these models a double disaster occurs. The anomaly free combination of P_1 and P_2 is B-L, which does not survive spontaneous symmetry breaking resulting in an exactly massless Goldstone boson. The orthogonal combination survives breaking at M but not λ , and gives rise to a Weinberg-Wilczek axion. Only a few values of N give the desired automatic symmetry; the smallest is $N = 9$. In this case the constraint from $\epsilon_9 \xi \xi H$ results in $U(1)_{PQ} = P_1 - 4P_2$. It is easy to check that this combination is anomalous, and that it is necessarily broken by the most general set of $SU(3) \times SU(2) \times U(1)$ invariant vevs, which are $O(M)$. Since the axions couplings are $O(\frac{1}{M})$ it decouples from the low energy effective theory, which is the standard model with $\bar{\theta} = 0$ to tree level.

While automatic symmetries give attractive alternatives to imposing global symmetries, there is a worrisome feature of the application to $U(1)_{PQ}$. The size of the term in the vacuum energy which aligns $\bar{\theta}$ to zero is $O(vm_q)$, which is very small compared with grand unified

scales. At the Planck mass (M_P) any new $U(1)_{PQ}$ violating interactions would appear in the GUT Lagrangian as non-renormalizable terms.

Although suppressed by powers of $(\frac{1}{M_P})$ these would surely be the dominant term aligning $\bar{\theta}$. The mechanism for resolving the strong CP problem would be destroyed. For example, adding a dimension six term $\epsilon_9 \xi H^5$ to \mathcal{L}_4 would align $\bar{\theta}$ to $\theta - (\omega_6 + \omega_7 + \omega_8 + \omega_9)$ where ω_i are the phases of $\langle H_i \rangle$, $i = 6 \dots 9$. We must assume that interactions at M_P also yield $U(1)_{PQ}$ automatically.

In this talk I have discussed two aspects of gauge theories which are considered by many to be undesirable. The first of these is the imposition of global symmetries. Why are gauge symmetries not sufficient? The second is the fine tuning of parameters of the theory. This is not only unnecessary, but amazingly perverse. Within the framework of grand unification I showed that it may not be necessary to impose $U(1)_{PQ}$ or to fine tune $\bar{\theta}$. Such a scheme does have drawbacks. The gauge group and representation content has been enlarged. The new axion has been constructed to be highly invisible; but maybe such a light particle is no worse a prediction of grand unification than that of superheavy particles. Finally the gauge hierarchy problem seems more acute than ever. One might hope that physics at the scale of the Planck mass will shed light on this hierarchy puzzle, as well as realizing the idea of an automatic Peccei-Quinn symmetry in an elegant way.

ACKNOWLEDGEMENTS

I would like to thank Howard Georgi and Mark Wise for teaching me much of what I know on this subject, and for sharing their ideas so freely. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

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