

Frictional Ignition of Methane by Continuous-Mining Machines in Underground Coal Mines



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FRictional IGNITION OF METHANE BY CONTINUOUS-MINING MACHINES IN UNDERGROUND COAL MINES

by

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ABSTRACT

This report presents information on frictional methane ignitions associated with continuous-mining machines, or continuous miners, as they are commonly called in the mining industry. Data indicate a gradual increase in frictional methane ignitions over the 6-year period 1971 through 1976. Ninety percent of these ignitions are attributed to continuous-mining machines. Factors affecting such frictional ignitions are analyzed and recommendations are submitted for reducing their incidence.

INTRODUCTION

Because methane gas liberated from coal and surrounding strata is explosive and ignites easily, it presents the most critical hazard in underground coal mining. As progressive increases in coal production are demanded, massive high-capacity continuous mining machines have emerged over the past three decades as the primary means of extracting coal. These machines can advance rapidly into virgin coal, causing increased liberation of methane and producing coal dust of finer particle size. In the presence of heat generated through frictional impact by the continuous miner, methane and fine coal dust provide the fuel for a potential propagating explosion. Further, demands for increased coal production to alleviate the energy shortage have caused the opening of new underground mines in highly gassy coal seams. This may further increase the frequency of frictional ignitions.

Data given in table 1 indicate that gas ignitions in underground coal mines are quite numerous and are attributed to a variety of causes. Flammable mixtures of gas may be ignited by frictional heating and sparking of cutter bits on continuous miners, longwall shearers, or cutting machines or drill bits on roof bolters; during cutting and welding operations; or by blown trailing cables, misuse of explosives, electrical arcing of trolley locomotives,

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breaking of rocks under stress during roof falls, open flames, smoking, or lightning in slopes and shafts. The data, presented graphically in figure 1, show an increase in the number of methane ignitions over the 6-year period 1971 through 1976.

TABLE 1. - Causes of methane ignitions in underground coal mines, 1971-1976

Cause of ignition	Number of methane ignitions							Total 1971-1976	
	1971	1972	1973	1974	1975	1976	Total	Injuries	Fatalities
Frictional ignitions:									
Continuous miner....	30	20	22	44	50	57	223	29	0
Cutting machine.....	3	1	0	1	1	1	7	0	0
Roof bolter.....	0	1	1	0	2	3	7	4	0
Roof fall.....	1	2	0	0	1	2	6	1	¹ 11
Longwall shearer....	0	0	0	0	0	4	4	0	0
Total.....	34	24	23	45	54	67	247	34	11
Non-frictional ignitions:									
Cutting and welding.	1	2	3	5	3	8	22	23	1
Electrical arc.....	1	2	2	3	5	4	17	5	² 22
Explosives.....	2	1	0	1	4	0	8	8	0
Smoking.....	1	0	0	0	0	0	1	2	0
Lightning.....	0	0	0	1	0	2	3	0	0
Total.....	5	5	5	10	12	14	51	38	23
Grand total....	39	29	28	55	66	81	298	72	34

¹Fatalities occurred in second Scotia Mine Explosion, March 11, 1976.

²Fifteen fatalities attributed to first Scotia Mine Explosion, March 9, 1976.

Over the 6-year reporting period, frictional gas ignitions attributed to continuous mining machines accounted for 223, or 75 percent, of the 298 reported ignitions. This report focuses on factors that contribute to and are associated with such frictional gas ignitions by continuous mining machines. Gas ignitions occurring prior to 1971 are excluded because reporting of ignitions was not mandatory and the information available is incomplete.

Examination of the occurrence of frictional ignitions on a geographic basis such as district-by-district, state-by-state, or mine-to-mine did not show a significant trend. Methane liberations, type of roof rock, type of bottom rock, type of seam inclusions, type of bits, type of continuous miners, etc., vary within a single mine and cannot be considered as parameters over a wide geographical area. Therefore, frictional methane ignitions attributed to continuous miners have been analyzed on an industry-wide basis.

This report is based on data derived from formal investigative reports prepared by inspection personnel of the Coal Mine Safety and Health activity of the Mine Safety and Health Administration during investigations of gas ignitions in accordance with the provisions of Title 30, Code of Federal Regulations, Part 80--Notification, Investigation, Reports, and Records of

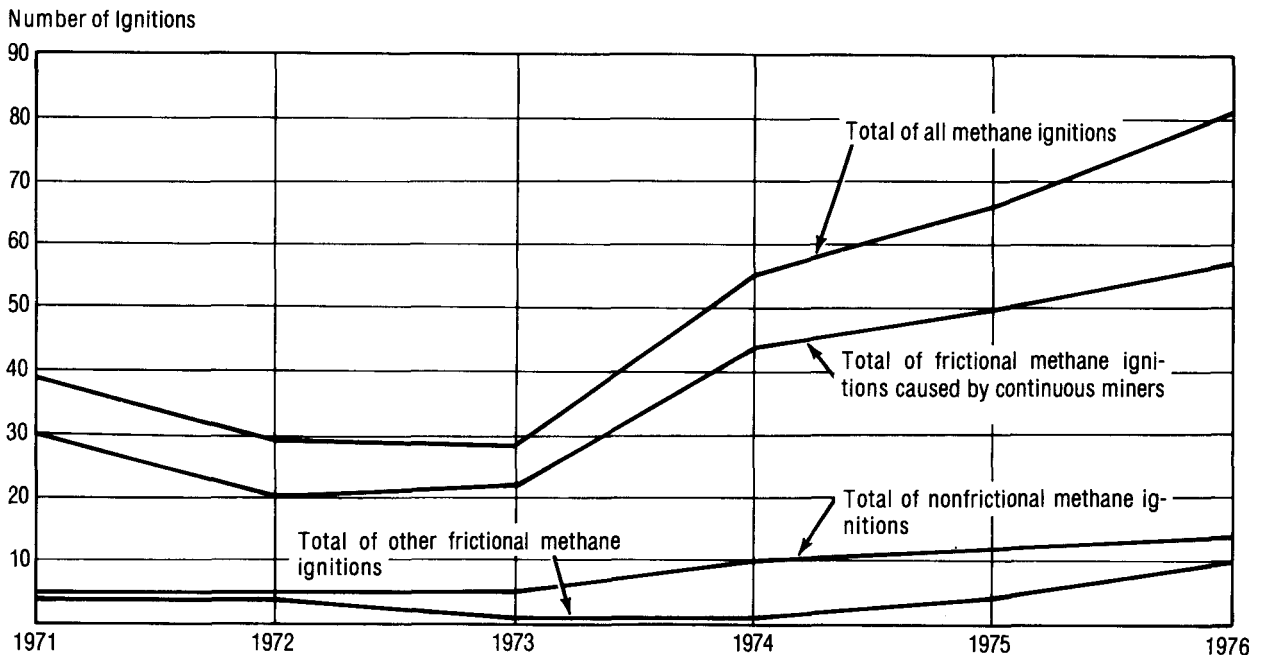


FIGURE 1. - Incidence of methane ignitions in underground coal mines, 1971-1976.

Accidents.⁴ In many instances certain information was not available to the inspector. For example, if length of flame travel during an ignition was not witnessed by miners, and if clear evidence of flame travel along the roof was not present, distances of flame travel could not be recorded and were, therefore, omitted. Thus, in the 298 gas ignition reports analyzed, flame lengths were recorded for 208 ignitions.

The purpose of this report is to inform mine workers, mining officials, and others affiliated with underground coal mines of the hazardous potential of frictional ignitions associated with continuous mining machines. Most of these ignitions involve only small, localized gas bodies and occur without injuries to miners and without property damage. However, it must be emphasized that under certain mine conditions, each ignition is potentially the source of a catastrophic coal dust explosion. With this fact confronting mine personnel, there is no excuse for complacency of indifference toward even these minor methane ignitions.

NATURE AND MECHANISM OF FRICTIONAL IGNITIONS

As previously mentioned, liberations of methane present the most serious explosion hazard in underground coal mines. The hazard is compounded by the physical characteristics of methane. Odorless, colorless, flammable, and lighter than air (specific gravity 0.55), methane has lower and upper

⁴Incorporated into Part 50 as of January 1, 1978.

explosive limits of 5 and 15 percent in air. The optimum or stoichiometric explosive mixture of methane in air is 9.5 percent.

When coal dust is suspended in a methane-air mixture, the lower explosive limits of coal dust and methane are decreased according to their relative concentrations in the mixture. As shown in figure 2,⁵ the minimum concentration of minus 200-mesh Pittsburgh coal dust required for ignition decreases linearly from 0.06 ounces per cubic foot to 0 as the methane concentration increases from 0 to 5.0 percent. The lower and upper explosive concentration limits of coal dust also vary according to its proximate analysis and particle size distribution. Under the test conditions applying in figure 2, a dust concentration corresponding to the lower explosive limit obscures a miners' cap lamp at 10 feet.

Less than 1.0 millijoule of energy is required to ignite a gas-air mixture.⁶ This amount of energy is only a fraction of the static energy that may be discharged by an average-sized person walking across a carpeted floor in a dry atmosphere. The ignition hazard is especially serious in some mines that liberate as much as 2000 cubic feet of methane per ton of coal mined.

As implied by their name, continuous mining machines, which extract coal from solid faces, are a radical change from equipment used in the cycle of conventional mining. Continuous miners are massive, high-powered machines that perform work by expending high energy with cutter bits striking coal

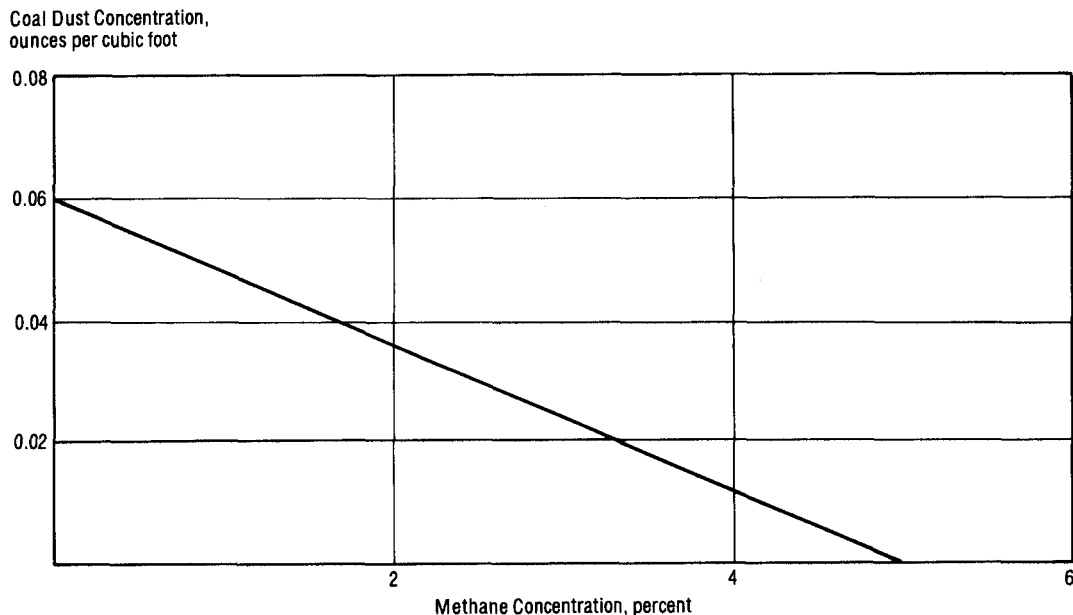


FIGURE 2. - Minimum explosive concentrations of float Pittsburgh coal dust in methane-air atmospheres.

⁵Nagy, John, and W. M. Portman. Explosibility of Coal Dust in an Atmosphere Containing a Low Percentage of Methane. BuMines RI 5815, 1961, 16 pp.

⁶Zabetakis, M. G. Flammability Characteristics of Combustible Gases and Vapors. BuMines Bull. 627, 1965, 121 pp.

faces. Moreover, hard pyritic-type inclusions and bands within seams, as well as hard-rock formations overlying and underlying seams, present serious ignition problems when struck by cutter bits. More methane is liberated and finer coal dust is produced with continuous than conventional mining techniques.

As reported by Hartmann,⁷ "... flammable mixtures of gas are ignited by frictional heating in two ways: (1) a small volume of gas is heated to the ignition temperature by direct contact with a hot part of one of the rubbing surfaces; (2) small heated particles, commonly termed frictional sparks, are projected into the gas mixture. These frictional sparks are small, hot particles of solid matter torn from one of the larger pieces in contact. The particles may be inert, in which case their temperature is limited by their melting point, or they may be chemically active, in which case their temperature is augmented by oxidation. The existence of frictional sparks should be considered as a warning in coal mines where a flammable atmosphere exists, but they do not necessarily mean that an ignition or explosion will occur. To ignite the gas, the sparks must have a high temperature and their heat content must be sufficient to impart the necessary amount of heat to a small volume of the gas mixture. The ability of sparks to ignite firedamp is greatly restricted by the short period of contact with the gas, owing to cooling by convection, and to the 'ignition lag' period of the gas, which varies with composition and with temperature. All these factors make flying sparks generally less dangerous than sparks whose paths are obstructed by a thermal insulator at an early stage."

Nagy⁸ has also pointed out that frictional sparks can occur in a gaseous atmosphere without causing an ignition. Frictional sparks are produced very readily with little expenditure of energy and are virtually impossible to eliminate in coal mining.

In considering cutter bits striking against hardrock formations, researchers⁹ have shown that quartz-bearing sandstones present the greatest frictional ignition hazard. Evidence points to the ignition of methane from flashing hotspots on the sandstone rather than from frictional sparks. Other studies¹⁰ have also demonstrated the ease of igniting methane by striking metal against sandstone surfaces.

It has been established that methane can be ignited by cutter bits striking pyritic nodules or bands.¹¹ The rubbing action of cutter bits on pyrites reduced the iron sulfides to a fine powder, which is heated to between 200 and 300° C. At this temperature, the powdered pyrite oxidizes rapidly, giving a

⁷Hartmann, Irving. Frictional Ignition of Gas by Mining Machines. BuMines IC 7727, 1955, 17 pp.

⁸Nagy, John and E. M. Kawenski. Frictional Ignition of Gas During A Roof Fall. BuMines RI 5548, 1960, 11 pp.

⁹Blickensderfer, R., D. K. Deardorff, and J. E. Kelley. Incendivity of Some Coal-Cutter-Materials by Impact-Abrasion in Air-Methane. BuMines RI 7930, 1974, 20 pp.

¹⁰Work cited in footnote 8.

¹¹Work cited in footnote 7.

flame of burning sulfur that ignites methane. The rubbing action that produces powdered pyrite and generates heat is much greater with dull cutting bits.

Researchers¹² have also shown that the rotational speed of cutter heads on continuous miners may affect the ease with which ignitions are produced. Rotational speeds not exceeding 450 feet per minute have been recommended to minimize the incidence of frictional ignitions.

SAFETY STANDARDS AFFECTING METHANE IGNITIONS

Current mandatory mine safety standards require many precautionary measures in face areas to prevent methane ignitions. Mandatory standards in Title 30, Code of Federal Regulations, Part 75, Subpart D, cover use of methane monitors, air volume requirements, periodic tests for methane, and specific locations for ventilation controls in underground coal mines. Notwithstanding these precautionary requirements, frictional ignitions attributed to continuous miners have increased annually during the latter 5-year period covered in this report.

VENTILATION CONTROLS

In order to determine the principal causes of frictional ignitions and factors contributing to their occurrence, statistical correlations were derived to associate contributing factors with ignitions. An important factor affecting face ignitions is the distance of ventilation controls (that is, line curtains or exhaust tubing) from the face. The location of these controls governs the flow of air to the face, and the primary function of the air is to dilute and carry away methane and coal dust. If the ventilation controls are not close enough to the face, the probability of methane accumulations extending outby along the roof increases, and so does the ignition potential. Consequently, it is meaningful to identify a possible correlation between two variables, the location of ventilation controls and the length of flame travel during an ignition. As shown in the appendix, a statistical analysis was made on data from investigation reports to determine the strength of a correlation between the two variables. This analysis indicated a very strong correlation, with evidence that 92 percent of the variation in length of flame travel can be attributed to the distance of ventilation controls from the face. A second statistical test, in which the observed value is compared with a tabulated value, discloses a direct linear correlation between flame length and ventilation control locations at a 95 percent confidence level.

POINTS OF IGNITION

Table 2 shows that of the 223 frictional ignitions attributed to continuous miners, 143, or 64 percent, were caused by bits striking roof rock, and 49, or 22 percent, were the result of bits striking inclusions within the coal seam. The balance of the frictional ignitions, 31, or 14 percent of the total, are attributed to bits striking the mine bottom. Because methane is less dense than air, it will accumulate at the roof in layers of varying

¹²Work cited in footnote 9.

concentrations. This phenomenon, and the inherent difficulty of controlling the cutter head of the continuous miner at an undulating roof line, account for the higher incidence of gas ignitions from bits hitting roof rocks, particularly the quartz-bearing sandstone types. Under certain conditions, methane layers may also be ignited when pyritic inclusions and bands within seams are contacted. For example, hot oxidized fragments of pyrite may be projected upward into the flammable gas layer, or hot particles adhering to cutter bits may be extended into and ignite the methane layers. If it is theorized that pyritic bands and inclusions cause ignition of methane roof layers in the manner described, then as many as 86 percent of the ignitions may actually have occurred near the roof. Records indicate that ignitions near the floor are generally gas fissures and result in pencil-like flame projections. This type of ignition is not self-extinguishing and requires application of an extinguishing agent. In contrast, roof layer and cavity ignitions are generally self-extinguishing, yet present the greatest potential for a catastrophic explosion.

TABLE 2. - Methane ignitions as affected by the location of materials contacted by cutter bits, 1971-1976

Surfaces contacted by bits	Number of ignitions recorded						Total 1971-1976
	1971	1972	1973	1974	1975	1976	
Roof.....	25	12	15	32	24	35	143
Inclusions in coal seam.....	4	8	5	8	9	15	49
Bottom.....	1	0	2	4	17	7	31
Total.....	30	20	22	44	50	57	223

CUTTER BITS

Researchers have differed as to the methane ignition hazard of cutter bits contacting rocks. Some have stated that it is the composition of the rock rather than the metallic composition of the cutter bits that affects ignition.¹³ Other researchers¹⁴ have shown that wide range of ignition potential exists between cutter bits and tips of varied composition as determined in impact tests on quartzitic sandstone.

Analysis of frictional ignition reports has indicated that conventional plumb-bob type cutter bits, severely worn and with the carbide tip broken off, are the major contributors to frictional ignitions. As can be seen from figure 3, dull bits are much more hazardous than sharp bits because they provide a greater surface area for generating heat. The surface cutting area of a bit may be increased by wear from a point contact to an elliptical area exceeding 1 square inch. The shank of a plumb-bob type bit fits loosely into the bit block, which permits rotation for distributing wear and for self-sharpening. When the bit tip becomes worn, increasing the surface area, rapid heat build-up occurs. Heat cannot be readily dissipated through the loose fitting shank, and consequently bit temperatures rise and gas ignition potential

¹³Work cited in footnote 7.

¹⁴Work cited in footnote 9.

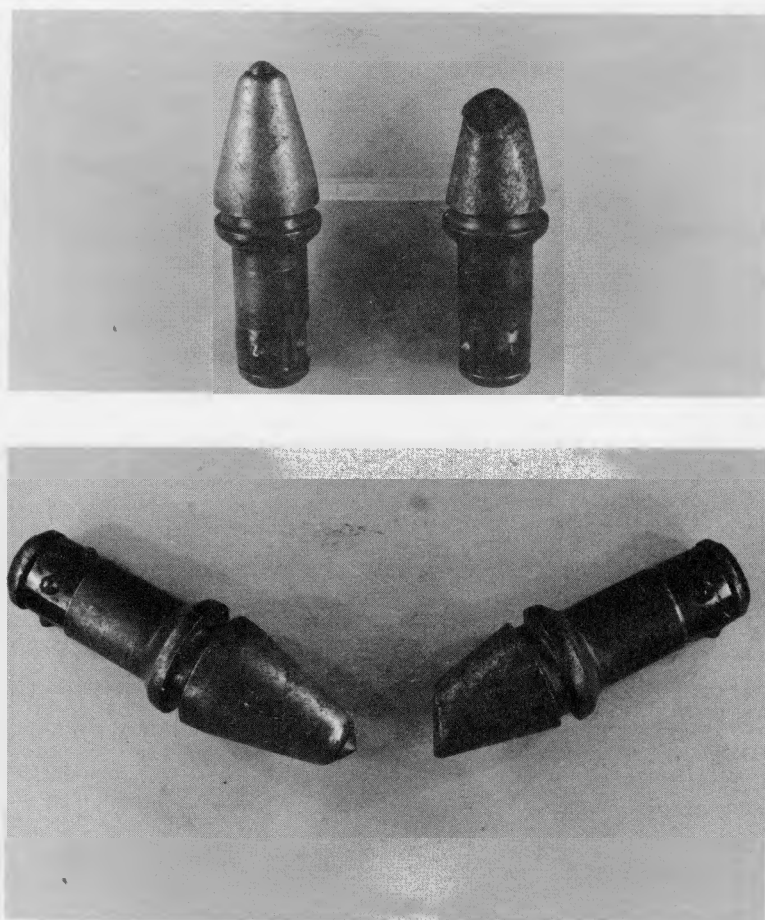


FIGURE 3. - Typical appearance of new and worn plumb-bob type cutter bits. Top photo, comparative surface cutting area of new and worn plumb-bob type cutter bits; bottom photo, profile of new and worn plumb-bob type cutter bits.

assume that a combination of higher bit speeds and the larger contacting surfaces of dull bits would certainly increase the ignition potential. This combination is evident in inspection reports of frictional ignitions, and is probably the major cause of most such ignitions.

INJURIES

Fortunately, no fatalities occurred in the 223 reported continuous miner frictional ignitions, as shown in table 1. The 29 reported injuries, mostly burns, give a frequency of one injury for about every eight ignitions. Data

increases. Blickensderfer¹⁵ has reported temperatures of hotspots on rock samples, caused by impacts with cutter bit steels, ranging from 1200 to 1430° C. In Guest's experiments,¹⁶ a 6 percent natural gas (93% methane)-air mixture was ignited by a metal bar heated to 933° C; with a coating of pyritic dust on the bar, ignitions were obtained at temperatures as low as 644° C.

In addition to the metallic composition of cutter bits, the same researchers¹⁷ have stressed the high ignition potential of the steel shank of a cutter bit when the bit is operated at elevated rotational speeds. Bit specimens involved in gas ignitions generally show excessive wear on shank steel from rock contact, which suggests the cause of such ignitions. Data indicate that with bits striking quartzitic sandstone at speeds exceeding 450 feet per minute, an ignition is practically assured even at relatively low expenditures of energy. It is logical to

¹⁵Work cited in footnote 9.

¹⁶Guest, P. G. Ignition of Natural Gas-Air Mixtures by Heated Surfaces. BuMines TP 475, 1930, 59 pp.

¹⁷Work cited in footnote 9.

given in table 3 indicate that an increase in injuries occurs with extensions of flame travel to and beyond the continuous miner operator's control position. Under normal conditions, the miner operator and his helper are exposed to any flame traveling beyond 20 feet. Table 3 shows that in the 21 ignitions when flame traveled beyond 20 feet, 24 men sustained injuries. Twenty-two injuries resulted from 89 ignitions that occurred when ventilation controls were more than 10 feet from the face, as shown in table 4.

CONCLUSIONS AND RECOMMENDATIONS

The basic means of preventing frictional ignitions is to remove or dilute methane by delivering an adequate supply of uncontaminated air to the face areas of continuous mining machine sections. Numerous publications¹⁸ have been written to inform the mining public of recommended procedures for ventilating these face areas. Despite the many recommended procedures, maintaining an adequate and continuous supply of air at the face during operation of bulky mining machines still presents a critical problem. As statistical calculations show, a direct correlation exists between the distance of ventilation controls from the face and the length of flame travel. Further, an obvious relationship exists between the distance of flame travel and the number of injuries sustained due to these ignitions. To decrease the incidence of frictional ignitions, the following recommendations are submitted to improve face ventilation:

1. Make proper use of diffuser fans, line curtains, check curtains, and ventilation tubing or combinations thereof.
2. Minimize obstructions to air flow such as coal piles and secondary loading behind continuous miners.
3. Minimize air leakage by proper construction and maintenance of ventilation controls.
4. Utilize Venturi principle of water sprays to increase flow of air between face and continuous miner cutter head.

The following additional safety measures may aid in preventing and/or quenching frictional ignitions:

1. Reduce speed of rotation of cutting heads.
2. Insure adequacy of water flow through spray heads to facilitate cooling of cutter bits.

¹⁸Luxner, J. V. Face Ventilation in Underground Bituminous Coal Mines.

BuMines RI 7223, 1969, 16 pp.

Schlick, D. P., and R. W. Dalzell. Ventilation of Continuous Miner Places in Coal Mines. BuMines IC 8161, 1963, 18 pp.

Stahl, R. W., and C. H. Hodge. Methane Build-Up During Cutting and Continuous Mining Operations. BuMines RI 5288, 1956, 22 pp.

TABLE 3. - Length of flame extent during frictional methane ignitions attributed to continuous miners as affecting the number of injuries sustained, 1971-1976

Extent of flame from coal face, ft	Number of ignitions							Number of injuries						
	1971	1972	1973	1974	1975	1976	Total 1971-1976	1971	1972	1973	1974	1975	1976	Total 1971-1976
Less than 10.....	18	6	11	21	36	38	130	0	0	0	0	0	0	0
11-20.....	9	7	6	15	9	11	57	1	0	1	1	2	1	6
21-50.....	2	4	0	3	3	6	18	3	5	1	2	2	7	20
Over 50.....	1	0	0	1	1	0	3	3	0	0	1	0	0	4
Total.....	30	17	17	40	49	55	208	7	5	2	4	4	8	30

TABLE 4. - Distance of ventilation controls from the coal face during frictional methane ignitions attributed to continuous miners as affecting the number of injuries sustained, 1971-1976

Distance of ventilation controls from coal face, ft	Number of ignitions ¹							Number of injuries						
	1971	1972	1973	1974	1975	1976	Total 1971-1976	1971	1972	1973	1974	1975	1976	Total 1971-1976
Less than 10.....	14	9	8	18	25	26	100	0	4	0	0	0	0	4
11-20.....	7	5	8	11	19	21	71	0	0	1	3	3	6	13
21-50.....	3	2	1	5	1	5	17	6	1	0	0	0	2	9
Over 50.....	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total.....	24	16	17	34	45	52	188	6	5	1	3	3	8	26

¹Locations of ventilation controls were omitted in reports of 4 frictional ignitions in which single injuries occurred.

3. Exercise caution with respect to possible methane liberations while mining through clay veins and taking roof or bottom rock.

4. Replace dull and broken cutter bits promptly.

5. Insure proper functioning, maintenance, and calibration of methane monitors.

6. Locate methane monitors as near as practicable to the front of the continuous mining machine.

7. Promote and encourage developments in technology leading to a system to detect and suppress methane ignitions caused by face-cutting equipment.

APPENDIX

In the statistical analysis¹ that follows, the relationship between the x and y variables becomes stronger as the calculated coefficient of correlation (r) approaches unity (1), and conversely, the relationship becomes weaker as the value approaches zero (0). A value of " r " equal to 0.96, as here, indicates a very strong correlation. Further, $100r^2$ here equals 92 percent, which implies that 92 percent of the variation in length of flame travel can be attributed to the distance of ventilation controls from the face.

A second statistical test² was applied to determine whether the observed " r ," 0.96, is larger than that value which would have been obtained accidentally in the absence of a correlation. When compared to a tabulated " r " value given by Brownlee,³ it is shown at a 95 percent confidence level that a direct correlation exists between flame length and locations of ventilation controls.

Data used in the statistical calculations following are the same as given in tables 3 and 4.

TABLE A-1. - Calculations for correlating the location of ventilation controls with the extent of flame travel in frictional methane ignitions

Distance from coal face, feet	x (number of ignitions with given distance of flame extension)	y (number of ignitions with given distance of ventilation controls)	x^2	y^2	xy
Less than 10...	130	100	16,900	10,000	13,000
11-20.....	57	71	3,249	5,041	4,047
21-50.....	18	17	324	289	306
Over 50.....	3	0	9	0	9
Total.....	208	188	20,482	15,330	17,353

Derivation of r :

$$\begin{aligned}
 N &= 4 & S_{xx} &= \text{dispersion of values corresponding to the flame extension.} \\
 \Sigma x &= 208 & S_{yy} &= \text{dispersion of values corresponding to the distance of} \\
 \Sigma y &= 188 & & \text{ventilation controls.} \\
 \Sigma x^2 &= 20,482 & S_{xy} &= \text{dispersion of values corresponding to both variables.} \\
 \Sigma y^2 &= 15,330 \\
 \Sigma xy &= 17,353 \\
 S_{xx} &= N\Sigma x^2 - (x)^2 = 4(20,482) - (208)^2 = 38,664 \\
 S_{yy} &= N\Sigma y^2 - (y)^2 = 4(15,330) - (188)^2 = 25,976 \\
 S_{xy} &= N(\Sigma xy) - xy = 4(17,353) - (208)(188) = 30,308
 \end{aligned}$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} = \frac{30,308}{\sqrt{(38,664)(25,976)}} = .96$$

¹Miller, Irwin, and John E. Freund. Probability and Statistics for Engineers. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965, 432 pp.

²Brownlee, K. A. Industrial Experimentation. Chemical Publishing Co., Inc., New York, 4th American ed., 1953, 194 pp.

³Work cited in footnote 2.