A Dynamical η'–Mass from an Infrared Enhanced Gluon Exchange

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Abstract. The pseudo–scalar flavor–singlet meson mixes with two gluons. A dimensional argument by Kogut and Susskind shows that this can screen the Goldstone pole of the chiral limit in this channel, if the gluon correlations are infrared enhanced. Using a gluon propagator as singular as σ/k^4 for $k^2 \to 0$ we relate the screening mass to the string tension σ. In the Witten–Veneziano action to describe the η–η' mixing this relation yields masses of about 810MeV for the η', 430MeV for the η and a mixing angle of about $-30°$ from the phenomenological value $\sigma \approx 0.18\text{GeV}^2$. The very weak temperature dependence of the string tension should make this mechanism experimentally distinguishable from exponentially temperature dependent instanton model predictions.

More than twenty years ago Kogut and Susskind pointed out that for dimensional reasons a non–vanishing contribution to the mass of the pseudo–scalar flavor–singlet meson in the chiral limit can result from its mixing with two non–perturbatively infrared enhanced gluons corresponding to a momentum space propagator $D(k) \sim \sigma/k^4$ for $k^2 \to 0$ [1]. Such infrared enhanced gluon correlations are known to lead to an area law in analogy to the Schwinger model in two dimensions. The identification of the string tension σ shows that effects due to infrared enhanced gluons can be expected to be complementary to instanton models.

In particular, a description of the η–η' mixing in terms of contributions to the topological susceptibility from infrared enhanced gluon correlations, thus driven by the string tension, provides an interesting alternative to the standard solution of the $U_A(1)$ problem by instantons.

Therefore we will not only test whether an anomalous mass of the right order of magnitude is generated but we will also try to shed some light on the question whether this mechanism leads to physically acceptable consequences, e.g. whether the experimental bounds for the η' decay constant will not be spoiled.

DISCLAIMER

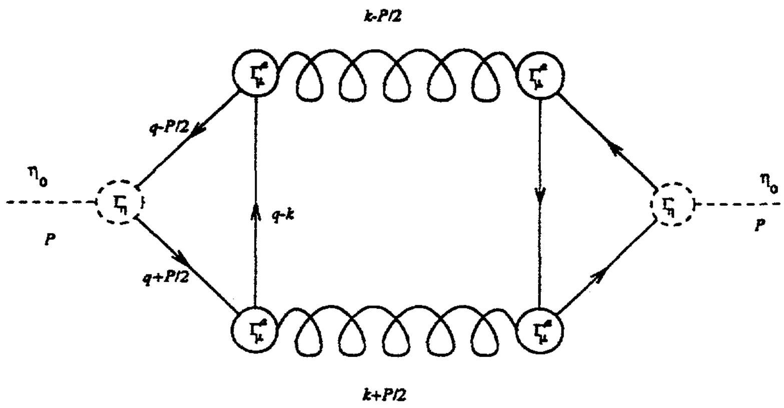

FIGURE 1. The diamond diagram.

We will calculate the anomalous mass contribution in two different ways. First, by evaluating the diamond diagram in fig. (1) directly. This is described in some detail in ref. [2], and the reader is refered hereto for the details of the calculation.

A second method consists of the use of the anomalous Ward identity for the axial current,

$$\partial_\mu j^{\mu 5} = 2i \sum_{f=1}^{N_f} m_f^0 j_f^5 - 2N_f \partial_\mu K^\mu, \tag{1}$$

where m_f^0 are the current quark masses, $j_f^5 = \bar{q}_f \gamma_5 q_f$, and

$$K_\mu = \frac{g^2}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}(A_\nu \partial_\rho A_\sigma + \frac{2}{3} A_\nu A_\rho A_\sigma)$$

is the anomalous gauge dependent current. The matrix element of the flavor singlet component of the axial current is defined as usual,

$$\langle 0 | \partial_\mu j^{\mu 5}(0) | \eta_0(p) \rangle = \sqrt{N_f} f_0 m_0^2. \tag{2}$$

To extract the anomalous mass contribution it is sufficient to consider the chiral limit. Using the reduction formula we obtain

$$\begin{aligned} \sqrt{N_f} f_0 m_0^2 &= \int d^4x e^{-ipx} i(\partial_x^2 + m_0^2) \langle T(\partial_\mu j^{\mu 5}(0) \eta_0(x)) \rangle \\ &= \frac{-i(p^2 - m_0^2)}{\sqrt{N_f} f_0 m_0^2} \int d^4x e^{-ipx} \langle T(\partial_\mu j^{\mu 5}(0) \partial_\nu j^{\nu 5}(x)) \rangle. \end{aligned} \tag{3}$$

The integral on the left hand side can be rewritten as

$$N_f^2 \left(\frac{g^2}{4\pi^2} \right)^2 \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} \int d^4x e^{-ipx} \langle T(\mathrm{tr}(\partial_\mu A_\nu \partial_\rho A_\sigma + \ldots) \mathrm{tr}(\partial_\alpha A_\beta \partial_\gamma A_\delta + \ldots)) \rangle$$

where keeping only the disconnected two gluon propagtor terms leads to

$$N_f^2 \left(\frac{g^2}{8\pi^2}\right)^2 \epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta} i \int \frac{d^4k}{(2\pi)^4}(k_+^\mu k_+^\alpha D_{\nu\beta}^{ab} k_-^\rho k_-^\gamma D_{\sigma\delta}^{ba} + k_+^\mu k_+^\gamma D_{\nu\delta}^{ab} k_-^\rho k_-^\alpha D_{\sigma\beta}^{ba})$$

with $k_\pm = k \pm P/2$. Collecting all factors and setting $g^2 D_{\nu\beta}^{ab}(k^2) = \delta^{ab}\delta_{\nu\beta} 8\pi\sigma/k^4$ we thus obtain in the limit $p^2 \to 0$

$$m_0^2 f_0^2 = N_f 4(N_c^2 - 1)\left(\frac{\sigma}{\pi}\right)^2 \lim_{p^2\to 0} i \int \frac{d^4k}{(2\pi)^4} \frac{(p^2k^2 - (pk)^2)}{k_+^4 k_-^4} = N_f 3\sigma^2/\pi^4. \quad (4)$$

Using $\sigma = 0.18\text{GeV}^2$ and $f_0 \approx f_\pi = 93\text{MeV}$ we obtain $m_0^2 \approx 0.346\text{GeV}^2$. This is plugged into the $\eta_8 - \eta_0$ mass matrix

$$\frac{1}{2}\begin{pmatrix}\eta_8 & \eta_0\end{pmatrix}\begin{pmatrix} \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 & \frac{2}{3}\sqrt{2}(m_\pi^2 - m_K^2) \\ \frac{2}{3}\sqrt{2}(m_\pi^2 - m_K^2) & \frac{2}{3}m_K^2 + \frac{1}{3}m_\pi^2 + m_0^2 \end{pmatrix}\begin{pmatrix}\eta_8 \\ \eta_0\end{pmatrix} \quad (5)$$

whose diagonalization yields the masses of the physical eigenstates, $m_{\eta'} \approx 810\text{MeV}$ and $m_\eta \approx 430\text{MeV}$ as well as their mixing angle $\theta \approx -30°$.

As these values are not too far from the experimental ones we conclude that the $U_A(1)$–anomaly might be encoded in the infrared behavior of QCD Green's functions. Furthermore, we found no evidence that the Kogut–Susskind mechanism leads to unacceptable results for other variables, for more details see ref. [2].

Finally, the question arises whether the Kogut–Susskind mechanism or the instanton based solution of the $U_A(1)$ problem is realized in nature. If instantons are the cause of the η' mass the $\eta - \eta'$ mixing angle is strongly varying function of temperature leading to a significant change of η and η' production rates in relativistic heavy ion collisions [3]. On the other hand, lattice calculations indicate that the string tension is almost temperature independent up to the confinement transition. Thus we conclude that studying η and η' production in heavy ion collisions is a suitable experiment to decide the issue of the underlying physics of the $U_A(1)$ anomaly.

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REFERENCES

1. J. Kogut and L. Susskind, Phys. Rev. **D10** 3468, (1974).
2. A. Mecke, L. v. Smekal and R. Alkofer, in preparation;
   A. Mecke, Diploma Thesis, Tübingen University, April 1997.
3. R. Alkofer, P. A. Amundsen and H. Reinhardt, Phys. Lett. **218B** 75 (1989).