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GENERAL ATOMIC COMPANY

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ABSTRACT

Several topics in the kinetic stability theory of flute-interchange modes in a hot electron plasma are discussed. The stability analysis of the hot-electron, curvature-driven flute-interchange mode, previously performed in a slab goemetry, is extended to a cylindrical plasma. The cold electron concentration necessary for stability differs substantially from previous criteria. The inclusion of a finite temperature background plasma in the stability analysis results in an ion curvature-driven flute-interchange mode which may be stabilized by either hot-electron diamagnetic effects, hot-electron plasma density, or finite (ion) Larmor radius effects.

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I. INTRODUCTION

The stability of flute-interchange modes in a hot electron plasma has recently been considered by several authors.[1-5] The stability analyses have emphasized either the kinetic hot electron plasma[1-3] or the MHD bulk plasma.[4,5] The stability analyses of hot electron driven modes in the presence of a cold, bulk plasma background concluded that an interchange instability exists with "bad" magnetic field curvature for both small and large hot electron precessional frequency, and this instability is stabilized by a sufficient concentration of cold electrons. The MHD stability analysis,[4] performed assuming an unperturbable (rigid) hot electron plasma, also concludes that cold plasma will stabilize the interchange instability. The kinetic stability analyses have necessarily employed a simplified geometry and considered only localized (within the hot electron region) modes while the MHD analysis considered a more realistic geometry, but did not employ a self-consistent model for the hot electron plasma.

In this work, we extend the stability analysis of the hot electron curvature driven flute-interchange mode[3] to a cylindrical geometry. In addition, the synthesis of localized kinetic hot electron and bulk plasma modes is considered by performing a stability analysis of flute-interchange modes including the effects of finite ion Larmor radius and ion precessional drifts associated with the bulk plasma.

Employing a sharp boundary pressure profile in the cylindrical plasma model,[6] we perform in Sec. II a non-local stability analysis of the hot electron curvature driven flute-interchange mode. The amount of cold plasma necessary for stability is shown to differ substantially from the criteria obtained from the local stability analysis.[3]

Finite ion temperature effects of the bulk plasma are more difficult to include in a non-local stability analysis. Hence, in the present work we have simplified the analysis by considering finite ion temperature effects only in the local approximation. In Sec. III we use a slab geometry to show that, in addition to the hot electron curvature driven interchange mode, an additional hybrid mode results from the inclusion of finite ion temperatures. This mode was discussed by Nelson[5] in a more restricted stability analysis. Unlike the hot electron curvature driven mode, the self-field generated by the hot electrons may have a strong stabilizing effect. Additional stabilizing effects arise from either the hot electron density or the finite ion Larmor radius.[7,8]

The electromagnetic equations used to perform the stability analysis in Sec. II are derived from the variational method for a cylindrical plasma,[6] which obtains the correct set of Maxwell equations. Stability plots are presented and a comparison of the local and non-local criteria for cold electron stabilization is made.

In Sec. III, we employ the results of the variational method for a slab geometry[9] to obtain the local Maxwell equations used in the local stability analysis.

II. HOT ELECTRON FLUTE INTERCHANGE MODE

The variational method[9] employed to obtain the Vlasov-Maxwell integral equations in Fourier space for a slab geometry has recently been extended to a cylindrical geometry.[6] From the Fourier space integral form of the Maxwell equations approximate Maxwell differential equations may be obtained in the long wavelength limit $k_\perp a_s < 1$ (a_s is the Larmor radius of species s = electrons, ions). The resulting set of Maxwell differential equations is manifestly Hermitian for real eigenvalues ω when Landau resonances are neglected.

As an application of the results of the variational method in a cylindrical geometry, we analyse the stability of finite β long wavelength flute-interchange modes in a hot electron plasma which contains cold electron and ion components. We present the relevant results of Ref. 6 without derivation and refer the reader to this reference for details.

In the cylindrical plasma model we shall assume a density variation in the radial direction, $N_s(r)$, and an unperturbed magnetic field $\underset{\sim}{B}_o = B_z(r)\hat{\underset{\sim}{z}}$. To simulate the curvature of $\underset{\sim}{B}_o$, we assume that the plasma is situated in an external species dependent gravitational field $\underset{\sim}{g}_s = \underset{\sim}{g}_s(r)\hat{\underset{\sim}{r}}$.

The plasma equilibrium is specified by the force balance and quasi-neutrality conditions

$$\frac{d}{dr}\left(N_h T + \frac{B_z^2}{8\pi}\right) = N_h m_e g \quad , \quad N_i = N_h + N_c \quad , \tag{1}$$

with the ion, cold electron and hot electron densities being given by $N_i = N(r)$, $N_e = (1 - \alpha)N(r)$, and $N_h = \alpha N(r)$. The magnitude of g is chosen as $g = (T/m_e R_c)(r/R_p)$ with T the hot electron temperature, R_c the radius of curvature of the external field, and R_p is a characteristic plasma scale length. To obtain analytically tractable results, we use an idealized model, the sharp boundary model, for the plasma density profile. We define the sharp boundary model by the equations

$$N(r) = \begin{cases} N_o & r < R \\ 0 & r \geq R \end{cases} , \tag{2a}$$

and

$$\alpha = \begin{cases} \alpha_o & R(1 - \varepsilon) \leq r < R \\ 0 & \text{otherwise} \end{cases} , \tag{2b}$$

where $\varepsilon \ll 1$. We restrict consideration to pure flute perturbations ($k_\parallel = 0$) generating the electric field

$$\underset{\sim}{E}(\underset{\sim}{r}) = - \nabla\phi(\underset{\sim}{r}) - \nabla x[a(\underset{\sim}{r})\hat{\underset{\sim}{z}}] , \tag{3}$$

where $\phi(\underset{\sim}{r}) = \phi_\ell(r)\exp i(\ell\psi - \omega t)$, ..., with ψ being the azimuthal angle and $\ell = 1, 2, \ldots$.

The Vlasov charge density and current response to the perturbation in Eq. (3) are the source terms in the Poisson equation and Ampère's Law. The resulting Maxwell differential equations[6] for long wavelength flute-interchange modes are

$$B'\phi_\ell(r) = \hat{\varepsilon}_{11}\phi_\ell(r) - i\hat{\varepsilon}_{12}a_\ell(r) \quad , \tag{4a}$$

$$B'\left(B' + \frac{\omega^2}{c^2}\right)a_\ell(r) = \frac{\omega^2}{c^2}[i\hat{\varepsilon}_{12}\phi_\ell(r) + \hat{\varepsilon}_{22}a_\ell(r)] \quad , \tag{4b}$$

where $B' = (1/r)(d/dr)[r(d/dr)] - \ell^2/r^2$ and $\hat{\varepsilon}_{\alpha\beta}$ are differential operators defined by

$$\hat{\varepsilon}_{11}\phi_\ell = \sum_s \frac{4\pi}{\omega}\int d\underset{\sim}{v}\left(\phi_\ell\left\{\left(1 - k_r^2\frac{a_s^2}{4}\right)\left[-\omega q_s^2\frac{\partial f_o^{(s)}}{\partial H} + \frac{D^{(s)}(\omega - k_\parallel v_\parallel)^2}{\tilde{\omega}}\right] + \frac{\ell}{r}\frac{d}{dr}\left[\frac{a_s^2}{2}D^{(s)}(\omega - k_\parallel v_\parallel)\frac{\tilde{\omega}\Omega_s}{\tilde{\omega}^2 - \Omega_s^2}\right]\right\} - \left(B'\phi_\ell + \frac{d\phi_\ell}{dr}\frac{d}{dr}\right)\left[\frac{a_s^2}{2}\frac{D^{(s)}(\omega - k_\parallel v_\parallel)^2}{\tilde{\omega}}\frac{\Omega_s^2}{\tilde{\omega}^2 - \Omega_s^2}\right]\right) \quad , \tag{5a}$$

$$\hat{\varepsilon}_{12}a = \sum_s \frac{4\pi}{\omega} \int d\underset{\sim}{v} \left\{ \frac{\ell}{r} a_\ell \frac{d}{dr} \left[\frac{v_\perp^2}{2\Omega_s} D^{(s)} (\omega - k_\| v_\|) \frac{\Omega_s}{\tilde{\omega}^2 - \Omega_s^2} \right] \right.$$

$$- \left(B' a_\ell + \frac{da_\ell}{dr} \frac{d}{dr} \right) \left(\frac{v_\perp^2}{2\Omega_s} D^{(s)} \frac{\omega - k_\| v_\|}{\tilde{\omega}} \frac{\Omega_s^2}{\tilde{\omega}^2 - \Omega_s^2} \right)$$

$$\left. - \frac{da_\ell}{dr} \frac{d}{dr} \left[\frac{v_\perp^2}{2\Omega_s} \left(D^{(s)} \frac{\omega_d}{\tilde{\omega}} + \frac{q_s^2}{M_s} \frac{\ell}{\omega \Omega_s r} \frac{\partial f_o^{(s)}}{\partial r} \right) \right] \right\} , \tag{5b}$$

$$\hat{\varepsilon}_{22} a_\ell = \sum_s \frac{4\pi}{\omega} \int d\underset{\sim}{v} \left[B' \left(\frac{v_\perp^2 a_s^2}{4\tilde{\omega}} D^{(s)} B' a_\ell \right) + a_\ell \frac{\ell}{r} \frac{d}{dr} \left(\frac{v_\perp^2}{2} D^{(s)} \frac{\Omega_s}{\tilde{\omega}^2 - \Omega_s^2} \right) \right.$$

$$\left. - \left(B' a_\ell + \frac{da_\ell}{dr} \frac{d}{dr} \right) \left(\frac{v_\perp^2}{2} D^{(s)} \frac{\tilde{\omega}}{\tilde{\omega}^2 - \Omega_s^2} \right) \right] , \tag{5c}$$

with $\tilde{\omega} = \omega - \omega_d - k_\| v_\|$, $\omega_d = (\ell/r) v_d$, $v_d = (v_\perp^2/2\Omega_s)(d\ell og\ B_z/dr) - (g_s/\Omega_s)$, $\Omega_s = q_s B_z / M_s c$, and g_s is the magnitude of the species dependent gravitational field (in the present case $g_i = 0$). For the flute mode analysis $k_\| = 0$ in Eq. (5).

The cold species response may be derived either from fluid equations or from Eq. (4) by choosing a distribution

$$f_{e,i}^{(c)} = F_{e,i}(H, R_G, v_\|) ,$$

such that $\int d\underset{\sim}{v}\, v_\perp^2 F_{e,i} = 0$. For the hot electrons, we choose the delta function distribution

$$f_h = \frac{\alpha}{2\pi v_o^2} N(R_G)\,\delta\!\left(\frac{H - m_e v^2/2}{H_o} - 1\right) F(v_\parallel) \quad ; \tag{6}$$

$$N_h = \int d\underset{\sim}{v}\, f_h \quad , \quad H_o = T = m_e v_o^2/2 \quad , \quad \int d\underset{\sim}{v}\; F(v_\parallel) = 1 \quad ,$$

in order to readily evaluate the hot electron resonant denominators in Eq. (4).

Using $f_{e,i}^{(c)}$ and f_h in Eq. (4) and performing some straightforward manipulations, we obtain the coupled differential equations

$$\Bigg\{\left(1 + \frac{\omega_{pi}^2}{\Omega_i^2 - \omega^2}\right) B'\phi_\ell + \frac{d}{dr}\left(1 + \frac{\omega_{pi}^2}{\Omega_i^2 - \omega^2}\right)\frac{d\phi_\ell}{dr}$$

$$- \frac{\ell}{r}\phi_\ell \frac{d}{dr}\left[\frac{\Omega_i}{\omega}\frac{\omega_{pi}^2}{\Omega_i^2 - \omega^2} - \frac{(1-\alpha)\omega_{pi}^2}{\omega\Omega_i}\right] - \frac{\alpha}{\lambda_e^2}\phi_\ell \frac{\omega_e}{\tilde{\omega}}\left(1 + \frac{\beta}{2}\frac{\omega - \omega_g}{\tilde{\omega}}\right)\Bigg\}$$

$$- i\left(\frac{\omega_{pi}^2}{\omega\Omega_i}\left\{\alpha\left[\frac{\omega_e}{\tilde{\omega}}\left(1 + \frac{\beta}{2}\frac{\omega - \omega_g}{\tilde{\omega}}\right) - \frac{\omega_b}{\omega}\right] - \frac{\omega^2}{\omega^2 - \Omega_i^2}\right\} B' a_\ell\right.$$

$$\left. + \frac{\ell}{r} a_\ell \frac{d}{dr}\left(\frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2}\right) - \frac{d}{dr}\left(\frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2}\frac{\omega}{\Omega_i}\right)\frac{da_\ell}{dr}\right) = 0 \quad , \tag{7a}$$

$$i\left(\frac{\omega_{pi}^2}{\omega\Omega_i}\left\{\alpha\left[\frac{\omega_e}{\tilde{\omega}}\left(1+\frac{\beta}{2}\frac{\omega-\omega_g}{\tilde{\omega}}\right)-\frac{\omega_b}{\tilde{\omega}}\right]-\frac{\omega^2}{\omega^2-\Omega_i^2}\right\}B'\phi_\ell\right.$$

$$+\frac{\ell}{r}\phi_\ell\frac{d}{dr}\left(\frac{\omega_{pi}^2}{\omega^2-\Omega_i^2}\right)-\frac{d}{dr}\left(\frac{\omega_{pi}^2}{\omega^2-\Omega_i^2}\frac{\omega}{\Omega_i}\right)\left.\frac{d\phi_\ell}{dr}\right)$$

$$+\frac{c^2}{\omega^2}B'\left(\left\{1+\frac{\beta}{2}\left[\frac{2(\omega-\omega_g)}{\tilde{\omega}}-\frac{\omega_e}{\tilde{\omega}}\left(1+\frac{\beta}{2}\frac{\omega-\omega_g}{\tilde{\omega}}\right)\right]\right\}B'a_\ell\right)=0\quad,\tag{7b}$$

with $\omega_e = (\ell/r)(T/m_e\Omega_e)[d\ell og\ N_h(r)]/dr$; $\omega_b = (\ell/r)(T/m_e\Omega_e)(d\ell og\ B_z)/dr$; $\omega_g = -(\ell/r)(g/\Omega_e)$; $\tilde{\omega} = \omega - \omega_d$; $\omega_d = \omega_b + \omega_g$; $T = m_e v_o^2/2$; and $\beta = 8\pi N_h T/B_z^2$. It is readily verified that Eq. (7) reduces to the hot electron dispersion relation of Dominguez and Berk[3] (with $\ell/r \to k_y$) when spatial derivatives of ϕ_ℓ and a_ℓ are neglected.

The eigenvalues, ω, for the sharp boundary model are obtained by integrating the coupled differential equations, Eq. (7), across the discontinuity at $r = R$. The detailed manipulations involve combining terms in Eq. (7) to obtain total derivatives and subsequently integrating by parts several times. Deleting these straightforward details, we find upon integrating Eq. (7) from $R_- = (1 - \varepsilon)R$ to $R_+ = (1 + \varepsilon)R$ ($\varepsilon << 1$) and using the continuity of ϕ_ℓ and a_ℓ, the system of algebraic equations

$$\left(\frac{\omega}{\omega+\Omega_i}+\frac{\alpha_o\omega_g}{\omega-\omega_g}\right)\phi_\ell(R)+i\,\frac{\omega}{\omega+\Omega_i}\,a_\ell(R)=0\quad,\tag{8a}$$

$$i \frac{\omega}{\Omega_i} \frac{\omega}{\omega + \Omega_i} \phi_\ell(R) + 2 \frac{\ell^2}{R^2} \frac{c^2}{\omega_{pi}^2} (1 + \beta) a_\ell(R) = 0 \quad , \tag{8b}$$

where $\omega_{pi}^2 = 4\pi N_o e^2/M_i$, $\Omega_i = eB_o/M_i c$ ($B_z = B_o$ for $r < R$), and $\omega_g = (\ell T/M_i\Omega_i)(1/R_c R)$ (R_p is replaced by R). In obtaining Eq. (8), we have employed the approximate form for the eigenfunctions $\phi_\ell(r)$, $a_\ell(r)$

$$\phi_\ell(r),\ a_\ell(r) = (\phi_o, a_o) \begin{cases} (r/R)^\ell & r < R \\ (r/R)^{-\ell} & r \geq R \end{cases} \quad . \tag{9}$$

It is readily verified that Eq. (9) is a solution of Eq. (7) for N(r) = const. The dispersion relation is obtained from Eq. (7) by requiring that the determinant of the coefficients of $\phi_\ell(R)$ and $a_\ell(R)$ vanish. We obtain the result

$$X(X - q) + \alpha_o q(X + 1) + \frac{\delta}{q} \frac{X^3(X - q)}{X + 1} = 0 \quad , \tag{10}$$

where $X = \omega/\Omega_i$, $\delta = (1/4\alpha_o\ell)(R/R_c)[\beta/(1 + \beta)]$, $q = (\ell T/M_i\Omega_i^2)(1/R_c R)$, $b_e = (1/2)k^2 a_e^2 = k^2 T/m_e\Omega_e^2$, and $k = \ell/R$.

The finite β correction term in Eq. (10) may be shown to yield corrections or order $\alpha_o\delta$ compared to unity. Neglecting this term, Eq. (10) is readily solved to obtain the root

$$X = \frac{q(1 - \alpha_o)}{2} \pm \sqrt{\left[\frac{q}{2}(1 - \alpha_o)\right]^2 - \alpha_o q} \quad , \tag{11}$$

In the presence of "bad" curvature, $q > 0$, an interchange instability is possible at both small and large precessional frequency. Cold electrons are stabilizing and stability results for $(1 - \alpha_o)^2 > 4\alpha_o/q$. Unlike finite ion Larmor radius stabilization of the low frequency flute-interchange mode, there is no cancellation of the cold electron stabilizing term for the lowest order $\ell = 1$ mode. We also note that Eq. (11) is precisely the result that would be obtained from a simple electrostatic theory.

From Eq. (11), we obtain the marginal stability condition

$$\frac{N_c}{N_h} = \frac{2}{q} \left(1 + \sqrt{1 + q}\right) \quad , \quad q = \frac{\ell T}{M_i \Omega_i^2} \frac{1}{RR_c} \equiv \ell q_o \quad . \tag{12}$$

Observe that, unlike the local criteria, there is no β dependence in Eq. (12). For $N_c/N_h > 2q^{-1}(1 + \sqrt{1 + q})$, we have stability. In Fig. 1 we plot the fraction of cold to hot electrons, $N_c/N_h = (1 - \alpha_o)/\alpha_o$, at marginal stability as a function of the parameter q_o for several poloidal mode numbers $\ell = 1$, 10, and 25. In the plots the regions above the curves are stable while those below are unstable. Clearly, the $\ell = 1$ mode requires the largest cold electron concentration for stability.

In Fig. 2 we plot the fraction N_c/N_h at marginal stability, obtained from the local stability analysis and Eq. (12), as a function of q_o for $\beta = 0.1$, $\ell = 10$, and $R_c/R = 20$. At small precessional frequency, $q << 1$, the critical fraction $N_c/N_h \approx 4/q$ is substantially larger than the result of the local stability analysis $N_c/N_h \simeq 4/\ell q$ for $\ell > 1$. At large precessional frequency, $q >> 1$, the critical fraction for stability $N_c/N_h \approx 2\, q^{-1/2}$. By comparison, the local stability analysis obtains the critical fraction $N_c/N_h \approx 2/\beta$ for $q \approx \ell > 1$ and for larger q, $q >> \ell > 1$, $N_c/N_h \approx 2(q/\ell)^{-1/2}$. Hence, the critical fraction of cold to hot electrons for stability of high frequency modes may be substantially smaller than local theory indicates.

III. FINITE ION TEMPERATURE FLUTE-INTERCHANGE MODE

The stability of localized flute-interchange modes in a hot electron plasma has previously been considered for the case of a cold electron-ion background plasma.[3] In this section, we extend the analysis to include finite ion temperature effects; finite electron temperature effects of the background plasma are negligible.

To perform the analysis, we use a slab geometry with density and magnetic field gradients in the x direction, an unperturbed magnetic field $\underset{\sim}{B}_o = B_z(x)\hat{\underset{\sim}{z}}$, and a species dependent gravitational field $\underset{\sim}{g}_s = - g_s \hat{\underset{\sim}{x}}$ that simulates the curvature of $\underset{\sim}{B}_o$ as seen by the hot electrons and ions.

The plasma equilibrium satisfies conditions analogous to Eq. (1)

$$\frac{d}{dx}\left(N_h T_h + N_i T_i + \frac{B_z^2}{8\pi}\right) = - N_h m_e g_e - N_h M_i g_i \quad , \tag{13a}$$

$$N_i = N_h + N_c \quad , \tag{13b}$$

where $N_i = N_o(x)$, $N_c = (1 - \alpha)N_o(x)$, and $N_h = \alpha N_o(x)$. The magnitude of $\underset{\sim}{g}_s$ is chosen as $g_s = T_s/M_s R_c$, R_c being the radius of curvature of the external field and T_s being the temperature of species s (s = hot electrons, ions). Perturbations of the equilibrium are of the form of Eq. (3) with $\phi(\underset{\sim}{r})$ replaced by $\phi(\underset{\sim}{x}) = \phi(x)\exp i(ky - \omega t)$, ... , where $\underset{\sim}{k} = k\hat{y}$.

To obtain the set of local Maxwell equations, we use the results of Ref. 9. In the local approximation, one assumes that

$$k^{-1} \frac{d\ell og\ \phi}{dx} \quad , \quad k^{-1} \frac{d\ell og\ a}{dx} << 1$$

and hence we obtain the system of algebraic equations

$$(1 + \varepsilon_{11})\phi - i\varepsilon_{12}a = 0 \quad , \tag{14a}$$

$$i\varepsilon_{12}\phi + \left(1 - \frac{k^2c^2}{\omega^2} + \varepsilon_{22}\right)a = 0 \quad , \tag{14b}$$

where

$$\varepsilon_{11} = \frac{4\pi}{k^2\omega} \sum_s \frac{q_s^2}{M_s} \int d\underset{\sim}{v} \left[- M_s\omega \frac{\partial f_s}{\partial H} + D^{(s)} \sum_N \frac{\omega^2 J_N^2}{\tilde{\omega}_s - N\Omega_s} \right.$$

$$\left. + \frac{1}{k} \frac{d}{dx} \left(D^{(s)} \sum_N \frac{N^2\ \omega\Omega_s\ J_N^2}{\tilde{\omega}_s - N\Omega_s} \right) \right] \quad , \tag{15a}$$

$$\varepsilon_{12} = \frac{4\pi}{k^2\omega} \sum_s \frac{q_s^2}{M_s} \int d\underset{\sim}{v} \left[D^{(s)} \sum_N \frac{\omega z_s\ \Omega_s\ J_N\ J_N'}{\tilde{\omega}_s - N\Omega_s} \right.$$

$$\left. + \frac{1}{k} \frac{d}{dx} \left(D^{(s)} \sum_N \frac{Nz_s\ \omega\Omega_s\ J_N J_N'}{\tilde{\omega}_s - N\Omega_s} \right) \right] \quad , \tag{15b}$$

$$\varepsilon_{22} = \frac{4\pi}{k^2\omega}\sum_s \frac{q_s^2}{M_s}\int d\underset{\sim}{v}\left[D^{(s)}\sum_N \frac{z_s^2\,\Omega_s^2\,J_N'^2}{\tilde{\omega}_s - N\Omega_s}\right.$$

$$\left. + \frac{1}{k}\frac{d}{dx}\left(D^{(s)}\sum_N \frac{N\,z_s^2\,\Omega_s^2\,J_N'^2}{\tilde{\omega}_s - N\Omega_s}\right)\right] \quad , \tag{15c}$$

in which $D^{(s)} = M_s\,\partial f_s/\partial H + (k/\omega\Omega_s)\partial f_s/\partial x$, $f_s = f_s(H_s,x,v_\parallel)$, $z_s = kv_\perp/\Omega_s$, $J_N = J_N(z_s)$, $J_N' = dJ_N/dz_s$, $\tilde{\omega}_s = \omega - \omega_{d_s}$, $\omega_{d_s} = (kv_\perp^2/2\Omega_s)(d\log B_z/dx) + (kg_s/\Omega_s)$, and $H_s = M_s v^2/2 + M_s g_s x$. [In Eq. (15), the summation over species includes the hot electrons, ions, and cold electrons.] The local dispersion relation results from the condition that Eq. (14) have a non-trivial solution.

The hot electron matrix elements in Eq. (15) typically contain the resonant denominators $\tilde{\omega}_e = \omega - \omega_{d_e}$. To readily evaluate these terms, we choose a delta function distribution for the hot electrons

$$f_h(H_h,x,v_\parallel) = \frac{\alpha\,N_o(x)\delta}{2\pi\,v_{oh}}\left(\frac{H_h - m_e v^2/2}{H_{oh}} - 1\right)F_h(v_\parallel) \quad , \tag{16}$$

where $H_{oh} = m_e v_{oh}^2/2 = T_h$ and $\int dv\,F_h(v_\parallel) = 1$. The ion matrix elements in Eq. (15) may be evaluated similiarly. For frequencies large compared to the ion precessional frequency, $\omega \gg \omega_{d_i}$, the $v_\parallel$ dependence of f_i is not crucial. Hence, we choose for the ion distribution

$$f_i(H_i, x, v_\parallel) = \frac{N_o(x)}{2\pi\, v_{oi}^2}\, \delta\left(\frac{H_i - M_i v_\parallel^2/2}{H_{oi}} - 1\right) F_i(v_\parallel) \quad , \tag{17}$$

with $H_{oi} = M_i v_{oi}^2/2 = T_i$ and $\int dv_\parallel F_i(v_\parallel)$. We note that in the present calculation, the results are independent of the form of $F_{h,i}(v_\parallel)$ in Eqs. (16) and (17). The cold electron response may be obtained from Eq. (15) by choosing a distribution $f_e = F_e(H_e, x, v_\parallel)$ such that $\int d\underset{\sim}{v}\, v_\perp^2 F_e = 0$.

Using Eqs. (16) and (17) in Eq. (15), we obtain from Eq. (14) the local dispersion relation for waves $\omega \ll \Omega_i$

$$D_{es}\, D_{em} + (CT)^2 = 0 \quad , \tag{18}$$

where

$$D_{es} = \frac{\omega_{pi}^2}{\Omega_i^2}\left[1 - \frac{\omega_i}{\tilde{\omega}_i}(1 + \beta) + \frac{\omega_{b_i}^2}{\tilde{\omega}_i^2}\right] + \frac{\delta\omega_{pi}^2}{\tilde{\omega}_i \Omega_i}\left(1 + \frac{\beta}{2}\,\frac{\omega - \omega_{g_i}}{\tilde{\omega}_i}\right)$$

$$- \frac{(1-\alpha)\delta\omega_{pi}^2}{\omega\Omega_i}\left(1 + \frac{\beta}{2}\right) + \frac{\alpha}{k^2\lambda_e^2}\,\frac{\omega_e}{\tilde{\omega}_e}\left(1 + \frac{\beta}{2}\,\frac{\omega - \omega_{g_e}}{\tilde{\omega}_e}\right) \quad , \tag{19a}$$

$$D_{em} = 1 + \frac{\beta_h}{2}\left[\frac{2(\omega - \omega_{g_e}) - \omega_e}{\tilde{\omega}_e} + \frac{\omega_{b_e}(\omega - \omega_{g_e})}{\tilde{\omega}_e^2}\right]$$

$$+ \frac{\beta_i}{2}\left[\frac{2(\omega - \omega_{g_i}) - \omega_i}{\tilde{\omega}_i} + \frac{\omega_{b_i}(\omega - \omega_{g_i})}{\omega_i^2}\right] \quad , \tag{19b}$$

$$CT = \left(\frac{\beta_h}{2k^2\lambda_e^2}\right)^{1/2}\left[\frac{\omega_e}{\tilde{\omega}_e}\left(1 + \frac{\beta}{2}\,\frac{\omega - \omega_{g_e}}{\tilde{\omega}_e}\right) - \frac{\omega_{b_e}}{\tilde{\omega}_e}\right]$$

$$- \left(\frac{\beta_i}{2k^2\lambda_i^2}\right)^{1/2}\left[\frac{\omega_i}{\tilde{\omega}_i}\left(1 + \frac{\beta}{2}\,\frac{\omega - \omega_{g_i}}{\tilde{\omega}_i}\right) - \frac{\omega_{b_i}}{\tilde{\omega}_i}\right] \quad , \tag{19c}$$

with $\omega_s = (kT_s/M_s\Omega_s)(d\ell og\ N_o/dx)$, $\omega_{b_s} = (kT_s/M_s\Omega_s)(d\ell og\ B_z/dx)$, $\delta = (kR_p^{-1})$, $\tilde{\omega}_s = \omega - \omega_{d_s}$, $\omega_{d_s} = \omega_{b_s} + \omega_{g_s}$, $\omega_{g_s} = (kg_s/\Omega_s)$, $\beta = \beta_h + \beta_i$, $\beta_h = 8\pi\alpha N_o T_h/B_z^2$, $\beta_i = 8\pi N_o T_i/B_z^2$, and $\lambda_s = (4\pi N_o e^2/T_s)^{-1/2}$. For higher frequency modes, $\omega \geq \Omega_i$, the finite ion temperature modifications are negligible and the results of Ref. 3 are obtained.

The low-frequency dispersion relation is simplified by the assumption $\omega >> \omega_{d_i}$. Further simplification, with no qualitative change in the results, is obtained by considering the moderate β regime $1 > \beta_h > \beta_i$. Employing these simplifications, we obtain the dispersion relation

$$(\omega - \omega_{g_e})(\omega - \omega_i) - \alpha\delta\Omega_i\omega_{g_e} + \delta\Omega_i\omega_{g_i}\frac{\tilde{\omega}_e}{\omega} = 0 \quad . \tag{20}$$

In the limit $\omega \approx \omega_{d_e} >> \omega_i$, Eq. (20) reduces to the results of Ref. 3. In the opposite limit (though not appropriate for localized modes in hot electron plasmas) $\omega \approx \omega_i >> \omega_{d_e}$, Eq. (20) yields the result of Pearlstein and Krall.[8]

Solutions of Eq. (20) may also be found in the frequency range $\omega_{d_e} >> \omega, \omega_i$. In this case, we obtain the root

$$\omega = \frac{\delta\Omega_i(b_i - \alpha)}{2} \pm \sqrt{\left[\frac{\delta\Omega_i(b_i - \alpha)}{2}\right]^2 + \delta\Omega_i \frac{T_i}{T_h} \omega_{d_e}} \quad , \qquad (21)$$

where $b_i = k^2 T_i / M_i \Omega_i^2$. In the presence of "bad" curvature $R_p/R_c > 0$, instability may result. However, the hot-electron self-field terms do not cancel; the hot-electron ∇B_z drift in the drifing term may reverse the sign of ω_{d_e} and stabilize the mode. The character of the additional stabilizing term in Eq. (21) depends upon the relative magnitude of α and b_i. Note that the hot-electron stabilizing effect, obtained for $\alpha > b_i$, may be offset by a corresponding loss of cold-plasma stabilization of the hot-electron curvature-driven flute-interchange mode.

Finally, we point out that the "rigid" hot electron approximation, investigated by Nelson[5] in connection with Eq. (21) and employed in the MHD stability analysis of Nelson and Hedrick,[4] may lead to substantially more optimistic stability criteria than the full stability analysis would indicate.

In kinetic theory, the "rigid" hot electron model sets $\alpha = 0$ but retains finite β_h. Hence, in this approximation Eq. (21) is replaced by

$$\omega = \frac{\delta\Omega_i b_i}{2} \pm \sqrt{\left(\frac{\delta\Omega_i b_i}{2}\right)^2 + \delta\Omega_i \frac{T_i}{T_h} \omega_{d_e}} \quad . \tag{22}$$

Comparing Eqs. (21) and (22), we observe that for $b_i - \alpha < b_i$ the "rigid" hot electron model may yield overly optimistic stability criteria. The "rigid" hot electron approximation is not generally a good approximation of the complete analysis.[10]

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FIGURE CAPTIONS

1. The critical value of the relative cold-electron density predicted by the non-local theory vs. the ratio of hot-electron precession frequency to the ion gyrofrequency for ℓ = 1, 10, and 25.
2. A comparison of the critical cold-electron density predicted by the localized theory and the corresponding values from the non-local theory.

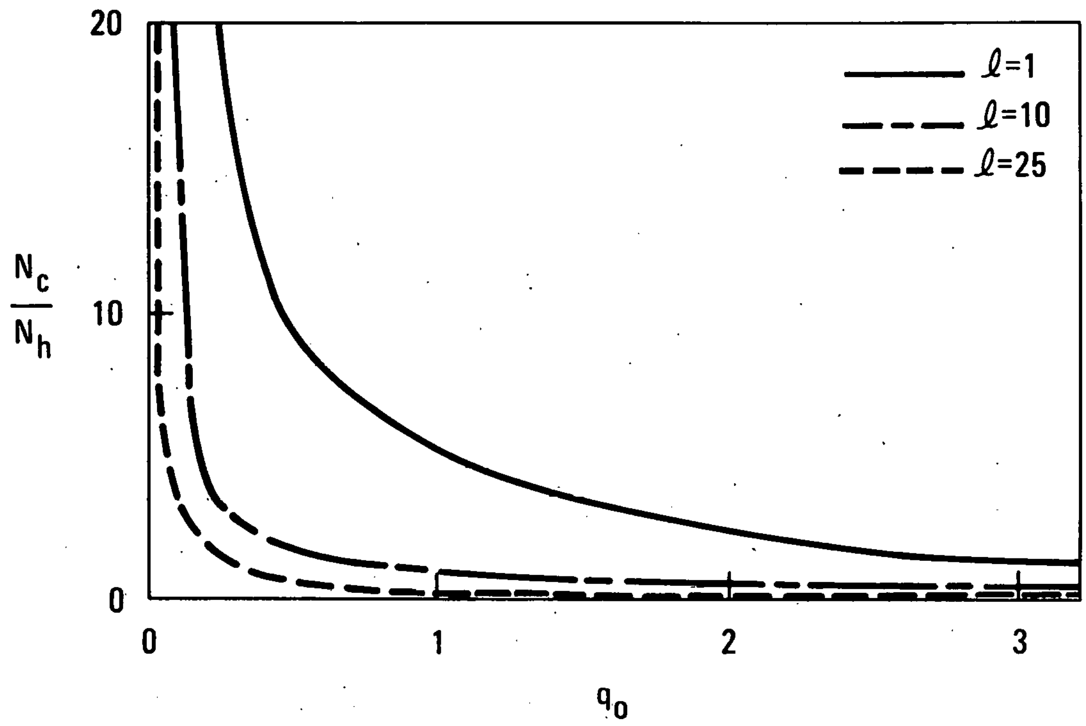

Fig. 1. The critical value of the realtive cold-electron density predicted by the non-local theory vs. the ratio of hot-electron precession frequency to the ion gyrofrequency for ℓ = 1, 10, and 25.

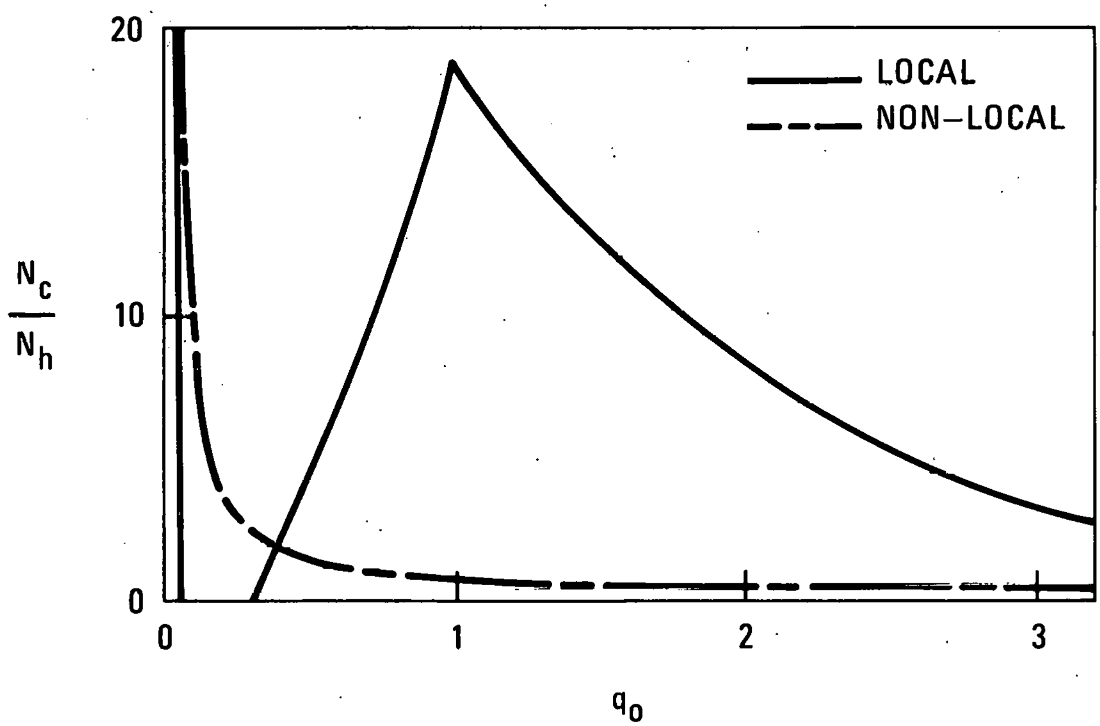

Fig. 2. A comparison of the critical cold-electron density predicted by the localized theory and the corresponding values from the non-local theory.

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