CONF-831047--59

DE83 015372

Conf-831047--59

Magnitude of Bias in Monte Carlo Eigenvalue Calculations

Prof. H. Bowsher*, E. M. Gelbard, P. Gemmel**, G. Back+

Most Monte Carlo eigenvalue calculations are based on power iteration methods, like those used in analytical algorithms. But if N_H, the number of histories in each generation is fixed, then such Monte Carlo calculations will be biased.(1,2) Various arguments lead to the conclusion that eigenvalue and shape biases are both proportional to $1/N_H$, but little more is known about their magnitudes. Numerical experiments on simple matrices suggest that the biases are small, but information more relevant to real reactor calculations is very sparse.

In fact to determine the bias in real reactor calculations is quite expensive. It seems worthwhile, therefore, to try to understand the Monte Carlo biases in systems more realistic than arbitrary matrices, but simpler than real reactors. For this reason we have computed biases in simple one-group model problems by the following procedure.

a. Each problem configuration is split into I subzones, and one calculates the zone-to-zone collision probabilities p^c_{ij}.

b. One then computes, deterministically, the probability, p^a_{ij}, that a neutron formed in subzone j will be absorbed in subzone i.

c. From p^c_{ij}, and the problem cross sections one gets the leading eigenvalue λ^D, and the corresponding eigenvector, ϕ_D, also deterministically.

d. Define the source vector $\underline{s}_D$, with components $s^i_D \equiv \nu\Sigma^i_f\phi^i_D/\sum_j \nu\Sigma^j_f\phi^j_D$, (where the superscripts are subzone numbers) and choose N_H neutron birth-sites from this source.

*Dept. of Chemistry & Physics, August College, Augusta, GA 30910
**4061 Linden Avenue, Western Springs, IL 60558
+1800 N. Noland Avenue, Independence, MO 64050

c. For each birth-site, j, choose an absorption site from the p.d.f. p^a_{ij}. Accumulate $\nu\Sigma^i_f$ in the i'th fission-source bin. After all N_H starters have been processed the total fission rate, summed over all source bins, is the first-generation eigenvalue, $\lambda^{(1)}$, and the fission rate per bin, divided by $\lambda^{(1)}$, is the p.d.f. for source sites for the second generation, etc.

In vection notation

$$1) \quad \underline{\underline{\Sigma}}_t \underline{\phi} = \underline{p}^c \left[\underline{\underline{\Sigma}}_s + 1/\lambda \ \underline{\underline{\nu\Sigma}}_f \right] \underline{\phi}.$$

Here $\underline{\underline{\Sigma}}_t$, $\underline{\underline{\Sigma}}_s$ *and* $\underline{\underline{\nu\Sigma}}_f$ *are diagonal matrices* whose elements are, respectively, Σ^i_t, Σ^i_s and $\nu\Sigma^i_f$. Further $\underline{p}^c$ is the matrix of collision probabilities and $\underline{\phi}$ the flux vector. It is easy to show, from Eq. (1), that $\lambda\underline{\alpha} = \underline{p}^a \underline{f}$, where

$$2) \quad \underline{p}^a \equiv \underline{\underline{\Sigma}}_a (\underline{\underline{\Sigma}}^c_t - p\underline{\underline{\Sigma}}_s)^{-1} \underline{p},$$

$\underline{f}$ is the fission source, $\underline{p}^a$ the matrix of absorption probabilities and $\underline{\alpha}$ the absorption rate. We use Eq. (2) to get the p^a_{ij}'s from the collision probabilities, p^c_{ij}.

Test problems have been run in various model problem configurations, with $N_H = 6$ and 12. Such low values of N_H were used to make the biases more easily detectable. For all test problems (a) in the core $\Sigma_s = 0.9$, $\Sigma_a = 0.1$, $\nu\Sigma_f = 0.1$, and (b) in the reflector $\Sigma_s = 1.98$, $\Sigma_a = 0.02$, $\nu\Sigma_f = 0$, all in inverse cm.

In test problems one through four, slab problems without control rods, there are, respectively 10, 20, 30 and 60 subzones in the core, all 1 cm wide.

All slab configurations have identical reflectors composed of six subzones. Of these the first subzone, nearest the core, is 1 cm wide, while all others are 3 cm wide.

Results for these configurations (and for other tests discussed below) are shown in Table I. The listed biases do seem to be proportional to $1/N_H$, though statistical uncertainties still leave some room for doubt. If biases are proportional to $1/N_H$, then eigenvalue biases would be negligible even for $N_H = 50$. A higher $N_H(\approx 200)$ would be required to reduce the shape-bias in the core to $\approx 5\%$, but $N_H = 200$ is still near the lower bound of normal batch sizes.

Similar slab computations also have been run with absorbing subzones used to simulate control rods. Biases seem to be reduced by the presence of such "rods" apparently because they increase dominance ratios.

Three X Y computations have been run in configurations shown in Fig. 1. These configurations differed only in the sizes of the core subzones which were, respectively, one, two and three cm squares in problems 5, 6 and 7. One sees, from, Table I, that the biases again seem to be proportional to $1/N_H$, and would be acceptable for $N_H \gtrsim 100$.

It seems, then, that it is permissable to use fairly small batch sizes in Monte Carlo eigenvalue calculations. Whether this is advantageous or not may depend on possible effects of N_H on the convergence rate. Preliminary results suggest, however, that the convergence rate does not depend on N_H.

References

1. R. C. Gast and N. R. Candelore, (W-BAPL, USA) "Monte Carlo Eigenvalue Strategies and Uncertainties".

2. E. M. Gelbard and R. E. Prael (ANL, USA), Monte Carlo Work at Argonne National Laboratory".

TABLE I.

Problem Number	Deterministic Calculation λ_0	Deterministic Calculation λ_1	Monte Carlo Calculations Based on 960,000 Histories: Percent Eigenvalue Bias, Histories/Generation 6	Percent Eigenvalue Bias, Histories/Generation 12	Percent Shape Bias, Core Center, Histories/Generation 6	Percent Shape Bias, Core Center, Histories/Generation 12	Percent Shape Bias, Outer Edge or Corner, Histories/Generation 6	Percent Shape Bias, Outer Edge or Corner, Histories/Generation 12
1	0.95135	0.64940	−0.276 ±0.030	−0.129 ±0.030	−4.2 ±0.5	−2.3 ±0.4	8.4 ±1.3	3.8 ±0.4
2	0.98329	0.86236	−0.417 ±0.029	−0.199 ±0.029	−12.2 ±2.3	−5.9 ±1.1	31.0 ±2.1	13.5 ±2.1
3	0.99163	0.92830	−0.406 ±0.029	−0.212 ±0.028	−17.69 ±1.6	−9.2 ±1.8	65.4 ±20.4	26.5 ±4.2
4	0.99762	0.97897	−0.385 ±0.029	−0.218 ±0.028	−25.1 ±3.8	−18.9 ±5.0	156.3 ±12.5	90.0 ±14.3
5	0.90749	0.62580	−0.540 ±0.049	−0.247 ±0.038	−8.1 ±1.4	−4.9 ±1.4	13.7 ±2.1	6.2 ±2.4
6	0.96008	0.81312	−0.662 ±0.037	−0.343 ±0.037	−16.7 ±1.5	−10.5 ±1.6	36.6 ±3.3	19.3 ±3.9
7	0.97447	0.87508	−0.602 ±0.032	−0.323 ±0.042	−21.2 ±1.7	−14.4 ±1.9	57.1 ±5.2	35.2 ±6.1

Eigenvalue and fission-source-shape biases in simple one-group test problems.

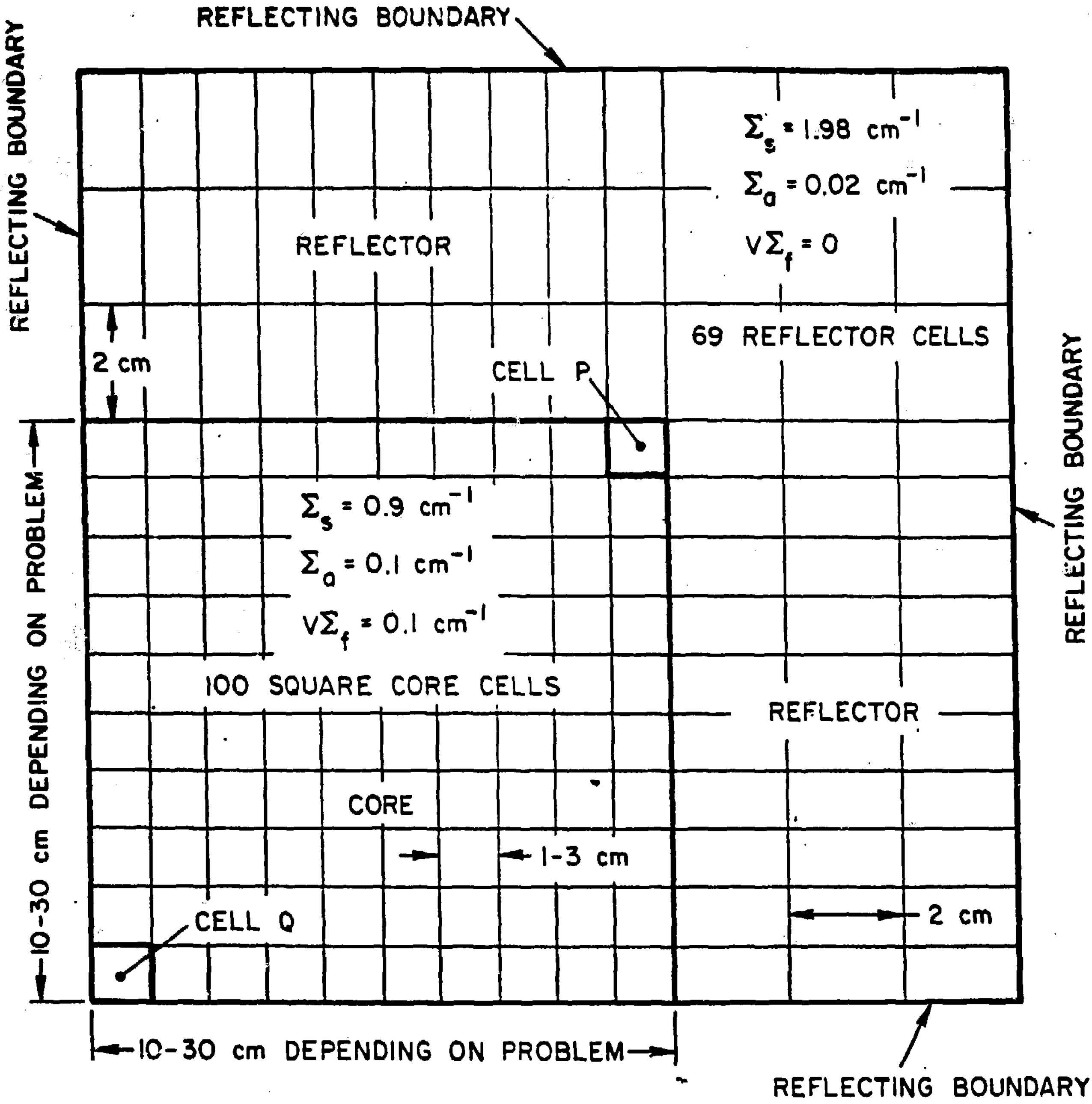

Fig. 1. Two-Dimensional Test Problem Configurations.

DISCLAIMER