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FULLY IMPLICIT THREE-DIMENSIONAL MHD STABILITY AND EVOLUTION IN TOROIDAL GEOMETRY*

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The FAR family of computer codes has been developed at Oak Ridge National Laboratory to solve ideal and resistive, linear and nonlinear, 3-D full MHD problems in toroidal geometry. Extensive application of an incompressible version of these codes has been made to the linear stability of tokamaks and stellarators,[1-4] and nonlinear studies have also been recently completed.[5-7] Present work includes generalization of the physical model in several respects to incorporate the effects of compressibility, long mean free path, and thermal conductivity.

A distinguishing feature of the FAR family of codes is the use of a fully implicit numerical technique to advance the linear part of the equations. The application and numerical properties of this method are discussed in great detail in the context of an incompressible linear MHD model in Ref. 8, and its application to stellarators is discussed in Ref. 9. The most interesting aspects of the fully implicit method as a linear eigenvalue solver are the abilities to obtain rapid convergence in a small number of iterations and to calculate instabilities other than the fastest growing mode, with both features realized through control of the relaxation parameter Δ (time step size in the small $\Delta\tau_{H_p} \ll \gamma_{\max}^{-1}$ limit).

In spite of proven favorable numerical stability properties in comparison with explicit and partially implicit techniques, two commonly held notions discourage the use of fully implicit methods: (1) that fully implicit methods are slow due to the inversion of large matrices, and (2) that programming for fully implicit methods is difficult. While the first objection is founded upon a point which is generally true, the iterations in MHD applications involve the repeated use of the same matrix so long as the relaxation parameter (for linear calculations) is not changed. Linear calculations typically involve several tens of iterations and nonlinear calculations involve thousands. In the adopted equilibrium magnetic flux coordinate system in toroidal geometry the matrix representation of the MHD equations is block tridiagonal. We solve this system using the band matrix solver BTMS,[10] which performs an LU decomposition of the matrix, and then solves by forward and back

substitution. The expensive portion of the procedure, namely the decomposition, is performed only once at the beginning of each calculation, and the results stored. Typically this step requires no more than a couple of minutes on the MFE CRAY-II Computer. The iterative portion of the calculation, which may require several hours of machine time for large nonlinear runs, requires only the solution portion of the band matrix solver, for which the operation count is comparable to that encountered in explicit schemes. Hence, except for a single decomposition performed at the beginning of each calculation, the fully implicit method employed in FAR is just as fast as an explicit method. Because of the large matrices, however, storage requirements are greater than for less implicit methods. Typical nonlinear calculations (300 radial grid points and 35 modes) require about 10 million words of storage.

The second objection to fully implicit methods, namely that the programming involved in setting up the matrix is difficult, has been overcome to a great extent by automating the matrix construction procedure. This automation is based upon our representation of the MHD equations in toroidal equilibrium magnetic geometry, which yields a coordinate system (ρ,θ,ζ) in which $0 \leq \rho \leq 1$ is a flux surface label and generalized minor radius, $0 \leq \zeta \leq 2\pi$ is the toroidal angle. In this system a finite difference representation is used in ρ, and a truncated Fourier series expansion is used for the θ and ζ dependence. Hence, dynamical quantities are written

$$f(\rho,\theta,\zeta,t) = \sum_{(m;n)} [f^{c}_{(m;n)}(\rho,t)\cos(m\theta + n\zeta) + f^{s}_{(m;n)}(\rho,t)\sin(m\theta + n\zeta)] \tag{1}$$

where the coefficients $f^{c,s}_{(m;n)}$ are represented by finite differences on a radial grid ρ_j, and the summation is truncated for some desired set of modes. In the angular dependence $m\theta + n\zeta$ the coefficients m and n are called the poloidal and toroidal mode numbers, respectively. For axisymmetric equilibria, equilibrium field quantities are independent of both t and ζ ($n = 0$ only), and for up/down symmetric equilibria only the cos or sin terms, not both, are present. The equations are formulated in a manner such that all radial derivatives are second order or lower. It is then possible to use second order accurate three point finite difference expressions to couple quantities at radius ρ_j with those at ρ_{j+1}. The terms in the linearized equations consist of constant coefficients times equilibrium quantities

$$g^{eq}(\rho,\theta) = \sum_{m} (g^{eq,c}_{m}(\rho)\cos m\theta + g^{eq,s}_{m}(\rho)\sin m\theta) \tag{2}$$

times dynamical quantities, so that there is coupling among the poloidal components m, in the equations, but each toroidal component n is not coupled to other toroidal components. Hence, each toroidal component n is described by a matrix in the fully implicit method. Each row of this matrix expresses a particular poloidal component m of one of the linearized dynamical equations ieq at a radius ρ_j. Each column multiplies the m' component of the dynamical variable $ivar$ at radius $\rho_{j'}$ by matrix elements which depend upon the equilibrium, the dynamical equation, and the role of the variable in the equation. By arranging the matrix in blocks of sequentially increasing radii, the radial differencing yields a block tridiagonal matrix. Within each radial block our convention is to make sub blocks for each dynamical equation (rows) and variable (columns), and each sub block contains the poloidal mode coupling of a particular variable into a particular equation. Using these conventions we have automated the matrix construction procedure by devising a subroutine, BLOCK, to fill the matrix. Subroutine BLOCK contains information about the equilibrium and dynamical mode truncations, and also the mode coupling coefficient due to the convolution of such quantities. Also, stored in BLOCK are the chosen forms of several of the most commonly used radial differencing operators. To construct the matrix a call is made, for each term in each equation to BLOCK supplying the toroidal mode number n, the equation and the dynamical variable entering the given term, the equilibrium related quantity multiplying the dynamical variable in given term, the order of the θ and ζ derivatives and the radial difference operator of the dynamical quantity in the given term, and a numerical coefficient. With this information BLOCK enters the contribution of the given term into the matrix, and the procedure of matrix construction is reduced to one of categorizing and tabulating the equations, variables, and terms.

We have described two features which render a fully implicit approach to toroidal 3-D MHD calculations tractable. The first is rapid computation speed which is attained by performing a single matrix decomposition at the outset, sot that only the fast solution portion of the inversion is performed during the many iterations. The second feature is the automation of the matrix construction, which reduces the job of coding to straightforward bookkeeping and facilitates changes in an equally straightforward manner.

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