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β-DECAY IN THE SKYRME-WITTEN REPRESENTATION OF QCD

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ABSTRACT

The renormalized coupling strength of the β-decay axial vector current is related to $\pi^{\pm}$ p cross sections through the Adler-Weisberger sum rule, that follows from chiral symmetry. We attempt to understand the Adler-Weisberger sum rule in the $1/N_c$ expansion in QCD, and in the Skyrme-Witten model that realizes the $1/N_c$ expansion in the low energy limit, using it to explicitly calculate both g_A and the $\pi^{\pm}$ p cross sections.

From QCD and electroweak theory, neutron β-decay proceeds through a $d^{-1/3}$ quark in a neutron making a transition to a $u^{2/3}$ quark in a proton, emitting a virtual W^-, which then converts to the e^- $\bar{\nu}$ pair. At the Lagrangian level, the W couples with the same strength, once accounting for the Cabbibo angle, to all quarks and leptons. Nevertheless, it is remarkable that there is approximate universality in β-decay — the parameters that describe this neutron decay process and nuclear β-decay processes are very close to those in μ-decay — despite the strong interaction gluon radiative corrections in the hadronic cases. The differential decay rate for neutron β-decay is [1]

$$d\Gamma = \cos^2\theta_c \frac{G_F^2}{32\pi^2} E_\nu^2 p_e^2 dp_e d\Omega_e d\Omega_\nu (g_V^2 + 3\,g_A^2)$$
$$\times\left[1 + \frac{g_V^2 - g_A^2}{g_V^2 + 3\,g_A^2}\hat{p}_e \cdot \hat{p}_\nu - 2\left(\frac{g_A^2 + g_V g_A}{g_V^2 + 3\,g_A^2}\right)\hat{S}_n \cdot \hat{p}_e + 2\left(\frac{g_A^2 - g_V g_A}{g_V^2 + 3\,g_A^2}\right)\hat{S}_n \cdot \hat{p}_\nu\right] ,$$

where $E_\nu \equiv M_n - M_p - E_e$. The renormalization of the hadronic vector current, g_V, is very close to 1 because isospin is such a good symmetry, and because the hadronic charged isospin currents are part of an isovector multiplet with the electromagnetic current. Low energy theorms for QED then guarantee the lack of renormalization corrections at low q^2 [2]. For the hadronic axial vector current, even though it too is almost conserved, there is no corresponding low energy theorm. There is, however, the remarkable Adler-Weisberger sum rule [3,4], that follows from chiral symmetry, that relates g_A to the cross sections of massless $\pi^{\pm}$ from protons,

$$1 - \frac{1}{g_A^2} = \frac{2M^2}{\pi g_{\pi NN}^2}\int_{(M+m_\pi)^2}^{\infty} \frac{ds}{s - M^2}\left(\sigma_{\pi^+p}(s) - \sigma_{\pi^-p}(s)\right) .$$

The integrated cross sections approximately cancel, leaving $g_A \equiv 1.24$.

I will argue here that this approximate cancellation is a reflection of the approximate validity of the $1/N_c$ expansion in QCD [5]. I will sketch the dual conceptualization of QCD of Witten [6,7] that follows from generalizing the number of quark colors from 3 to N_c, and expanding in $1/N_c$. I will indicate how one can analyse low energy hadronic interactions, and nuclei and nuclear interactions, from the Skyrme -Witten model, an effective Lagrangian for massless pseudoscalar degrees of freedom, in which baryons and nuclei are topological solitons, skyrmions [8]. This is a radical conceptualization of nuclear physics, which I will attempt to justify. Skyrme introduced many of these ideas in a visionary way 30 years ago. His work was largely ignored until they were shown by Witten to fit into QCD. It is perhaps fitting that Stu Bloom, who's career we celebrate today, who's contributions to β-decay range from both fundamental experiments on parity violation to shell model calculations of β-decay processes in complex nuclei, should be about to participate in exploring this old problem in a very new way. It is within this framework that we will attempt to understand the renormalization of the Gamow-Teller strength.

First, a brief review of QCD. QCD describes the interactions of quarks through a generalization of electromagnetism,

$$L = \bar{q}\left(\partial - \frac{g}{\sqrt{N_c}} G^a \frac{\lambda^a}{2}\right) q - \frac{1}{4} G_{\mu\nu}^{a\,2} \ ,$$

where $q=(u, d, s, ...)^t$, and where λ^a , $a = 1, \cdots , N^2-1$ are the NxN matrix generators of SU(N). Each quark is in the complex N-dimensional representation of SU(N) on which the λ^a act. Thus the SU(3) theory has been embedded into a class of similar theories. There are also mass terms for the quarks, which arise from the electroweak interactions, and which are perturbations on the dynamics of this theory. For the lightest u, d, and s quarks, the quarks play an important dynamical role; the heavier c, b, and t quarks have little effect on the dynamics of the color fields in which they move. There are primarily three kinds of basic physical excitations of the theory—mesons, baryons, and glueballs. The mesons are quark-antiquark pairs,

$$\sum_{c=1}^{N} \bar{q}^c q_c \rightarrow \bar{q}^c(x')P\exp\left(i \frac{g}{\sqrt{N_c}} \int_x^{x'} G_\mu^a \frac{\lambda^a}{2} dx^\mu\right) q_c(x) \ ,$$

where the operator on the left is a local color singlet, and where Dirac or flavor matrices or derivative operators could be inserted between the fields to define meson operators of different Lorentz or flavor quantum numbers; on the right, P is the path ordering symbol, and the separated quark fields are connected by a string of color electric flux created by the exponential of the line integral of the vector potential. Such an operator for the extended meson is gauge invariant. Baryons are created by the local operator,

$$\varepsilon^{c_1 c_2 \cdots c_{N_c}} q_{c_1} q_{c_2} \cdots q_{c_{N_c}} \ ,$$

anti-symmetric in color indices, and where the corresponding flavor and non-local generalizations are analogous to the meson case. Glueballs are created by the operator,

$$\mathrm{tr}\, P \exp\left(i \frac{g}{\sqrt{N_c}} \oint_C G_\mu^a \frac{\lambda^a}{2} dx^\mu\right) \ ,$$

where the line integral is now around a closed curve, C; this operator creates loops (closed strings) of color electric flux. There are also exotic states, such as baryonium, color singlet states of N_c-1 quarks and N_c-1 antiquarks that are both antisymmetric in color.

In contrast to QED. this theory actually has no coupling constant. The effective coupling, corrected by quantum fluctuations, is asymptotically free [9],

$$\bar{g}^2(P/\mu) \approx \frac{g^2/N_c}{1+\frac{g^2}{N_c} b_0 \frac{1}{2}\ln\left(\frac{-P^2}{\mu^2}\right)} = \frac{1}{\frac{b_0}{2}\ln\left(\frac{-P^2}{\Lambda^2}\right)} ,$$

where the middle expression is valid for $-P^2 >> \mu^2$, where μ is an arbitrary subtraction scale needed to define renormalized Green's functions, and

$$b_0 = \frac{1}{24\,\pi^2}(11\,N_c - 2\,n_f) ,$$

where n_f is the number of quarks of mass lighter than μ. The second form follows from the definition,

$$1 + \frac{g^2}{N_c} b_0 \frac{1}{2}\ln\frac{1}{\mu^2} = \frac{g^2}{N_c} b_0 \frac{1}{2}\ln\frac{1}{\Lambda^2} ,$$

or

$$\Lambda = \mu \exp - N_c/b_0 g^2 .$$

Λ sets the mass scale, and $1/\Lambda$ sets the size for all particles in the theory. If μ is chosen differently, g^2 must also be readjusted to keep Λ fixed. This trading of g for Λ is called dimensional transmutation. All mass ratios are pure numbers. In the SU(3) theory, there are no parameters; by embedding the theory in the SU(N) class of theories, there is now a parameter, $1/N_c$.

With massless quarks there is a large global symmetry of the theory, chiral symmetry. The left and right handed flavor multiplet of quarks can be independently transformed into one another,

$$q_L \to A\, q_L ,$$

$$q_R \to B\, q_R .$$

At the classical level, the matrices A and B are $U(n_f)$ matrices, but quantum mechanically one of the symmetry generators is "anomalous,"

$$\partial^\mu \bar{q}\gamma_\mu\gamma_5 q = 2n_f \frac{g^2/N_c}{16\,\pi^2} G^a_{\mu\nu}\tilde{G}^{\mu\nu a} .$$

This lack of current conservation of the flavor singlet axial current reduces the exact chiral symmetry to [10]

$$SU_L(n_f)xSU_R(n_f)xU_B(1)xZ_A(n_f),$$

where $U_B(1)$ is the baryon number symmetry, and $Z_A(n_f)$ is a discrete n_f-fold subgroup of the axial $U_A(1)$ symmetry that is still left unbroken despite the anomaly. The dynamics of the theory will lead to a vacuum that spontaneously breaks the chiral $SU_L(n_f)xSU_R(n_f)$ symmetry down to $SU(n_f)$; that is, the vacuum is infinitely degenerate, the axial $SU(n_f)$ charges generate transformations from one equivalent vacuum to another, and each vacuum state is invariant under a vector $SU(n_f)$ symmetry, so physical particles in the theory will be in irreducible representations of $SU(n_f)$. (The degenerate vacua correspond to the points of a coset manifold, $SU_L(n_f)xSU_R(n_f)/SU(n_f)$.) As we mentioned above, when the quark masses are turned on, $n_f=3$, although even for the heavier quarks, the particles' quantum numbers will be those of SU(6) representations. Quark masses lift the degenerancy of the vacuum and pick out a single direction in the vacuum coset manifold.

In order to calculate masses, scattering amplitudes, or nuclear structure from this theory, some kind of approximation scheme is required. Since the theory has no expansion paramerer one must create one — lattice gauge theory and the $1/N_c$ expansion are the possibilities so far. On the lattice there is a coupling, g, and in the leading approximation of the strong coupling expansion one has a perturbation expansion in the correct physical degrees of freedom — quark confinement and spontaneous chiral symmetry breaking are features of the strong coupling limit. (While quark confinement can be understood in strong coupling perturbation theory [11], spontaneous chiral symmetry breaking is nonperturbative [12].) However, the continuum limit, taking the lattice spacing $a \rightarrow 0$, requires $g \rightarrow 0$ as well, keeping

$$\Lambda = \frac{1}{a}\exp - \frac{1}{b_0 g^2}$$

fixed. Monte Carlo methods have been very useful for studying the hadron mass spectrum and thermodynamic quantities, but physical calculations with dynamical quarks are still beyond the computing capabilities of even special purpose supercomputers [13]. Hamiltonian variational methods (such as the t - expansion [14] that Stu and Grant and I [15] have worked on) may hold promise for calculating similar quantities.

Physical processes for which there is factorization of short and long distance physics have perturbatively calculable short distance parts, and may also have calculable matrix elements by lattice methods.

The $1/N_c$ expansion [5,6,16], while much more qualitative thus far, holds great promise as a way of understanding low energy scattering processes and nuclear structure. Instead of focusing on the quark and gluon degrees of freedom of the theory, there is an (in principle) exact rewriting of the theory in terms of meson fields, and in which baryons and nuclei [6] are topological solitons.

The $1/N_c$ expansion is a topological expansion of Feynman diagrams. That is, each order in $1/N_c$ sums an ∞ class of Feynman diagrams involving gluon exchanges, but such that the diagrams fit on 2-dimensional surfaces of different topology, with the number of holes or handles increasing to each order in $1/N_c$ [16]. A quark line is a boundary, and gluon lines are expressed as quark-antiquark lines. Each closed quark closed loop gives a factor of N_c. The topological expansion is exactly analogous to similar expansions in the dual resonance model and string theory. In a string theory one generalizes Feynman path integrals from trajectories to surfaces, and sums over all surfaces of increasing topological complexity, with more holes or handles to each higher order of approximation. In the leading order of the $1/N_c$ expansion, there are ∞ Regge trajectories of noninteracting mesons. To order $1/\sqrt{N_c}$, these mesons interact.

Baryons are antisymmetric in color, and have totally symmetric spatial wavefunctions. For $N_c \to \infty$, the state of the quark spatial wavefunction becomes macroscopically occupied; that is, the wave function becomes a classical field. The baryon mass is of order N_c, or like $1/(1/N_c)$. This behavior is characteristic of solitons [6].

If $1/N_c$ can be considered small, then mesons are weakly interacting, and baryons and nuclei are semiclassical solitons of this theory of mesons. The semiclassical approximation to this quantum field theory is a generalization of the WKB approximation of quantum mechanics [17], and $1/N_c$ corresponds to $\hbar$.

The string theory that corresponds to the meson theory of the $1/N_c$ expansion is so far unknown. However, for understanding nuclei and low energy nuclear scattering, one does not really need to have a meson theory with an ∞ tower of increasing mass mesons. Even for large nuclei, one does not need to start from an effective theory that includes excitations comparable to the nuclear mass. The relevant point is how sensitive is the physics of nuclei to short distance degrees of freedom. Those higher mass excitations just renormalize the parameters in the effective Lagrangian for the lightest mesons. We will in fact include only massless π and K and η degrees of freedom. The ρ and higher mass particles are only necessary as explicit degrees of freedom in the effective Lagrangian if the pion fields are changing in space or time on scales of order $1/m_\rho \sim 1/4$ f. Our effective Lagrangian will therefore not be sufficient to describe the annhilation of the $N\bar{N}$ system, even though it may describe heavy nuclei. Also, the exact chiral Lagrangian describing pseudoscalar dynamics involves all powers of a field describing fluctuations in the vacuum coset space; higher powers of the field involve higher derivatives and dimensional couplings. The effective Lagrangian we will consider is a truncation of this derivative expansion series. Since higher derivative terms of the chiral fields have dimensional compensating factors of inverse masses, and these masses will correspond to more massive mesons with the order of the term, only a few terms may be necessary.

The effective Lagrangian for the low energy dynamics of massless pseudoscalar mesons, for 3 quark flavors, is

$$L = \frac{f_\pi}{16} \mathrm{tr}\, \partial_\mu U \partial^\mu U^{-1} + N_c \Gamma(U) + \cdots .$$

where $U = \exp(2i\pi^a(x,t)\lambda^a/f_\pi)$ is an element of SU(3), and where $f_\pi \sim 186$ MeV is the pion decay constant measured in $\pi^- \to \mu^- \bar{\nu}_\mu$. The first term in this effective Lagrangian, the nonlinear sigma model, was shown by Weinberg [18,19] to reproduce standard current algebra. Expanding the exponential gives interactions of arbitrary numbers of mesons. Γ is the Wess-Zumino [20] term introduced originally to explain anomalous processes like $K^+K^- \to \pi^+\pi^-\pi^0$. Witten [7] showed that the Wess-Zumino term makes the π fields pseudoscalar, and that N_c must be an integer, and corresponds to color. (We will return to the $\cdots$ terms below.) This effective lagrangian has all the same symmetries and topology of QCD. It can be deduced directly from QCD [21]. Not only does this effective Lagrangian describe pseudoscalar meson dynamics, but it also describes baryon dynamics. All field configurations U(x,t) fit into discrete classes characterized by

$$B = \frac{1}{24\pi^2}\varepsilon^{ijk}\int d^3x \; tr\, U^{-1}\nabla_i U \; U^{-1}\nabla_j U \; U^{-1}\nabla_k U \; .$$

B is an integer and corresponds to baryon number [8].

Since many features of strong interaction dynamics follow from the symmetry and topology of the $1/N_c$ expansion, this kind of effective chiral lagrangian may be a good approximation to describing the physics of hadrons and nuclei, especially when quark mass symmetry breaking effects that mix representations are included [19]. One may first ask, though, why an SU(3) chiral model with K's and η's should be required to describe nuclear physics. It is only the SU(3) chiral model with Wess-Zumino term that properly correlates color and flavor, and has all the symmetries and topology of QCD [7]. The quantum numbers that come from allowed symmetrized wavefunctions of spin 1/2 colored quarks of given isospin and hypercharge are exactly reproduced from the topology of meson field configurations; baryons are fermions if N_c is odd, bosons if N_c is even, and the Gell-Mann SU(3) representations for baryons, a spin 1/2 octet and spin 3/2 decuplet are correlated with N_c=3. For the SU(2) model, there is an additional symmetry not present in QCD [7]; ther is a naive parity, $x \rightarrow -x$, and there is also the symmetry, $U \rightarrow U^{-1}$, while only the product is a symmetry of QCD corresponding to pions being pseudoscalar. Also, color does not explicitly enter the soliton quantum number determination; the freedom to quantize the soliton as either a fermion or boson may be interpreted as odd or even color, but the isospin representations are the same for all N_c , although the number of allowed representations increases with N_c. In the model, since there is no constraint on the maximum isospin, this suggests $N_c \rightarrow \infty$.

One may also object that with $m_K \sim 1/2$ GeV, the pseudoscalar masses are far from zero, so how is it possible to dynamically neglect them. Despite the relatively large K and η (and even π) masses, they are in fact perturbations on the QCD dynamics, just as the Coulomb interaction is a perturbation on conventional nuclear interactions. Both quark masses and Coulomb interactions should be considered perturbations of the same magnitude. The Coulomb interaction breaks isospin symmetry in nuclei; the Coulomb interaction is an isospin magnetic field — just as an external magnetic field on atoms produces the Zeeman splitting of angular momentum multiplets, the Coulomb interaction produces the isospin Zeeman effect splitting of isospin multiplets. Similarly, the quark masses are like an external magnetic field on a ferromagnet. (Actually, the chiral vacuum is more like an antiferromagnet.) Just as the spin waves in a ferromagnet become higher energy excitations due to the external field, so do the pseudoscalar mesons. The pseudoscalar masses, while large (because they are proportional to $\sqrt{\Lambda}$), are anomalously light compared to their spin flip vector partners. Their masses are properly explained in this way. Also, the sucess of current algebra in explaining the dynamics of pseudoscalar mesons is comparable to Gell-Mann SU(3) Clebsch-Gordan relations; chiral symmetry is as good a dynamical symmetry as unitary symmetry, which is almost as good a symmetry as isospin [32].

Returning to the effective chiral Lagrangian, if the Skyrme term [8],

$$\frac{1}{32g^2_{\rho\pi\pi}}\, tr\left[U^{-1}\partial_\mu U \; , \; U^{-1}\partial_\nu U\right]^2 \; ,$$

is added, one of two terms of the next order in the derivative expansion, stable finite energy soliton solutions of the classical field equations exist with integer B. $g_{\rho\pi\pi} \propto m_\rho/f_\pi$ [22] is the $\rho\pi\pi$

coupling that determines the ρ width. In the $1/N_c$ expansion, $f_\pi \sim \sqrt{N_c}$, and $g_{\rho\pi\pi} \sim 1/\sqrt{N_c}$ [6], so this term is of the same order in N_c as the first terms. In perturbation theory in the fields π, this term gives the low energy limit of π-π scattering through the ρ meson.

For B=1, the soliton solution is of the form

$$U_A = A \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix} A^{-1} ,$$

where $U_0(x) = \exp(iF(r)\hat{\mathbf{r}}.\tau)$, and where $F(r{=}\infty) = 0$, and $F(r{=}0) = \pi$, and A is a constant SU(3) matrix. This field configuration has localized energy of size $1/g_{\rho\pi\pi}f_\pi \sim (N_c)^0$. The Gell'Mann SU(3) invariance of a vacuum state, corresponding to different values of A, leads, in the semiclassical quantization [7, 23], to excited states as excited rotational modes. Restrictions on the SU(3) rotor quantum numbers, mentioned above, linking color and flavor representations, follows from symmetry of the skyrmion and topology.

The π fluctuations about the soliton,

$$U = \exp\left(iF(r)\hat{r}_i D^{ij}(A)\lambda^j + 2i\delta\pi^a(x,t)\lambda^a/f_\pi\right) ,$$

correspond to vibrational modes. The action for the fluctuating field is of the form [25],

$$\mathcal{L}(U) = \mathcal{L}(U_A) + \frac{1}{2}\partial_\mu\delta\pi \cdot \partial^\mu\delta\pi \left(1-2\delta\pi^2/f_\pi^2 + \cdots\right) + \frac{1}{f_\pi}\partial_\mu\delta\pi \cdot J_5^\mu(U_A) + \partial_\mu\delta\pi \times \delta\pi \cdot J^\mu(U_A) + \cdots .$$

Since U_A is a solution to the equations of motion, the axial current $J_{5\mu}{}^a(U_A)$ is conserved. The term quadratic in pion fluctuation fields, is of the form

$$\int \delta\pi \cdot D^{-1}(U_A) \cdot \delta\pi + (\dot{A}\ \text{terms}) ,$$

where D^{-1} is the inverse pion propagator in the background field of a Skyrmion. A determinant of the fluctuation operator arises from summing over all fluctuations $\delta\pi$. From $\det^{-1/2}[D^{-1}] = \exp[-1/2\ \mathrm{tr}\log[D^{-1}]$, the log, which is the sum of all continuum eigenvalues times the time, can be expanded in an infinate sum of one-loop Feynman diagrams of the pion with insertions of the external classical field. Divergences, which arise from the first few graphs, are absorbed into a renormalization of f_π (seagull and self-energy graphs of the pion propagator), of $g_{\rho\pi\pi}$, plus a renormalization of the coefficient of

$$\left[\mathrm{tr}\left(\partial_\mu U \partial^\mu U^{-1}\right)\right]^2 ,$$

a term of the same order in derivatives as the Skyrme term. This term contains momentum corrections to the low energy limit of σ exchange for π-π scattering. The renormalization of this term and the Skyrme term arise from the one-loop 2-pion exchange graphs of π-π scattering [19] , with σ or ρ-exchange quantum numbers. (Because the effective field theory is nonrenormalizable, additional terms in the effective Lagrangian must be included to each order in perturbation theory to absorb the divergences. The higher loop graphs with multiple pion exchange correspond to higher mass meson resonances; their large masses suppress the contribution of the higher derivative counterterms.) The divergent renormalizations for B>0 have exactly the same counterterms as the B=0 sector [16, 24]. After renormalizing the divergences, the sum of eigenvalues of the fluctuation operator gives a finite renormalization of the soliton mass.

While the continuous eigenvalues of the fluctuation operator give the renormalizations, the asymptotic behavior of the corresponding eigenfunctions give the scattering phase shifts for π-N scattering. In the lowest approximation to π-N scattering [26], of order $(N_c)^0$, the terms in dA/dt are neglected,

$$\mathcal{D}^{-1}(U_A) = D^1(A)\,\mathcal{D}^{-1}(U_0)\,D^1(A^{-1}) \ ,$$

The operator **K** = **L**(pion) + **I**(pion) commutes with $D^{-1}(U_0)$, and so the eigenfunctions are of the form

$$\delta\pi(x,t) = \sum_{K,L} \pi_{KL}(r)\, e^{-iEt}\, \mathbf{X}_{KLK_3}(\theta,\phi) \ ,$$

where **X** are the vector spherical harmonics, and the orbital angular momentum has the values: L=K, K±1. For L=K, for example,

$$\pi_{KK}(r) \rightarrow A(\omega)\, j_K(kr) + B(\omega)\, n_K(kr) \ ,$$

the S-matrix is

$$S_{KKK}(\omega) = \frac{A + iB}{A - iB} = e^{2i\delta_{KKK}(\omega)} \ .$$

The other channels, with L=K±1 are similar. From these amplitudes the physical S-matrix with conserved total angular momentum and isospin (or flavor SU(3)) are constructed by angular momentum recoupling methods [26, 27].

Neutron β-decay is a function of g_V and g_A. In this model, g_V is calculated, in lowest order, from the matrix elements of the vector current,

$$\int dA\, \chi^{1/2^*}_{1/2\,1/2}(A)\, J_\mu(U_A)\, \chi^{1/2}_{-1/2,1/2}(A)\, e^{iq\cdot x}\, d^3x \ ,$$

where $\chi^{1/2^*}{}_{i_3\, s_3}(A)$ is the wavefunction for the isospin 1/2, spin 1/2 nucleon. For $\mathbf{q} \rightarrow 0$, only the time component of the vector current is nonzero, and there is no spin flip. Explicit calculation of this matrix element gives $g_V = 1$. This is a consequence of exact isospin invariance of the model. When pion electromagnetic mass splitting is included, g_V will differ from 1 by effects of order α/π.

As for g_A, if it is calculated in the analogous way to g_V, only the spatial components of the axial current contribute for a nucleon spin flip, as it should for a nonrelativistic Gamow-Teller transition. The resulting value of g_A [28] is, however, too small. Furthermore, it is of order N_c, making its small value even harder to understand. The Adler-Weisberger sum rule should contain the solution to this mystery. We will argue that it implies the lowest order wrong value of g_A should be related to a lowest order wrong value for the integrated cross sections for π-N scattering.

From the elastic π-N scattering phase shifts, δ_{LIJ}, with $J=L\pm1/2$, one calculates

$$\sigma_{\pi^+p} - \sigma_{\pi^-p} = \frac{2\pi}{k^2}\sum_{L,J} (2J+1)\left[\sin^2 \delta_{L,3/2,J} - \left(\frac{1}{3}\sin^2 \delta_{L,3/2,J} + \frac{2}{3}\sin^2 \delta_{L,1/2,J}\right)\right] .$$

From the lowest order phase shifts, of order $(N_c)^0$, numerical evaluation of the spectral integral gives a small result [28]; its value being sensitive to the numerical accuracy. To see what this implies about g_A, consider the Adler-Wessberger relation re-expressed

$$g_A^2 = 1 + f_\pi^2 \frac{1}{2\pi}\int_{m_\pi}^{\infty} \frac{d\omega}{\omega}(\sigma_{\pi^+p}(\omega) - \sigma_{\pi^-p}(\omega)) .$$

We have expressed the spectral integral in the lab frame, and have used the Goldberger-Treiman relation [30], $2Mg_A/g_{\pi NN} = f_\pi$, which is satisfied in the model [28]. Now the lowest order calculation of g_A is of order N_c, making the left hand side of order N_c^2, and $f_\pi \sim \sqrt{N_c}$, making the coefficient of the spectral integral of order N_c. Rewriting the spectral integral

$$g_A^2 \approx 1 + f_\pi^2 \int_{m_\pi}^{\infty} \frac{d\omega}{\omega^3}\left(\sum_{L,J} (2J+1)\frac{2}{3}\left[\sin^2 \delta_{L,3/2,J} - \sin^2 \delta_{L,1/2,J}\right]\right) ,$$

it is not clear how the integral can be of order N_c to match the N_c dependence of the left hand side, since the phase shifts are of order $(N_c)^0$. One seems forced to assume that the integral is dominated by $\omega \sim 1/N_c^{1/3}$ (which I do not understand, although to lowest order, the Δ resonance is a threshold bound state [26], and once past the baryon resonance regime the $\pi^\pm$ cross sections rapidly become equal). If this is true, though, then, equating terms of order N_c^2, the small value of g_A is related to the small value of the lowest order difference of the π-N scattering cross sections. The 1 term is still missing; however, it is contained in the order $1/N_c$ contribution to the π-N scattering cross sections. The excitation of the Δ resonance, the dominant feature of the π-N scattering cross section, and of $\sigma_{\pi+p} - \sigma_{\pi-p}$, is a $1/N_c$ effect [26]. Without the Δ resonance, the integrated difference of cross sections almost cancel, explaining the small value of g_A found by Adkins, Nappi, and Witten [28]. Therefore, the $1/N_c$ corrections should be responsible for determining the correct value of g_A.

The proper way to confirm this $1/N_c$ analysis is to return to the derivation of the Adler-Weisberger sum rule. It follows from the proton matrix elements of [3]

$$\left[Q_5^+ , Q_5^-\right] = 2I_3 .$$

A complete set of states is inserted, and the evaluation is carried out in the infinite momentum frame. It is interesting that the time components of the axial charges enter here, and as we have mentioned, in lowest order in β-decay only the space components contribute. However, the charge is a limit of the Fourier transform of the current, and the time components will contribute to the terms of order $\mathbf{q}$. (By Lorentz invariance, g_A must be the same coefficient of the nucleon matrix elements of either the space or time components.) The evaluation of the commutator in the

$1/N_c$ expansion is analogous to the evaluation of the soliton expectation value of the canonical commutation relations that Goldstone and Jackiw [31] evaluated in models in one spatial dimension. Inelastic processes in π-N scattering are higher order in $1/N_c$, just as states with a soliton and many mesons are suppressed in the semiclassical expansion. This would then explain the rapid convergence of the Adler-Weisberger sum rule. The implementation of this analysis is in progress.

The $1/N_c$ corrections to π-N scattering are of three kinds. The first corresponds to the Δ resonance pole contribution, and arises as the classical Skyrmion background field contribution to the pion propagator. The second and third correspond respectively to the recoil of the Skyrmion in the scattering process and to the vibrational-rotational mode correction to the inverse pion propagator in the background (iso)rotating Skyrmion field. The addition of the terms in dX/dt and dA/dt should correct the S, P, and D waves of the analyses of Mattis, Peskin, and Karliner [26,27]. It is this problem that, with Stu, we are currently engaged in understanding.

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