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# ANALYTICAL FRAMEWORK FOR THE LIFE2 COMPUTER CODE\*

*by*

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## ABSTRACT

The LIFE2 computer code is a fatigue/fracture analysis code specifically designed for the analysis of wind turbine components. It is a PC-compatible Fortran code that is written in a top-down modular format. In this numerical formulation, an "S-n" fatigue analysis is used to describe the initiation, growth and coalescence of micro-cracks into macro-cracks. A linear, "da/dn" fracture analysis is used to describe the growth of a macro-crack. This paper presents the numerical formulations used to implement these analyses.

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## TABLE OF CONTENTS

INTRODUCTION .....	1
NUMERICAL PHILOSOPHY .....	2
Memory Allocation .....	3
Computation Times.....	3
Units.....	4
Alternating Stress Levels.....	4
NUMERICAL FRAMEWORK .....	5
WIND REGIME.....	7
Round-Off Error.....	8
Tabular Input .....	9
Weibull and Rayleigh Distributions .....	9
Plotting Routines.....	10
Interpolation.....	11
Summary Statement .....	11
CONSTITUTIVE FORMULATION.....	11
S-n Fatigue Constitutive Relations.....	12
Tabular Input .....	12
Constitutive Relations .....	12
Maximum and Minimum Values .....	13
Interpolation .....	13
da/dn Fracture Mechanics Relations.....	14
Tabular Input .....	14
Constitutive Relations .....	14
Interpolation .....	15
Plotting Routines.....	15
Summary Statement .....	15
STRESS STATES.....	16
Operational Stresses .....	16
Tabular Input .....	16
Narrow-Band Gaussian Model .....	17
Parked Stresses .....	18
Start-Stop Stresses .....	18

Stress Interval.....	19
Summary Statement .....	19
OPERATIONAL PARAMETERS.....	19
Operation Regime.....	19
Parked Stress Cycles.....	19
Start-Stop Cycles.....	20
Stress Concentration Factors.....	20
DAMAGE CALCULATION.....	20
Annual Average Cycle Count.....	22
Operational and Parked Cycle Count Matrices .....	22
Start-Stop Cycle Count Matrices .....	22
Alternating Stress Parameter.....	23
Operational Wind Speeds .....	23
Fatigue Analysis.....	23
Stress Concentration Factor.....	23
Cycles to Failure.....	24
Damage Calculation .....	24
Fracture Analysis.....	25
Current Crack Length.....	25
Stress Intensity Factor .....	26
Crack Growth Rate.....	27
Damage Calculation for the Current Crack Length.....	27
Summary Statement .....	28
SUMMARY.....	28
ACKNOWLEDGMENTS.....	29
REFERENCES.....	30
APPENDIX. Forman Constitutive Model .....	32

## **LIST OF TABLES**

<b>TABLE I.</b>	<b>Constitutive Parameters for the S-n Formulations .....</b>	<b>13</b>
<b>TABLE II.</b>	<b>Specialized Values for the Exponents in the Forman Constitutive Equation.....</b>	<b>15</b>
<b>TABLE A-I.</b>	<b>Cross-Reference for the Forman Constitutive Constants.....</b>	<b>34</b>

## LIST OF SYMBOLS

$a$	Crack length.
$a_i$	The initial crack length.
$a_f$	The final crack length.
$a_w$	The final value of the crack length used in the computations, see Eq. 42.
$a_w$	The maximum crack length in the $w^{\text{th}}$ crack interval.
$\hat{a}_w$	The average crack length in the $w^{\text{th}}$ crack interval, see Eq. 43.
$a(\Delta t)$	The current crack length.
$a_0$	The initial value of the crack length used in the computations, see Eq. 41.
$A(\cdot)$	The cumulative density function for the annual wind speed distribution, see Eq.18.
$A_0$	Forman calculational constant, see Eq. A-7.
$A_1$	Forman calculational constant, see Eq. A-8.
$A_2$	Forman calculational constant, see Eq. A-10.
$A_3$	Forman calculational constant, see Eq. A-9.
$A_k$	Forman calculational constant, see Eq. A-4.
$B_k$	Forman calculational constant, see Eq. A-3.
$C$	Forman material parameter, see Eq. 24.
$C(\cdot)$	The complementary cumulative density function for the annual wind speed distribution, see Eq. 17..
$d$	Forman calculational constant, see Eq. A-16.
$da/dn$	The crack growth rate at the stress intensity state $(\kappa_m, \kappa_a)$ .
$\hat{da}/dn$	The adjusted crack growth rate at the adjusted stress intensity state $[(\kappa_m)_r, (\kappa_a)_s]$ , see Eq. 37.
$D(\cdot)$	The average annual damage rate for the turbine.
$f_0$	The dominate frequency of blade vibration or the rate of mean level crossings, see Eq. 28. The LIFE2 code assumes units of Hz for this variable.
$G(i)$	Constants for determining the value of the $r$ function, see Eq. 15.
$i, I$	Index.
$j, J$	Index.
$k, K$	Index for the mean stress terms of the constitutive formulation.
$K_c$	Forman calculational constant, see Eq. A-3.
$K_{Ic}$	Critical stress intensity factor for mode I failure.
$L, l$	Index for the alternating stress terms of the constitutive formulation.
$n(\cdot)$	The number of stress cycles imposed on the turbine component in time $\Delta t$ with a wind velocity $V$ and at the stress state $(S_m, S_a)$ .

$(\hat{n}_{rs})_q$	The annual average cycle count imposed on the turbine component for the $q^{\text{th}}$ wind velocity interval and for the $(S_m, S_a)$ stress state interval, see Eq. 33 and 34.
$N(\cdot)$	The total number of cycles required to fail the material at the stress state $(S_m, S_a)$ .
$\hat{N}(\cdot)$	The adjusted number of cycles required to fail the material at the adjusted stress state $[(\hat{S}_m)_r, (\hat{S}_a)_s]$ , see Eq. 37.
$ps v$	The conditional probability density function for the vibratory stress, see Eq. 26.
$p_v$	The probability density function for the annual wind speed distribution.
$(P_S v)_s$	The conditional probability density function for the vibratory stress integrated over the $s^{\text{th}}$ stress interval, see Eq. 27.
$(P_V)_i$	The probability density function for the annual wind speed distribution in the $i^{\text{th}}$ interval, see Eq. 16.
$q, Q$	Index for the summation over wind speed.
$r, R$	Index for the summation over mean stresses.
$R$	Stress ratio, see Eq. A-1.
$s, S$	Index for the summation over alternating stresses.
$S$	Stress state.
$S_a$	The alternating stress.
$(\hat{S}_a)_s$	The adjusted alternating stress in the $s^{\text{th}}$ interval, see Eq. 37.
$S_c$	The component stress state.
$S_m$	The mean stress.
$(\hat{S}_m)_r$	The adjusted mean stress in the $r^{\text{th}}$ interval, see Eq. 37.
$S_{\max}$	The maximum stress in a stress cycle.
$[S_{\max}/\sigma_o]$	Forman calculational constant, set equal to 0.3, see Eq. A-7.
$S_{\min}$	The minimum stress in a stress cycle.
$[S_o/S_{\max}]$	Forman calculational constant, see Eqs. A-11 and A-15.
$[S_o/S_{\max}]_R$	Forman calculational constant, see Eq. A-12.
$t$	Thickness of the component, see Eq. A-4.
$t_o$	Forman calculational constant, see Eq. A-5.
$T$	The total number of seconds in one year. It equals $365.25 \text{ day/yr} \cdot 24 \text{ hr/day} \cdot 60 \text{ min/hr} \cdot 60 \text{ sec/min} = 31,557,600 \text{ sec}$ .
$(T_{aa})_q$	The annual average time the turbine is subjected to the $q^{\text{th}}$ wind velocity interval, see Eq. 32.
$T_f$	The service lifetime of the component.
$v, V$	The wind speed.
$V_m$	The mean wind speed for the Weibull distribution, see Eq. 14..
$V_{ci}$	The cut-in wind speed.
$V_{co}$	The cut-out wind speed.
$V_q$	The maximum wind speed in the $q^{\text{th}}$ interval.
$w$	Forman calculational constant, see Eq. A-4.
$w, W$	Index for the summation over crack length.

$Y_o$	The geometric shape factor for the stress intensity factor, see Eqs. 44 and 45.
$\alpha$	The shape factor for the Weibull distribution, see Eqs. 13 and 14.
$\alpha_F$	Forman calculational constant, see Eqs. A-7 and A-8.
$\beta$	The amplitude factor for the Weibull distribution, see Eqs. 13 and 14.
$\gamma$	The stress concentration factor, see Eqs. 30 and 31.
$\gamma_A$	The range/amplitude multiplier, see Eqs. 36 and 48.
$\gamma_f$	The stress concentration factor used for fatigue analysis, see Eqs. 35 and 36.
$\gamma_G$	The exponent for Goodman's or Gerber's rule, see Eq. 22 and Table I.
$\gamma_i$	The $i^{\text{th}}$ stress concentration factor, see Eq. 29.
$\gamma_r$	The stress concentration factor used for fracture analysis, see Eqs. 46, 47 and 48.
$\gamma_{SF}$	The safety factor.
$\gamma_t$	The total stress concentration factor resulting from various phenomena, see Eq. 29.
$\Gamma(\cdot)$	The gamma function.
$\eta_q$	The number of start-stop events counted for the stress cycle matrix that describes the $q^{\text{th}}$ class of start-stop events, see Eq. 34.
$\bar{n}_q$	The annual average number of the $q^{\text{th}}$ class of start-stop events, see Eq. 34.
$\Delta a_w$	Crack extension in the $w^{\text{th}}$ interval, see Eqs. 6 and 40.
$\Delta \kappa$	The range of the stress intensity factor for a stress cycle, see Eq. 24.
$[\Delta \kappa]_{\text{eff}}$	Forman calculational constant, see Eq. A-13.
$\Delta \kappa_o$	Forman calculational constant, see Eq. A-6.
$\Delta \kappa_{th}$	Forman material parameters, see Eq. 24.
$\Delta \kappa_1$	Forman calculational constant, see Eq. A-13.
$\Delta t$	Time period.
$\Delta t_q$	The $q^{\text{th}}$ time interval.
$\Delta V_q$	The $q^{\text{th}}$ wind speed interval, see Eq. 25.
$\kappa_a$	The alternating stress intensity factor.
$\kappa_c$	Forman material parameter, see Eq. 24.
$\kappa_m$	The mean stress intensity factor.
$\mu$	Forman material parameter, see Eq. 24 and Table II.
$\eta$	Forman material parameter, see Eq. 24 and Table II.
$\pi$	Pi.
$\sigma_e$	The effective cyclic stress amplitude for Goodman's or Gerber's rule, see Eq. 22 and Table I.



$\sigma_i$	The stress in the $i^{\text{th}}$ interval, see Eq. 23.
$\sigma_{\text{ref}}$	The reference stress for Goodman's or Gerber's rule, see Eq. 22 and Table I.
$\sigma_y$	The yield stress of a material.
$\sigma_u$	The ultimate stress of a material.
$\sigma_V$	The standard deviation of the vibratory stresses, see Eq. 26.
$\nu$	Forman material parameter, see Eq.24.
$\psi$	Forman material parameter, see Eq.24 and Table II.

# INTRODUCTION

The LIFE2 computer code is a fatigue/fracture analysis code specifically designed for the analysis of wind turbine components. The code was originally written by Sutherland, Ashwill and Naassan,<sup>1</sup> and it was subsequently modified by Sutherland and Schluter.<sup>2</sup> The code is written in a top-down modular format using Fortran. It is designed to run on an IBM personal micro-computer (PC) or compatible.

The service lifetime of a wind turbine component may be divided into three phases: 1) micro-crack initiation, 2) growth and coalescence of micro-cracks into macro-cracks and 3) growth of a macro-crack. In the LIFE2 formulation, an "S-n" fatigue analysis is used to describe the first two phases and a linear, "da/dn" fracture mechanics analysis is used to describe the third phase.

The code is divided into five main sections. The first four describe the wind resource, the constitutive properties of the turbine material, the stress state in which the turbine operates and operational parameters for the turbine system. The fifth uses the data files written by the first four sections to calculate the service lifetime of a turbine component. In addition to the main sections, auxiliary sections are included to permit the storage of input data and code calculations and to permit the plotting of results.

The wind resource is described in the LIFE2 code as the probability density function for the yearly wind speed distribution. Rayleigh and Weibull distributions<sup>3</sup> may be calculated directly by the code. Other distributions may be entered in tabular form.

Two sets of constitutive properties are used by the code. The first is the classical "S-n" fatigue characterization that describes the number of stress cycles required to fail the component.<sup>4</sup> Each stress cycle is described as a function of applied stress state. The stress state is characterized by both its mean stress level and its alternating stress level. This functional relation may be entered in tabular form or by using a constitutive description. The constitutive relations include Goodman's rule (using either the yield or the ultimate stress), Gerber's rule and the modified Gerber rule.<sup>4</sup> The second set of constitutive properties describes the rate of crack growth as a function of the cyclic change in the stress intensity factor. The growth rate is assumed to be dependent on the mean stress intensity factor and the alternating stress intensity factor. Again, the relationship between these variables may be entered in tabular form. A Forman constitutive model is also included in the code.<sup>5</sup>

The third section of the code describes the cyclic content of the stress state imposed on the turbine component. Classes of stress states include operational stresses, parked stresses (i.e., the stresses due to the wind buffeting a parked blade) and start-stop stresses. The first two are taken to be functions of the annual wind speed distribution and the third is taken to be a function of specific events. For each of these stress states, the code accepts a cycle count matrix in which the number of stress cycles (for a specified time period) is characterized as a function of the mean stress and the alternating stress. This matrix, commonly called a "rain-flow" matrix, is accepted in

tabular form. A narrow-band Gaussian model for operational stresses<sup>6</sup> is also included in the code.

The fourth section of the code describes the operational parameters for the turbine. This section records the miscellaneous parameters required for the calculations. Typical inputs include such parameters as the cut-in wind speed, the cut-out wind speed and the stress concentration factor(s).

The fifth section of the code has two calculational modules. The S-n fatigue analysis module uses Miner's Rule<sup>4</sup> to determine the initial portion of the service lifetime of a turbine component; namely, this module describes the initiation of micro-cracks and their growth and coalescence into macro-cracks. In this formulation, the damage rule is integrated over all stress cycles to determine the annual average rate at which damage is being accumulated. The lifetime of the component is inversely proportional to this damage accumulation rate.

In the second calculational module, the crack propagation characteristics of a pre-existing macro-crack are analyzed using a  $da/dn$  constitutive rule.<sup>4</sup> In this formulation, the crack growth rate,  $da/dn$ , is integrated over all stress cycles. As  $da/dn$  is a function of the crack length, the calculation is divided into a finite number of steps. For each step, the crack length is taken to be a constant, and an average growth rate is determined and the time required for the crack to grow across that integration segment is calculated. The sum over all of the integration segments yields the time of growth for the crack from its initial size to its final size.

This report describes the computational framework used in the LIFE2 code to evaluate the damage rules cited above. It is designed to be a companion report to the "User's Manual" that was written by Schluter and Sutherland.<sup>7</sup> Example problems to illustrate its capabilities are presented in the Ref. 7.

## NUMERICAL PHILOSOPHY

From its inception, the LIFE2 code has been designed to run on an IBM personal micro-computer (PC) or compatible. This decision was made to insure that it could be used by the maximum number of wind turbine designers. To implement the numerical formulations discussed below onto this relatively small computer required that special formulations and techniques be incorporated into the code. The main differences between this code and a code written for a larger, faster machine are dictated because the smaller machines have limited memory, have comparatively slow processors and have comparatively few significant digits.

## MEMORY ALLOCATION

For a PC, the available memory for a program and all of its data is limited to a total of 640K of operational random-access memory (RAM). This relatively small memory placed a rather severe limitation on the program and forced several programming compromises to be incorporated into the code. The first compromise was to use program overlays. In this technique, the program resides in external memory (e.g., a hard disk, a floppy disk, a RAM disk in extended memory, etc.) and only those subprograms required by the current calculations are loaded into the operational RAM. When another subprogram is required, it is automatically loaded into the operational RAM and "overlays" the memory locations of a subprogram that is no longer required. *These operations are transparent to the operator.*

Further reductions in total memory requirements were achieved by limiting the size of the matrices used in the code. We have tried to make the matrices as large as possible and still have the program fit within the 640K memory limit of a standard PC. Also, data overlays are used throughout the program; namely, one matrix is used to store several different variables at different times during the course of the computations. This numerical philosophy requires extensive read/write statements to disk that significantly increase the time for the code to complete a task. However, these compromises are required for the program to maintain PC compatibility and a reasonable degree of accuracy.

The program does not use expanded memory directly, because this feature is not available in all PC's. However, expanded memory may be used in this formulation by using a "virtual disk" for reading and writing computational data files. In this manner, the I/O (input/output) time for data files can be reduced significantly.

## COMPUTATION TIMES

As the PC uses a relatively slow processor, computational times may become excessively long when all computations are preformed at one time. To permit the use and storage of previous calculations and to permit a calculation to be terminated in mid-stream without the loss of the calculations that have been completed to that point, extensive "library" functions (i.e., data storage to permanent files) have been incorporated into the code. The transfer of data between calculational modules is accomplished by read/write statements to permanent files.

This "open architecture" for data transfer between modules has several benefits. First, this formulation inherently yields calculational modules that are independent of one another. This aspect of the program implies that new calculational modules may be added to the code with relative ease. When a new module is added, only that module needs to be validated, rather than the whole code. Second, data from sources other than the code may be incorporated into the calculations by simply placing them in the proper format and then adding them to the code's library. Third, parameter studies may be conducted relatively quickly because a large portion of the computations do not

have to be duplicated. And, finally, all computations may be archived for future reference.

The major problem with this formulation is that the program uses tables (matrices) to transfer computations between modules. This formulation implies that the numerical framework relies heavily on interpolations. Thus, the accuracy of the calculations decreases. Unfortunately, the decision to make this code PC compatible has severely limited our ability to address this problem. Our primary technique for addressing it is to keep all data transfer tables (matrices) as large as possible and to have them cover only those ranges required by the calculations. Additional techniques for increasing accuracy have been incorporated into the code. These techniques are discussed below in the sections that described the individual computation modules.

## UNITS

With the one exception of variables with units of time, the LIFE2 code assumes that a consistent set of units are being analyzed by it. Namely, *units are not checked for consistency by the code; no units are changed by the calculational modules*. Thus, all variables with equivalent units must be entered with the same units (e.g., if stresses are entered in KPa in one place than they must be entered in KPa everywhere). *If mixed units are entered into the code, the code will not preform correctly*. To help alleviate the problem of mixed units, subroutines are provided in the code to change units, as required. These subroutines are invoked at the discretion of the operator.

Time variables are the one exception to this rule. As the service lifetime of a turbine component (i.e., the ultimate answer produced by the code) has units of time, the code must know the time units entered into it. *The LIFE2 code always assumes that all time variables (i.e., those variable that have exclusive units of time) have units of seconds*. If other units are used, they must be changed to seconds (using the appropriate subroutines) before calculations can proceed. The code prompts have been written to specify units of seconds for the appropriate time variables and Hertz for the appropriate frequency variables.

## ALTERNATING STRESS LEVELS

Throughout this paper, the oscillating portion of the stress state,  $S_a$ , is described as the alternating stress level. Two definitions may be used for the alternating stress level: the range of the stress oscillation or the amplitude of the stress oscillation.<sup>4</sup> The range is equal to the maximum stress in the cycle,  $S_{max}$ , minus the minimum stress in the cycle,  $S_{min}$ , and the amplitude equals half the range.

As the numerical formulations are, in general, not dependent on the definition used, either may be used by the operator. *However, the operator must insure the consistency of the calculation*. Namely, if the constitutive equation is defined using the range, then the operating stresses must also be defined using the range; and if the constitutive equation

is defined using the amplitude then the operating stresses must also be defined using the amplitude. Typically, the S-n constitutive relations are defined using the amplitude of the oscillating stress. And, the "da/dn" vs stress intensity factor are based on the range. Thus, the operator must be careful when entering the alternating stress data into the code.

Notes have been placed throughout the program input prompts to inform the operator when a particular formulation assumes that the alternating stress is an amplitude or a range variable. The operator is given the opportunity to change the alternating stress variable between these two definitions in the calculational module.

## NUMERICAL FRAMEWORK

The LIFE2 computer code has been developed to compute the fatigue lifetime of a turbine component. It uses either Miner's Rule<sup>4</sup> or linear fracture mechanics<sup>4</sup> to analyze the behavior of the component. In the case of the former, the damage law used in the computations is given by the relation

$$D(\Delta t) = \int_{V_{ci}}^{V_{co}} p_V \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{n(S_m, S_a, V, \Delta t)}{N(S_m, S_a)} dS_m dS_a dV, \quad (1)$$

where  $p_V$  is the probability density function for the annual wind speed distribution,  $S_m$  is the mean stress,  $S_a$  is the alternating stress,  $N(\cdot)$  is the total number of cycles required to fail the material at the stress state  $(S_m, S_a)$  and  $n(\cdot)$  is the number of stress cycles imposed on the turbine component in time  $\Delta t$  with a wind velocity  $V$ .  $V_{ci}$  and  $V_{co}$  are the cut-in and the cut-out wind speeds, respectively. In this form,  $D$  is the average annual damage rate for the turbine. When the damage rate  $D$  integrates over time to one, the component is expected to fail. As this formulation is linear, the service lifetime  $T_f$  of the component is given by:

$$1 = \int_0^{T_f} D(\Delta t) dt = D(\Delta t) T_f, \quad (2)$$

or,

$$T_f = \frac{1}{D(\Delta t)}. \quad (3)$$

We have assumed apriori that the stress cycles are a function of the velocity of the wind impinging on the turbine and that the annual wind speed distribution may be described by a probability density function. Eq. 1 has been discretized to the following finite summation:

$$D(\Delta t) = \sum_{q=1}^Q \sum_{r=1}^R \sum_{s=1}^S (P_V)_q \frac{n_{qrs}[(S_m)_r, (S_a)_s, V_q, \Delta t]}{N_{rs}[(S_m)_r, (S_a)_s]} \quad (4)$$

The index  $q$  is for the summation over wind speed,  $r$  is for the summation over mean stresses and  $s$  for the summation over alternating stresses. The exact formulation of each term in this sum will be defined later in this report.

For the linear fracture mechanics model, the propagation of a pre-existing, finite length crack is determined by integrating the crack growth rate  $da/dn$  over all stress cycles  $N$ . As the growth rate is a function of the current crack length  $a(\Delta t)$ , this integration is typically divided into a finite number of integration steps of the form:

$$a_f = a_i + \sum_{w=1}^W \Delta a_w \quad (5)$$

where  $a_i$  is the initial length of the crack and  $a_f$  is the "final" length of the crack produced by the  $N$  stress cycles occurring in time  $\Delta t$ . The growth rate at each step is determined by integrating the annual average growth rate over the unknown number of stress cycles  $N$  required to grow the crack an incremental length of  $\Delta a_w$ ; namely:

$$\Delta a_w = \int_0^N \text{avg}[(da/dn)_w] dn = \text{avg}[(da/dn)_w] N \quad (6)$$

Expanding this integral to illustrate its functional dependence,

$$\Delta a_w = \int_{V_{cl}}^{V_{co}} P_V \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{da(\kappa_m, \kappa_a)}{dn} n(S_m, S_a, V, \Delta t) dS_m dS_a dV \quad (7)$$

(Note: this form of the crack growth equation is parallel in construction to the damage accumulation integral for fatigue shown in Eq. 1.)

As seen in this formulation, the crack growth rate  $da/dn$  is a function of the stress intensity state  $(\kappa_m, \kappa_a)$  and the cycle count  $n$  is a function of the stress state  $(S_m, S_a)$ . To map the stress state into the stress intensity state requires a knowledge of the current

crack length  $a$ . Thus, the integrand is a function of the independent variable  $a$ . To integrate this equation numerically, let us assume that the crack length  $a$  may be divided into  $W$  discrete intervals (as discussed above) and that the crack length,  $a_w$ , in each interval may be considered to be a constant. Then, Eqs. 5 and 7 may be combined to yield the following:

$$a(\Delta t) = a_f = a_i + \sum_{w=1}^W \int_{V_{ci}}^{V_{co}} p_V \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \frac{da(\kappa_m, \kappa_a)}{dn} \right] \cdot \left[ n[(\kappa_m)_w, (\kappa_a)_w, V, \Delta t] \right] d\kappa_m d\kappa_a dV \quad (8)$$

where the two components of the stress state have been mapped into the current stress intensity state using the current crack length  $a_w$ . In this formulation, the final crack length  $a_f$  is equivalent to the current crack length  $a(\Delta t)$  because the integration is conducted over the stress cycles occurring in a time  $\Delta t$ . Discretizing Eq. 8 for numerical evaluation yields:

$$a(\Delta t) = a_i + \sum_{w=1}^W \sum_{q=1}^Q \sum_{r=1}^R \sum_{s=1}^S (P_V)_q \left[ \frac{da[(\kappa_m)_{wr}, (\kappa_a)_{ws}]}{dn} \right] \cdot \left[ n[(\kappa_m)_{wr}, (\kappa_a)_{ws}, V_q, \Delta t] \right] \quad (9)$$

The index  $w$  is for the summation over crack length,  $q$  is for the summation over wind speed,  $r$  is for the summation over mean stress intensity factors and  $s$  is for the summation over alternating stress intensity factors. Again, each term will be defined in detail later in this report.

## WIND REGIME

The annual wind speed distribution which acts upon the turbine is assumed to be described by a probability density function  $p_V$ , see Eqs. 4 and 9. This function may take the form of a table or it may be described with an equation(s). If we assume that the stress state terms in Eqs. 1 and 8 are independent of this distribution, then the integration over the wind speed distribution may take the form:



$$(P_V)_i = \int_{V_{i-1}}^{V_i} p_V dv \quad , \quad (10)$$

where the integration has been divided into I intervals. An alternative formulation of this equation is given by:

$$(P_V)_i = C(V_{i-1}) - C(V_i) \quad , \quad (11)$$

where  $C(\cdot)$  is the complementary cumulative density function defined by:

$$C(V) = \int_V^{\infty} p_V dv \quad . \quad (12)$$

$C(V)$  was chosen as the storage parameter for transferring this term to the calculation module because this form permits the calculational module to interpolate a tabular  $C(V)$  function rather than numerically integrating a probability density tabular function. This procedure decreases computation times and it permits the yearly wind speed distribution to be entered in tabular form.

### ROUND-OFF ERROR

To illustrate the problems with PC compatibility and accuracy, Eq. 11 is analyzed in detail here. Eq. 11 takes the difference of two numbers that are typically small (in comparison to one) and close together at high wind speeds. As most PC's are limited to 6 or 8 significant digits in floating points operations and approximately 15 digits in double precision,<sup>8</sup> this difference can be significantly influenced by numerical round-off if not handled carefully.

To minimize round-off errors caused by this term in the computation of total damage rates, let us consider the stress states of a wind turbine. In general, the stress state (and, thus, the damage) imposed on a wind turbine increases with increasing wind velocity. Therefore, the numerical formulation chosen to evaluate  $P_V$  should have high accuracy at large wind speeds. For the form chosen in Eq. 11,  $P_V$  is evaluated as the difference between two numbers that are relatively close to zero for high wind conditions; and, thereby, maximizes the accuracy of the calculation.

Also, the accuracy of this numerical formulation has been increased by taking the  $C(V_i)$  matrix to be double precision.

## TABULAR INPUT

To handle wind distributions that are not described by a functional relation supported by the code, tabular inputs may be used. The LIFE2 code will accept the complementary cumulative density function  $C(V)$  in tabular form. The function  $C(V)$  has a range that is limited to  $0.0 \leq C(V) \leq 1.0$ . To insure that this range is not violated, the code checks  $C(V)$  to insure that it always falls in the proper range. If it does not, the operator is asked to re-enter the data.

## WEIBULL AND RAYLEIGH DISTRIBUTIONS

Two mathematical descriptions of the wind regime are supported by the code at this time. They are the Weibull and the Rayleigh distributions. The formulation for the Weibull distribution is given by:

$$p_V = \left[ \frac{\alpha}{\beta} \right] \left[ \frac{V}{\beta} \right]^{\alpha-1} \exp \left\{ - \left[ \frac{V}{\beta} \right]^\alpha \right\} , \quad (13)$$

where  $p_V$  is the probability density function,  $\alpha$  and  $\beta$  are the shape factor and the amplitude factor of the distribution, respectively. The Rayleigh distribution is a special case of the Weibull distribution, with  $\alpha = 2$ .

Typically, the shape factor  $\alpha$  and the average wind speed  $V_m$  are used to describe the annual wind speed distribution at turbine sites (e.g., see Ref. 9).  $\alpha$  and  $\beta$  are related to the average wind speed (velocity)  $V_m$  through the gamma function  $\Gamma$ :<sup>3</sup>

$$V_m = \beta \Gamma \left[ 1 + \frac{1}{\alpha} \right] . \quad (14)$$

To maintain consistency,  $\alpha$  and  $V_m$  are used as input variables. Eq. 14 is used to change them into the computational appropriate framework of  $\alpha$  and  $\beta$ .

For this analysis, the gamma function  $\Gamma$  is evaluated using the following expression:<sup>10</sup>

$$\Gamma(1 + X) = 1 + \sum_{i=1}^5 G(i) X^{i-1} , \quad (15)$$

where  $G(i)$ ,  $i = 1$  to  $5$ , are constants defined to be  $-0.5748646$ ,  $0.9512363$ ,  $-0.6998588$ ,  $0.4245549$  and  $-0.1010678$ , respectively. This expansion yields an absolute error that is always less than  $5 \times 10^{-5}$  for  $0 \leq X \leq 1$ .

$C(V)$  is determined by numerically integrating the probability density function  $p_V$ . Let the  $i$ th wind velocity intervals be defined as the interval between  $V_i$  and  $V_{i-1}$ , then the total probability for that interval  $(P_V)_i$  may be approximated by:

$$(P_V)_i = \int_{V_{i-1}}^{V_i} p_V dv \approx (p_V)_i (V_i - V_{i-1}) \quad , \quad (16)$$

where  $(p_V)_i$  is evaluated at the midpoint of the velocity interval, i.e., at  $[(V_i + V_{i-1}) / 2]$ .  $C(V_i)$  is then determined using:

$$C(V_i) = 1.0 - \sum_{k=1}^i (P_V)_k \quad . \quad (17)$$

Here, we assume that  $(P_V)_1$  covers the entire interval from  $V = 0$  to  $V = V_1$ .

## PLOTING ROUTINES

The code supports two classes of plots for the wind regime. They are the cumulative density function  $A(V)$  and the probability density function  $p_V$ . The first plot uses the relation:

$$C(V) = 1.0 - A(V) \quad , \quad (18)$$

to determine  $A(V)$  from  $C(V)$ .

For the second plot,  $C(V)$  is numerically differentiated using the relation:

$$(p_V)_i = \frac{[C(V_{i-1}) - C(V_i)]}{V_i - V_{i-1}} \quad . \quad (19)$$

This term is centered at the average velocity of  $(V_i + V_{i-1}) / 2$ . This form is numerically equivalent to that used in Eqs. 10, 11, 16 and 17 to determine  $C(V_i)$  from  $p_V$ .

## INTERPOLATION

The computational modules use linear interpolation to determine  $(P_V)_q$  in Eqs. 4 and 9. To evaluate each term in Eq. 11, the set of  $I$  ordered pairs  $[V_i, C(V_i)]$  are interpolated using

$$C(V) = \frac{V - V_{i-1}}{V_i - V_{i-1}} \left[ C(V_i) - C(V_{i-1}) \right] + C(V_{i-1}) \quad (20)$$

Here, the value of  $i$  is chosen using the relation:

$$i = \begin{cases} 2 & \text{for } V \leq V_2 \\ j & \text{for } V_{j-1} < V \leq V_j \text{ and } j=2,3,\dots, \text{ or } I-1. \\ I & \text{for } V_{I-1} < V \end{cases} \quad (21)$$

## SUMMARY STATEMENT

The wind resource is described in the LIFE2 code as the probability density function,  $p_V$ , for the annual wind speed distribution. To increase the accuracy of the calculations and to accommodate the PC framework of the code, a tabular representation of the complementary cumulative density function has been chosen to transfer the annual wind speed distribution to the computation modules. Rayleigh, Weibull and tabular distributions are supported by the code. The code has the capability of displaying the annual wind speed distribution in graphical form.

## CONSTITUTIVE FORMULATION

As discussed above, the LIFE2 code accepts two classes of constitutive formulations. The first is the classical "S-n" fatigue characterization that describes the number of stress cycles required to fail the component and the second is a linear fracture mechanics analysis that describes the rate of crack growth as a function of the cyclic change in the stress intensity factor.

## S-n FATIGUE CONSTITUTIVE RELATIONS

The classical S-n fatigue characterization of a material describes the number of stress cycles  $N$  required to "fail" the material at a particular stress state. Each stress cycle is described by two terms, its mean stress  $S_m$  and its alternating stress  $S_a$ .<sup>4</sup> The mean stress is defined to be the sum of the maximum and minimum values of the stress cycle divided by two, and the alternating stress is defined to be the range or the amplitude of the stress cycle (see discussion above).

As discussed above, the functional form chosen for the description of the constitutive material property  $N(S_m, S_a)$  is tabular. Namely, the ranges of the mean stress and the alternating stress are divided into  $K$  and  $L$  intervals, respectively. At each ordered pair  $[(S_m)_k, (S_a)_l]$ , the table contains the number of cycles to failure at this stress state,  $N_{kl}$ . This two dimensional (2D) matrix may be entered into the code as a table or it may be calculated using standard formulations.

### Tabular Input

To input the S-n constitutive table into the LIFE2 code, the values of  $(S_m)_k$ ,  $k=1, K$  and  $(S_a)_l$ ,  $l=1, L$  are entered. The code then prompts the user for each  $N_{kl}$ . *The program assumes that all points in this matrix are defined.*

If the S-n properties of a particular material are independent of mean stress, only the dependence of  $N$  on alternating stress must be defined in the input table.

### Constitutive Relations

Four constitutive relationships for defining the S-n fatigue characterization matrix are supported by the LIFE2 code at this time. They are Goodman's rule using the yield stress, Goodman's rule using the ultimate stress, Gerber's rule and the modified Gerber rule.<sup>4</sup> The general formulation for these rules is given by:

$$S_a = \sigma_e \left\{ 1 - \left[ \frac{S_m}{\sigma_{ref}} \right]^{\gamma_G} \right\} \quad (22)$$

where,  $\sigma_e$  is the effective cyclic stress amplitude, and the reference stress  $\sigma_{ref}$  and the exponent  $\gamma_G$  are constitutive parameters that are listed in Table I for the four different formulations. In this table,  $\sigma_y$  and  $\sigma_u$  are the yield stress and the ultimate stress of the material, respectively.

The number of cycles to failure at the effective stress  $\sigma_e$  is equal to the number of cycles to failure at stress state  $S_m = 0$  and  $S_a = \sigma_e$ ; i.e.,  $N(0, \sigma_e)$ . Thus, the S-n material characterization for zero mean stress must be known. For the LIFE2 code, this relation is entered in tabular form. The set of ordered pairs that form this table are  $[\sigma_i, N_i]$  for  $i=1, I$ .

**TABLE I. Constitutive Parameters for the S-n Formulations.**

Constitutive Relations	Constitutive Parameter	
	$\sigma_{ref}$	$\gamma_G$
Goodman (Yield)	$\sigma_y$	1
Goodman (Ultimate)	$\sigma_u$	1
Gerber	$\sigma_u$	2
Modified Gerber	$\sigma_u$	material constant

This constitutive table that is transferred to the computational modules is set up automatically. First, a set of values for  $(S_m)_k$  and  $(S_a)_l$  are chosen. Then,  $\sigma_e$  is computed for each ordered pair  $[(S_m)_k, (S_a)_l]$  using Eq. 22 and the appropriate constants from Table I.  $N_{kl}$  is determined by interpolating on the tabular values at zero mean stress. For this interpolation, an exponential (logarithmic) interpolation is used. The form used here is given by:

$$\log(N_{kl}) = \frac{\log(\sigma_e) - \log(\sigma_{i-1})}{\log(\sigma_i) - \log(\sigma_{i-1})} \left[ \log(N_i) - \log(N_{i-1}) \right] + \log(N_{i-1}) \quad .(23)$$

Here, the value of  $i$  is chosen using an equivalent relation to that described in Eq. 21.

#### Maximum and Minimum Values

For this code to be PC compatible, special attention must be given to the calculations described in Eq. 23. In particular, the range of single-precision real numbers that can be handled by a PC is approximately  $-1E-38$  to  $3E38$ .<sup>8</sup> As these values could be exceeded by the interpolation, the code automatically limits the maximum value of  $N_{kl}$  to 1E37. The minimum value is limited to 1; i.e., one stress cycle is always required to fail the material.

#### Interpolation

The computational modules use logarithmic interpolation to determine  $N_{rs}(\cdot)$  at  $[(S_m)_r, (S_a)_s]$ , see Eq. 4. To evaluate  $N_{kl}$  at  $[(S_m)_k, (S_a)_l]$  from the constitute matrix transferred by the code, a two dimensional interpolation scheme must be used. The technique is equivalent to that cited in Eq. 23.

## da/dn FRACTURE MECHANICS RELATIONS

The classical da/dn fracture mechanics characterization of a material describes the crack growth rate of a macro-crack as a function of the stress intensity factor. The relationship between the stress state and the crack length is discussed below. Typically, the stress intensity factor is taken to have two components, a mean and an alternating stress intensity factor,  $\kappa_m$  and  $\kappa_a$ , respectively. As discussed above, the functional form chosen to describe the constitutive material property da/dn( $\kappa_m, \kappa_a$ ) is tabular. Namely, the ranges of the mean and the alternating stress intensity factors are divided into K and L intervals, respectively. The table contains the crack growth rate at each ordered pair  $[(\kappa_m)_k, (\kappa_a)_l]$  of stress intensity factors. This matrix may be entered as a table or it may be calculated using standard formulations.

### Tabular Input

To input the da/dn constitutive table into the LIFE2 code, the values of  $(\kappa_m)_k$ ,  $k=1, K$  and  $(\kappa_a)_l$ ,  $l=1, L$  are entered; the code then prompts the user for each  $(da/dn)_{kl}$ . *The program assumes that all points in this matrix are defined.*

If the da/dn properties of a particular material are independent of mean stress, only the dependence of da/dn on alternating stress must be defined in the input table.

### Constitutive Relations

Many constitutive equations have been proposed for the description of crack propagation in metallic materials. One general form proposed by Forman<sup>5,11</sup> is supported by the LIFE2 code at this time. The governing equation for this formulation is given by:

$$\frac{da}{dn} = \frac{C (1 - R)^\mu \Delta\kappa^\nu [\Delta\kappa - \Delta\kappa_{th}]^\eta}{[(1 - R) \kappa_c - \Delta\kappa]^\psi}, \quad (24)$$

where a is the current crack length; n is the number of stress cycles; R is the stress ratio<sup>4</sup>;  $\Delta\kappa$  is the change in the stress intensity factor  $\kappa$  at the crack tip for cycle n; and C,  $\mu$ ,  $\nu$ ,  $\eta$ ,  $\psi$ ,  $\Delta\kappa_{th}$  and  $\kappa_c$  are material parameters (constants) defined by Forman.<sup>5</sup>

As noted by Forman<sup>5</sup> this formulation may be reduced to other commonly used forms by setting the exponents  $\mu$ ,  $\eta$  and  $\psi$  to the values listed in Table II.

In Table II,  $m_w$  is another material constant. Ref. 11 lists a set of constants, for various materials, that may be used in this formulation. The appendix describes the procedure for obtaining the constitutive table required here from these parameters.

**TABLE II. Specialized Values for the Exponents in the Forman Constitutive Equation.**

Proposed Constitutive Relation	Exponent		
	$\mu$	$\eta$	$\psi$
Paris	0	0	0
Forman	0	0	1
Walker	$(m_w - 1)\nu$	0	0

### Interpolation

The computational modules use logarithmic interpolation to determine  $(da/dn)_{rs}$  at  $[(\kappa_m)_r, (\kappa_a)_s]$ . The technique is equivalent to that cited in Eq. 23. To evaluate this term, from the 2D matrix transferred from the code,  $(da/dn)_{kl}$  at  $[(\kappa_m)_k, (\kappa_a)_l]$ , a two dimensional interpolation scheme must be used. This scheme is identical to that used to determine  $N_{rs}$  at  $[(S_m)_r, (S_a)_s]$  from  $N_{kl}$  at  $[(S_m)_k, (S_a)_l]$ . As the interpolation scheme is discussed above, it is not duplicated here.

### PLOTTING ROUTINES

The code supports plots of these constitutive relations. For the S-n formulations, a family of curves is presented. Each curve is a plot of the cycles-to-failure versus the alternating stress level for a constant mean stress. For the da/dn formulations, two classes of plots are supported. The first class is a family of curves of da/dn versus  $\kappa_a$  at constant  $\kappa_m$  and the second is a family of curves of da/dn versus  $\kappa_a$  at constant R. *The latter class of plots is for illustration purposes only. As noted to the operator by a prompt from the code, these data must not be used for the calculation of the service lifetime.*

### SUMMARY STATEMENT

The constitutive properties of a material are described in the LIFE2 code using the classical S-n fatigue characterization and a da/dn fracture mechanics characterization. The form chosen for the transfer of this information to the computational modules is a 2D matrix. For the S-n characterization, the cycles to failures are specified over a range of mean stresses and alternating stress. Four constitutive relations are supported. For the da/dn characterization, the crack growth rate is specified over a range of mean



and alternating stress intensity factors. A generalized constitutive relation is supported. Any constitutive characterization may be input in tabular form.

## STRESS STATES

The LIFE2 code accepts three classes of stress states for the turbine. They are the operational stresses, parked stresses (i.e., the stresses due to the wind buffeting a parked blade) and start-stop stresses. The first two are taken to be functions of time and the last as specific events. All three stress states are characterized using three dimensional (3D) cycle count matrices of the form  $n_{qrs}$ . Each of these matrices contains the stress cycle count associated with a fixed time period (or a fixed number of events) and a particular operating condition of the turbine. Thus, they describe, on the average, the stress cycles imposed upon the turbine whenever it is in this particular operating condition.

In each matrix (commonly called a "rain flow" matrix), each stress cycle is characterized as a function of its mean stress  $(S_m)_r$  and its alternating stress  $(S_a)_s$ . To save memory in the operational RAM, the 3D cycle count matrix  $n_{qrs}$  is defined as a set of Q 2D-matrices,  $(n_{rs})_q$ , and only one of these matrices is held in the operational RAM at any one time.

It should be noted here that the stress states  $[(S_m)_r, (S_a)_s]$  are not required to be equivalent to the state states  $[(S_m)_k, (S_a)_l]$  or the stress intensity factor states  $[(\kappa_m)_r, (\kappa_a)_s]$  that are used to describe the constitutive properties of the material, see above.

## OPERATIONAL STRESSES

The operational stress states are input into the LIFE2 code as a series of cyclic count matrices. Each matrix is defined for a specific wind interval  $\Delta V_q$  and a time interval  $\Delta t_q$ , where:

$$\Delta V_q = V_q - V_{q-1} \quad (25)$$

The code assumes that the first interval covers the wind interval from zero to  $V_1$ .

For each of these velocity intervals, the code accepts a cycle count matrix  $n_{rs}$  that is determined from the stress-time history of the component when the wind turbine is being operating with an average wind speed of  $(V_q + V_{q-1})/2$  for a time of  $\Delta t_q$ .

### Tabular Input

To input the operational stress matrices into the LIFE2 code, the code prompts the user for  $V_q$ , the time  $\Delta t_q$ ,  $(S_m)_r$  for  $r=1,R$  and  $(S_a)_s$  for  $s=1,S$ . Then the values for  $n_{rs}$  are

entered. This process is repeated for each wind speed interval. *The program assumes that all points in each one of these matrices are defined. The values of  $(S_m)_r$  and  $(S_a)_s$  may be different for each velocity interval  $V_q$ .*

### Narrow-Band Gaussian Model

One technique for modeling the operational stresses is supported by the LIFE2 code at this time. This technique was used by Veers<sup>6</sup> to model the operational stress states of a vertical axis wind turbine (VAWT) component. The technique is based on the assumption that the amplitude of the vibrational stresses at any wind speed follows the distribution of peak values of a narrow-band Gaussian process; i.e., a Rayleigh distribution. The conditional probability density function  $p_{S|V}$  of the vibratory stress  $S_a$  occurring at a particular wind speed  $V$  may be written as:

$$p_{S|V} = \left[ \frac{S_a}{\sigma_V^2} \right] \exp \left[ -\frac{S_a^2}{2\sigma_V^2} \right] \quad , \quad (26)$$

where  $\sigma_V$  is the standard deviation of the vibratory stresses. Here, we assume that  $\sigma_V$  is only a function of the wind speed  $V$ .

The functional relationship between the wind speed and  $\sigma_V$  is input in tabular form as a set of ordered pairs of the form  $[V_i, (\sigma_V)_i]$ . For values of  $V$  that lie between the tabulated points, linear interpolation, see Eq. 20 and 21, is used to determine the value of  $\sigma_V$ .

To determine the cyclic count matrix from this formulation, the probability density function in Eq. 26 is integrated over a series of  $S$  cyclic count intervals, at each wind speed  $V_q$ . Namely,

$$(P_{S|V})_s = \int_{(S_a)_{s-1}}^{(S_a)_s} \left[ \frac{S_a}{\sigma_V^2} \right] \exp \left[ -\frac{S_a^2}{2\sigma_V^2} \right] dS_a \quad . \quad (27)$$

The integration is centered in the wind speed interval by evaluating  $\sigma_V$  at the midpoint between  $V_{q-1}$  and  $V_q$ .

This formulation assumes that the mean stress  $S_m$  is a constant at all wind speeds, i.e., the turbine is operated at constant rotational speed (the magnitude of this mean stress is an input parameter to the code).

Thus, a probability matrix has been determined for the stress cycles at each wind speed interval  $V_q$ . Each one of these matrices is composed of one column of  $S$  elements. The  $s^{\text{th}}$  element is the probability of occurrence of a stress cycle with a mean stress of  $S_m$  and an alternating stresses in the range between  $(S_a)_{s-1}$  and  $(S_a)_s$ .

To convert these probabilities into cycle count matrices, the rate at which the stress cycles occur must be known. Veers assumes that the stress cycle rate is, on the average, equal to the dominant frequency of blade vibration  $f_o^6$  or the rate of mean level crossings.<sup>12</sup> Thus, the time  $\Delta t$  for the average stress cycle is given by:

$$\Delta t = \frac{1}{f_o} \quad (28)$$

*To maintain correct units of time, the LIFE2 code assumes that the frequency  $f_o$  has units of Hz.*

Thus, the probability matrices become cycle count matrices by setting their count period  $\Delta t_q$  to  $\Delta t$ .

### PARKED STRESSES

The parked stress states are also input as a series of cyclic count matrices. Each matrix is defined for a specific wind interval  $\Delta V_q$ . For each of these velocity intervals, the code accepts a cycle count matrix  $n_{qrs}$  in which the number of stress cycles are characterized as a function of the mean stress and the alternating stress. Each stress cycle count matrix is based on the count from a specific time period  $\Delta t_q$ . At this time, only tabular inputs may be used to define these cyclic count matrices.

### START-STOP STRESSES

The start-stop stress states are also input as a series of 2D cyclic count matrices. Each matrix is based on the count from a specific number of start-stop events,  $\eta_q$ . For each state, the code accepts a cycle count matrix  $n_{rs}$  in which the number of stress cycles (for a specified number of starts or stops) are characterized as a function of the mean stress and the alternating stress. Each input matrix is defined for a class of starts and/or stops, e.g. normal start, high wind start, normal stop, high wind stop, emergency stop, "lost of grid stop", etc. At this time, only tabular inputs may be used to define these cyclic count matrices.

## STRESS INTERVAL

One should note that in the formulation cited here, a stress cycle that is counted in the interval described by its end points  $[(S_m)_r, (S_a)_s]$  has an actual stress state anywhere between  $[(S_m)_{r-1}, (S_a)_{s-1}]$  and  $[(S_m)_r, (S_a)_s]$ . To assure conservative results, all stress cycles are assumed to occur at the maximum stress levels for that interval; i.e., at  $[(S_m)_r, (S_a)_s]$ .

## SUMMARY STATEMENT

The operational stress states are described in the LIFE2 code as a series of cycle count matrices. Three classes of stress states are analyzed by the code. They are the operational stress states, the parked stress states, and the start-stop stress states. The first two states are taken to be functions of wind speed and the last state is taken to be a series of discrete events. A narrow band Gaussian model<sup>6</sup> is supported for the determination of operational stresses. Tabular data is accepted for all three stress states.

## OPERATIONAL PARAMETERS

The forth section of the LIFE2 code describes the miscellaneous operational parameters for the turbine that are required for the calculations.

### OPERATION REGIME

The first two parameters are the minimum and maximum wind speed at which the turbine is operated, commonly called the "cut-in" wind speed  $V_{ci}$  and the "cut-out" wind speed  $V_{co}$ .

### PARKED STRESS CYCLES

The next set of parameters describes the annual average time spent in each interval of the parked stress matrices. The parameter  $P_V$  is the annual average fraction of time for the  $q^{th}$  parked stress matrix. *This number is entered by the operator rather than being calculated from the wind speed distribution.)*

## START-STOP CYCLES

The next set of parameters describes the number of start-stop events for each cycle count matrix recorded in the start-stop stress matrices. The parameter  $n_q$  is the yearly average number of events for the  $q^{\text{th}}$  start-stop matrix.

## STRESS CONCENTRATION FACTORS

The final two parameters relate the stress state used in the cycle count matrices to the actual stress state anticipated in the turbine component. These two parameters are used to describe the stress concentration factor  $\gamma_t$  and the safety factor  $\gamma_{SF}$  used in the analysis. Typically, the  $\gamma_t$  is divided into its contributing factors through the relation<sup>11</sup>

$$\gamma_t = \gamma_1 * \gamma_2 * \dots * \gamma_n \quad (29)$$

Each  $\gamma_i$  is the stress concentration factor due to a particular phenomenon. Typical stress concentration factors that should be considered for turbine applications include the type of loading, size effects, surface roughness, surface treatment, penetrations, notch sensitivity, etc. A description of various factors is given in Refs. 4, 13, 14, and 15.

If we define the total "stress concentration factor"  $\gamma$  to be:

$$\gamma = \gamma_t * \gamma_{SF} \quad (30)$$

Then, the nominal stress state  $S$  is related to the component stress state  $S_c$  by:

$$S_c = \gamma S \quad (31)$$

As  $\gamma$  is different for a fatigue and fracture analyses, see below, two parameters are defined here,  $\gamma_f$  for the fatigue analysis and  $\gamma_r$  for the fracture analysis. The implementation of Eq. 31 into the service lifetime calculation is described below in Eqs. 35, 36, 47 and 48.

## DAMAGE CALCULATION

The fifth section of the code uses the inputs from the other four sections to determine the service lifetime of the turbine component. At this time, the service lifetime may be computed using either one of two damage rules. The first is Miner's rule,<sup>4</sup> see Eq. 4, and the second is based on linear fracture mechanics, see Eq. 9.

The computations for each technique proceed along similar lines. The computational steps are:

1. The code reads into current memory the files generated by the other four modules and creates a "summary file" to record the input parameters that are being used for this particular calculation.
2. The code reads in each cycle count matrix  $(n_{rs})_q$  and adjusts it to an equivalent annual cycle count. The operational and the parked cycle count matrices are adjusted to an annual average using the time parameter  $\Delta t_q$  and the fraction of the time,  $(P_V)_q$ , at which the turbine operates in the wind velocity interval  $V_{q-1}$  to  $V_q$ . For operational stresses, the time fraction is determined from the wind speed distribution. For the parked stresses, the time fraction is set explicitly by the operator. The start-stop cycle count matrices are adjusted to an annual average using the count parameters  $\eta_q$  and  $\hbar_q$ .
3. The nominal stress levels in each matrix are adjusted to the component stress levels (stress intensity factors) by using the appropriate "stress concentration factor"; i.e.,  $\gamma_f$  or  $\gamma_r$  and the range-to-amplitude or amplitude-to-range variable (see below).
4. The constitutive relation is interpolated to determine the damage rate per cycle at each adjusted stress (or stress intensity) state.
5. The damage rule and adjusted damage rate per cycle are used with the cycle count matrix to determine the annual average damage rate for each stress (or stress intensity) state in that cycle count matrix.
6. The damage rates are summed for all stress (or stress intensity) states in that cycle count matrix.
7. Steps 2 through 6 are repeated for each of the operational, parked and start-stop cycle count matrix.
8. Each damage rate is summed over the wind velocity interval from  $V_{ci}$  to  $V_{co}$  for the operational stress; for all velocities greater than  $V_{co}$  for the parked stresses; and for all start-stop stresses.
9. Fatigue calculation:  
The sum computed in step 8 is the total annual average damage rate for the turbine. This average damage rate is inversely proportional to the service lifetime of the component.

Fracture analysis:

- a. The sum computed in step 8 is the annual average growth rate for the current crack length. This crack growth rate defines the time for the crack to grow from  $a_{w-1}$  to  $a_w$ .
- b. Steps 2 through 8 are repeated for the each interval of crack growth.

- c. The time for the crack to grow across each interval is summed to determine the time required for the crack to grow from its initial length  $a_0$  to its specified final length  $a_f$ .

Thus, the code computes the triple sum described in Eq. 4 for the fatigue analysis and the quadruple sum in Eq. 9 for the fracture analysis. The particular order of calculations described above was chosen to facilitate archiving intermediate calculations for future reference.

## ANNUAL AVERAGE CYCLE COUNT

### Operational and Parked Cycle Count Matrices

The adjustment for the operational and parked cycle count matrices to the annual average cycle count is identical for both damage rules. In the procedure used by the LIFE2 code, the annual average time,  $(T_{aa})_q$ , that the turbine is subjected to a wind velocity interval  $V_{q-1}$  to  $V_q$  is determined first. This time equals:

$$(T_{aa})_q = T (P_V)_q \quad , \quad (32)$$

where  $T$  is the total time contained in one year and  $(P_V)_q$  is the annual average probability for this wind interval. For operational stress states, the later parameter is determined using the interpolating technique on  $C(V)$  described in Eqs. 20 and 21. For parked stress states,  $P_V$  is an input variable.

To determine the total number of stress cycles for one year of operation, each cycle count  $(n_{rs})_q$  is multiplied by  $T_{aa}$  and divided by  $\Delta t_q$  (the time length for the cycle count matrix). Thus, the annual average cycle count for a operational and parked stress cycle count matrices,  $(\hat{n}_{rs})_q$

$$(\hat{n}_{rs})_q = \frac{(T_{aa})_q}{\Delta t_q} (n_{rs})_q = (P_V)_q \frac{T}{\Delta t_q} (n_{rs})_q \quad . \quad (33)$$

The implementation of this relation requires that the units of  $\Delta t_q$  be known. *The LIFE2 code assumes that  $\Delta t_q$  has the units of seconds.* Thus,  $T = 365.25 \text{ day/yr} \cdot 24 \text{ hr/day} \cdot 60 \text{ min/hr} \cdot 60 \text{ sec/min} = 31,557,600 \text{ sec}$ .

### Start-Stop Cycle Count Matrices

The adjustment for each start-stop cycle count matrix is the equivalent relationship:

$$(\hat{n}_{rs})_q = \frac{\hat{n}_q}{\eta_q} (n_{rs})_q \quad . \quad (34)$$

## ALTERNATING STRESS PARAMETER

As discussed above, the alternating stress may be described as either a range variable or an amplitude variable. During the initial set-up of both damage calculations, the operator is given the option of changing the operational stresses, parked stresses and start-stop stresses between the two. The range/amplitude multiplier  $\gamma_A$  is set equal to 1.0 for no change, to 0.5 for changing from range to amplitude, or to 2.0 for changing from amplitude to range. This variable is then incorporated into the damage calculation as describe in Eqs. 36 and 48.

## OPERATIONAL WIND SPEEDS

The damage calculations for the operational stress states, described below in Eqs. 38 and 50, are evaluated for each operational-stress cycle count matrix. If one operational stress matrix does not correspond to the cut-in wind speed  $V_{ci}$ , another to the cut-out wind speed  $V_{co}$ , then the linear interpolation is used to determine the damage rates at these end points.

This numerical technique for calculating the end points of these summations was chosen to permit the damage rates for all defined stress matrices to be examined by the operator. However, this choice can produce significant numerical errors in the damage calculation if the stress matrices cover relatively large velocity increments. *To alleviate this problem, the stress matrices entered by the operator should have relatively small velocity increments when compared to the cut-in wind speed.*

## FATIGUE ANALYSIS

The S-n fatigue analysis module in the LIFE2 code uses Miner's Rule<sup>4</sup> to describe the initiation, growth and coalescence of micro-cracks into macro-cracks.

As described above, the cycle count matrices are read into operational RAM, one at a time, and then adjusted to the annual average cycle count, see Eqs. 32 and 33. At this time the nominal stress levels for each cycle count are adjusted to the component stress levels using the stress concentration factor  $\gamma_f$  and the range/amplitude factor  $\gamma_A$ .

### Stress Concentration Factor

The LIFE2 code offers two options for changing the nominal stresses to the component stresses using the stress concentration factor  $\gamma_f$ . The choices are (1) applying  $\gamma_f$  to only the alternating stress; and (2) applying  $\gamma_f$  to both the alternating and mean stress.

For the latter case, the nominal mean stresses  $(S_m)_r$  in each cycle count matrix (i.e., all operational stress matrices, all parked stress matrices, and all start-stop matrices) is



multiplied by the fatigue stress concentration factor  $\gamma_f$  to obtain the component mean stress levels  $(\hat{S}_m)_s$ ; namely,

$$(\hat{S}_m)_r \Big|_q = \gamma_f (S_m)_r \Big|_q \quad (35)$$

And for both cases, the nominal alternating stresses  $(S_a)_s$  in each matrix is multiplied by the same fatigue stress concentration factor  $\gamma_f$  and the range/amplitude multiplier  $\gamma_A$  to obtain the component stress levels  $(\hat{S}_a)_s$ ; namely,

$$(\hat{S}_a)_s \Big|_q = \gamma_A \gamma_f (S_a)_s \Big|_q \quad (36)$$

### Cycles to Failure

After the cycle count and the stress states in the cycle count matrices have been adjusted, the S-n fatigue characterization matrix (i.e., the fatigue constitutive relation matrix) is then interpolated to determine the number of cycles to failure at each stress state in the current cycle count matrix. Thus, the constitutive matrix is mapped into the cycle count matrix's stress states; namely,

$$N[(S_m)_k, (S_a)_l] \Rightarrow \hat{N}[(\hat{S}_m)_r, (\hat{S}_a)_s] \quad (37)$$

If all of the cycle count matrices have *exactly* the same stress states  $[(\hat{S}_m)_r, (\hat{S}_a)_s]$ , then this interpolation needs to be conducted only once. The code permits the operator to specify this class of operation. *The operator must be careful when using this option because the program does not verify that the stress states are equivalent.*

### Damage Calculation

With the completion of the mapping described in Eq. 37, the sum described in Eq. 4 can be obtained. As the program holds only one cycle count matrix in current memory at any one time, the summation is conducted in steps. First, the damage for each cycle count matrix is determined. Namely, for each operational, parked and start-stop matrix, the sum

$$D_q(T) = \sum_{r=1}^R \sum_{s=1}^S \frac{\hat{n}_{qrs}[(\hat{S}_m)_r, (\hat{S}_a)_s, T]}{\hat{N}_{rs}[(\hat{S}_m)_r, (\hat{S}_a)_s]} \quad (38)$$

is computed. Then, the average damage accumulated in one year  $D(T)$  is determined by summing over all of these operational conditions (i.e., operational stress states from  $V_{ci}$  to  $V_{co}$ , all parked stress states and all start-stop stress states) using the relation:

$$D(T) = \sum_{q=1}^Q D_q(T) \quad . \quad (39)$$

The predicted service lifetime in years is determined using  $D(T)$  and Eq. 3.

## FRACTURE ANALYSIS

The linear fracture mechanics analysis module in the LIFE2 code uses a "da/dn" analysis<sup>4</sup> to describe the propagation of a pre-existing macro-crack. As da/dn is a function of the current crack length, the calculation is divided into a finite number of steps. For each step, the crack length is taken to be a constant, and an average growth rate is determined. This average rate is then used to determine the time required for the crack to grow across that integration segment. The sum over all of the integration segments yields the time of growth for the crack from its initial size to its final size.

As described above, the cycle count matrices are read into operational RAM, one at a time, and then adjusted to the annual average cycle count, see Eqs. 32 and 33. At this time the nominal stress levels for each cycle count are mapped into the component stress intensity factors using the current crack length, the fracture stress concentration factor  $\gamma_f$  and the range/amplitude factor  $\gamma_A$ .

### Current Crack Length

As the crack growth rate da/dn is a function of the crack length, the current crack length must be known before the calculation can proceed. The numerical solution to this dependence of the integrand on the independent variable is to divide the calculation into a finite number of steps with the crack length held constant for each step. A logarithmic variation is used in the LIFE2 code for this series. For I equal to the number of intervals per decade, the crack length at the completion of step w,  $a_w$ , is given by:

$$\log_{10}(a_w) = \log_{10}(a_0) + \frac{w}{I} \quad . \quad (40)$$

The initial value of this series,  $a_0$ , is defined to be:

$$\log_{10}(a_0) = \text{AINT}[\log_{10}(a_i)] \quad , \quad (41)$$

where,  $\text{AINT}(\cdot)$  is the greatest integer function that truncates its argument to an integer. The final term in the series is chosen such that:

$$\log_{10}(a_w) = \text{AINT}[\log_{10}(a_f) + 1] \quad . \quad (42)$$

This choice for the initial and final values of the crack length used in the calculations is based on keeping the calculations as versatile as possible.

For each interval, the current crack size is taken to be at the center of interval:

$$\hat{a}_w = \frac{a_w + a_{w-1}}{2} \quad (43)$$

### Stress Intensity Factor

The stress intensity factor  $\kappa$  is a function of the stress and the current crack length. The relation between the three is:

$$\kappa = Y_o \sqrt{\pi \hat{a}_w} S \quad (44)$$

where  $Y_o$  is a geometric shape factor.  $Y_o$  may be a constant, or it may be a function of the crack length  $a$ .<sup>14,15</sup> At this time, the LIFE2 code only permits  $\kappa$  to depend upon  $\sqrt{a}$ . Therefore, the "fracture stress concentration factor"  $\gamma_r$  is defined to be:

$$\gamma_r = Y_o \sqrt{\pi} \quad (45)$$

And, the LIFE2 code maps the current stress states into the the stress intensity factors through the relation

$$\kappa = \gamma_r \sqrt{\hat{a}_w} S \quad (46)$$

The relations that are used to map the mean stresses and the alternating stresses into the mean and alternating stress intensity factors are:

$$(\kappa_m)_r \Big|_q = \gamma_r \sqrt{\hat{a}_w} (S_m)_r \Big|_q \quad (47)$$

and,

$$(\kappa_a)_s \Big|_q = \gamma_A \gamma_r \sqrt{\hat{a}_w} (S_a)_r \Big|_q \quad (48)$$

respectively.

## Crack Growth Rate

After the cycle count and the stress states in the cycle count matrices have been adjusted, the  $da/dn$  crack growth rate matrix (i.e., the fracture mechanics constitutive relation matrix) is then interpolated to determine the crack growth rate at each stress intensity state in the current cycle count matrix. Thus, the constitutive matrix is mapped into the crack growth rate matrix's stress intensity states:

$$\frac{da}{dn} [(\kappa_m)_k, (\kappa_a)_l] \Rightarrow \frac{\hat{da}}{dn} [(\kappa_m)_r, (\kappa_a)_s] \quad (49)$$

## Damage Calculation for the Current Crack Length

With the completion of the mapping described in Eq. 49, the sum described in Eq. 9 can be obtained. The summation is conducted in steps, because the program holds only one cycle count matrix in current memory at any one time. First, the extension of crack for each cycle count matrix is determined for each operational, parked and start-stop matrix by computing the sum

$$\Delta a_{wq}(T) = \sum_{r=1}^R \sum_{s=1}^S \frac{\hat{da}_{wrs}(\kappa_m, \kappa_a)}{dn} \hat{n}_{wqrs}[(\kappa_m)_{wr}, (\kappa_a)_{ws}, V_q, T] \quad (50)$$

Then, the total extension of the crack for one year  $\Delta a_w(T)$  is determined by summing over the operational conditions (i.e., operational stress states from  $V_{ci}$  to  $V_{co}$ , all parked stress states and all start-stop stress states) using the relation

$$\Delta a_w(T) = \sum_{q=1}^Q \Delta a_{wq}(T) \quad (51)$$

As this crack extension calculation assumes that the current crack length is growing from  $a_{w-1}$  to  $a_w$ , then the time in years for the crack across this interval is given by:

$$\Delta t_w = \frac{a_w - a_{w-1}}{\Delta a_w(T)} \quad (52)$$

Thus, the total time, in years, required for the crack to grow from  $a_i$  to  $a_f$  is the sum of the these times. If  $a_i$  does not correspond to  $a_0$  and  $a_f$  to  $a_w$ , see Eqs. 41 and 42, then linear interpolation is used to determine the initial and the final time steps in this series.

## **SUMMARY STATEMENT**

The damage calculations performed by the LIFE2 code use the tabular input from the four other modules to determine the service lifetime of a turbine component. Two damage rules are currently available. The first rule uses a classical S-n fatigue analysis that is based on Miner's Rule<sup>4</sup> to sum the damage over all of the imposed stress states. This formulation is used to describe the initiation of micro-cracks and their growth and coalescence into macro-cracks. The second damage rule uses a linear fracture mechanics analysis<sup>4</sup> to predict the growth of a pre-existing macro-crack from its initial size to its final size.

## **SUMMARY**

The LIFE2 computer code is a fatigue/fracture analysis code specifically designed for the analysis of wind turbine components. This paper describes the numerical formulations that have been implemented in the code. It is meant to be the companion paper for the user's manual that is published under separate cover<sup>7</sup>.

From its inception, this code has been designed for a PC or compatible computer. This decision has required that the numerical implementation of the analysis techniques used here be formulated to fit on this comparatively small machine. Extensive overlays and read/write statements have been implemented in the code to permit the effective use of available operational RAM. And, as this class of machine has comparatively few significant digits, numerical algorithms had to be chosen carefully to prevent large numerical round-off errors from invalidating the results of the analysis.

The code is written in a top-down modular format and data transfer between modules is handled using data transfer files. This open architecture was chosen here because it permits the code to be modified and subroutines to be added to it with relative ease and because it greatly simplifies the validation of the modified code. An additional benefit of this formulation is that calculations are easily archived for future reference. However, this formulation does present a significant problem in that it requires the extensive use of tabular data and interpolation schemes. Thus, the accuracy of the calculations is decreased. Numerical techniques are employed in the code to insure that the decreased accuracy does not become significant.

Two constitutive formulations are currently available in the code. The first uses a conventional S-n fatigue analysis and the second uses linear da/dN fracture mechanics analysis. The former is used to describe the initiation, growth and coalescence of micro-cracks into macro-cracks and the latter is used to describe the growth of macro-cracks.

## **ACKNOWLEDGMENTS**

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# APPENDIX

## FORMAN CONSTITUTIVE MODEL

The constitutive equation proposed by Forman<sup>5,11</sup> is presented in Eq. 24 of this report. In his tabulation of materials<sup>11</sup>, Forman presents the formulation based on a different set of material parameters (see Table A-I for a list of this parameter set). The implementation of the latter set of parameters into the LIFE2 code is described here.

For the LIFE2 code, the fracture constitutive formulation requires that  $da/dn$  be written as a function of the mean stress intensity factor  $\kappa_m$  and the alternating stress intensity factor  $\kappa_a$ . As Eq. 24 is written in terms of the stress ratio  $R$ , each set of the stress parameters were mapped into their equivalent  $R$  value using:

$$R = \frac{\kappa_m - [\kappa_a / 2]}{\kappa_m + [\kappa_a / 2]} \quad (A-1)$$

Note that this formulation *assumes that  $\kappa_a$  is a range variable* as is the convention for this class of constitutive relations.

To simplify the numerical implementation of this constitutive formulation, Eq. 24 is simplified into three regions:  $R < 0$ ,  $R = 0$  and  $R > 0$ . For  $R = 0$ , Eq. 24 becomes:

$$\frac{da}{dn} = \frac{C \Delta \kappa^\nu [\Delta \kappa - \Delta \kappa_{th}]^\eta}{[\kappa_c - \Delta \kappa]^\psi} \quad (A-2)$$

In terms of Forman's input parameter set:

$$\kappa_c = \kappa_{Ic} \left[ 1 + B_k \exp(-w) \right] \quad (A-3)$$

$$w = \left[ A_k \frac{t}{t_o} \right]^2 \quad (A-4)$$

$$t_o = 2.5 \left[ \frac{\kappa_{Ic}}{\sigma_y} \right]^2 \quad (A-5)$$

and,

$$\Delta\kappa_{th} = \Delta\kappa_o \quad (A-6)$$

In these relations,  $\sigma_y$  is the yield strength of the material and  $t$  is the thickness of the component being analyzed.

For non-zero values of  $R$ , define the following to be:

$$A0 = (0.085 - 0.34 \alpha_F + 0.05 \alpha_F^2) \cos \left[ \frac{\pi [S_{max}/\sigma_o]}{2} \right]^{1/\alpha_F}, \quad (A-7)$$

$$A1 = (0.415 - 0.071 \alpha_F) [S_{max}/\sigma_o], \quad (A-8)$$

$$A3 = 2 A0 + A1 - 1.0, \quad (A-9)$$

and,

$$A2 = 1.0 - A0 - A1 - A3, \quad (A-10)$$

where  $\alpha_F$  is a Forman input variable and  $[S_{max}/\sigma_o]$  is a material variable that is assumed to be equal to 0.3.

Then, for  $R < 0$  define:

$$[S_o/S_{max}] = A0 + A1 R, \quad (A-11)$$

$$[S_o/S_{max}]_R = A0, \quad (A-12)$$

and,

$$[\Delta\kappa]_{eff} = [\Delta\kappa]_1 \frac{[S_o/S_{max}]}{[S_o/S_{max}]_R} (1 - R). \quad (A-13)$$

The crack growth rate for  $R < 0$  is then given by:

$$\frac{da}{dn} = \frac{C (\Delta\kappa)_{eff}^\nu [(\Delta\kappa)_{eff} - \Delta\kappa_{th}]^\eta}{\left[ \kappa_c - (\Delta\kappa)_{eff} \right]^\psi}. \quad (A-14)$$

For  $R > 0$ , define,

$$[S_o/S_{\max}] = \max\{ R, A_0 + A_1 R + A_2 R^2 + A_3 R^3 \} , \quad (A-15)$$

where  $\max(\cdot)$  chooses the maximum argument,

$$\Delta\kappa_{th} = \left[ 1 - C_o R \right]^d \Delta\kappa_o , \quad (A-16)$$

and,  $(\Delta\kappa)_{eff}$  using the definition given in Eq. A-13.

Then, the crack growth rate for  $R > 0$  is given by the equivalent formulation of Eq. A-14, namely:

$$\frac{da}{dn} = \frac{C (\Delta\kappa)_{eff}^\nu [(\Delta\kappa)_{eff} - \Delta\kappa_{th}]^\eta}{\left[ \kappa_c - (\Delta\kappa)_{eff} \right]^\psi} \quad (A-17)$$

Table A-I cross references the symbols used in this report with the symbols used by Forman in Ref. 11. *The LIFE2 code asks for these constitutive constants using the symbols that were used by Forman in Ref. 11.*

**TABLE A-I. Cross-Reference for the Forman Constitutive Constants.**

Forman Symbols	LIFE2 Symbols	Forman Symbols	LIFE2 Symbols	Forman Symbols	LIFE2 Symbols
YS	$\sigma_y$	C	C	Co	$C_o$
KIo	$\kappa_{Io}$	n	$\nu$	d	d
KIc	$K_{Ic}$	p	$\eta$	DK1	$\Delta\kappa_1$
Ak	$A_k$	q	$\psi$	$\alpha$	$\alpha_F$
Bk	$B_k$	DKo	$\Delta\kappa_o$		

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