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MASTER

Mixing Angles and CP Violation in SU(2)$_L$ × U(1) Gauge Model

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Abstract

Expressions for the mixing parameters are obtained in terms of mass ratios in the standard Weinberg-Salam model with permutation symmetry S_3 for six quarks. The CP violating phase δ is taken into account and there are no arbitrary parameters except for the quark masses. In the lowest order, the angles defined by Kobayashi-Maskawa are $\sin\theta_1 \approx (m_d/m_d + m_s)^{\frac{1}{2}}$, $\sin\theta_2 \approx -[(m_s/m_b)^2 - (m_c/m_t)^2]^{\frac{1}{2}}$, and $\sin\theta_3 = (m_s/m_b)^2/\cos\theta_1$. The model can be consistent with the observed magnitude of CP violation. The b-quarks decay predominantly into c-quarks with lifetime of $\tau_b \approx 10^{-11}$ sec.

DISCLAIMER

Recently, several authors have computed the mixing angles in gauge models for six quarks in terms of quark mass ratios.[1-7] The $SU(2)_L \times SU(2)_R \times U(1)$ gauge models are usually supplemented by discrete symmetries and left-right symmetry[2-5] whereas the standard Weinberg-Salam $SU(2)_L \times U(1)$ gauge models[8] are supplemented by permutation symmetry[7] and sometimes also by an additional discrete symmetry.[6] Our purpose is to supplement the $SU(2)_L \times U(1)$ gauge model with permutation symmetry S_3, a simple extension that enables one to compute all three mixing angles. The CP violating phase is taken into account.

In the present model the quark mass matrices are generated by a quark Higgs-Yukawa interaction,

$$\mathcal{L} = \sum_{i,j,k=1,2,3} [g^k_{ij}(\bar{Q}_{iL}\phi_k n_{jR}) + h^k_{ij}(\bar{Q}_{iL}\tilde{\phi}_k p_{jR})], \qquad (1)$$

where Q_{iL} represent the quark doublets $Q_{1L} = (u_o, d_o)_L$, $Q_{2L} = (c_o, s_o)_L$, $Q_{3L} = (t_o, b_o)_L$, and n_{jR} and p_{iR} represent the quark singlets $n_{jR} = (d_o, s_o, b_o)_R$ and $p_{iR} = (u_o, c_o, t_o)_R$. The coupling constants g^k_{ij} and h^k_{ij} are taken to be real. Three Higgs fields (ϕ_1, ϕ_2, ϕ_3) with vacuum expectation values (v_1, v_2, v_3) are required to implement S_3 symmetry, and $\tilde{\phi}_k = i\sigma_2\phi^\dagger_k$.

Let each combination $\bar{Q}_{iL}n_{jR}$ couple to only one Higgs field and the Higgs field that couples to $\bar{Q}_{1L}n_{1R}$ is defined to be ϕ_1. The application of S_3 symmetry that we use is the permutations $12 \to 21$, $13 \to 31$, and $23 \to 32$. This leads to only three types of down quark mass matrices m^o_2 (and also similarly for the up quark mass matrix, m^o_1) after spontaneous symmetry breakdown, depending on whether the term $\bar{Q}_{1L}n_{2R}$ is coupled to ϕ_1, ϕ_2, or ϕ_3. For the case ϕ_2, we obtain the transpose of the mass matrix m^o_1. The quark mass terms take the form

$$\sum_{i=1,2} \bar{\psi}^o_{iL} m^o_i \psi^o_{iR} + \text{h.c.},$$

where $\psi^o_1 = (u_o, c_o, t_o)$, $\psi^o_2 = (d_o, s_o, b_o)$,

$$m_2^o = \begin{pmatrix} fv_1 & gv_3 & gv_2 \\ gv_3 & fv_2 & gv_1 \\ gv_2 & gv_1 & fv_3 \end{pmatrix} \quad , \quad m_1^o = \begin{pmatrix} kv_1 & jv_1 & jv_1 \\ jv_2 & kv_2 & jv_2 \\ jv_3 & jv_3 & kv_3 \end{pmatrix} . \qquad (2)$$

The alternative assignment of the mass matrices, namely, taking the down quark mass matrix to be m_1^o and the up quark mass matrix to be m_2^o does not lead to interesting physical results. We adjust the parameters in the Higgs potential so that $v_1 = 0$ and divide all matrix elements as well as the quark masses by v_3 and obtain

$$m_1^o = \begin{pmatrix} 0 & 0 & 0 \\ jre^{-i\delta} & kre^{-i\delta} & jre^{-i\delta} \\ j & j & k \end{pmatrix} , \quad m_2^o = \begin{pmatrix} 0 & g & gre^{-i\delta} \\ g & fre^{-i\delta} & 0 \\ gre^{-i\delta} & 0 & f \end{pmatrix} , \qquad (3)$$

where $v_2/v_3 = re^{-i\delta}$. Since m_1^o and m_2^o are not hermitian, we diagonalize the square of the mass matrices $m_1^o m_1^{o\dagger}$ and $m_2^o m_2^{o\dagger}$. For the case of $m_1^o m_1^{o\dagger}$, we find that when $v_1 = 0$, $m_u = 0$. Further, when $m_u = 0$, $v_1 = 0$ is a solution. The results presented below survive when v_1 is treated as small but non-zero. We obtain from (3)

$$m_1^o m_1^{o\dagger} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (k^2+2j^2)r^2 & (2kj+j^2)re^{-i\delta} \\ 0 & (2kj+j^2)re^{i\delta} & (k^2+2j^2) \end{pmatrix} ,$$

$$m_2^o m_2^{o\dagger} = \begin{pmatrix} g^2(1+r^2) & fgre^{i\delta} & fgre^{-i\delta} \\ fgre^{-i\delta} & f^2r^2+g^2 & g^2re^{i\delta} \\ fgre^{i\delta} & g^2re^{-i\delta} & f^2+g^2r^2 \end{pmatrix} . \qquad (4)$$

The characteristic equation for $m_1^o m_1^{o\dagger}$ with eigenvalues $(0, m_c^2, m_t^2)$ yields

$$m_t^2(1+q^2)/(1+r^2) = k^2+2j^2, \quad q = m_c/m_t,$$

$$m_t^2[(r^2-q^2)(1-r^2q^2)]^{\frac{1}{2}}/(1+r^2) = r(2kj+j^2), \tag{5}$$

and for $m_2^o m_2^{o\dagger}$ with eigenvalues (m_d^2, m_s^2, m_b^2) yields

$$m_d^2+m_s^2+m_b^2 = (f^2+2g^2)(1+r^2),$$

$$m_d^2m_s^2+m_s^2m_b^2+m_d^2m_b^2 = g^2(2f^2+g^2) + r^2(f^4+2g^4) + g^2r^4(2f^2+g^2),$$

$$m_d^2m_s^2m_b^2 = f^2g^4(1+2r^3\cos 3\delta+r^6). \tag{6}$$

The mass matrices squared given by (4) are approximately diagonalized by the following unitary transformations[9] $\psi_1^o = U_1\psi_1$, $\psi_2^o = U_2\psi_2$

$$U_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -se^{i\delta} & ce^{i\delta} \end{pmatrix},$$

$$U_2 = \begin{pmatrix} e^{i\delta}\cos\theta & e^{i\delta}\sin\theta & e^{i\delta}\tan\theta(m_s/m_b)^2 \\ -\sin\theta & \cos\theta & 0 \\ -e^{2i\delta}\sin\theta(m_s/m_b)^2 & -e^{2i\delta}\sin\theta\tan\theta(m_s/m_b)^2 & e^{2i\delta} \end{pmatrix}, \tag{7}$$

where

$$c = [(1-q^2r^2)/(1+r^2)(1-q^2)]^{\frac{1}{2}}, \quad s = [(r^2-q^2)/(1+r^2)(1-q^2)]^{\frac{1}{2}},$$

$$\sin\theta = (m_d/m_s+m_d)^{\frac{1}{2}}. \tag{8}$$

The charged current coupled to physical quarks are $J_{\mu L} = \bar{\psi}^o_{1L}\gamma_\mu\psi^o_{2L} = \bar{\psi}_{1L}\Gamma\gamma_\mu\psi_{2L}$ where a phase transformation $\psi_{1L} \to P_1\psi_{1L}$ is introduced to bring Γ to the form

$$\Gamma = P_1^\dagger U_1^\dagger U_2 =$$

$$\begin{pmatrix} \cos\theta & \sin\theta & \tan\theta(m_s/m_b)^2 \\ -c\sin\theta + s\sin\theta e^{i\delta}(m_s/m_b)^2 & c\cos\theta + s\sin\theta\tan\theta e^{i\delta}(m_s/m_b)^2 & -se^{i\delta} \\ -s\sin\theta - c\sin\theta e^{i\delta}(m_s/m_b)^2 & s\cos\theta - c\sin\theta\tan\theta e^{i\delta}(m_s/m_b)^2 & ce^{i\delta} \end{pmatrix}, \quad (9)$$

where

$$P_1 = \begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

This Γ can be identified with the Kobayashi-Maskawa[10] form

$$\Gamma = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & -c_1s_2c_3 - c_2s_3e^{i\delta} & -c_1s_2s_3 + c_2c_3e^{i\delta} \end{pmatrix}, \quad (10)$$

where $c_1 = \cos\theta_1$, $s_1 = \sin\theta_1$, etc., θ_1, θ_2, and θ_3 are the weak mixing angles. Comparison of (9) and (10) yields

$$s_1 = \sin\theta = (m_d/m_s + m_d)^{\frac{1}{2}}, \quad s_2 \cong -[s + c(m_s/m_b)^2e^{i\delta}], \quad s_3 \cong (m_s/m_b)^2/\cos\theta. \quad (11)$$

If we require the mixing angles to be real when $\delta = 0$ we obtain $r \geq q$ which leads to

$$m_tm_s > m_bm_c = (4.75\ \text{GeV}) \times (1.5\ \text{GeV}) = 7.1(\text{GeV})^2 \quad (12)$$

or $m_t \geq 24$ GeV for $m_s = 0.3$ GeV. For definiteness, we choose $m_t = 25$ GeV in which case s_2 of (11) is $|s_2| \approx s = 1.97 \times 10^{-2}$.

The CP violation in $K^o - \bar{K}^o$ system is proportional to $\mathrm{Im}(\Gamma_{31}^* \Gamma_{32})$ and is the ratio of the imaginary part of the off-diagonal element in the $K^o - \bar{K}^o$ mass matrix and the $K_L - K_S$ mass difference. It is expressible in terms of the mixing angles and δ by[11]

$$|\varepsilon| = 2|c_1 c_2 c_3 s_2 s_3 \sin\delta| [c_2^2 (m_t^2/m_t^2 - m_c^2) \ell n (m_t^2/m_c^2) - c_2^2 + s_2^2 (m_t^2/m_c^2)] . \quad (13)$$

We obtain from Eqs.(11) and (13)

$$|\varepsilon| \cong 2(m_s/m_b)^2 [(m_s/m_b)^2 - (m_c/m_t)^2]^{\frac{1}{2}} [\ell n (m_t^2/m_c^2) + (m_s m_t/m_b m_c)^2 - 2] \sin\delta .$$

The value of $|\varepsilon| \approx 0.7 \times 10^{-3} \sin\delta$ is consistent with that obtained in Ref. 6.

We note from (8) and (9) that $\Gamma(b \to u + X)/\Gamma(b \to c + X) = \tan^2\theta (m_s/m_b)^4/s^2 \cong 10^{-3}$ so that b-quarks decay predominantly into c-quarks. The b-quark lifetime τ_b can be obtained from $\tau_b = \tau_\mu (m_\mu/m_b)^5/s^2$, where $\tau_\mu = 2.2 \times 10^{-6}$ sec is the muon lifetime of mass m_μ. One obtains $\tau_b \cong 10^{-11}$ sec for $m_t = 25$ GeV. When m_t is increased, $|\varepsilon|$ increases and τ_b decreases.

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