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MATHEMATICAL METHODOLOGY FOR EVALUATING SIMULATIONS OF FLOW IN POROUS MEDIA

Final Report, February 1, 1980—August 31, 1983

By
Wayne T. Ford
Ronald M. Anderson

January 1984
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Texas Tech University
Lubbock, Texas

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MATHEMATICAL METHODOLOGY FOR EVALUATING SIMULATIONS OF FLOW IN POROUS MEDIA

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I. ABSTRACT

This report concerns development of a methodology for evaluation of reservoir simulators. Although validity and usefulness of reservoir simulations might be questioned on various grounds, this study involves tests of resolution of numerical computational difficulties and of compatibility of correlations used in representation of fluid properties and phase flow interference.

A similarity concept is used which allows testing of simulators to be based on solution of a system of ordinary differential equations, referred to as a Similarity Porous Media System (SPMS), rather than on the usual system of partial differential equations, referred to as a Porous Media System (PMS). The SPMS can be solved with the same computational effort as would be required for a single time step in the usual application of the PMS itself.

The SPMS consists of study of possible level curves, also called similarity curves, on which all the unknowns in the PMS are constant. The curves can be chosen to be either a family of coaxial, common vertex parabolas or of parallel lines for evaluation of a simulator restricted to a linear reservoir. Moreover, the SPMS can be used, on a family of parabolas, for a simulator restricted to porous media problems with circular symmetry.

Simulation of an actual reservoir problem imposes field conditions on a simulator's solution of the PMS, but use of the similarity concept in evaluation of a simulator involves imposition of selected samples from a solution of the SPMS as conditions on a solution of the PMS by the simulator. Then, this solution of the PMS can be checked against a solution predicted by the solution of the SPMS. Examples have been tested and data is available on request.

II. INTRODUCTION

A. Project Background

1. Practical Motivation

Simulators consisting of computer software designed to numerically predict fluid flow in porous media are referred to as reservoir simulators or simply as simulators within this report. Since such simulators could also be used for study of flow of water and air in aquifers, porous medium or medium is used to include both applications. This report concerns development of a methodology for evaluation of the mathematical and computational capabilities of simulators offered for use in simulation of performance of field tests of enhanced oil recovery from hydrocarbon reservoirs.

Validity and usefulness of reservoir simulations in enhanced oil recovery might be questioned with respect to representation of reservoir heterogeneities, completeness of the physical laws used in formulation of a simulator, etc. This study, however, involves tests of resolution of numerical computational difficulties and of compatibility and accuracy of correlations used in representation of fluid properties and phase flow interference such as might be used in computing relative permeabilities and capillary pressures.

This project involves usage of a similarity concept for testing of numerical difficulties as well as possible incompatibilities of fluid and flow correlations. This similarity concept allows such testing to be based on the solution of a system of ordinary differential equations, referred to as a Similarity Porous Media System (SPMS), rather than on the usual system of partial differential

equations, referred to as a Porous Media System (PMS). The SPMS can be solved with the same computational effort as would be required for a single time step in the usual application of the PMS itself.

2. Project History

Faculty and students at Texas Tech University have been interested in the uses of mathematics in petroleum production and processing for some years. A list of our related publications and presentations is appended to this report. Some of this list's entries also appear in the list of cited references* for convenience.

Our interests became directed toward study of accuracy of reservoir simulators in prediction of field results in enhanced oil recovery. It was found that new mathematical descriptions were needed, and an effort was made to satisfy this need [6]*. Since the system of partial differential equations in the PMS is too complicated for analysis at the present time, work was then directed toward development of usable processes for testing solutions of the PMS [7]*. This led to formulation of the similarity concept for evaluation of simulators, which is the basis for the work reported here. Outline of progress beyond this basis requires some schematic detail below.

Figure 1 is a schematic representation of an (x,t) domain wherein a simulator of a linear porous medium must evaluate the various unknowns of a PMS. The figure shows a coarse grid of a type typical of a finite-difference method of solution of a PMS. However, the grid and the nodes at intersection points in the grid are for discussion and do not imply any necessity for use of finite-differences in simulation of a PMS.

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* Cited references are indicated by number in brackets.

Figure 2 shows superposition of several curves on a copy of Figure 1 with node marks removed. The curves are coaxial, common vertex parabolas. The SPMS consists of a flow problem which restricts all unknowns to constancy on each curve of a family such as that partially illustrated in Figure 2. In other words, the curves of the family are level curves, also called similarity curves, for all the unknowns in the PMS, and solution of the SPMS involves solution of the PMS as a unidimensional movement from curve to curve rather than as a bidimensional areal problem as suggested by Figure 1.

The original proposal leading to this report did not include the knowledge that the similarity curves in Figure 1 could consist of an arbitrary family of parallel lines. It also did not include the fact that the entire similarity approach applies to porous media problems with circular symmetry, referred to below as radial similarity applied in a radial medium.

3. Boundary Conditions

Simulation of an actual field reservoir problem and study of the validity of a reservoir simulator are quite different with respect to the fundamental nature of their boundary and initial conditions. Simulation of an actual problem imposes field physical conditions on a solution of the PMS by the simulator. Evaluation of a simulator by use of the similarity concept involves imposition of selected samples from a solution of the SPMS as conditions on a solution of the PMS by the simulator. Then, this solution of the PMS can be checked against a solution predicted by the solution of the SPMS.

The first initial-boundary value problem for the PMS consists of its solution on the rectangle, where $0 < x < X$ and $0 < t < T$, subject to specified initial data at zero time and boundary data at the ends, $x = 0$ and $x = X$, of the

medium, represented in Figure 1 by its solid base and its solid sides, respectively. This specified data, initial and boundary conditions, would be given by measurements in actual simulation of a field application and the results would be compared with field tests. Discussion of initial and boundary conditions is part of the introduction of this report to emphasize that this project seeks a methodology for evaluation of the capabilities of simulators which might be available or offered by someone. In other words, simulation of field tests is not part of this project.

B. Report Format

Chapters, tagged with Roman numerals, are partitioned into sections which are tagged with Latin letters. Formula numbers are consecutive within each section and include the Latin letter of the section but not the Roman numeral of the chapter. A Roman numeral appears in a formula reference only when the reference and the referenced formula are in different chapters. Thus, (III.C.1) here would refer to the PMS given in Section C of Chapter III, while (C.1) here would refer to the PMS just below in Section C.

C. Flow Equations

1. Basic Formulation

The mathematical formulation of the system of partial differential equations describing two-phase, immiscible, compressible, fluid flow in a homogeneous porous medium can be written as a PMS of the form,

$$\frac{\partial(\phi\rho_i S_i)}{\partial\tau} = \frac{\partial}{\partial x}\left(k\,\frac{k_i\rho_i}{\mu_i}\,\frac{\partial P_i}{\partial x}\right) \quad , \qquad \text{(C.1a)}$$

for the linear horizontal medium, and

$$\frac{\partial(\phi\rho_i S_i)}{\partial\tau} = \frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,k\,\frac{k_i\rho_i}{\mu_i}\,\frac{\partial P_i}{\partial r}\right) \quad , \qquad \text{(C.1b)}$$

for the radial medium with radial symmetry. The subscript, i, used in (C.1), takes on the values, 1 and 2, in all of its appearances throughout this report.

2. Time Scales

Usage of τ for time in (C.1) anticipates use of the constancy of k and Φ implied by the assumed homogeneity of the porous medium in the change of time scale given by

$$\tau = \Phi t/k \quad \text{or} \quad t = \tau k/\Phi \quad , \tag{C.2}$$

which transforms (C.1) into a PMS of the form,

$$\frac{\partial(\rho_i S_i)}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k_i \rho_i}{\mu_i}\frac{\partial P_i}{\partial x}\right) \quad , \tag{C.3a}$$

for the linear medium, and

$$\frac{\partial(\rho_i S_i)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{k_i \rho_i}{\mu_i}\frac{\partial P_i}{\partial r}\right) \quad , \tag{C.3b}$$

for the radial medium.

D. Similarity Principles

The similarity concept, as used in this project, requires definition of a coordinate transformation, from (x,t) to (ξ,η) for the linear medium or from (r,t) to (ξ,η) for the radial medium, wherein we assume that all dependent variables are to be independent of η. This assumption transforms both the linear and radial medium to the same SPMS as given by the ordinary differential equations,

$$\frac{d}{d\xi}\left(\frac{k_i \rho_i}{\mu_i}\frac{dP_i}{d\xi}\right) + F(\xi)\,\frac{k_i \rho_i}{\mu_i}\frac{dP_i}{d\xi} + G(\xi)\,\frac{d(\rho_i S_i)}{d\xi} = 0 \quad , \tag{D.1}$$

although the conditions on $F(\xi)$ and $G(\xi)$ differ for the two media.

E. Application

This project has developed conditions on $F(\xi)$ and $G(\xi)$ in (D.1) and solutions satisfying them so that the SPMS, as given in (D.1), can be used for evaluation and validation of reservoir simulators in several ways, two of which have been specifically identified and tested as part of the project. They can be briefly described as follows:

1. Various methods of computer representation and/or smoothing of density, viscosity, relative permeability, and capillary data can be used in the SPMS for the purpose of comparison of difficulties in solution of the ordinary differential equations defining the SPMS.

2. A SPMS can be solved and its solution can be used to impose initial-boundary conditions on a PMS, which can then be solved by an existing simulator using these imposed conditions. Since the inverse coordinate transformation, (ξ,η) to (x,t) or (r,t), predicts the values the PMS simulator should obtain under the imposed initial-boundary conditions, the PMS simulator can be checked for errors as well as for inefficiencies.

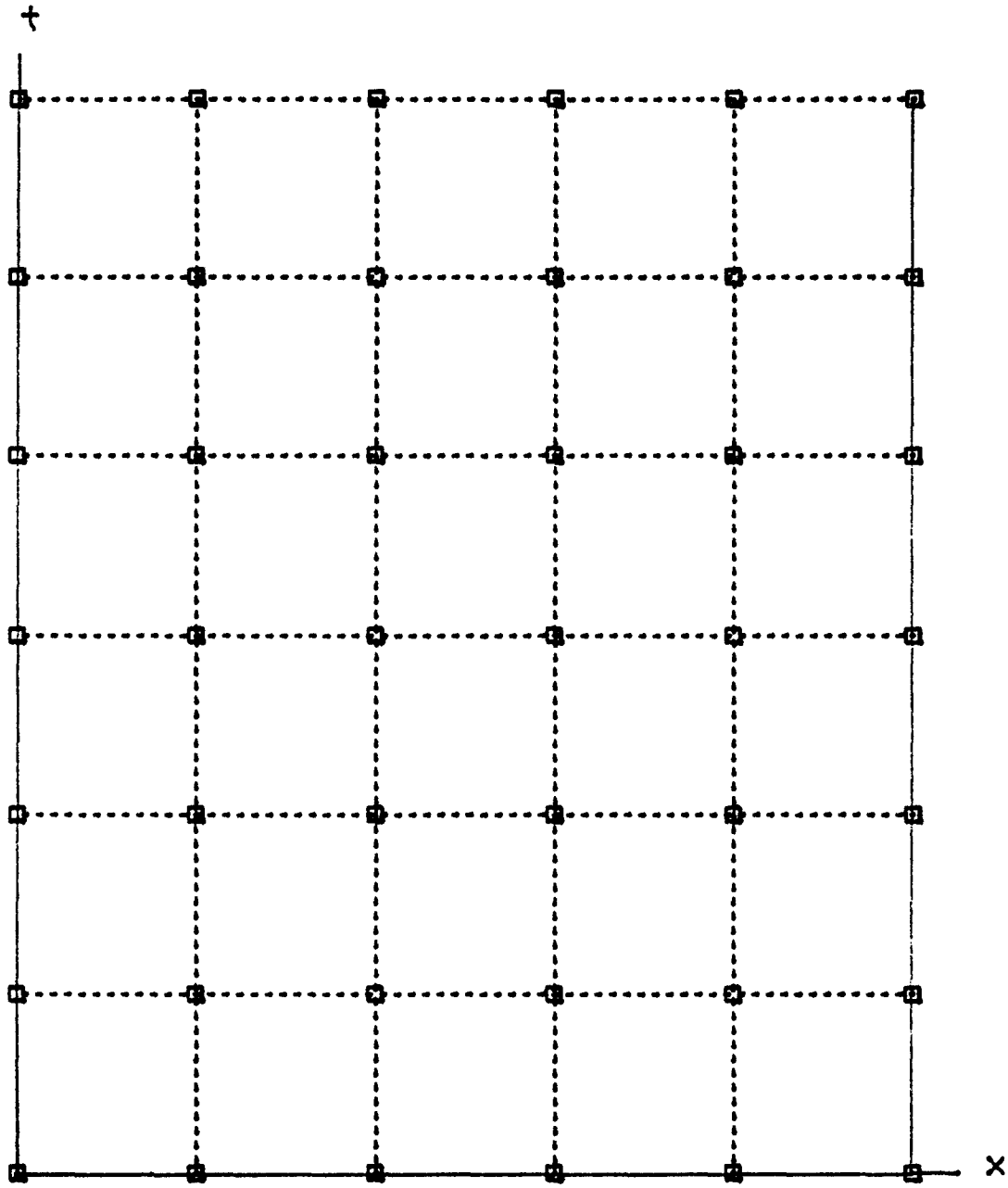

Figure 1: A Finite-Difference (x,t) Domain

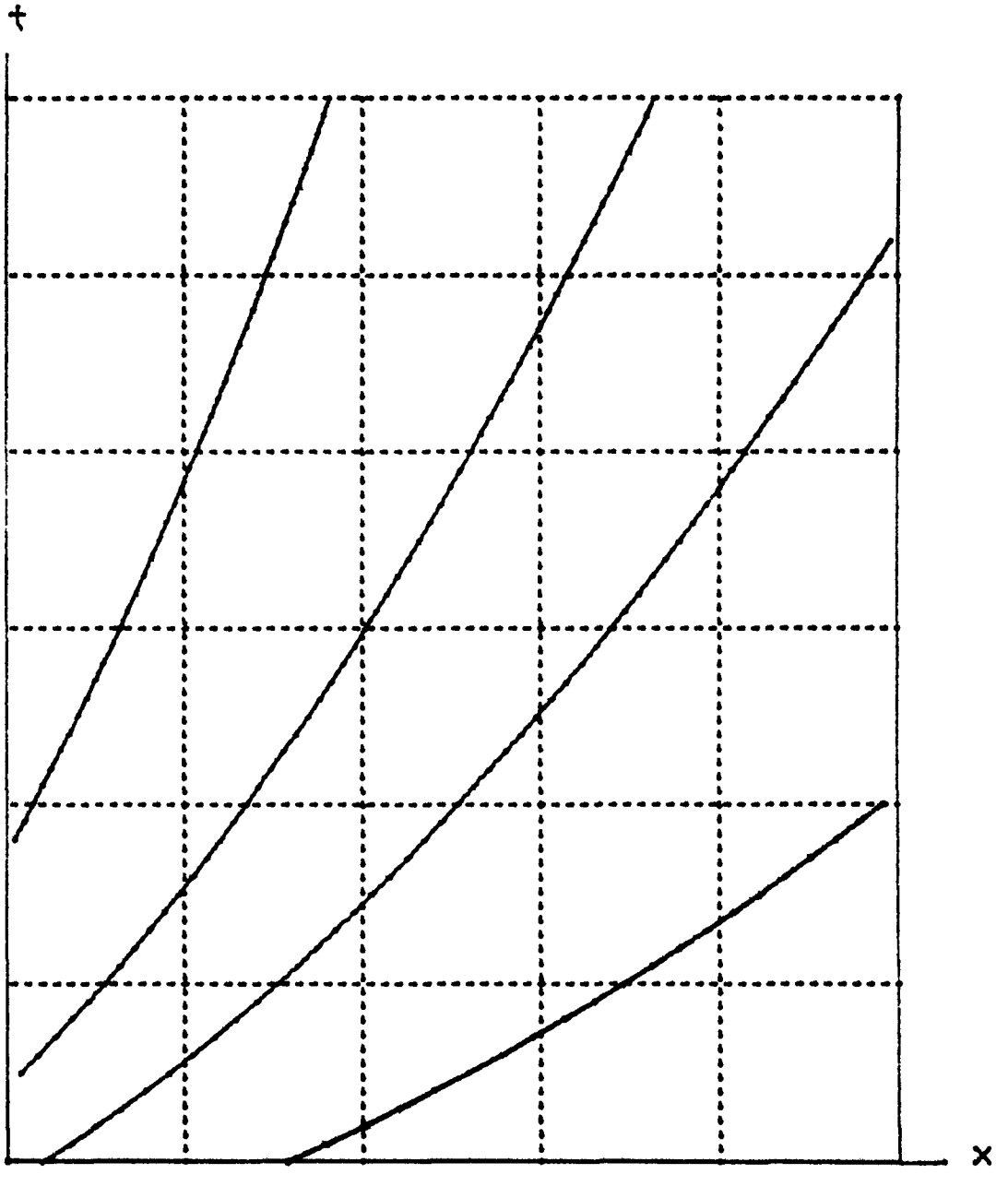

Figure 2: Typical Coaxial, Common Vertex Parabolas

III. MOTIVATION AND TERMINOLOGY

A. Conventional Nomenclature

Engineering convention requires that introduction of the name of a physical quantity implies a particular choice of a mathematical symbol corresponding to that quantity. For example, pressure, density, and temperature are denoted by P, ρ, and T, respectively. This report conforms to the standard notation of the Society of Petroleum Engineers except as noted in the text.

Since certain quantities are generally understood to be related to other quantities, qualifiers must be included if the general situation is to be restricted. For example, we use the adjective, isothermal, to convey the idea that temperature is constant throughout this report.

Certain adjectives convey corresponding restrictions on the partial differential equations of fluid flow in porous media, or on fluid properties and on relationships between fluids. In particular, "source-free" means that the equations will contain no additive undifferentiated terms, "horizontal" means that gravity plays no role in them, and "laminar" means that they will be based on Darcy's Law and its generalizations alone. Similarly, "immiscible" implies more than one phase pressure.

B. Darcy's Law

The differential form of Darcy's Law states that source-free, laminar, isothermal flow of a compressible fluid in a linear, horizontal, porous medium is described by the nonlinear partial differential equation,

$$\frac{\partial[\Phi(x,\tau)\rho(P)]}{\partial\tau} = \frac{\partial}{\partial x}\left[k(x,\tau)\ \frac{\rho(P)}{\mu(P)}\ \frac{\partial P}{\partial x}\right] \quad , \qquad \text{(B.1)}$$

where the porosity of the medium is Φ, its effective permeability is k, and the dynamic viscosity of the fluid is μ.

Since Φ and k are constants for the homogeneous media considered in this report, we will generally ignore their possible dependence on (x,τ). Moreover, it is frequently convenient to suppress arguments. Thus, (B.1) becomes

$$\frac{\partial(\Phi\rho)}{\partial\tau} = \frac{\partial}{\partial x}\left[k\ \frac{\rho}{\mu}\ \frac{\partial P}{\partial x}\right] \quad . \qquad \text{(B.2a)}$$

Darcy's Law has been generalized by empirical analogy to higher space dimensions. Source-free laminar isothermal flow of a compressible fluid in an areal horizontal porous medium is described by an extension of (B.2a) to the form,

$$\frac{\partial(\Phi\rho)}{\partial\tau} = \frac{\partial}{\partial x}\left[k\ \frac{\rho}{\mu}\ \frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[k\ \frac{\rho}{\mu}\ \frac{\partial P}{\partial y}\right] \quad . \qquad \text{(B.2b)}$$

Use of our fundamental similarity concept in two dimensions will require the assumption of radial circular symmetry, and transformation of space variables from (x,y) to (r,θ) causes (B.2b) to become

$$\frac{\partial(\Phi\rho)}{\partial\tau} = \frac{1}{r}\ \frac{\partial}{\partial r}\left[r\ k\ \frac{\rho}{\mu}\ \frac{\partial P}{\partial r}\right] \quad , \qquad \text{(B.2c)}$$

which is verified below in Section D.

Different basic equations of flow describe the linear medium in (B.2a) and the radial medium in (B.2c). Certain formulas, however, are identical in form for both types of media. In this is so, the symbol, s, may be used to be interpreted as applicable to both r and x.

C. Immiscible Fluids

Darcy's Law has also been empirically generalized to

describe source-free, laminar, isothermal flow of two immiscible fluid phases in a horizontal, homogeneous, porous medium as the system of partial differential equations in the PMS given by

$$\frac{\partial(\phi\rho_i S_i)}{\partial\tau} = \frac{\partial}{\partial x}\left(k\,\frac{k_i\rho_i}{\mu_i}\,\frac{\partial P_i}{\partial x}\right) \qquad \text{(C.1a)}$$

for the linear medium, and by

$$\frac{\partial(\phi\rho_i S_i)}{\partial\tau} = \frac{1}{r}\,\frac{\partial}{\partial r}\left(r\;k\,\frac{k_i\rho_i}{\mu_i}\,\frac{\partial P_i}{\partial r}\right) \qquad \text{(C.1b)}$$

for the radial medium, wherein the subscript, i, takes on the values, 1 and 2, above as well as in all of its usages throughout this report, k_1 and k_2 are phase relative permeabilities, and S_1 and S_2 are phase saturations satisfying

$$S_1 + S_2 = 1 \quad . \qquad \text{(C.1c)}$$

Each immiscible fluid has its own phase pressure P_i, density ρ_i, viscosity μ_i, and saturation S_i. Densities and viscosities are assumed to be given as correlations in terms of their respective pressures.

It is conventional that the capillary pressure P_c is given as an empirical correlation of the form,

$$P_c(S_1) = P_1 - P_2 \qquad \text{(C.2)}$$

Moreover, each relative permeability k_i is assumed to be correlated in terms of its respective saturation S_i.

The assumed homogeneity of the porous medium motivates replacement of the use of τ for time, in (B.1), (B.2), and (C.1), by the use of t, as given in (II.C.2), through the balance of this report. Also, frequent use of the product, $\rho_i S_i$, and mathematical convenience suggest the notation,

$$R_i \equiv \rho_i S_i \quad . \qquad \text{(C.3)}$$

The PMS in (C.1) then becomes

$$\frac{\partial R_i}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k_i \rho_i}{\mu_i}\frac{\partial P_i}{\partial x}\right) \qquad (C.4a)$$

for the linear medium, and

$$\frac{\partial R_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{k_i \rho_i}{\mu_i}\frac{\partial P_i}{\partial r}\right) \qquad (C.4b)$$

for the radial medium, and the SPMS in (II.D.1) becomes

$$\frac{d}{d\xi}\left(\frac{k_i \rho_i}{\mu_i}\frac{dP_i}{d\xi}\right) + F(\xi)\,\frac{k_i \rho_i}{\mu_i}\frac{dP_i}{d\xi} + G(\xi)\,\frac{dR_i}{d\xi} = 0 \quad . \qquad (C.5)$$

The balance of this report is based on (C.4) as the PMS and on (C.5) as the SPMS.

D. Mathematical Appendix (Chapter III)

1. Circular Symmetry

Verification that circular symmetry transforms a porous media equation such as that in (B.2b) to a form such as that in (B.2c) begins with the definition of a linear differential operator by the equation,

$$L(U) = = \frac{\partial}{\partial x}\left(M(r)\,\frac{\partial U}{\partial x}\right) + \frac{\partial}{\partial y}\left(M(r)\,\frac{\partial U}{\partial y}\right) \quad , \qquad (D.1)$$

where

$$r^2 = x^2 + y^2 \quad \text{and} \quad \theta = \tan^{-1}(y/x) \qquad (D.2a)$$

below. Thus,

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta \quad . \qquad (D.2b)$$

Denote partial differentiation by subscripts to obtain

$$\theta_x = -y/r^2 = -\sin\theta / r \quad , \quad r_x = x/r = \cos\theta \quad , \qquad (D.3a)$$

$$\theta_y = x/r^2 = \cos\theta / r \quad \text{and} \quad r_y = y/r = \sin\theta \quad . \qquad (D.3b)$$

If V is a function which is independent of θ, then

$$V_x = r_x V_r = (x/r) V_r = V_r \cos\theta \quad , \qquad (D.4a)$$

and

$$V_y = r_y V_r = (y/r) V_r = V_r \sin\theta \quad . \qquad (D.4b)$$

Assume U to be independent of θ in (D.1) and compute

$$
\begin{aligned}
L(U) &= [M(r)\cos\theta\, U_r]_x + [M(r)\sin\theta\, U_r]_y \qquad \text{(D.5)}\\
&= M(r)U_r(\cos\theta)_x + \cos^2\theta\,[M(r)\,U_r]_r\\
&+ M(r)U_r(\sin\theta)_y + \sin^2\theta\,[M(r)\,U_r]_r\\
&= M(r)U_r\,[-\theta_x\sin\theta + \theta_y\cos\theta] + [M(r)\,U_r]_r\\
&= M(r)U_r\,[\sin^2\theta + \cos^2\theta]\,/\,r + [M(r)\,U_r]_r\\
&= M(r)U_r\,/\,r + [M(r)\,U_r]_r \quad .
\end{aligned}
$$

Thus,

$$
L(U) = \frac{1}{r}\,\frac{\partial}{\partial r}\left[r\,M(r)\,\frac{\partial U}{\partial r}\right] \quad . \qquad \text{(D.6)}
$$

IV. SYSTEM FORMULATIONS

A. Classification of Variables

As a motivational example for this Chapter, take viscosity to be a unit constant, and apply the change of scale in (II.C.2) to write the single phase problem in (III.B.2a) in the form,

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial x}\left(\rho \frac{\partial P}{\partial x}\right) \quad , \quad \mu = 1 \quad . \tag{A.1a}$$

Since (A.1a) is a single equation in the two unknowns, ρ and P, auxiliary information is required. For illustration, assume a slightly compressible fluid with density expressed in the correlation,

$$\rho = \alpha \, e^{\beta P} \quad \text{so that} \quad \frac{d\rho}{dP} = \beta \rho \ . \tag{A.1b}$$

Although (A.1a) can be put in either of the forms,

$$\frac{\partial \rho}{\partial t} = \frac{1}{\beta} \frac{\partial}{\partial x}\left(\frac{\partial \rho}{\partial x}\right) \quad \text{or} \quad \frac{\partial P}{\partial t} = \frac{1}{\beta} e^{-\beta P} \frac{\partial}{\partial x}\left(e^{\beta P} \frac{\partial P}{\partial x}\right) \quad , \tag{A.2}$$

the linearity of the first one implies that it should be chosen and solved for ρ. Then, (A.1b) can be used to calculate P.

The independent variables are x and t in (A.1). Although the dependent variables are ρ and P, they have different natures in that one of them is the solution of the chosen equation from (A.2) while the other is to be calculated from that solution.

Solutions of a system of differential equations, or a single equation as in the example chosen from (A.2), will be referred to as integrated variables, while quantities

calculated from them will be referred to as algebraic variables. A choice of the first equation in (A.2) implies that ρ is the integrated variable and P is the algebraic variable.

B. Porous Media Systems

The form of (III.C.4) suggests that pressures might be the integrated variables for a PMS. Our original proposal suggested that densities might be the integrated variables for the PMS which would require (III.C.4) to be in the form,

$$\frac{\partial R_i}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k_i \rho_i}{\mu_i}\frac{dP_i}{d\rho_i}\frac{\partial \rho_i}{\partial x}\right) \qquad (B.1a)$$

and

$$\frac{\partial R_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{k_i \rho_i}{\mu_i}\frac{dP_i}{d\rho_i}\frac{\partial \rho_i}{\partial r}\right) \quad . \qquad (B.1b)$$

Another useful formulation, which was considered in a recent publication [2], can be based on either

$$\Psi_i = \int^{\rho_i} \frac{\rho_i}{\mu_i}\frac{dP_i}{d\rho_i}\,d\rho_i \quad \text{or} \quad \Psi_i = \int^{P_i} \frac{\rho_i}{\mu_i}\,dP_i \qquad (B.2)$$

to transform (III.C.4) to

$$\frac{\partial R_i}{\partial t} = \frac{\partial}{\partial x}\left(k_i\frac{\partial \Psi_i}{\partial x}\right) \qquad (B.3a)$$

and

$$\frac{\partial R_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r k_i\frac{\partial \Psi_i}{\partial r}\right) \quad . \qquad (B.3b)$$

Whether a representation of the PMS is based on either (III.C.4), (B.1), or (B.3); it uses one of the equivalent forms in

$$\frac{k_i \rho_i}{\mu_i} \frac{\partial P_i}{\partial s} = k_i \frac{\partial \psi_i}{\partial s} = \frac{k_i \rho_i}{\mu_i} \frac{dP_i}{d\rho_i} \frac{\partial \rho_i}{\partial s} \quad , \tag{B.4}$$

where $s = x$ or $s = r$. Indeed, each such representation of the PMS implies a corresponding representation of the SPMS which involves one of these forms with partial derivatives replaced by ordinary ones, and $s = \xi$ or $s = \zeta$.

Since simulation models may use various choices of the integrated variables, such as P_a and S_w for an air-water case, each form in (B.4) should be written as a product of a diagonal matrix and a differentiated column vector, as illustrated by writing the respective forms in (B.4) as

$$\begin{bmatrix} \frac{k_1 \rho_1}{\mu_1} & 0 \\ 0 & \frac{k_2 \rho_2}{\mu_2} \end{bmatrix} \frac{d}{ds} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \frac{d}{ds} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} \tag{B.5}$$

$$= \begin{bmatrix} \frac{k_1 \rho_1}{\mu_1} \frac{dP_1}{d\rho_1} & 0 \\ 0 & \frac{k_2 \rho_2}{\mu_2} \frac{dP_2}{d\rho_2} \end{bmatrix} \frac{d}{ds} \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$

C. Changing Integrated Variables

We believe that a major tool for understanding fluid flow in porous media is obtained by denoting the integrated variables by new symbols, which are U_1 and U_2 for the problem considered in this report.

The matrices in (B.5) are related to each other through the partial derivatives of the functions defining each of the pairs, (P_1, P_2), (ψ_1, ψ_2), and (ρ_1, ρ_2), in terms of chosen integrated variables. Let (U_1, U_2) be the vector denoting the chosen integrated variables for the

PMS and the SPMS. Then, the discussion of Jacobians and Jacobian matrices in Section D allows the forms in (B.5) to become

$$K \frac{d\vec{\psi}}{d\vec{P}} \frac{d\vec{P}}{ds} = K \frac{d\vec{\psi}}{ds} = K \frac{d\vec{\psi}}{d\vec{\rho}} \frac{d\vec{\rho}}{ds} \quad , \tag{C.1}$$

where K is the diagonal matrix of relative permeabilities in the middle form in (B.5). Similarly, any PMS in (III.C.4a), (B.1a), or (B.3a) can be written in the form,

$$\frac{\partial \vec{R}}{\partial t} = \frac{\partial}{\partial x}\left[M(\vec{U}) \frac{\partial \vec{U}}{\partial x}\right] \quad , \tag{C.2a}$$

where M is a diagonal matrix chosen from (B.5) for use in the PMS in (III.C.4a), (B.1a), or (B.3a),respectively. The analogous formulation for the radial case is given by

$$\frac{\partial \vec{R}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r}\left[r\, M(\vec{U}) \frac{\partial \vec{U}}{\partial r}\right] \quad . \tag{C.2b}$$

D. Mathematical Appendix (Chapter IV)

1. Jacobian Matrices

The matrices in (B.5) are related to each other through the partial derivatives of the functions defining each of the pairs, (P_1, P_2), (ψ_1, ψ_2), and (ρ_1, ρ_2), in terms of chosen integrated variables. To provide details, let the vector (U_1, U_2) be the chosen integrated variables for the PMS and the SPMS; then the Jacobian matrix relating (U_1, U_2) to (P_1, P_2) is defined by

$$\frac{d\vec{U}}{d\vec{P}} \equiv \frac{d(U_1, U_2)}{d(P_1, P_2)} = \begin{bmatrix} \frac{\partial U_1}{\partial P_1} & \frac{\partial U_1}{\partial P_2} \\ \frac{\partial U_2}{\partial P_1} & \frac{\partial U_2}{\partial P_2} \end{bmatrix} \quad . \tag{D.1a}$$

The notation would perhaps be improved if $\partial(U_1, U_2)/\partial(P_1, P_2)$

were used for the above matrix, but unfortunately this is not possible because Jacobi used the latter notation for the determinant of the matrix in (D.1a). This determinant is now called the Jacobian of the coordinate transformation from (P_1,P_2) to (U_1,U_2). The matrix is called a Jacobian matrix in this report. The familiar result of Jacobi is that the determinant of a transformation and that of its inverse are reciprocal numbers, e.g.,

$$\frac{\partial(U_1,U_2)}{\partial(P_1,P_2)}\,\frac{\partial(P_1,P_2)}{\partial(U_1,U_2)} = 1 \quad . \tag{D.1b}$$

But it is also true that the respective Jacobian matrices are inverse to each other, e.g.,

$$\frac{d(U_1,U_2)}{d(P_1,P_2)}\,\frac{d(P_1,P_2)}{d(U_1,U_2)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad . \tag{D.1c}$$

The same concepts are intrinsic to a discussion of similarity transformations. The Jacobian matrix of the transformation from (s,t) to (ξ,η), is given by

$$\frac{d(\xi,\eta)}{d(s,t)} = \begin{pmatrix} \frac{\partial\xi}{\partial s} & \frac{\partial\xi}{\partial t} \\ \frac{\partial\eta}{\partial s} & \frac{\partial\eta}{\partial t} \end{pmatrix} = \begin{pmatrix} \xi_s & \xi_t \\ \eta_s & \eta_t \end{pmatrix} \quad , \tag{D.2a}$$

where the latter equality illustrates abbreviation of partial derivatives by use of subscripts. The inverse transformation, from (ξ,η) to (s,t), has the Jacobian matrix,

$$\frac{d(s,t)}{d(\xi,\eta)} = \begin{pmatrix} s_\xi & s_\eta \\ t_\xi & t_\eta \end{pmatrix} = \begin{pmatrix} \xi_s & \xi_t \\ \eta_s & \eta_t \end{pmatrix}^{-1} = \frac{1}{J}\begin{pmatrix} \eta_t & -\xi_t \\ -\eta_s & \xi_s \end{pmatrix} \quad , \tag{D.2b}$$

where J is the Jacobian in

$$J \equiv \frac{\partial(\xi,\eta)}{\partial(s,t)} = \begin{vmatrix} \xi_s & \xi_t \\ \eta_s & \eta_t \end{vmatrix} \quad . \tag{D.2c}$$

Equality of the components in the matrices obtained by

using (D.2b) to write

$$\begin{pmatrix} s_\xi J & s_\eta J \\ t_\xi J & t_\eta J \end{pmatrix} = \begin{pmatrix} \eta_t & -\xi_t \\ -\eta_s & \xi_s \end{pmatrix} \tag{D.3}$$

will be a basic tool in the solution of the constraints on the coefficients of the SPMS in Chapter V.

V. SIMILARITY

A. The Similarity Concept

1. The General SPMS

The similarity concept, as used in this project, requires definition of a coordinate transformation, from (x,t) to (ξ,η) or from (r,t) to (ξ,η), for the linear or for the radial model, respectively, wherein we assume that all dependent variables are to be independent of η. We shall see that this assumption transforms both PMS models in (III.C.4) to the same SPMS appearing in the system of ordinary differential equations,

$$\frac{d}{d\xi}\left(\frac{k_i \rho_i}{\mu}\frac{dP_i}{d\xi_i}\right) + F(\xi)\frac{k_i \rho_i}{\mu}\frac{dP_i}{d\xi_i} + G(\xi)\frac{dR_i}{d\xi} = 0 \quad , \qquad (A.1)$$

although the conditions on $F(\xi)$ and $G(\xi)$ differ for the two models. Specifically, validity of (A.1) requires ξ to be such that $F(\xi)$ and $G(\xi)$ exist satisfying the constraints,

$$\left(\frac{\partial\xi}{\partial x}\right)^2 F(\xi) = \frac{\partial}{\partial x}\left(\frac{\partial\xi}{\partial x}\right) \quad \text{and} \quad \left(\frac{\partial\xi}{\partial x}\right)^2 G(\xi) = -\frac{\partial\xi}{\partial t} \qquad (A.2a)$$

for the linear model, and

$$\left(\frac{\partial\xi}{\partial r}\right)^2 F(\xi) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\xi}{\partial r}\right) \quad \text{and} \quad \left(\frac{\partial\xi}{\partial r}\right)^2 G(\xi) = -\frac{\partial\xi}{\partial t} \quad , \qquad (A.2b)$$

for the radial model.

The mathematical nature of coordinate transformations is such that use of any part of (A.2) to express $F(\xi)$ or $G(\xi)$ as a ratio of partial derivatives is expected to result in a function of the independent variables, (x,t) or (r,t), rather than a function of ξ. Restriction of the

coordinate transformation to obtain validity of the notation, $F(\xi)$ and $G(\xi)$, is a key step in the use of the similarity concept.

Solution of the constraints in (A.2) leads to expression of the similarity variable, ξ, in the form,

$$\xi = f[Y(s,t)] \quad , \tag{A.3}$$

where $s = x$ for the linear model and $s = r$ for the radial model, the function, Y, is fixed by the constraints except for certain parameters, and the function, f, is arbitrary. In other words, the function, Y, defines the geometric form of the curves of constant ξ, while the function, f, defines values of constant ξ to be used as labels for the curves.

Figure 2 shows portions of four coaxial parabolas with common vertex at $(-3,-1)$ where the distance between dashed lines is one unit, both horizontally and vertically. If the labels for the parabolas in Figure 2 were 1, 2, 3, and 4 with $f(y) = y$, then the same curves would be labelled 1, 4, 9, and 16 with $f(y) = y^2$.

2. SPMS Formulations

Multiplication by the integrating factor,

$$H(\xi) \equiv \exp\left[\int^{\xi} F(\xi)\, d\xi \right] \quad , \tag{A.4a}$$

simplifies the SPMS in (A.1) to the form,

$$\frac{d}{d\xi}\left[H(\xi)\, \frac{k_i \rho_i}{\mu_i}\, \frac{dP_i}{d\xi} \right] + H(\xi) G(\xi)\, \frac{dR_i}{d\xi} = 0 \quad . \tag{A.4b}$$

Suppress the argument, ξ, and normalize (A.4b) in the form,

$$\frac{H}{G_1}\, \frac{d}{d\xi}\left[\frac{k_i \rho_i}{\mu_i}\, \frac{H}{G_1}\, \frac{dP_i}{d\xi} \right] + \frac{HG}{G_1}\, \frac{H}{G_1}\, \frac{dR_i}{d\xi} = 0 \quad , \quad G_1 \equiv G(1) \quad , \tag{A.5a}$$

to motivate definition of an independent variable, ζ, and functions, $Z(\xi)$ and $B(\zeta)$, by

$$\zeta = Z(\xi) = G_1 \int^{\xi} \frac{d\xi}{H(\xi)} \quad \text{and} \quad B(\zeta) = \frac{H(\xi)G(\xi)}{G_1} \quad . \qquad \text{(A.5b)}$$

Then, simplify the SPMS in (A.5a) to the form,

$$\frac{d}{d\zeta}\left(\frac{k_i \rho_i}{\mu_i} \frac{dP_i}{d\zeta}\right) + B(\zeta) \frac{dR_i}{d\zeta} = 0 \quad . \qquad \text{(A.5c)}$$

Calculations of F, G, H, ζ, and B are listed in Table 1 for simple choices of the arbitrary function, f, in (A.3).

The type of vector analysis leading to (IV.C.2) allows the SPMS in (III.C.5) to be written in the form,

$$\frac{d}{d\xi}\left(M(\vec{U}) \frac{d\vec{U}}{d\xi}\right) + F(\xi)\, M(\vec{U}) \frac{d\vec{U}}{d\xi} + G(\xi) \frac{d\vec{R}}{d\xi} = 0 \quad , \qquad \text{(A.6)}$$

which will turn out to be the form directly developed in application of the similarity concept to the PMS in (III.C.4).

B. The Linear Reservoir SPMS

1. Coefficient Constraints

It is convenient to suppress arrows on vectors to use the PMS for a linear medium from (IV.C.2a) in the form,

$$\frac{\partial R}{\partial t} = \frac{\partial}{\partial x}\left(M \frac{\partial U}{\partial x}\right) \quad . \qquad \text{(B.1)}$$

Similarity requires definition of a coordinate transformation, from (x,t) to (ξ,η), wherein we assume that all dependent variables are to be independent of η. This assumption transforms (B.1) as follows:

$$\frac{\partial R}{\partial t} = \frac{\partial \xi}{\partial t}\frac{\partial R}{\partial \xi} = \frac{\partial}{\partial x}\left(M \frac{\partial U}{\partial x}\right) = \frac{\partial}{\partial x}\left(M \frac{\partial \xi}{\partial x}\frac{\partial U}{\partial \xi}\right) \qquad \text{(B.2)}$$

$$= M \frac{\partial^2 \xi}{\partial x^2}\frac{\partial U}{\partial \xi} + \left(\frac{\partial \xi}{\partial x}\right)^2 \frac{\partial}{\partial \xi}\left(M \frac{\partial U}{\partial \xi}\right)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \xi}{\partial r}\right) M \frac{\partial U}{\partial \xi} + \left(\frac{\partial \xi}{\partial r}\right)^2 \frac{\partial}{\partial \xi}\left(M \frac{\partial U}{\partial \xi}\right) \quad .$$

which transforms (IV.C.2a) to the form shown in (A.6),

where (A.2a) constrains the coefficients, $F(\xi)$ and $G(\xi)$, for the linear model.

2. Similarity Curves

Development of the effects of (A.2a) on $F(\xi)$ and $G(\xi)$, for the linear medium, appears in a reprint of a published paper [2] which is appended to this report. The results are that

$$\xi = f[Y(x,t)] \quad \text{with} \quad Y(x,t) = xA(t) + B(t) \quad , \qquad (B.3)$$

where f is arbitrary. There are certain conditions on the A(t) and B(t) defining Y(x,t), which are modified from the published conditions by observing that f[Y] can be used to alter Y by a additive constant or by a constant multiplier. It follows that Y(x,t) is such that constants a and b exist such that either

$$A(t) = (t + b)^{-1/2} \quad \text{and} \quad B(t) = aA(t) \quad , \qquad (B.4a)$$

or

$$A(t) = a \neq 0 \quad \text{and} \quad B(t) = bt \quad . \qquad (B.4b)$$

Use of (B.3) and (B.4) to define the similarity variable, ξ, involves the alternative forms,

$$Y(x,t) = \frac{x + a}{(t + b)^{1/2}} \quad \text{or} \quad Y(x,t) = ax + bt \quad , \qquad (B.5)$$

to be used in (A.3). Full definition of ξ involves selection of the following: the parameters, a and b, in (B.5); the first or second form in (B.5); the arbitrary function, f, as shown in (A.3); and a range of values for the variable, ξ, itself. The alternative forms of the similarity curves are coaxial parabolas with common vertex at (-b,-a) or parallel lines corresponding to use of the first or second form in (B.5), respectively.

Table 1

Similarity Variables

Notation:

$A = A(t) \equiv (t+b)^{-1/2}$, $\frac{dA}{dt} = -\frac{1}{2}(t+b)^{-3/2} = -\frac{1}{2}A^3$

Variable	Alternative Formulations		
s	x	x	r
ξ	$as + bt$	$(s+a)A$	sA
$\frac{\partial \xi}{\partial t}$	b	$-\frac{1}{2}(s+a)A^3$	$-\frac{1}{2}sA^3$
$\frac{\partial \xi}{\partial s}$	a	A	A
$\frac{\partial^2 \xi}{\partial s^2}$	0	0	
$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial \xi}{\partial s}\right)$			$\frac{A}{s}$
$F(\xi)$	0	0	$\frac{1}{\xi}$
$G(\xi)$	$-\frac{b}{a^2}$	$\frac{\xi}{2}$	$\frac{\xi}{2}$
$H(\xi)$	1	1	ξ
ζ	$-\frac{b\xi}{a^2}$	$\frac{\xi}{2}$	$\frac{\ln \xi}{2}$
$B(\zeta)$	1	2ζ	$e^{4\zeta}$

C. The Radial Reservoir SPMS

1. Coefficient Constraints

It is convenient to suppress arrows on vectors to use the PMS for a radial medium from (IV.C.2b) in the form,

$$\frac{\partial R}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\ M\ \frac{\partial U}{\partial r}\right) \quad . \tag{C.1}$$

Similarity requires definition of a coordinate transformation, from (r,t) to (ξ,η), wherein we assume that all dependent variables are to be independent of η. This assumption transforms (C.1) as follows:

$$\begin{aligned}\frac{\partial R}{\partial t} &= \frac{\partial \xi}{\partial t}\frac{\partial R}{\partial \xi} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\ M\ \frac{\partial U}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\ M\ \frac{\partial \xi}{\partial r}\frac{\partial U}{\partial \xi}\right) \\ &= \frac{1}{r}\,M\,\frac{\partial \xi}{\partial r}\frac{\partial U}{\partial \xi} + M\,\frac{\partial^2 \xi}{\partial r^2}\frac{\partial U}{\partial \xi} + \left(\frac{\partial \xi}{\partial r}\right)^2\frac{\partial}{\partial \xi}\left(M\,\frac{\partial U}{\partial \xi}\right) \\ &= \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial \xi}{\partial r}\right) M\,\frac{\partial U}{\partial \xi} + \left(\frac{\partial \xi}{\partial r}\right)^2\frac{\partial}{\partial \xi}\left(M\,\frac{\partial U}{\partial \xi}\right) \quad .\end{aligned} \tag{C.2}$$

which transforms (IV.C.2b) to the form shown in (A.6), where (A.2b) constrains the coefficients, $F(\xi)$ and $G(\xi)$.

2. Similarity Curves

Development of the effects of (A.2b) on $F(\xi)$ and $G(\xi)$, for the radial medium, appears in a mathematical appendix in Section D of this Chapter. The results imply that the similarity variable, ξ, can be written in the form,

$$\xi = f[Y(r,t)] \quad \text{with} \quad Y(r,t) = \frac{r^2}{t+b} \ ; \tag{C.3}$$

where f is arbitrary, and b is constant. Full definition of ξ involves the following selections: the parameter, b, in (C.3); the arbitrary function, f, as shown in (A.3); and a range of values for the variable, ξ, itself. Regarding r as a typical space variable, the similarity curves are

coaxial parabolas with a common vertex at $(r,t) = (0,-b)$.

D. Mathematical Appendix (Chapter V)

1. Solving Radial Media Coefficient Constraints

Development of the effects of (A.2b) on $F(\xi)$ and $G(\xi)$, for the radial medium, begins with the use of H as defined in (A.4a) to express the first part of (A.2b) in the form,

$$\left[\frac{\partial\xi}{\partial r}\right] F(\xi) = \frac{\partial}{\partial r}\left[r \frac{\partial\xi}{\partial r}\right] \div \left[r \frac{\partial\xi}{\partial r}\right] , \qquad (D.1a)$$

which is equivalent to the form,

$$\frac{\partial}{\partial r}[\ell n\ H(\xi)] = \frac{\partial}{\partial r}\left[\ell n\left[r \frac{\partial\xi}{\partial r}\right]\right] . \qquad (D.1b)$$

Integration of (D.1b) yields

$$\ell n\ H(\xi) = \ell n(r\ \partial\xi/\partial r) - \ell n\ A(t) \qquad (D.2a)$$

or

$$A(t)H(\xi) = r\ \partial\xi/\partial r , \qquad (D.2b)$$

where $A(t)$ is an arbitrary positive constant of integration with respect to r. Thus,

$$\frac{A(t)}{r} = \frac{\partial}{\partial r}\left[Z(\xi)\right] \quad \text{where} \quad Z(\xi) \equiv \int^{\xi} \frac{d\xi}{H(\xi)} , \qquad (D.3a)$$

which implies

$$A(t)\ \ell n\ r + B(t) = Z(\xi) , \qquad (D.3b)$$

where $B(t)$ is an arbitrary constant of integration with respect to r.

Rewrite the first part of (D.3a) as

$$\frac{A}{r} = \frac{dZ}{d\xi} \frac{\partial\xi}{\partial r} \qquad (D.4a)$$

and differentiate (D.3b) with respect to t to obtain

$$\frac{dA}{dt}\ \ell n\ r + \frac{dB}{dt} = \frac{dZ}{d\xi} \frac{\partial\xi}{\partial t} . \qquad (D.4b)$$

Then, use (D.4) to eliminate the partial derivatives of ξ from the second part of (A.2b). The result is

$$-G(\xi)/Z'(\xi) = r^2A^{-2}(A'\ell n\ r + B') . \qquad (D.5)$$

Since the left side of (D.5) is independent of η, the derivative of the right side of (D.5), with respect to η,

must vanish. Thus,

$$r^2[(A''\ln r + B'')A^{-2} - 2(A'\ln r + B')A^{-3}A'] \partial t/\partial\eta + A^{-2}[r^2(A'/r) + 2r(A'\ln r + B')] \partial r/\partial\eta = 0 \quad , \qquad (D.6a)$$

and multiplication by A^3/r yields

$$r[(A''\ln r + B'')A - 2(A'\ln r + B')A'] \partial t/\partial\eta + A[A' + 2(A'\ln r + B')] \partial r/\partial\eta = 0 \quad . \qquad (D.6b)$$

Use the differential of the transformation from (r,t) to (ξ,η), by analogy with the linear medium [2, eqn. 2.8], to eliminate η-derivatives in favor of ξ-derivatives to obtain

$$r[(A''\ln r + B'')A - 2(A'\ln r + B')A'] \partial\xi/\partial r - A[A' + 2(A'\ln r + B')] \partial\xi/\partial t = 0 \quad . \qquad (D.6c)$$

Finally, multiply by $dZ/d\xi$, use (D.4) to eliminate all reference to ξ in (D.6c), and divide through by A to obtain

$$\begin{aligned} 0 &= (A''\ln r + B'')A - 2(A'\ln r + B')A' \\ &\quad - [A' + 2(A'\ln r + B')](A'\ln r + B') \\ &= (A''\ln r + B'')A - 3(A'\ln r + B')A' \\ &\qquad - 2(A'\ln r + B')^2 \\ &= (B''A - 3B'A' - 2B'^2) \\ &\quad + (A''A - 3A'^2 - 4A'B')\ln r - 2A'^2(\ln r)^2 \quad . \end{aligned} \qquad (D.7)$$

Validity of (D.7), for some interval in r, implies that A' must be zero. Thus, A(t) = a, for some constant a, and (D.7) reduces to

$$0 = aB'' - 2B'^2 \quad , \qquad (D.8)$$

which has the first integral,

$$2B' = -a/(b + t) \quad , \qquad (D.9)$$

where b is a constant. Thus,

$$B(t) = (a/2)\ln[c^2/(t + b)] \quad , \qquad (D.10)$$

where c is a constant.

Finally, use (D.10) in (D.3b) to obtain

$$\begin{aligned} Z(\xi) &= a\ln r + a\ln[c/(t + b)^{1/2}] \\ &= (a/2)\ln[r^2/(t + b)] + a\ln c \quad . \end{aligned} \qquad (D.11)$$

Since similarity deals with an arbitrary function of the $Z(\xi)$ in (D.11), no generality is lost by choosing a = 2 and c = 1.

VI. DEMONSTRATION PROGRAMS

A. Test Procedures

Three programs, referred to as the Texas Tech Porous Media Validation Demonstration Package, are appended to this report. The programs are designed to illustrate the steps involved in testing a user supplied model. A testing methodology and a description of the place of each program in the testing methodology are introduced in this Chapter. Further details needed for use of the programs are given in Chapter IX.

The development of a computer model for simulation of flow in a porous media involves two distinct aspects. The first involves the method used in describing the relations between the various physical quantities required for the model. The second aspect is the method involved in solving the system of partial differential equations associated with the model.

A procedure for testing a model should be capable of evaluating the correlations independent of the testing of the accuracy of the numerical method used in solving the partial differential equations in the model. Therefore, several steps must be implemented in order to properly carry out our testing procedure. The required steps are outlined below.

1. Compiling the Model

If the model is to be tested on a computer differing from that on which the model was developed, modification of the code may be required. For example, IBM FORTRAN and CDC FORTRAN have some minor differences.

2. Executing the model

The model developer should supply sample input and output data for the model. The program should be executed with the given input values and the results compared with the supplied output values. Results may differ due to differences in word length between the development computer and the computer used in testing. Also, changes in the computer code may inadvertently be introduced in moving from computer to computer and these changes may need to be redone.

3. Testing Correlations

The program SPMSTEST, to be described in Section C, is used for testing the methods of evaluation of the physical relationships involved in the model. SPMSTEST uses the model's correlations in the SPMS as a two-point boundary value problem. It can be run several times, as desired, to test the correlations over a wide range of parameter values independent of the model's method of solution of the actual PMS. This allows timing and solvability analysis to be carried out for the correlations.

4. Testing the Model

The testing of a given model's method of solution of a PMS begins with a merger of the model with the method of solution of a SPMS as found in SPMSTEST. The combined code, called PMSTEST in the appended programming, performs a number of tasks. First, a SPMS is solved. Then, its solution is interpolated to impose initial and boundary conditions on a PMS. Next, the resulting PMS is solved using the model's methods. Finally, the solution of the SPMS is interpolated to the grid used in the solution of the PMS, and the two solutions are compared.

B. Model for Evaluation (MODEL)

This program was developed as a simple porous media model to be used in testing the porous media evaluation methodology. The model is based on a perfect gas and a slightly compressible fluid.

The program, MODEL, is a simplified mathematical model for flow in a unidimensional porous media. This model was developed strictly as a tool for illustrating the steps involved in testing a model. An outline of the mathematical formulation used in the model is given in Section E.

The main program, MAIN, serves as the driver for the model. It contains statements for reading physical and mathematical parameters for assigning initial and boundary values. It also contains a loop for calling the subroutine for the numerical solution of the partial differential equations and printing the results.

C. Similarity Porous Media System (SPMSTEST)

The program, SPMSTEST, is used to test the model's method of generation of pressures, saturations, and relative permeabilities. The mathematical formulas for these functions are obtained from the model to be tested and are contained in the functions, PRESA, PRESB, DPHI, DPSI, SIG, TAU, PC, and PCINV.

The program numerically solves the two-point boundary value problem for the Similarity Porous Media System (SPMS). The program accepts as input the method (parabolas or straight lines), the parameters for the correlations and boundary values for the densities U and V. The output of the program gives solution curves for the system together with information concerning the number of iterations required to obtain a solution.

The main program, MAIN, allows processing several sets of boundary conditions and altering the method for each set

of data. The control of the steps in solving the SPMS is carried out by the subprogram INITSP.

The SPMS is generally a stiff two-point boundary value system. Therefore, accurate estimates of the initial slopes of U and V at the left end point are needed to successfully carry out a solution by the method of shooting. This is accomplished by first finding a finite difference solution to the system and using the result to obtain a first estimate for the shooting technique.

D. Porous Media System (PMSTEST)

The program, PMSTEST, combines all the implications of the similarity concept in a single package. It solves the SPMS for given boundary data. This solution of the SPMS is then interpolated by use of cubic splines to impose initial and boundary values on the PMS. The resulting PMS is then solved by finite-differences, and this solution is compared to the values implied by interpolation of the solution of the SPMS.

E. Mathematical Notation

This project was originally proposed with the PMS and the SPMS formulated in terms of densities as the integrated variables. Moreover, the proposed work considered only the linear medium to the exclusion of any remarks on possible study of the radial medium. The enclosed programming was based on the original proposal.

The appended FORTRAN programming is formulated and names are chosen based on writing the PMS in (IV.B.1a) in the form,

$$\frac{\partial(yu)}{\partial t} = \frac{\partial}{\partial x}\left[\sigma(y)\ \Phi'(u)\ \frac{\partial u}{\partial x}\right] \qquad \text{(E.1a)}$$

and

$$\frac{\partial(zv)}{\partial t} = \frac{\partial}{\partial x}\left[\tau(z)\ \Psi'(v)\ \frac{\partial v}{\partial x}\right] \quad ; \qquad \text{(E.1b)}$$

where u and v are densities, y and z are saturations, σ and τ are relative permeabilities, and (IV.B.2) has been used in the form,

$$\Phi'(u) = \frac{u}{\mu_1}\frac{dP_1}{du_1} \quad \text{and} \quad \Psi'(v) = \frac{v}{\mu_2}\frac{dP_2}{dv_2} \quad . \qquad \text{(E.1c)}$$

The notation in (E.1) leads to expression of the SPMS in (V.A.1) in the equations,

$$\frac{d}{d\xi}\left[\sigma\ \Phi'\ \frac{du}{d\xi}\right] + F(\xi)\ \sigma\ \Phi'\ \frac{du}{d\xi} + G(\xi)\ \frac{d(yu)}{d\xi} = 0 \qquad \text{(E.2a)}$$

and

$$\frac{d}{d\xi}\left[\tau\ \Psi'\ \frac{dv}{d\xi}\right] + F(\xi)\ \tau\ \Psi'\ \frac{dv}{d\xi} + G(\xi)\ \frac{d(zv)}{d\xi} = 0 \quad , \qquad \text{(E.2b)}$$

as the SPMS corresponding to the PMS in (E.1). Of course, the integrating factor in (V.A.4a) can be used, as it was used in producing (V.A.4b), to transform (E.2) to the forms,

$$\frac{d}{d\xi}\left[H(\xi)\ \sigma\ \Phi'\ \frac{du}{d\xi}\right] + H(\xi)G(\xi)\ \frac{d(yu)}{d\xi} = 0 \qquad \text{(E.3a)}$$

and

$$\frac{d}{d\xi}\left[H(\xi)\ \tau\ \Psi'\ \frac{dv}{d\xi}\right] + H(\xi)G(\xi)\ \frac{d(zv)}{d\xi} = 0 \quad . \qquad \text{(E.3b)}$$

Suppressing arguments in G and H, add [yu d(HG)/dξ] to both sides of (E.3a) to obtain

$$\frac{d}{d\xi}\left[H\ \left[\sigma\ \Phi'\ \frac{du}{d\xi} + G\ yu\right]\right] = yu\ \frac{d(HG)}{d\xi} \quad , \qquad \text{(E.4a)}$$

and add [zv d(HG)/dξ] to both sides of (E.3b) to obtain

$$\frac{d}{d\xi}\left[H\ \left[\tau\ \Psi'\ \frac{dv}{d\xi} + G\ zv\right]\right] = zv\ \frac{d(HG)}{d\xi} \quad , \qquad \text{(E.4b)}$$

which is exactly the form used for the SPMS in the appended programming.

F. Mathematical Appendix (Chapter VI)

1. Prototypic Correlations

Application of the similarity concept in evaluation of a simulator of flow in porous media has been programmed and tested for inclusion in this report. Formulation of usable correlations is an essential part of a complete example for testing. This Section is a discussion of the correlations which were found to be mathematically convenient to be used in the programming included in this report.

The correlations in this Section would be replaced by those from an independent simulator to be evaluated. Thus, the discussion below involves temporary correlations for testing of the similarity concept.

Actual computer representations relating capillary pressure to a saturation, phase relative permeabilities to respective phase saturations, and densities and viscosities to pressures require careful attention so error messages and termination of execution do not result without physical necessity. One view is that it is a mistake to continually test for physically impossible values throughout a computer simulation. This view dictated the design of the functions used as correlations in the enclosed programming.

2. Capillary Pressure

Immiscibility implies consideration of more than one pressure in formulations of the PMS. It is conventional to correlate the difference between P_1 and P_2, for a two-phase problem, in terms of one saturation as shown in (III.C.2). A generalized formulation of capillary pressure in terms of saturation, which increases with increasing saturation and which can be inverted to solve for the saturation based on any value of capillary pressure, is given by

$$\frac{dP_c}{dS_1} = \sum_{n=1}^{\infty} \left[\frac{a_n}{(1 - S_1)^n} + \frac{b_n}{S_1^n} \right] , \; a_n \geq 0 , \; b_n \geq 0 , \qquad (F.1)$$

where $a_n b_n > 0$ for at least one value of n. It should be sufficient, however, to test the similarity method using only one term from (F.1). The enclosed programming assumes that this would be normal and includes a message which reports the exponent, n, in the denominators of the term defining dP_c/dS_1.

The example which is actually used in the appended computer programming consists of setting $a_n = b_n = 0$ for every n except that $a_1 = b_1 = 1$ to use (F.1) to obtain

$$\frac{dP_c}{dS_1} = \frac{1}{(1 - S_1)^2} + \frac{1}{S_1^2} \quad \text{and} \quad P_c = \frac{1}{1 - S_1} - \frac{1}{S_1} , \qquad (F.2a)$$

which can be inverted in the form,

$$S_1 = \frac{(4 + P_c^2)^{1/2} + P_c - 2}{2P_c} = \frac{2}{(4 + P_c^2)^{1/2} - P_c + 2} . \qquad (F.2b)$$

This equation can be used to obtain,

$$S_1 S_2 = S_1(1 - S_1) = \frac{1}{(4 + P_c^2)^{1/2} + 2} , \qquad (F.2c)$$

which shows that neither S_1 nor S_2 can become either zero or one from computations based on (F.2).

A second example is merely of mathematical interest and is not actually used in the enclosed programming. It consists of setting $a_n = b_n = 0$ for every n except that $a_1 = b_1 = 1/2$ to use (F.1) to obtain

$$\frac{dP_c}{dS_1} = \frac{1}{2(1 - S_1)} + \frac{1}{2S_1} \quad \text{and} \quad P = \frac{1}{2} \ell n \frac{1}{1 - S_1} , \qquad (F.3a)$$

which can be inverted in the form,

$$S_1 = \frac{1}{1 + e^{-2P_c}} = \frac{1}{2}\,(1 + \tanh P_c) \quad . \qquad \text{(F.3b)}$$

It follows that

$$S_2 = \frac{1}{1 + e^{2P_c}} = \frac{1}{2}\,(1 - \tanh P_c) \quad , \qquad \text{(F.3c)}$$

and neither S_1 nor S_2 can become either zero or one from computations based on (F.3).

3. Relative Permeabilities

Each relative permeability is calculated as the square of its respective phase saturation in the programming used in this report. At least this approach yields the expected positivity of derivatives and curvatures in both phases.

4. Fluid Properties

The appended programming uses fluid properties based on certain modifications of the normal formulations of a slightly compressible fluid and an ideal gas. The former can be defined by choices of constants, k and α, for use in the equations,

$$k\rho_1 = \exp(P_1/\alpha) \quad \text{or} \quad P_1 = \alpha\, \ell n(k\rho_1) \quad , \qquad \text{(F.4a)}$$

which leads to

$$\rho_1\, dP_1/d\rho_1 = \alpha \quad \text{so that} \quad \phi'(\rho_1) = \alpha/\mu_1 \quad . \qquad \text{(F.4b)}$$

Since an iterative estimate of ρ_1 might be negative even though further iterations should lead to convergence to a positive value, (F.4) is modified to the form,

$$k\rho_1 = \sinh(P_1/\alpha) \qquad \text{(F.5a)}$$

$$\text{or} \quad P_1 = \alpha\, \ell n\{k\rho_1 + [(k\rho_1)^2 + 1]^{1/2}\} \quad , \qquad \text{(F.5b)}$$

which leads to

$$\rho_1\, dP_1/d\rho_1 = \alpha\, k\rho_1 [(k\rho_1)^2 + 1]^{-1/2} = \alpha \tanh(P_1/\alpha) \quad , \qquad \text{(F.5c)}$$

where the latter equality is based on the identity,

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\sinh x}{(1 + \sinh^2 x)^{1/2}} \quad . \qquad (F.6)$$

Mathematical convenience in illustration of the methodology of the report motivates the definition of a constant, μ_1, in the assumption of the correlation,

$$\mu_1 = \mu_a \tanh(P_1/\alpha) \quad \text{so that} \quad \phi'(\rho_1) = \alpha/\mu_a \quad . \qquad (F.7)$$

The ideal gas is described by use of a constant, β, in the equation,

$$P_2 = \beta \rho_2 \qquad (F.8a)$$

which leads to

$$\rho_2 \, dP_2/d\rho_2 = \beta \rho_2 \quad \text{and} \quad \Psi'(\rho_2) = \beta \rho_2/\mu_2 \quad . \qquad (F.8b)$$

Mathematical convenience, similar to that leading to (F.7), motivates the use of a constant, μ_b, in the correlation,

$$\mu_2 = \mu_b \rho_2 \quad \text{so that} \quad \Psi'(\rho_2) = \beta/\mu_b \quad . \qquad (F.9)$$

VII. CONCLUSIONS

A. Summary

The mathematical formulation of the system of partial differential equations describing two-phase, immiscible, compressible, fluid flow in a homogeneous porous medium can be written as a PMS of the form,

$$\frac{\partial(\rho_i S_i)}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_i \rho_i}{\mu_i}\frac{\partial P_i}{\partial x}\right] \quad , \tag{A.1a}$$

for the linear medium, and

$$\frac{\partial(\rho_i S_i)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\,\frac{k_i \rho_i}{\mu_i}\frac{\partial P_i}{\partial r}\right] \quad , \tag{A.1b}$$

for the radial medium.

The similarity concept, as used in this project, requires definition of a coordinate transformation, from (x,t) to (ξ,η) for the linear medium or from (r,t) to (ξ,η) for the radial medium, wherein we assume that all dependent variables are to be independent of η. This assumption transforms both media to the same SPMS as given by the ordinary differential equations,

$$\frac{d}{d\xi}\left[\frac{k_i \rho_i}{\mu_i}\frac{dP_i}{d\xi}\right] + F(\xi)\,\frac{k_i \rho_i}{\mu_i}\frac{dP_i}{d\xi} + G(\xi)\,\frac{d(\rho_i S_i)}{d\xi} = 0 \quad , \tag{A.2}$$

although the conditions on $F(\xi)$ and $G(\xi)$ differ for the two media.

B. Application

This project has developed conditions on $F(\xi)$ and $G(\xi)$

in (A.1) and solutions satisfying them so that the SPMS, as given in (A.2), can be used for evaluation and validation of reservoir simulators in several ways, two of which have been specifically identified and tested as part of the project. They can be briefly described as follows:

1. Various methods of computer representation and/or smoothing of density, viscosity, relative permeability, and capillary data can be used in the SPMS for the purpose of comparison of difficulties in solution of the ordinary differential equations defining the SPMS.

2. A SPMS can be solved and its solution can be used to impose initial-boundary conditions on a PMS, which can then be solved by an existing simulator using these imposed conditions. Since the inverse coordinate transformation, (ξ,η) to (x,t) or (r,t), predicts the values the PMS simulator should obtain under the imposed initial-boundary conditions, the PMS simulator can be checked for errors as well as for inefficiencies.

VIII. FUTURE POSSIBILITIES

A. SPMS Test Solutions

Use the integrated variables, ψ_i, as in (IV.B.2) and parallel lines in the linear medium as formulated in Table 1 to write the SPMS in (V.A.5c) in the form,

$$\frac{d}{d\xi}\left[k_i \frac{d\psi_i}{d\xi}\right] + \frac{dR_i}{d\xi} = 0 \quad , \tag{A.1a}$$

which is actually an abbreviation of the form,

$$\frac{d}{d\xi}\left[k_i(\psi_1,\psi_2) \frac{d\psi_i}{d\xi}\right] + \frac{d}{d\xi} R_i(\psi_1,\psi_2) = 0 \quad . \tag{A.1b}$$

A first integral for (A.1a) is given by

$$k_i \frac{d\psi_i}{d\xi} = A_i - R_i \quad , \tag{A.2a}$$

where A_i is a constant of integration. Next, divide the version of (A.2a) with i = 1 by the version with i = 2 to obtain the differential equation,

$$\frac{k_1}{k_2} \frac{d\psi_1}{d\psi_2} = \frac{R_1 - A_1}{R_2 - A_2} \quad . \tag{A.2b}$$

Traditional study of an equation of the form in (A.2b) begins with its revision to the form,

$$(R_2 - A_2)k_1 \, d\psi_1 - (R_1 - A_1)k_2 \, d\psi_2 = 0 \quad . \tag{A.3}$$

Suppose (A.3) can be solved in the implicit form,

$$g(\psi_1,\psi_2;A_1,A_2)) = B_1 \quad . \tag{A.4a}$$

If (A.4a) can be solved for one ψ in terms of the other in the form

$$\psi_2 = G_2(\psi_1;A_1,A_2,B_1) \quad , \tag{A.4b}$$

then the notation in

$$\tilde{k}_1(\psi_1) \equiv k_1[\psi_1, G_2(\psi_1; A_1, A_2, B_1)] \qquad \text{(A.5a)}$$

and

$$\tilde{R}_1(\psi_1) \equiv R_1[\psi_1, G_2(\psi_1; A_1, A_2, B_1)] \quad , \qquad \text{(A.5b)}$$

can be used with i = 1 in (A.2a) to imply

$$\tilde{k}_1(\psi_1) \frac{d\psi_1}{d\xi} = A_1 - \tilde{R}_1(\psi_1) \quad . \qquad \text{(A.5c)}$$

Write the general solution of (A.5c) in the form,

$$\psi_1(\xi) = G_1(\xi; A_1, A_2, B_1, B_2) \quad , \qquad \text{(A.6a)}$$

and use (A.4b) to obtain

$$\psi_2(\xi) = G_2[\psi_1(\xi); A_1, A_2, B_1, B_2] \quad . \qquad \text{(A.6b)}$$

Further study of these equations is in progress.

B. Compositional Models

Coats [3] formulates compositional flow of oil, gas, and water as a PMS which reduces, for the linear medium, to the forms,

$$\frac{\partial(\tilde{R}_o x_j + \tilde{R}_g y_j)}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\rho_o k_o x_j}{\mu_o}\frac{\partial P_o}{\partial x} + \frac{\rho_g k_g y_j}{\mu_g}\frac{\partial P_g}{\partial x}\right) \qquad \text{(B.1a)}$$

and

$$\frac{\partial \tilde{R}_w}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\rho_w k_w}{\mu_w}\frac{\partial P_w}{\partial x}\right) \quad , \qquad \text{(B.1b)}$$

where

$$\tilde{R}_i = \rho_i S_i \quad \text{for} \quad i = o, g, w \quad . \qquad \text{(B.1c)}$$

Application of similarity to the compositional problem simply involves the observation that there is one partial differential equation per flowing component rather than one per phase. Effects of this observation should be studied.

IX. PROGRAMMER'S GUIDE

Three programs, MODEL, SPMSTEST, and PMSTEST, are appended to this report. This Chapter supplements the discussion in Chapter VI for using the appended programs in test computations. Relations between subprograms are shown in Tables 2, 3, and 4, and input variables are summarized in Table 5. The tables can be used with appended sample input and corresponding sample output for preparation of test computations using the appended programs.

The variable, IRQT, was originally used, in MODEL and PMSTEST but never in SPMSTEST, as the number of time steps between outputs of densities on a grid in (x,t). It now appears in all three programs, but is actually used in PMSTEST only. For example, IRQT = 2 would mean that printed output occurs at alternate time steps.

A. Model for Evaluation (MODEL)

This program was developed as a simple porous media model to be used in testing the porous media evaluation methodology. The model is based on a perfect gas and a slightly compressible fluid.

The program, MODEL, is a simplified mathematical model for flow in a unidimensional porous medium. This model was developed strictly as a tool for illustrating the steps involved in testing a model. An outline of the mathematical formulation used in the model is given in Section VI.E.

The main program reads mathematical and physical parameters for assigning initial and boundary values. It also contains a loop to call for the numerical solution of the partial differential equations and printing of results.

B. Similarity Porous Media System (SPMSTEST)

The program, SPMSTEST, is used to test the model's correlations giving pressures, saturations, and relative permeabilities from densities. Coding of these functions is moved from the model to be tested to the functions, PRESA, PRESB, DPHI, DPSI, SIG, TAU, PC, and PCINV.

The program numerically solves the two-point boundary value problem for the SPMS. The program accepts as input the method (parabolas or straight lines), the parameters for the correlations, and boundary values of the densities, U and V. The program reports solution curves for the system and the number of iterations required to obtain a solution.

The main program, MAIN, allows processing several sets of boundary conditions and altering the method for each set of data. The control of the steps in solving the SPMS is carried out by the subprogram INITSP.

Since the SPMS is generally a stiff two-point boundary value system, accurate estimates of the initial slopes of U and V are needed. They are obtained by means of a finite difference solution of the SPMS. This solution is refined by a shooting technique using a Newton-Raphson iteration of a Runge-Kutta formulation. The control of this procedure is carried out in INITSP.

C. Porous Media System (PMSTEST)

The program, PMSTEST, combines all the implications of the similarity concept in a single package. It solves the SPMS for given boundary data. This solution of the SPMS is then interpolated by use of cubic splines to impose initial and boundary values on the PMS. The resulting PMS is then solved by finite-differences, and this solution is compared to the values implied by interpolation of the solution of the SPMS.

Table 2

Main Programs

Name	M	S	P	Transfer Vector
MODEL	M			BOUNDU,BOUNDV,OUTPUT PAGE ,SOLVE ,TITLE
SPMSTEST		S		HEADER,INITSP,SIMPLE
PMSTEST			P	INITPM,OUTSPS,SOLVE SPSVAL,TITLE

Table 3

Functions and Function-Entries*

Name	M	S	P	Computation
BOUNDU	M			User MODEL Boundary Condition, i = 1
BOUNDV	M			User MODEL Boundary Condition, i = 2
DPHI*	M	S	P	Integrated Variable Derivative of Φ
DPSI*	M	S	P	Integrated Variable Derivative of Ψ
HEADER	M	S	P	Print Fluid Correlation Descriptions
PC*	M	S	P	Capillary Pressure from Saturation
PCINV*	M	S	P	Saturation from Capillary Pressure
PRESA*	M	S	P	Pressure from Density, i = 1
PRESB*	M	S	P	Pressure from Density, i = 2
SIG*	M	S	P	Relative Permeability, i = 1
SIMILE		S	P	Similarity Variable, ξ, from (x,t)
SIMPLE*		S	P	Print Similarity Type and Parameters
SIMSEG*		S	P	Similarity Coefficient, [G(ξ)H(ξ)]
SIMSEX*		S	P	Similarity Integrating Factor, H(ξ)
SIMSID*		S	P	Derivative, d[G(ξ)H(ξ)]/dξ
TAU*	M	S	P	Relative Permeability, i = 2

Remark: No function above has a transfer vector

Table 4

Subroutines and Subroutine-Entries*

Name	M	S	P	Transfer Vector
BURINV*	M		P	
BURTON	M		P	PC ,PCINV ,PRESA ,PRESB
COEFFS		S	P	DCOUIN,DPHI ,DPSI ,SIG SIMSEG,SIMSEX,SIMSID,TAU
DCOUIN*		S	P	
DCOUPL		S	P	PC ,PCINV ,PRESA ,PRESB
INITPM			P	DCOUIN,HEADER,NEWTON,SIMILE SIMPLE,SPLINT,SPMSFD,SPMSIV
INITSP		S		DCOUIN,NEWTON,SIMILE,SPMSFD,SPMSIV
NEWTON		S	P	PROPS ,RUNGE
OUTPUT	M			PCINV ,PRESA ,PRESB
OUTSPS			P	SPSVAL
PAGE	M			
PROPS		S	P	DCOUIN,DPHI ,DPSI ,SIG SIMSEG,SIMSEX,SIMSID,TAU
RUNGE		S	P	PROPS
SOLVE	M		P	BURINV,BURTON,DPHI DPSI ,SIG ,TAU
SPLINE			P	
SPLINT			P	TRIPLE
SPMSFD		S	P	COEFFS,TRIPLE
SPMSIV		S	P	PROPS ,RUNGE
SPSVAL			P	SIMILE,SPLINE
TITLE	M			HEADER,OUTPUT,PAGE
TITLE			P	HEADER,OUTSPS
TRIPLE		S	P	

Table 5

Input Data

Columns	Format	Name	Definition and Restrictions
Card Type 1 : Density and Viscosity Parameters			
1- 5	F5.1	ALFA	
6-10	F5.1	VISA	
11-15	F5.1	QUE	- Physical
16-20	F5.1	BETA	- Parameters
21-25	F5.1	VISB	
26-30	F5.1	ELL	
Card Type 2 : Stability Parameter			
1- 5	F5.1	RAMBDA	Stability Parameter
Card Type 3 : (x,t) Domain Variation			
1- 5	F5.1	XO	Left X Endpoint
6-10	F5.1	X1	Right X Endpoint
Card Type 4 : Similarity Curve Type Selection			
1- 5	I5	LNORPR	Lines or Parabolas
Card Type 5 : Space and Time Increment Controls			
1- 5	I5	NDX	No. of X intervals (<=128)
6-10	I5	NDT	Number of T intervals
11-15	I5	IRQT	Time steps per print
Card Type 6 : Similarity Parameters			
1- 5	F5.1	UA	Left boundary value, u
6-10	F5.1	VA	Left boundary value, v
11-15	F5.1	UB	Right boundary value, u
16-20	F5.1	VB	Right boundary value, v
21-25	F5.1	XMUL	Similarity Parameter
26-30	F5.1	TMUL	Similarity Parameter
MODEL uses neither Card Type 4 nor Card Type 6 SPMSTEST iterates Card Types 5 and 6 until NDX = 0			

SYMBOL GLOSSARY

Roman Alphabet

k	Phase Relative Permeability
P	Phase Pressure [lbf/in^2]
r	Radial Distance [ft]
R	Saturation x Density [lbm/ft^3]
s	Distance [ft]
S	Phase Saturation
t	Permeability x Time / Porosity [$in^2 sec$]
x	Independent Space Variable [ft]
y	Independent Space Variable [ft]

Greek Alphabet

η	Similarity Variable, along a Curve of Constant ξ
θ	Polar Coordinate Angle
κ	Effective Absolute Permeability [in^2]
μ	Phase Viscosity [$lbf.sec/ft^2$]
ξ	Similarity Variable, from Curve to Curve
ρ	Phase Density [lbm/ft^3]
τ	Time [sec]
Φ	Media Porosity
ψ	[$lbm/(ft.in^2 sec)$]

Subscripts

c	Capillarity
i	Phase

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SAMPLE INPUT : PROGRAM MODEL

```
   10 FORMAT(6F5.1)
   11 FORMAT(5I5)
C                    PROGRAM NAME : MODEL
      READ(5,10) ALFA,VISA,QUE,BETA,VISB,ELL
      READ(5,10) RAMBDA
      READ(5,10) X0,X1
      READ(5,11) NDX,NDT,IRQT
C
COLUMNS  1          2          3
C23456789012345678901234567890
C
//GO.SYSIN DD *
  2.9   1.    1.    1.    1.    1.
   .2
   0.    1.
   10    40     1
//
```

```
            ============================
            DEMONSTRATION OUTPUT: MODEL
            ============================

FUNCTIONS DEFINING PHYSICAL QUANTITIES
--------- -------- -------- ----------
P(U) = ALFA*INVSINH(QUE*U)
Q(V) = BETA*V
DPHI(U) = ALFA/VISA
DPSI(V) = BETA/VISB
SIG(Y) = Y*Y
TAU(Z) = Z*Z
DPC/DS EXPONENT IS TWO

VISCOSITY AND DENSITY PARAMETERS
---------- --- ------- ----------
ALFA =    2.9     VISA =    1.0     QUE =    1.0
BETA =    1.0     VISB =    1.0     ELL =    1.0

NUMERICAL INPUT VALUES
--------- ----- ------
DT = 0.0020       DX = 0.1000     LAMBDA = 0.2000
XO =    0.0       X1 =    1.0
NT =     41       NX =     11
```

DEMONSTRATION OUTPUT: MODEL

COMPUTATIONS

X

	0.0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
T = 0.0											
U	1.00	2.75	3.74	4.14	4.17	4.00	3.83	3.86	4.26	5.25	7.00
V	2.00	2.48	2.84	3.00	2.91	2.60	2.14	1.65	1.24	1.02	1.04
Y	0.57	0.74	0.77	0.78	0.78	0.79	0.80	0.82	0.84	0.86	0.87
T = 0.00200											
U	1.00	2.62	3.52	3.99	4.09	4.00	3.91	4.03	4.54	5.63	7.00
V	2.00	2.41	2.77	2.95	2.88	2.59	2.16	1.68	1.28	1.05	1.04
Y	0.57	0.74	0.76	0.77	0.78	0.79	0.80	0.82	0.84	0.86	0.87
T = 0.00400											
U	1.00	2.49	3.36	3.84	4.01	4.00	4.00	4.21	4.82	5.78	7.00
V	2.00	2.35	2.72	2.90	2.86	2.59	2.18	1.71	1.31	1.07	1.04
Y	0.57	0.74	0.76	0.77	0.78	0.79	0.81	0.83	0.85	0.86	0.87
T = 0.00600											
U	1.00	2.39	3.22	3.72	3.93	4.00	4.09	4.40	5.00	5.92	7.00
V	2.00	2.30	2.67	2.86	2.83	2.59	2.20	1.75	1.34	1.08	1.04
Y	0.57	0.73	0.76	0.77	0.78	0.79	0.81	0.83	0.85	0.86	0.87
T = 0.00800											
U	1.00	2.30	3.10	3.61	3.87	4.01	4.19	4.54	5.16	6.01	7.00
V	2.00	2.26	2.62	2.82	2.81	2.59	2.22	1.77	1.36	1.10	1.04
Y	0.57	0.73	0.75	0.77	0.78	0.79	0.81	0.83	0.85	0.86	0.87
T = 0.01000											
U	1.00	2.23	3.00	3.51	3.82	4.03	4.27	4.67	5.28	6.09	7.00
V	2.00	2.23	2.58	2.79	2.79	2.59	2.23	1.79	1.37	1.11	1.04
Y	0.57	0.72	0.75	0.76	0.78	0.79	0.81	0.83	0.85	0.86	0.87
T = 0.01200											
U	1.00	2.16	2.91	3.44	3.78	4.05	4.35	4.78	5.39	6.15	7.00
V	2.00	2.20	2.55	2.76	2.78	2.59	2.25	1.81	1.39	1.12	1.04
Y	0.57	0.72	0.75	0.76	0.77	0.79	0.81	0.83	0.85	0.86	0.87
T = 0.01400											
U	1.00	2.11	2.84	3.37	3.75	4.07	4.41	4.87	5.47	6.20	7.00
V	2.00	2.17	2.51	2.73	2.76	2.59	2.26	1.82	1.40	1.12	1.04
Y	0.57	0.72	0.75	0.76	0.77	0.79	0.81	0.83	0.85	0.86	0.87

```
============================
DEMONSTRATION OUTPUT: MODEL
============================

                COMPUTATIONS
                ------------

                                X
   0.0   0.10  0.20  0.30  0.40  0.50  0.60  0.70  0.80  0.90  1.00

T = 0.01600
U 1.00  2.06  2.78  3.32  3.73  4.09  4.47  4.94  5.54  6.24  7.00
V 2.00  2.16  2.49  2.71  2.75  2.59  2.27  1.84  1.41  1.13  1.04
Y 0.57  0.72  0.74  0.76  0.77  0.79  0.81  0.83  0.85  0.86  0.87

T = 0.01800
U 1.00  2.02  2.72  3.27  3.71  4.10  4.52  5.01  5.60  6.28  7.00
V 2.00  2.14  2.46  2.69  2.74  2.60  2.28  1.85  1.42  1.14  1.04
Y 0.57  0.71  0.74  0.76  0.77  0.79  0.81  0.84  0.85  0.86  0.87

T = 0.02000
U 1.00  1.99  2.68  3.24  3.70  4.12  4.56  5.07  5.65  6.31  7.00
V 2.00  2.13  2.44  2.67  2.73  2.60  2.28  1.86  1.43  1.15  1.04
Y 0.57  0.71  0.74  0.76  0.77  0.79  0.81  0.84  0.85  0.86  0.87

T = 0.02200
U 1.00  1.96  2.64  3.21  3.69  4.14  4.60  5.12  5.69  6.33  7.00
V 2.00  2.12  2.42  2.66  2.72  2.60  2.29  1.87  1.44  1.15  1.04
Y 0.57  0.71  0.74  0.76  0.77  0.79  0.82  0.84  0.85  0.86  0.87

T = 0.02400
U 1.00  1.94  2.61  3.18  3.68  4.15  4.63  5.16  5.73  6.35  7.00
V 2.00  2.11  2.41  2.64  2.72  2.60  2.30  1.88  1.45  1.16  1.04
Y 0.57  0.71  0.74  0.76  0.77  0.79  0.82  0.84  0.85  0.86  0.87

T = 0.02600
U 1.00  1.92  2.59  3.16  3.68  4.17  4.66  5.19  5.76  6.37  7.00
V 2.00  2.11  2.40  2.63  2.71  2.60  2.30  1.88  1.46  1.16  1.04
Y 0.57  0.71  0.74  0.76  0.77  0.79  0.82  0.84  0.85  0.86  0.87

T = 0.02800
U 1.00  1.90  2.57  3.15  3.68  4.18  4.69  5.22  5.79  6.39  7.00
V 2.00  2.10  2.38  2.62  2.70  2.59  2.30  1.89  1.47  1.17  1.04
Y 0.57  0.70  0.74  0.76  0.77  0.79  0.82  0.84  0.85  0.86  0.87

T = 0.03000
U 1.00  1.89  2.55  3.14  3.68  4.19  4.71  5.25  5.81  6.40  7.00
V 2.00  2.10  2.38  2.61  2.70  2.59  2.31  1.89  1.47  1.18  1.04
Y 0.57  0.70  0.74  0.76  0.77  0.79  0.82  0.84  0.85  0.86  0.87
```

```
==========================
DEMONSTRATION OUTPUT: MODEL
==========================

            COMPUTATIONS
            ------------

                                X
  0.0   0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00

T = 0.03200
U 1.00 1.88 2.54 3.13 3.68 4.21 4.73 5.27 5.83 6.41 7.00
V 2.00 2.10 2.37 2.60 2.69 2.59 2.31 1.90 1.48 1.18 1.05
Y 0.57 0.70 0.74 0.76 0.77 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.03400
U 1.00 1.87 2.53 3.12 3.68 4.22 4.75 5.29 5.85 6.42 7.00
V 2.00 2.10 2.36 2.60 2.69 2.59 2.31 1.90 1.49 1.19 1.05
Y 0.57 0.70 0.74 0.76 0.77 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.03600
U 1.00 1.87 2.52 3.12 3.68 4.23 4.77 5.31 5.87 6.43 6.99
V 2.00 2.10 2.36 2.59 2.68 2.59 2.31 1.91 1.49 1.19 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.03800
U 1.00 1.86 2.52 3.12 3.69 4.24 4.78 5.33 5.88 6.44 6.99
V 2.00 2.10 2.35 2.58 2.68 2.59 2.31 1.91 1.50 1.20 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.04000
U 1.00 1.86 2.51 3.12 3.69 4.24 4.79 5.34 5.89 6.44 6.99
V 2.00 2.10 2.35 2.58 2.67 2.58 2.31 1.92 1.50 1.20 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.04200
U 1.00 1.85 2.51 3.12 3.69 4.25 4.80 5.35 5.90 6.45 6.99
V 2.00 2.10 2.34 2.57 2.67 2.58 2.31 1.92 1.51 1.21 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.04400
U 1.00 1.85 2.51 3.12 3.70 4.26 4.81 5.36 5.91 6.45 6.99
V 2.00 2.10 2.34 2.57 2.67 2.58 2.31 1.92 1.51 1.21 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.04600
U 1.00 1.85 2.51 3.12 3.70 4.27 4.82 5.37 5.92 6.46 6.99
V 2.00 2.10 2.34 2.56 2.66 2.58 2.31 1.92 1.52 1.21 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87
```

```
==========================
DEMONSTRATION OUTPUT: MODEL
==========================

                COMPUTATIONS
                ------------

                          X
  0.0   0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00

T = 0.04800
U 1.00 1.85 2.51 3.12 3.70 4.28 4.83 5.38 5.92 6.46 6.99
V 2.00 2.11 2.34 2.56 2.66 2.58 2.31 1.93 1.52 1.22 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.05000
U 1.01 1.85 2.51 3.12 3.71 4.28 4.84 5.39 5.93 6.46 6.99
V 2.00 2.11 2.33 2.56 2.65 2.57 2.31 1.93 1.53 1.22 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.05200
U 1.01 1.85 2.51 3.12 3.71 4.29 4.85 5.40 5.93 6.47 6.99
V 2.00 2.11 2.33 2.55 2.65 2.57 2.31 1.93 1.53 1.23 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.05400
U 1.01 1.85 2.51 3.13 3.72 4.29 4.85 5.40 5.94 6.47 6.99
V 2.00 2.11 2.33 2.55 2.65 2.57 2.31 1.93 1.53 1.23 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.05600
U 1.01 1.85 2.51 3.13 3.72 4.30 4.86 5.41 5.94 6.47 6.99
V 2.00 2.11 2.33 2.54 2.64 2.56 2.31 1.94 1.54 1.23 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.05800
U 1.01 1.85 2.52 3.13 3.73 4.30 4.87 5.41 5.95 6.47 6.99
V 2.00 2.11 2.33 2.54 2.64 2.56 2.31 1.94 1.54 1.24 1.05
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.06000
U 1.01 1.85 2.52 3.14 3.73 4.31 4.87 5.42 5.95 6.47 6.99
V 2.00 2.11 2.33 2.54 2.63 2.56 2.31 1.94 1.55 1.24 1.06
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87

T = 0.06200
U 1.01 1.86 2.52 3.14 3.73 4.31 4.88 5.42 5.95 6.47 6.98
V 2.00 2.12 2.33 2.53 2.63 2.56 2.31 1.94 1.55 1.25 1.06
Y 0.57 0.70 0.74 0.76 0.78 0.80 0.82 0.84 0.85 0.86 0.87
```

DEMONSTRATION OUTPUT: MODEL

COMPUTATIONS

X

	0.0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
T = 0.06400											
U	1.01	1.86	2.52	3.14	3.74	4.32	4.88	5.42	5.96	6.47	6.98
V	2.00	2.12	2.33	2.53	2.63	2.55	2.31	1.94	1.55	1.25	1.06
Y	0.57	0.70	0.74	0.76	0.78	0.80	0.82	0.84	0.85	0.86	0.87
T = 0.06600											
U	1.01	1.86	2.53	3.15	3.74	4.32	4.88	5.43	5.96	6.48	6.98
V	2.00	2.12	2.33	2.53	2.62	2.55	2.31	1.95	1.56	1.25	1.06
Y	0.57	0.70	0.74	0.76	0.78	0.80	0.82	0.84	0.85	0.86	0.87
T = 0.06800											
U	1.01	1.86	2.53	3.15	3.75	4.33	4.89	5.43	5.96	6.48	6.98
V	2.00	2.12	2.32	2.52	2.62	2.55	2.31	1.95	1.56	1.26	1.06
Y	0.57	0.70	0.74	0.76	0.78	0.80	0.82	0.84	0.85	0.86	0.87
T = 0.07000											
U	1.01	1.86	2.53	3.15	3.75	4.33	4.89	5.43	5.96	6.48	6.98
V	2.00	2.12	2.32	2.52	2.61	2.54	2.31	1.95	1.57	1.26	1.06
Y	0.57	0.70	0.74	0.76	0.78	0.80	0.82	0.84	0.85	0.86	0.87
T = 0.07200											
U	1.01	1.86	2.53	3.16	3.75	4.33	4.89	5.44	5.96	6.48	6.98
V	2.00	2.12	2.32	2.52	2.61	2.54	2.31	1.95	1.57	1.26	1.06
Y	0.57	0.70	0.74	0.76	0.78	0.80	0.82	0.84	0.85	0.86	0.87
T = 0.07400											
U	1.01	1.87	2.54	3.16	3.76	4.34	4.90	5.44	5.96	6.48	6.98
V	2.00	2.12	2.32	2.52	2.61	2.54	2.31	1.95	1.57	1.27	1.06
Y	0.57	0.70	0.74	0.76	0.78	0.80	0.82	0.84	0.85	0.86	0.87
T = 0.07600											
U	1.01	1.87	2.54	3.16	3.76	4.34	4.90	5.44	5.97	6.48	6.98
V	2.00	2.12	2.32	2.51	2.60	2.54	2.30	1.95	1.58	1.27	1.06
Y	0.57	0.70	0.74	0.76	0.78	0.80	0.82	0.84	0.85	0.86	0.87
T = 0.07800											
U	1.01	1.87	2.54	3.16	3.76	4.34	4.90	5.44	5.97	6.48	6.98
V	2.00	2.12	2.32	2.51	2.60	2.53	2.30	1.96	1.58	1.27	1.07
Y	0.57	0.70	0.74	0.76	0.78	0.80	0.82	0.84	0.85	0.86	0.87

```
============================
DEMONSTRATION OUTPUT: MODEL
============================

                COMPUTATIONS
                ------------

                                X
   0.0   0.10  0.20  0.30  0.40  0.50  0.60  0.70  0.80  0.90  1.00

T = 0.08000
U 1.01  1.87  2.54  3.17  3.77  4.34  4.90  5.45  5.97  6.48  6.97
V 1.99  2.12  2.32  2.51  2.60  2.53  2.30  1.96  1.58  1.28  1.07
Y 0.57  0.70  0.74  0.76  0.78  0.80  0.82  0.84  0.85  0.86  0.87
```

DEMONSTRATION LISTING: MODEL

```
C
C                    PROGRAM NAME : MODEL
C       OIL AND GAS POROUS MEDIA DEMONSTRATION MODEL
C             MODIFIED : W. T. FORD : 10/17/83
C
      DIMENSION XPTS(129),UOLD(129),VOLD(129),
     1                    UNEW(129),VNEW(129)
      COMMON /ARAYS/ XPTS,UOLD,VOLD
      COMMON /PARAMS/ ALFA,VISA,QUE,BETA,VISB,ELL
      COMMON /INPUT/ DT,DX,XO,X1,NX,NT,RAMBDA
C
   10 FORMAT(6F5.1)
   11 FORMAT(5I5)
C
C   VISCOSITY AND DENSITY PARAMETERS
C
      READ(5,10) ALFA,VISA,QUE,BETA,VISB,ELL
C
C STABILIZING CONTROL PARAMETER
C
      READ(5,10)RAMBDA
C
C (X,T) DOMAIN VARIATION
C
      READ(5,10) XO,X1
      READ(5,11) NDX,NDT,IRQT
C
C OPERATIONAL CONTROL
C
      NX = NDX + 1
      NT = NDT + 1
      DX = (X1 - XO)/NDX
      DT = RAMBDA * (DX**2)
C
C INITIAL VALUES
C
      ZERO = 0.0
      X = DX
      NIX = NX - 1
      DO 20 I = 2,NIX
         UOLD(I) = BOUNDU(X,ZERO)
         VOLD(I) = BOUNDV(X,ZERO)
         X = X + DX
   20 CONTINUE
C
C BOUNDARY VALUES
C
      UOLD(1) = BOUNDU(XO,ZERO)
      VOLD(1) = BOUNDV(XO,ZERO)
      UOLD(NX) = BOUNDU(X1,ZERO)
      VOLD(NX) = BOUNDV(X1,ZERO)
```

```
              ============================
              DEMONSTRATION LISTING: MODEL
              ============================

C
C  PRINT TITLE PAGE, FUNCTIONS, INPUT VALUES
C
      CALL TITLE
C
C SOLVE PROBLEM
C
      LINENO= 1
      T = DT
      DO 21 I = 2,NT
         IF (MOD(LINENO,8) .EQ. 0) CALL PAGE(NX)
         CALL SOLVE(DT,DX,NT,NIX,UOLD,VOLD,UNEW,VNEW)
         DO 22 J = 2,NIX
            UOLD(J) = UNEW(J)
            VOLD(J) = VNEW(J)
   22    CONTINUE
         UOLD(1) = BOUNDU(XO,T)
         VOLD(1) = BOUNDV(XO,T)
         UOLD(NX) = BOUNDU(X1,T)
         VOLD(NX) = BOUNDV(X1,T)
         CALL OUTPUT(T,NX,UOLD,VOLD)
         LINENO = LINENO + 1
         T = T + DT
   21 CONTINUE
C
      END
```

```
============================
DEMONSTRATION LISTING: MODEL
============================

      SUBROUTINE BURTON (YU,ZV,U,V,Y,Z)
C THE PRODUCTS YU AND ZV, BEING GIVEN, BURTON FINDS THE
C FACTORS, Y, U, Z AND V, CONSISTENT WITH THE PRODUCTS.
      DATA E,TOL,KMAX/.001,.00001,20/
C PROGRAM CHANGE: 7/23/81: SOLVE GIVES INITIAL Y VALUE
C     Y=.5
      K=0
   20 Z = 1.  -Y
      F = PC(Y) - PRESA(YU/Y) + PRESB(ZV/Z)
      IF (ABS(F).LE.TOL) GO TO 30
      K = K + 1
      IF (K.LT.KMAX) GO TO 25
   22   CONTINUE
        P1=PC(Y)
        P2=PRESA(YU/Y)
        P3=PRESB(ZV/Z)
        U1=YU/Y
        V1=ZV/Z
        WRITE(6,24)
        WRITE(6,23) F,G,Y,P1,P2,P3
        WRITE(6,36)
        WRITE(6,35) U1,V1,K,YU,ZV
   35   FORMAT(' ',10X,2(1X,E9.3),2X,I2,2(1X,E9.3))
   36   FORMAT(16X,'U',9X,'V',5X,'K',
     1             8X,'YU',8X,'ZV')
   23   FORMAT(' ',10X,6(1X,E9.3))
   24   FORMAT(16X,'F',9X,'G',9X,'Y',7X,'PC(Y)',
     1       3X,'PRESA(YU/Y)',3X,'PRESB(ZV/Z)',/)
        STOP
   25 CONTINUE
      D = SIGN(E,F)
      S = Y - D
      T = 1. - S
      G = PC(S) - PRESA(YU/S) + PRESB(ZV/T)
      IF (ABS(G-F) .LT. TOL) GOTO 22
      DY = D*F/(G-F)
      Y = Y + DY
      IF (Y*(1.-Y) .LT. 0.) GOTO 70
      GO TO 20
   30 U = YU/Y
      V=ZV/Z
      RETURN
   70 WRITE(6,71)
   71 FORMAT(' ',10X,'OUT OF INTERVAL')
      ENTRY BURINV (YU,ZV,U,V,Y,Z)
C THE QUANTITIES U AND V, BEING GIVEN, BURINV FINDS
C Y, Z, AND THE PRODUCTS, YU AND ZV.
      Y=PCINV(PRESA(U)-PRESB(V))
      Z=1.-Y
      YU=Y*U
      ZV=Z*V
```

```
RETURN
END
```

DEMONSTRATION LISTING: MODEL

```
      FUNCTION BOUNDU (X,T)
C
      IF (X .EQ. 0.) GOTO 10
      IF (X .EQ. 1.) GOTO 11
      IF (T .EQ. 0.) GOTO 12
C
   10 BOUNDU = 1. + 2.1*T*T
      RETURN
C
   11 BOUNDU = 7. - 4.*T*T
      RETURN
C
   12 BOUNDU = 32.*X**3 - 48.*X*X + 22.*X + 1.
      RETURN
C
      END
```

DEMONSTRATION LISTING: MODEL

```
      FUNCTION BOUNDV (X,T)
C
      IF (X .EQ. 0.) GOTO 10
      IF (X .EQ. 1.) GOTO 11
      IF (T .EQ. 0.) GOTO 12
C
   10 BOUNDV = 2. - .8*T*T
      RETURN
C
   11 BOUNDV = 2. + SIN(5.) + 4.*T*T
      RETURN
C
   12 BOUNDV = SIN(5.*X) + 2.
      RETURN
C
      END
```

```
            ============================
            DEMONSTRATION LISTING: MODEL
            ============================

      FUNCTION HEADER (X)
      COMMON /PARAMS/ ALFA,VISA,QUE,BETA,VISB,ELL
   10 FORMAT('1',10X,'FUNCTIONS DEFINING PHYSICAL',
     1            ' QUANTITIES',/,
     2       10X,' --------- -------- --------',
     3            ' ----------',/,
     4     11X,31HP(U) = ALFA*INVSINH(QUE*U)      ,/,
     5     11X,31HQ(V) = BETA*V                   ,/,
     6     11X,31HDPHI(U) = ALFA/VISA             ,/,
     7     11X,31HDPSI(V) = BETA/VISB             ,/,
     8     11X,31HSIG(Y) = Y*Y                    ,/,
     9     11X,31HTAU(Z) = Z*Z                    ,/,
     A     11X,31HDPC/DS EXPONENT IS TWO          ,/)
C
CALCULATIONS OF PHYSICAL QUANTITIES FROM FUNCTIONS,
CORRELATIONS, ETC. ARE TO BE THRU THIS SUBPROGRAM ONLY.
CHANGE IN FORMULATION BELOW REQUIRES
CORRESPONDING CHANGE IN FORMAT ABOVE.
CONSISTENCY IS REQUIRED IN PC AND PCINV ENTRIES.
C
      WRITE(6,10)
      HEADER=X
      RETURN
C
      ENTRY PRESA (X)
      Z=QUE*X
      PRESA=ALFA*ALOG(Z+SQRT(Z*Z+1))
      RETURN
C
      ENTRY PRESB (X)
      PRESB=BETA*X
      RETURN
C
      ENTRY DPHI  (X)
      DPHI=ALFA/VISA
      RETURN
C
      ENTRY DPSI  (X)
      DPSI=BETA/VISB
      RETURN
C
      ENTRY SIG   (X)
      SIG=X*X
      RETURN
C
      ENTRY TAU   (X)
      TAU=X*X
      RETURN
C
      ENTRY PCINV (X)
      Z=2.+(-X+SQRT(4.+X*X))
```

```
      PCINV=2./Z
      RETURN
C
      ENTRY PC (X)
      PC=1./(1.-X)-1./X
      RETURN
      END
```

```
==============================
DEMONSTRATION LISTING: MODEL
==============================

      SUBROUTINE OUTPUT (T,NX,UOLD,VOLD)
C
      DIMENSION UOLD(129),VOLD(129),Y(15)
   11 FORMAT('0',10X,'T = ',F7.5)
   13 FORMAT('0',10X,'U',11(1X,F4.2))
   14 FORMAT(' ',10X,'V',11(1X,F4.2))
   15 FORMAT(' ',10X,'Y',11(1X,F4.2)//)
C
      WRITE(6,11) T
C
C PRINT U VALUES
C
      WRITE(6,13) (UOLD(I),I=1,NX)
C
C PRINT V VALUES
C
      WRITE(6,14) (VOLD(I), I=1,NX)
C
C COMPUTE Y VALUES
C
      DO 16 I=1,NX
        Y(I)=PCINV(PRESA(UOLD(I)) - PRESB(VOLD(I)))
   16 CONTINUE
C
C PRINT Y VALUES
C
      WRITE(6,15) (Y(I), I=1,NX)
C
      END
```

DEMONSTRATION LISTING: MODEL

```
      SUBROUTINE PAGE (NX)
C
      DIMENSION XPTS(129)
      COMMON /ARAYS/ XPTS,UOLD,VOLD
C
   10 FORMAT('0',35X,'X',/,12X,11(1X,F4.2),/)
   11 FORMAT('1',30X,'COMPUTATIONS',/,
     1       '0',30X,'------------',/)
C
      WRITE(6,11)
      WRITE(6,10) (XPTS(I), I = 1,NX)
C
      END
```

DEMONSTRATION LISTING: MODEL

```
      SUBROUTINE SOLVE (DT,DX,NT,NIX,UOLD,VOLD,UNEW,VNEW)
      DIMENSION UOLD(129),UNEW(129),VOLD(129),VNEW(129)
      RAMBDA=DT/DX**2
      J=0
      GOTO 15
   11 DQ=EQ
      DR=ER
   13 J=J+1
      UC=UR
      VC=VR
      YC=YR
      ZC=ZR
   15 UR=UOLD(J+1)
      VR=VOLD(J+1)
      CALL BURINV (YU,ZV,UR,VR,YR,ZR)
      IF (J.LT.1) GOTO 13
      UBR=(UC+UR)/2.
      VBR=(VC+VR)/2.
      YBR=(YC+YR)/2.
      ZBR=1.-YBR
      EQ=SIG(YBR)*DPHI(UBR)*(UR-UC)
      ER=TAU(ZBR)*DPSI(VBR)*(VR-VC)
      IF (J.EQ.1) GOTO 11
      YU=YC*UC+RAMBDA*(EQ-DQ)
      ZV=ZC*VC+RAMBDA*(ER-DR)
C PROGRAM CHANGE: 7/23/81: INITIALIZE Y FOR BURTON
      Y = YR
      CALL BURTON (YU,ZV,UNEW(J),VNEW(J),Y,Z)
      IF (J.LT.NIX) GOTO 11
      RETURN
      END
```

DEMONSTRATION LISTING: MODEL

```
      SUBROUTINE TITLE
C
      DIMENSION XPTS(129),UOLD(129),VOLD(129)
      COMMON /ARAYS/ XPTS,UOLD,VOLD
      COMMON /INPUT/ DT,DX,XO,X1,NX,NT,RAMBDA
      COMMON /PARAMS/ ALFA,VISA,QUE,BETA,VISB,ELL
C
    9 FORMAT('0',10X,'VISCOSITY AND DENSITY PARAMETERS',
     1        /,10X,' --------- --- ------- ----------',
     2        /,11X,7HALFA = ,F6.1,5X,7HVISA = ,
     3        F6.1,5X,6HQUE = ,F6.1,
     4        /,11X,7HBETA = ,F6.1,5X,7HVISB = ,
     5        F6.1,5X,6HELL = ,F6.1/)
   10 FORMAT('0',10X,'NUMERICAL INPUT VALUES',
     1        /,10X,' --------- ------ ------')
   11 FORMAT('0',10X,'DT = ',F6.4,7X,'DX = ',
     1       F6.4,5X,'LAMBDA = ',F6.4)
   12 FORMAT('0',10X,'XO = ',F6.1,7X,'X1 = ',F6.1)
   13 FORMAT('0',10X,'NT = ',I6,7X,'NX = ',I6)
C
C PRINT INFORMATION ON PHYSICAL QUANTITIES
C
      PRNT = HEADER(XO)
      WRITE(6,9) ALFA,VISA,QUE,BETA,VISB,ELL
C
C PRINT NUMERICAL INPUT VALUES
C
      WRITE(6,10)
      WRITE(6,11) DT,DX,RAMBDA
      WRITE(6,12) XO,X1
      WRITE(6,13) NT,NX
C
C STORE X POINTS FOR USE IN SUBROUTINE PAGE
C
      X = XO
      DO 30 I = 1,NX
         XPTS(I)=X
         X = X + DX
   30 CONTINUE
C
C PRINT HEADING ON A NEW PAGE
C
      CALL PAGE(NX)
C
C PRINT INITIAL VALUES
C
      T = 0.0
      CALL OUTPUT(T,NX,UOLD,VOLD)
C
      END
```

SAMPLE INPUT : PROGRAM SPMSTEST

```
   10 FORMAT (6F5.1)
   11 FORMAT (5I5)
C                      PROGRAM NAME: SPMSTEST
      READ(5,10) ALFA,VISA,QUE,BETA,VISB,ELL
      READ(5,10) RAMBDA
      READ(5,10) XO,X1
      READ(5,11) LNORPR
 500  READ(5,11) NDX,NDT,IRQT
      IF (NDX .EQ. 0) GOTO 600
      READ(5,10) UA,VA,UB,VB,XMUL,TMUL
C
COLUMNS  1          2          3
C234567890123456789012345678901
C
//GO.SYSIN DD *
  2.9   1.   1.   1.   1.   1.
   .5
   0.   1.
    0
    10  500   50
   1.   4.   3.   1.   1.-  1.
   10  500  100
   1.   2.  10.   5.   1.-  1.
   10  500   50
   1.   4.   3.   1.   2.-  1.
   10  500  100
   1.   2.  10.   5.   2.-  1.
    10  500   50
   1.   4.   3.   1.   1.-  2.
   10  500  100
   1.   2.  10.   5.   1.-  2.
     0    0    0
```

```
==============================
DEMONSTRATION OUTPUT: SPMSTEST
==============================


    UA          VA          UB          VB
    1.00        4.00        3.00        1.00
GEOMETRIC FACTORS
XMUL       1.00000
TMUL      -1.00000
PARALLEL LINES
P(U)=ALFA*INVSINH(QUE*U)        Q(V)=BETA*V
DPHI(U)=ALFA/VISA               DPSI(V)=BETA/VISB
SIG(Y)=Y*Y                      TAU(Z)=Z*Z
DPC/DS EXPONENT IS TWO
ALFA =    2.9      VISA =    1.0      QUE  =    1.0
BETA =    1.0      VISB =    1.0      ELL  =    1.0
XI EXTREMES    -2.499997     1.000000
DELTA XI        0.054687
                        SUCCESSIVE SUBSTITUTION
  U(HI+DH)   HV(HI+DH)    U(HM-DH)   V(HM-DH)    IT    MAX.ABS INCR.
   1.03125    3.95312     2.96875    1.04687     0
   1.28097    3.70335     3.00851    0.98545     1    0.220712E 01
   1.30604    3.68569     2.98988    1.00381     2    0.848126E 00
   1.29051    3.68458     2.99198    0.99759     3    0.177077E 00
   1.29416    3.68516     2.99173    0.99901     4    0.282540E-01
   1.29360    3.68502     2.99174    0.99874     5    0.381660E-02
   1.29364    3.68504     2.99174    0.99878     6    0.131017E-02
   1.29363    3.68504     2.99174    0.99877     7    0.336230E-03
   1.29363    3.68504     2.99174    0.99877     8    0.907183E-04
   1.29363    3.68504     2.99174    0.99877     9    0.219345E-04
   1.29363    3.68504     2.99174    0.99878    10    0.352859E-04
                        NEWTON'S METHOD
   WO        XO        UR        VR        WR        XR        UPA       VPA

  2.1210    0.1254    2.4018    0.6557    2.1210    0.1254    5.3692   -5.7592
  2.1710    0.1254    2.4479    0.6611    2.1710    0.1254    5.5198   -5.7592
  2.1210    0.1754    2.4249    0.8837    2.1210    0.1754    5.3692   -5.6450

  2.7389    0.1863    2.9989    1.0166    2.7389    0.1863    7.2303   -5.6200
  2.7889    0.1863    3.0444    1.0229    2.7889    0.1863    7.3809   -5.6200
  2.7389    0.2363    3.0267    1.2442    2.7389    0.2363    7.2303   -5.5058

  2.7424    0.1825    3.0000    0.9993    2.7424    0.1825    7.2408   -5.6286
```

DEMONSTRATION OUTPUT: SPMSTEST

ETA	Y	U	V	W	X
-2.50000	0.33836	1.00000	4.00000	2.74244	0.18254
-2.39062	0.50853	1.47477	3.35574	2.74244	0.18254
-2.28125	0.62465	1.69021	2.69470	2.74244	0.18254
-2.17187	0.69615	1.82566	2.09754	2.74244	0.18254
-2.06250	0.73784	1.92741	1.63182	2.74244	0.18254
-1.95312	0.76195	2.01189	1.31347	2.74244	0.18254
-1.84375	0.77615	2.08607	1.11739	2.74244	0.18254
-1.73437	0.78483	2.15332	1.00622	2.74244	0.18254
-1.62500	0.79043	2.21538	0.94787	2.74244	0.18254
-1.51562	0.79430	2.27325	0.92022	2.74244	0.18254
-1.40625	0.79717	2.32754	0.90957	2.74244	0.18254
-1.29687	0.79945	2.37864	0.90796	2.74244	0.18254
-1.18750	0.80136	2.42686	0.91096	2.74244	0.18254
-1.07812	0.80300	2.47241	0.91616	2.74244	0.18254
-0.96875	0.80447	2.51548	0.92228	2.74244	0.18254
-0.85937	0.80580	2.55624	0.92865	2.74244	0.18254
-0.75000	0.80701	2.59482	0.93495	2.74244	0.18254
-0.64062	0.80813	2.63137	0.94100	2.74244	0.18254
-0.53125	0.80916	2.66601	0.94675	2.74244	0.18254
-0.42187	0.81011	2.69883	0.95218	2.74244	0.18254
-0.31250	0.81100	2.72995	0.95728	2.74244	0.18254
-0.20312	0.81183	2.75946	0.96208	2.74244	0.18254
-0.09375	0.81260	2.78746	0.96658	2.74244	0.18254
0.01563	0.81332	2.81402	0.97082	2.74244	0.18254
0.12500	0.81399	2.83922	0.97480	2.74244	0.18254
0.23438	0.81462	2.86315	0.97854	2.74244	0.18254
0.34375	0.81521	2.88586	0.98206	2.74244	0.18254
0.45313	0.81576	2.90742	0.98538	2.74244	0.18254
0.56250	0.81628	2.92789	0.98851	2.74244	0.18254
0.67188	0.81677	2.94733	0.99146	2.74244	0.18254
0.78125	0.81723	2.96580	0.99424	2.74244	0.18254
0.89063	0.81766	2.98334	0.99686	2.74244	0.18254
1.00000	0.81807	3.00001	0.99934	2.74244	0.18254

```
                ============================
                DEMONSTRATION OUTPUT: SPMSTEST
                ============================

    UA          VA          UB          VB
    1.00        2.00       10.00        5.00
GEOMETRIC FACTORS
XMUL       1.00000
TMUL      -1.00000
PARALLEL LINES
P(U)=ALFA*INVSINH(QUE*U)        Q(V)=BETA*V
DPHI(U)=ALFA/VISA               DPSI(V)=BETA/VISB
SIG(Y)=Y*Y                      TAU(Z)=Z*Z
DPC/DS EXPONENT IS TWO
ALFA =    2.9      VISA =    1.0      QUE  =    1.0
BETA =    1.0      VISB =    1.0      ELL  =    1.0
XI EXTRESqjh   -2.499997     1.000000
DELTA XI        0.054687
                        SUCCESSIVE SUBSTITUTION
  U(HI+DH)   HV(HI+DH)   U(HM-DH)   V(HM-DH)    IT    MAX.ABS INCR.
   1.14063    2.04688    9.85937    4.95312     0
   1.49237    2.05009    9.95254    5.00345     1   0.243118E 01
   1.45026    2.08474    9.94730    4.98433     2   0.115222E 01
   1.44154    2.07611    9.95706    4.99721     3   0.134865E 01
   1.46330    2.07709    9.94428    4.99292     4   0.149835E 01
   1.43572    2.08013    9.95924    4.98674     5   0.151861E 01
   1.46488    2.07355    9.94400    5.00459     6   0.146602E 01
   1.43797    2.08353    9.95754    4.97443     7   0.131750E 01
   1.46022    2.07099    9.94686    5.01492     8   0.109801E 01
   1.44619    2.08514    9.95277    4.96733     9   0.829185E 00
   1.45092    2.07118    9.95195    5.01585    10   0.904826E 00
                        NEWTON'S METHOD
    WO        XO         UR         VR         WR         XR        UPA       VPA

  8.2883   1.1063   8.8843   5.0455   8.2883   1.1063   8.2454   1.3016
  8.3383   1.1063   8.9313   5.0532   8.3383   1.1063   8.2988   1.3016
  8.2883   1.1563   8.9529   5.1672   8.2883   1.1563   8.2454   1.5698

  9.6278   1.0030   9.9795   4.9641   9.6278   1.0030   9.6762   0.7475
  9.6778   1.0030  10.0254   4.9706   9.6778   1.0030   9.7296   0.7475
  9.6278   1.0530  10.0549   5.0995   9.6278   1.0530   9.6762   1.0157

  9.6285   1.0162  10.0000   5.0005   9.6285   1.0162   9.6768   0.8183
```

DEMONSTRATION OUTPUT: SPMSTEST

ETA	Y	U	V	W	X
-2.50000	0.56820	1.00000	2.00000	9.62846	1.01617
-2.39062	0.68321	1.80859	2.23537	9.62846	1.01617
-2.28125	0.71460	2.43267	2.59907	9.62846	1.01617
-2.17187	0.72875	2.98698	2.94711	9.62846	1.01617
-2.06250	0.73769	3.49522	3.24009	9.62846	1.01617
-1.95312	0.74460	3.96619	3.47842	9.62846	1.01617
-1.84375	0.75043	4.40469	3.67219	9.62846	1.01617
-1.73437	0.75552	4.81413	3.83168	9.62846	1.01617
-1.62500	0.76001	5.19726	3.96513	9.62846	1.01617
-1.51562	0.76398	5.55641	4.07863	9.62846	1.01617
-1.40625	0.76751	5.89362	4.17661	9.62846	1.01617
-1.29687	0.77066	6.21067	4.26227	9.62846	1.01617
-1.18750	0.77346	6.50911	4.33795	9.62846	1.01617
-1.07812	0.77598	6.79033	4.40541	9.62846	1.01617
-0.96875	0.77825	7.05558	4.46597	9.62846	1.01617
-0.85937	0.78030	7.30596	4.52068	9.62846	1.01617
-0.75000	0.78216	7.54247	4.57036	9.62846	1.01617
-0.64062	0.78385	7.76603	4.61566	9.62846	1.01617
-0.53125	0.78540	7.97746	4.65714	9.62846	1.01617
-0.42187	0.78681	8.17752	4.69524	9.62846	1.01617
-0.31250	0.78811	8.36689	4.73035	9.62846	1.01617
-0.20312	0.78930	8.54624	4.76277	9.62846	1.01617
-0.09375	0.79040	8.71614	4.79279	9.62846	1.01617
0.01563	0.79142	8.87715	4.82063	9.62846	1.01617
0.12500	0.79236	9.02977	4.84652	9.62846	1.01617
0.23438	0.79323	9.17449	4.87061	9.62846	1.01617
0.34375	0.79405	9.31174	4.89308	9.62846	1.01617
0.45313	0.79480	9.44195	4.91406	9.62846	1.01617
0.56250	0.79550	9.56549	4.93367	9.62846	1.01617
0.67188	0.79616	9.68273	4.95203	9.62846	1.01617
0.78125	0.79677	9.79401	4.96923	9.62846	1.01617
0.89063	0.79735	9.89965	4.98537	9.62846	1.01617
1.00000	0.79788	9.99996	5.00051	9.62846	1.01617

```
                    ==============================
                    DEMONSTRATION OUTPUT: SPMSTEST
                    ==============================

     UA          VA          UB          VB
     1.00        4.00        3.00        1.00
GEOMETRIC FACTORS
XMUL       2.00000
TMUL      -1.00000
PARALLEL LINES
P(U)=ALFA*INVSINH(QUE*U)        Q(V)=BETA*V
DPHI(U)=ALFA/VISA               DPSI(V)=BETA/VISB
SIG(Y)=Y*Y                      TAU(Z)=Z*Z
DPC/DS EXPONENT IS TWO
ALFA =    2.9      VISA =    1.0      QUE  =    1.0
BETA =    1.0      VISB =    1.0      ELL  =    1.0
XI EXTREMES    -2.499997     2.000000
DELTA XI        0.070312
                          SUCCESSIVE SUBSTITUTION
  U(HI+DH)   HV(HI+DH)    U(HM-DH)   V(HM-DH)    IT    MAX.ABS INCR.
   1.03125    3.95312     2.96874    1.04688     0
   1.13863    3.90218     2.99433    0.99293     1    0.991584E 00
   1.17219    3.89981     2.98591    1.00009     2    0.368733E 00
   1.17015    3.89942     2.98562    0.99905     3    0.175600E-01
   1.16989    3.89944     2.98571    0.99902     4    0.374699E-02
   1.16996    3.89944     2.98571    0.99903     5    0.275612E-03
   1.16994    3.89945     2.98570    0.99903     6    0.177383E-03
   1.16994    3.89944     2.98571    0.99903     7    0.629425E-04
   1.16994    3.89945     2.98570    0.99903     8    0.543594E-04
   1.16994    3.89944     2.98571    0.99903     9    0.476837E-04
   1.16993    3.89945     2.98570    0.99904    10    0.753403E-04
                          NEWTON'S METHOD
    WO        XO          UR          VR          WR          XR          UPA         VPA

  0.8870    0.0356    2.7447    0.7794    0.8870    0.0356    2.4168   -1.4301
  0.9370    0.0356    2.8452    0.7894    0.9370    0.0356    2.5674   -1.4301
  0.8870    0.0856    2.8306    1.6050    0.8870    0.0856    2.4168   -1.3159

  1.0038    0.0475    3.0001    1.0284    1.0038    0.0475    2.7685   -1.4028
  1.0538    0.0475    3.0994    1.0409    1.0538    0.0475    2.9190   -1.4028
  1.0038    0.0975    3.0949    1.8290    1.0038    0.0975    2.7685   -1.2886

  1.0054    0.0457    3.0001    0.9958    1.0054    0.0457    2.7734   -1.4069
```

DEMONSTRATION OUTPUT: SPMSTEST

ETA	Y	U	V	W	X
-2.50000	0.33836	1.00000	4.00000	1.00542	0.04574
-2.35937	0.41949	1.30382	3.79543	1.00542	0.04574
-2.21875	0.48649	1.50402	3.57945	1.00542	0.04574
-2.07812	0.54271	1.65091	3.35521	1.00542	0.04574
-1.93750	0.58976	1.76626	3.12640	1.00542	0.04574
-1.79687	0.62887	1.86137	2.89729	1.00542	0.04574
-1.65625	0.66118	1.94270	2.67241	1.00542	0.04574
-1.51562	0.68777	2.01422	2.45616	1.00542	0.04574
-1.37500	0.70961	2.07852	2.25240	1.00542	0.04574
-1.23437	0.72754	2.13732	2.06417	1.00542	0.04574
-1.09375	0.74226	2.19184	1.89350	1.00542	0.04574
-0.95312	0.75438	2.24295	1.74144	1.00542	0.04574
-0.81250	0.76437	2.29128	1.60812	1.00542	0.04574
-0.67187	0.77262	2.33730	1.49295	1.00542	0.04574
-0.53125	0.77948	2.38138	1.39480	1.00542	0.04574
-0.39062	0.78519	2.42379	1.31220	1.00542	0.04574
-0.25000	0.78997	2.46475	1.24348	1.00542	0.04574
-0.10937	0.79400	2.50444	1.18694	1.00542	0.04574
0.03125	0.79741	2.54299	1.14092	1.00542	0.04574
0.17188	0.80032	2.58050	1.10388	1.00542	0.04574
0.31250	0.80283	2.61707	1.07439	1.00542	0.04574
0.45313	0.80500	2.65277	1.05123	1.00542	0.04574
0.59375	0.80691	2.68765	1.03330	1.00542	0.04574
0.73438	0.80858	2.72178	1.01967	1.00542	0.04574
0.87500	0.81007	2.75519	1.00957	1.00542	0.04574
1.01563	0.81142	2.78792	1.00232	1.00542	0.04574
1.15625	0.81263	2.81999	0.99738	1.00542	0.04574
1.29687	0.81374	2.85144	0.99430	1.00542	0.04574
1.43749	0.81476	2.88229	0.99269	1.00542	0.04574
1.57811	0.81570	2.91256	0.99226	1.00542	0.04574
1.71873	0.81658	2.94227	0.99276	1.00542	0.04574
1.85935	0.81740	2.97144	0.99398	1.00542	0.04574
1.99997	0.81818	3.00008	0.99576	1.00542	0.04574

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                    ==============================
                    DEMONSTRATION OUTPUT: SPMSTEST
                    ==============================

     UA          VA          UB          VB
     1.00        2.00       10.00        5.00
GEOMETRIC FACTORS
XMUL       2.00000
TMUL      -1.00000
PARALLEL LINES
P(U)=ALFA*INVSINH(QUE*U)        Q(V)=BETA*V
DPHI(U)=ALFA/VISA               DPSI(V)=BETA/VISB
SIG(Y)=Y*Y                      TAU(Z)=Z*Z
DPC/DS EXPONENT IS TWO
ALFA =    2.9      VISA =    1.0      QUE  =    1.0
BETA =    1.0      VISB =    1.0      ELL  =    1.0
XI EXTREMES    -2.499997     2.000000
DELTA XI        0.070312
                          SUCCESSIVE SUBSTITUTION
  U(HI+DH)   HV(HI+DH)    U(HM-DH)   V(HM-DH)    IT    MAX.ABS INCR.
   1.14063    2.04688     9.85935    4.95311     0
   1.30995    2.01806     9.90435    4.99527     1    0.111977E 01
   1.29878    2.02703     9.89987    4.98232     2    0.679875E 00
   1.28725    2.02794     9.90733    4.97568     3    0.694808E 00
   1.30107    2.02282     9.90018    4.99544     4    0.496565E 00
   1.29319    2.02835     9.90365    4.97534     5    0.231609E 00
   1.29401    2.02535     9.90388    4.98554     6    0.252902E 00
   1.29778    2.02515     9.90155    4.98718     7    0.242076E 00
   1.29365    2.02713     9.90368    4.97946     8    0.151585E 00
   1.29556    2.02530     9.90293    4.98607     9    0.769348E-01
   1.29614    2.02591     9.90236    4.98418    10    0.984879E-01
                          NEWTON'S METHOD
   WO        XO        UR        VR        WR        XR        UPA       VPA

  4.0854   0.2846   9.3139   5.1038   4.0854   0.2846   4.2118   0.3685
  4.1354   0.2846   9.4072   5.1187   4.1354   0.2846   4.2652   0.3685
  4.0854   0.3346   9.6084   5.5558   4.0854   0.3346   4.2118   0.6367

  4.5362   0.2583   9.9874   4.9629   4.5362   0.2583   4.6932   0.2273
  4.5862   0.2583  10.0786   4.9759   4.5862   0.2583   4.7466   0.2273
  4.5362   0.3083  10.3010   5.4584   4.5362   0.3083   4.6932   0.4955

  4.5297   0.2622   9.9997   5.0026   4.5297   0.2622   4.6862   0.2483
```

DEMONSTRATION OUTPUT: SPMSTEST

ETA	Y	U	V	W	X
-2.50000	0.56820	1.00000	2.00000	4.52968	0.26220
-2.35937	0.66313	1.54336	2.07338	4.52968	0.26220
-2.21875	0.70352	1.97973	2.20863	4.52968	0.26220
-2.07812	0.72491	2.37231	2.38062	4.52968	0.26220
-1.93750	0.73767	2.74014	2.56901	4.52968	0.26220
-1.79687	0.74597	3.09136	2.76013	4.52968	0.26220
-1.65625	0.75178	3.43008	2.94580	4.52968	0.26220
-1.51562	0.75614	3.75859	3.12158	4.52968	0.26220
-1.37500	0.75963	4.07826	3.28545	4.52968	0.26220
-1.23437	0.76256	4.38996	3.43681	4.52968	0.26220
-1.09375	0.76514	4.69430	3.57592	4.52968	0.26220
-0.95312	0.76749	4.99170	3.70347	4.52968	0.26220
-0.81250	0.76966	5.28250	3.82035	4.52968	0.26220
-0.67187	0.77171	5.56696	3.92753	4.52968	0.26220
-0.53125	0.77366	5.84532	4.02598	4.52968	0.26220
-0.39062	0.77552	6.11779	4.11659	4.52968	0.26220
-0.25000	0.77730	6.38455	4.20021	4.52968	0.26220
-0.10937	0.77901	6.64579	4.27760	4.52968	0.26220
0.03125	0.78065	6.90166	4.34941	4.52968	0.26220
0.17188	0.78223	7.15234	4.41625	4.52968	0.26220
0.31250	0.78374	7.39797	4.47864	4.52968	0.26220
0.45313	0.78519	7.63871	4.53704	4.52968	0.26220
0.59375	0.78659	7.87468	4.59185	4.52968	0.26220
0.73438	0.78793	8.10603	4.64343	4.52968	0.26220
0.87500	0.78921	8.33289	4.69209	4.52968	0.26220
1.01563	0.79045	8.55538	4.73809	4.52968	0.26220
1.15625	0.79163	8.77361	4.78168	4.52968	0.26220
1.29687	0.79276	8.98770	4.82306	4.52968	0.26220
1.43749	0.79385	9.19776	4.86242	4.52968	0.26220
1.57811	0.79490	9.40389	4.89992	4.52968	0.26220
1.71873	0.79590	9.60620	4.93571	4.52968	0.26220
1.85935	0.79687	9.80478	4.96991	4.52968	0.26220
1.99997	0.79780	9.99972	5.00265	4.52968	0.26220

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DEMONSTRATION OUTPUT: SPMSTEST
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    UA          VA          UB          VB
    1.00        4.00        3.00        1.00
GEOMETRIC FACTORS
XMUL       1.00000
TMUL      -2.00000
PARALLEL LINES
P(U)=ALFA*INVSINH(QUE*U)        Q(V)=BETA*V
DPHI(U)=ALFA/VISA               DPSI(V)=BETA/VISB
SIG(Y)=Y*Y                      TAU(Z)=Z*Z
DPC/DS EXPONENT IS TWO
ALFA =    2.9      VISA =    1.0      QUE  =    1.0
BETA =    1.0      VISB =    1.0      ELL  =    1.0
XI EXTREMES    -4.999994     1.000000
DELTA XI        0.093750
                        SUCCESSIVE SUBSTITUTION
  U(HI+DH)   HV(HI+DH)   U(HM-DH)   V(HM-DH)    IT    MAX.ABS INCR.
   1.03125    3.95312    2.96875    1.04687     0
   1.98372    3.07053    3.01433    0.98651     1    0.350860E 01
   1.52232    2.87935    2.99592    1.00465     2    0.252706E 01
   1.57470    2.92079    3.00070    0.99851     3    0.499782E 00
   1.57279    2.91700    2.99918    1.00037     4    0.121087E 00
   1.57138    2.91699    2.99960    0.99980     5    0.309934E-01
   1.57207    2.91716    2.99949    0.99997     6    0.719184E-02
   1.57184    2.91709    2.99952    0.99992     7    0.161403E-02
   1.57191    2.91711    2.99951    0.99993     8    0.342250E-03
   1.57188    2.91710    2.99951    0.99993     9    0.715256E-04
   1.57191    2.91711    2.99951    0.99993    10    0.829697E-04
                        NEWTON'S METHOD
   WO        XO        UR        VR        WR        XR        UPA       VPA

  2.7022   0.2365   1.7298   0.5390   2.7022   0.2365   6.1004-11.5509
  2.7522   0.2365   1.7581   0.5431   2.7522   0.2365   6.2510-11.5509
  2.7022   0.2865   1.7389   0.6412   2.7022   0.2865   6.1004-11.4367

  4.9039   0.3753   2.9952   1.0258   4.9039   0.3753  12.7319-11.2339
  4.9539   0.3753   3.0231   1.0297   4.9539   0.3753  12.8825-11.2339
  4.9039   0.4253   3.0078   1.1414   4.9039   0.4253  12.7319-11.1197

  4.9178   0.3637   3.0000   0.9994   4.9178   0.3637  12.7736-11.2605
```

DEMONSTRATION OUTPUT: SPMSTEST

ETA	Y	U	V	W	X
-4.99999	0.33836	1.00000	4.00000	4.91776	0.36366
-4.81249	0.70789	1.79382	1.89683	4.91776	0.36366
-4.62499	0.77738	2.02673	1.01550	4.91776	0.36366
-4.43749	0.79187	2.19367	0.88645	4.91776	0.36366
-4.24999	0.79793	2.32848	0.89095	4.91776	0.36366
-4.06249	0.80195	2.43963	0.90888	4.91776	0.36366
-3.87499	0.80501	2.53182	0.92494	4.91776	0.36366
-3.68749	0.80740	2.60856	0.93805	4.91776	0.36366
-3.49999	0.80932	2.67257	0.94871	4.91776	0.36366
-3.31249	0.81086	2.72605	0.95741	4.91776	0.36366
-3.12499	0.81212	2.77081	0.96456	4.91776	0.36366
-2.93749	0.81315	2.80831	0.97046	4.91776	0.36366
-2.74999	0.81399	2.83975	0.97534	4.91776	0.36366
-2.56249	0.81469	2.86612	0.97939	4.91776	0.36366
-2.37499	0.81526	2.88827	0.98276	4.91776	0.36366
-2.18749	0.81574	2.90687	0.98557	4.91776	0.36366
-1.99999	0.81614	2.92250	0.98792	4.91776	0.36366
-1.81249	0.81647	2.93563	0.98988	4.91776	0.36366
-1.62499	0.81675	2.94668	0.99153	4.91776	0.36366
-1.43749	0.81698	2.95596	0.99291	4.91776	0.36366
-1.24999	0.81718	2.96377	0.99406	4.91776	0.36366
-1.06249	0.81734	2.97034	0.99503	4.91776	0.36366
-0.87499	0.81748	2.97587	0.99584	4.91776	0.36366
-0.68749	0.81759	2.98052	0.99652	4.91776	0.36366
-0.49999	0.81769	2.98444	0.99710	4.91776	0.36366
-0.31250	0.81777	2.98773	0.99758	4.91776	0.36366
-0.12500	0.81784	2.99050	0.99799	4.91776	0.36366
0.06250	0.81789	2.99284	0.99833	4.91776	0.36366
0.25000	0.81794	2.99480	0.99862	4.91776	0.36366
0.43750	0.81798	2.99645	0.99886	4.91776	0.36366
0.62500	0.81801	2.99784	0.99906	4.91776	0.36366
0.81250	0.81804	2.99901	0.99923	4.91776	0.36366
1.00000	0.81807	3.00000	0.99938	4.91776	0.36366

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                    DEMONSTRATION OUTPUT: SPMSTEST
                    ==============================

     UA          VA          UB          VB
     1.00        2.00       10.00        5.00
GEOMETRIC FACTORS
XMUL       1.00000
TMUL      -2.00000
PARALLEL LINES
P(U)=ALFA*INVSINH(QUE*U)        Q(V)=BETA*V
DPHI(U)=ALFA/VISA               DPSI(V)=BETA/VISB
SIG(Y)=Y*Y                      TAU(Z)=Z*Z
DPC/DS EXPONENT IS TWO
ALFA =    2.9      VISA =    1.0      QUE  =    1.0
BETA =    1.0      VISB =    1.0      ELL  =    1.0
XI EXTREMES    -4.999994     1.000000
DELTA XI        0.093750
                          SUCCESSIVE SUBSTITUTION
  U(HI+DH)   HV(HI+DH)    U(HM-DH)   V(HM-DH)    IT    MAX.ABS INCR.
   1.14063    2.04688     9.85938    4.95312     0
   2.34038    2.15085     9.99940    5.00510     1    0.579405E 01
   2.06656    2.38801     9.99432    4.99564     2    0.260961E 01
   2.12080    2.30718    10.00366    5.00175     3    0.304748E 01
   2.11979    2.32885     9.99062    5.00029     4    0.366350E 01
   2.09455    2.32644    10.01024    4.99599     5    0.400798E 01
   2.13360    2.31965     9.98591    5.00793     6    0.453465E 01
   2.07737    2.33235    10.01973    4.98695     7    0.479978E 01
   2.14557    2.31297     9.98036    5.01953     8    0.522049E 01
   2.05613    2.33861    10.03266    4.97367     9    0.540600E 01
   2.15922    2.30438     9.97445    5.03597    10    0.570794E 01
                          NEWTON'S METHOD
   WO        XO        UR        VR        WR        XR        UPA       VPA

 12.7136   2.3325    8.2324    5.0779   12.7136   2.3325   12.3650    3.2467
 12.7636   2.3325    8.2621    5.0833   12.7636   2.3325   12.4184    3.2467
 12.7136   2.3825    8.2578    5.1346   12.7136   2.3825   12.3650    3.5149

 16.0146   1.9512    9.9627    4.9046   16.0146   1.9512   15.8906    1.2015
 16.0645   1.9512    9.9917    4.9087   16.0645   1.9512   15.9440    1.2015
 16.0145   2.0012    9.9916    4.9734   16.0145   2.0012   15.8906    1.4697

 16.0093   2.0208   10.0000    4.9996   16.0093   2.0208   15.8851    1.5749
```

DEMONSTRATION OUTPUT: SPMSTEST

ETA	Y	U	V	W	X
-4.99999	0.56820	1.00000	2.00000	16.00932	2.02082
-4.81249	0.72089	2.88419	2.96975	16.00932	2.02082
-4.62499	0.74460	4.17940	3.62587	16.00932	2.02082
-4.43749	0.75847	5.20073	3.99617	16.00932	2.02082
-4.24999	0.76787	6.02391	4.23187	16.00932	2.02082
-4.06249	0.77452	6.69566	4.39650	16.00932	2.02082
-3.87499	0.77940	7.24824	4.51798	16.00932	2.02082
-3.68749	0.78308	7.70526	4.61072	16.00932	2.02082
-3.49999	0.78593	8.08468	4.68314	16.00932	2.02082
-3.31249	0.78816	8.40056	4.74063	16.00932	2.02082
-3.12499	0.78994	8.66408	4.78680	16.00932	2.02082
-2.93749	0.79137	8.88427	4.82422	16.00932	2.02082
-2.74999	0.79253	9.06849	4.85476	16.00932	2.02082
-2.56249	0.79347	9.22276	4.87982	16.00932	2.02082
-2.37499	0.79425	9.35206	4.90048	16.00932	2.02082
-2.18749	0.79489	9.46049	4.91758	16.00932	2.02082
-1.99999	0.79541	9.55147	4.93175	16.00932	2.02082
-1.81249	0.79585	9.62784	4.94354	16.00932	2.02082
-1.62499	0.79622	9.69196	4.95337	16.00932	2.02082
-1.43749	0.79652	9.74583	4.96157	16.00932	2.02082
-1.24999	0.79677	9.79108	4.96842	16.00932	2.02082
-1.06249	0.79698	9.82911	4.97415	16.00932	2.02082
-0.87499	0.79716	9.86107	4.97895	16.00932	2.02082
-0.68749	0.79731	9.88793	4.98296	16.00932	2.02082
-0.49999	0.79743	9.91052	4.98633	16.00932	2.02082
-0.31250	0.79754	9.92950	4.98916	16.00932	2.02082
-0.12500	0.79762	9.94547	4.99153	16.00932	2.02082
0.06250	0.79770	9.95889	4.99353	16.00932	2.02082
0.25000	0.79776	9.97018	4.99520	16.00932	2.02082
0.43750	0.79781	9.97967	4.99661	16.00932	2.02082
0.62500	0.79785	9.98765	4.99779	16.00932	2.02082
0.81250	0.79789	9.99436	4.99878	16.00932	2.02082
1.00000	0.79792	10.00001	4.99961	16.00932	2.02082

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            DEMONSTRATION LISTING: SPMSTEST
            ================================

C
C                    PROGRAM NAME: SPMSTEST
C             DEMONSTRATION OF TESTING OF CORRELATIONS
C                 MODIFIED : W. T. FORD : 10/17/83
C
      COMMON/NOTICE/ITM,NI,NMT,KUT,NOW
      COMMON/PARAMS/ALFA,VISA,QUE,BETA,VISB,ELL
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      COMMON/SMILES/XMUL,TMUL,LNORPR
      COMMON/VECTOR/U(129),V(129),UU(129),VV(129)
      COMMON/WORKER/AU(127),BU(127),CU(127),
     1              AV(127),BV(127),CV(127)
      DATA YZM /.01/
      CALL ERRSET (208,256,-1)
   10 FORMAT(6F5.1)
   11 FORMAT(5I5)
C
C PARAMETERS FOR CALCULATIONS OF PHYSICAL QUANTITIES
C
      READ(5,10) ALFA,VISA,QUE,BETA,VISB,ELL
C
C STABILIZING CONTROL PARAMETER
C
      READ(5,10)RAMBDA
C
C (X,T) DOMAIN VARIATION
C
      READ(5,10) XO,X1
C
C SIMILARITY - LINES OR PARABOLAS
C           PARALLEL LINES IF (LNORPR.EQ.0)
C           COAXIAL PARABOLAS IF (LNORPR.NE.0)
C
      READ(5,11)LNORPR
C
C MULTIPLE CASE RE-ENTRY : TERMINATE IF NDX.EQ.0
C
  500 READ(5,11) NDX,NDT,IRQT
      IF (NDX .EQ. 0) GOTO 600
C
C SIMILARITY PARAMETERS
C
      READ(5,10) UA,VA,UB,VB,XMUL,TMUL
   15 FORMAT ('1',4X,'UA',9X,'VA',9X,'UB',9X,'VB')
      WRITE (6,15)
   16 FORMAT ('0',4(F8.2,3X))
      WRITE (6,16) UA,VA,UB,VB
C
C OPERATIONAL CONTROL
C
      NI=64
```

```
      ================================
      DEMONSTRATION LISTING: SPMSTEST
      ================================

      KUT=2
      NMT=NI-1
      ITM=10
      DX = (X1-XO)/NDX
      DT = RAMBDA*DX**2
      TC=SIMPLE(TC)
      TC=HEADER(TC)
      TC = NDT*DT
      CALL INITSP (XOT,TP,TC,L)
  550 GOTO 500
  600 STOP
C
C       REQUIRED SUBPROGRAMS
C
C     SUBROUTINE DCOUPL (YU,ZV,U,V,Y,Z)
C     SUBROUTINE COEFFS (N)
C     FUNCTION HEADER (X)
C     SUBROUTINE INITSP (XP,TP,TF,L)
C     SUBROUTINE NEWTON (U,UP,YZM,L)
C     SUBROUTINE PROPS (U,UP,H,Y,IT)
C     SUBROUTINE RUNGE (U,UP,H,Y,DH)
C     FUNCTION SIMILE (X,T)
C     SUBROUTINE SPMSFD
C     SUBROUTINE SPMSIV (U,UP,UX,VX,YZM)
C     SUBROUTINE TRIPLE (A,B,C,D,N)
      END
```

```
================================
DEMONSTRATION LISTING: SPMSTEST
================================

      SUBROUTINE DCOUPL (YU,ZV,U,V,Y,Z)
C THE PRODUCTS YU AND ZV, BEING GIVEN, DCOUPL FINDS THE
C FACTORS, Y, U, Z AND V, CONSISTENT WITH THE PRODUCTS.
      DATA E,TOL,KMAX/.001,.00001,20/
C INITIAL ESTIMATE OF Y PRESUMED GIVEN IN CALL STATEMENT
      IF ((YU.LE.0.).OR.(ZV.LE.0.)) GOTO 90
      K=0
   20 Z = 1. - Y
      F = PC(Y) - PRESA(YU/Y) + PRESB(ZV/Z)
      IF (ABS(F).LE.TOL) GO TO 30
      K = K + 1
      IF (K.GE.KMAX) GO TO 30
      D = SIGN(E,F)
      S = Y - D
      T = 1. - S
      G = PC(S) - PRESA(YU/S) + PRESB(ZV/T)
      DY = D*F/(G-F)
      Y = Y + DY
      GO TO 20
   30 U = YU/Y
      V=ZV/Z
      RETURN
C
   89 FORMAT ('0PROBABLE INSTABILITY - FATAL ERROR')
   90 WRITE(6,89)
      STOP
C
      ENTRY DCOUIN (YU,ZV,U,V,Y,Z)
C THE QUANTITIES U AND V, BEING GIVEN, DCOUIN FINDS
C Y, Z, AND THE PRODUCTS, YU AND ZV.
      Y=PCINV(PRESA(U)-PRESB(V))
      Z=1.-Y
      YU=Y*U
      ZV=Z*V
      RETURN
      END
```

```
================================
DEMONSTRATION LISTING: SPMStest
================================

      SUBROUTINE COEFFS (N)
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      COMMON/VECTOR/U(129),V(129),UU(129),VV(129)
      COMMON/WORKER/AU(127),BU(127),CU(127),
     1              AV(127),BV(127),CV(127)
CREATE COEFFICIENTS FOR PAIR OF TRI-DIAGONAL SYSTEMS.
      NMT=N-2
      HC=HI
      I=-2
      GOTO 210
  200 PUL=PUR
      PVL=PVR
      QUL=QUR
      QVL=QVR
      HC=HC+DH
  205 UC=UR
      VC=VR
      YC=YR
      ZC=ZR
  210 I=I+1
      UR=U(I+2)
      VR=V(I+2)
      CALL DCOUIN (YU,ZV,UR,VR,YR,ZR)
      IF (I.LT.0) GOTO 205
      UBR=(UC+UR)/2.
      VBR=(VC+VR)/2.
      YBR=(YC+YR)/2.
      ZBR=(ZC+ZR)/2.
      HR=HC+DH/2.
      Q=SIMSEX(HR)
      PUR=Q*SIG(YBR)*DPHI(UBR)
      PVR=Q*TAU(ZBR)*DPSI(VBR)
      Q=SIMSEG(HR)
      QUR=Q*DH*YBR/2.
      QVR=Q*DH*ZBR/2.
      IF (I.EQ.0) GOTO 200
      Q=SIMSID(HC)
      BU(I)=QUR-QUL-PUL-PUR-Q*DH*DH*YC
      BV(I)=QVR-QVL-PVL-PVR-Q*DH*DH*ZC
      UU(I)=0.
      VV(I)=0.
      IF (I.NE.1) GOTO 220
      AU(1)=0.
      AV(1)=0.
      UU(1)=U(1)*(QUL-PUL)
      VV(1)=V(1)*(QVL-PVL)
  230 CU(I)=PUR+QUR
      CV(I)=PVR+QVR
      GOTO 200
  220 AU(I)=PUL-QUL
      AV(I)=PVL-QVL
```

DEMONSTRATION LISTING: SPMSTEST

```
IF (I.LT.NMT) GOTO 230
CU(I)=0.
CV(I)=0.
UU(I)=-U(N)*(PUR+QUR)
VV(I)=-V(N)*(PVR+QVR)
RETURN
END
```

```
            ================================
            DEMONSTRATION LISTING: SPMSTEST
            ================================

      FUNCTION HEADER (X)
      COMMON/PARAMS/ALFA,VISA,QUE,BETA,VISB,ELL
   10 FORMAT (30HOP(U)=ALFA*INVSINH(QUE*U)      ,
     1         30H Q(V)=BETA*V                   ,/,
     2         30H DPHI(U)=ALFA/VISA             ,
     3         30H DPSI(V)=BETA/VISB             ,/,
     4         30H SIG(Y)=Y*Y                    ,
     5         30H TAU(Z)=Z*Z                    ,/,
     6         30H DPC/DS EXPONENT IS TWO        )
C
CALCULATIONS OF PHYSICAL QUANTITIES FROM FUNCTIONS,
CORRELATIONS, ETC. ARE TO BE THRU THIS SUBPROGRAM ONLY.
CHANGE IN FORMULATION BELOW REQUIRES
CORRESPONDING CHANGE IN FORMAT ABOVE.
CONSISTENCY IS REQUIRED IN PC AND PCINV ENTRIES.
C
   11 FORMAT(7HOALFA =,F6.1,5X,6HVISA =,F6.1,5X,6HQUE  =,F6.1,
     1      /,7H BETA =,F6.1,5X,6HVISB =,F6.1,5X,6HELL  =,F6.1)
      WRITE(6,10)
      WRITE(6,11)ALFA,VISA,QUE,BETA,VISB,ELL
      HEADER=X
      RETURN
C
      ENTRY PRESA (X)
      Z=QUE*X
      PRESA=ALFA*ALOG(Z+SQRT(Z*Z+1))
      RETURN
C
      ENTRY PRESB (X)
      PRESB=BETA*X
      RETURN
C
      ENTRY DPHI  (X)
      DPHI=ALFA/VISA
      RETURN
C
      ENTRY DPSI  (X)
      DPSI=BETA/VISB
      RETURN
C
      ENTRY SIG   (X)
      SIG=X*X
      RETURN
C
      ENTRY TAU   (X)
      TAU=X*X
      RETURN
C
      ENTRY PCINV (X)
      Z=2.+(-X+SQRT(4.+X*X))
      PCINV=2./Z
```

```
==============================
DEMONSTRATION LISTING: SPMSTEST
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      SUBROUTINE NEWTON (U,UP,YZM,L)
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      DIMENSION U(4),UP(4),UT(3),VT(3)
      DATA CV,DWX/.00002,.05/
   15 FORMAT (1HO,23X,15HNEWTON'S METHOD,/,4X,
     1        2HWO,6X,2HXO,6X,2HUR,6X,2HVR,6X,
     2        2HWR,6X,2HXR,5X,3HUPA,5X,3HVPA)
   17 FORMAT (1H ,8F8.4)
      U(1)=UA
      U(2)=VA
      CALL PROPS (U,UP,HI,Y,-1)
      WRITE(6,15)
      L=0
      WA=U(3)
      XA=U(4)
  220 K=0
      WRITE(6,17)
  230 H=HI
      U(3)=WA
      U(4)=XA
      CALL PROPS (U,UP,H,Y,1)
      UPA=UP(1)
      VPA=UP(2)
  240 CALL RUNGE (U,UP,H,Y,DH)
      IF (Y*(1.-Y).LE.YZM) GOTO 38
      IF (H.LT.HS) GOTO 240
      K=K+1
      WRITE(6,17)WA,XA,U,UPA,VPA
      UT(K)=U(1)
      VT(K)=U(2)
      U(1)=UA
      U(2)=VA
      IF (K-2) 260,270,280
  260 L=L+1
      IF (L.EQ.11) GOTO 290
      Q=(UB-UT(1))**2+(VB-VT(1))**2
      IF (Q.LE.CV) GOTO 290
      WA=WA+DWX
      GOTO 230
  270 WA=WA-DWX
      XA=XA+DWX
      GOTO 230
  280 XA=XA-DWX
      A1=(UT(2)-UT(1))/DWX
      A2=(UT(3)-UT(1))/DWX
      A3=(VT(2)-VT(1))/DWX
      A4=(VT(3)-VT(1))/DWX
      D=A1*A4-A2*A3
      WA=WA+(A4/D)*(UB-UT(1))-(A2/D)*(VB-VT(1))
      XA=XA-(A3/D)*(UB-UT(1))+(A1/D)*(VB-VT(1))
      GOTO 220
```

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================================
DEMONSTRATION LISTING: SPMSTEST
================================

 38 L=-1
290 U(3)=WA
    U(4)=XA
    RETURN
    END
```

DEMONSTRATION LISTING: SPMSTEST

```
      SUBROUTINE PROPS (U,UP,H,Y,IT)
C
C  SUBROUTINE PROPS CALCULATES VALUES CORRESPONDING
C  TO U, V, X, W, Y, AND/OR Z DEPENDING ON THE VALUE
C  OF THE PARAMETER, IT. GENERALLY, WE ARE FINDING
C  STARTING OR INITIAL VALUES FOR RUNGE.  WE FIND
C  Y AND Z AND CERTAIN OTHER VALUES AT SOME NODE.
C
C  PROPS CALLS THE BUILT-IN FUNCTIONS
C  SIG(X), TAU(X), DPHI(X), DPSI(X) FROM HEADER.
C                  LIST OF VARIABLES
C  IT: VARIABLE TELLING US WHICH WAY TO GO.
C     IT<=-1 => WE KNOW U(1),U(2),UP(1),UP(2);
C             SO WE NEED  U(3),U(4),UP(3),UP(4).
C       WHERE (U(3)=W, U(4)=X, UP(3)=W', UP(4)=X')
C     IT=0,1 => WE KNOW U(1),U(2),U(3),U(4)
C             SO WE NEED UP(1),UP(2),UP(3),UP(4).
C  Y:  SATURATION OF OUR POROUS MEDIA. Y:(U,V)=>R
C  Z:  PART NOT SATURATED (OF THE POROUS MEDIA).
C  U:  VECTOR OF VALUES FOR U,V,W,AND X AT SOME H
C  UP: VECTOR OF VALUES FOR U',V',W',AND X' AT SOME H
C  H:  X-VALUE WHERE WE HAVE U AND V VALUES (IT<=1).
C
      DIMENSION U(4),UP(4)
      CALL DCOUIN (YU,ZV,U(1),U(2),Y,Z)
      Q=SIMSEX(H)
      E=Q*SIG(Y)*DPHI(U(1))
      F=Q*TAU(Z)*DPSI(U(2))
      Q=SIMSID(H)
      UP(3)=Q*YU
      UP(4)=Q*ZV
      HR=SIMSEG(H)
      IF (IT.GE.0) GOTO 10
      U(3)=E*UP(1)+HR*YU
      U(4)=F*UP(2)+HR*ZV
      GOTO 20
   10 UP(1)=(U(3)-HR*YU)/E
      UP(2)=(U(4)-HR*ZV)/F
   20 RETURN
      END
```

```
================================
DEMONSTRATION LISTING: SPMSTEST
================================

      SUBROUTINE RUNGE (U,UP,H,Y,DH)
      DIMENSION U(4),UP(4),UO(4),DU(4)
      DATA UMX/3600./
      DO 50 I=1,4
      UO(I)=U(I)
      DU(I)=UP(I)
   50 U(I)=UO(I)+DH*UP(I)/2.
      H=H+DH/2.
      CALL PROPS (U,UP,H,Y,1)
      DO 60 I=1,4
      DU(I)=DU(I)+2.*UP(I)
   60 U(I)=UO(I)+DH*UP(I)/2.
      CALL PROPS (U,UP,H,Y,1)
      DO 70 I=1,4
      DU(I)=DU(I)+2.*UP(I)
   70 U(I)=UO(I)+DH*UP(I)
      H=H+DH/2.
      CALL PROPS (U,UP,H,Y,1)
      DO 80 I=1,4
      DU(I)=DU(I)+UP(I)
   80 U(I)=UO(I)+DH*DU(I)/6.
      CALL PROPS (U,UP,H,Y,1)
      Q=U(1)*U(1)+U(2)*U(2)
      IF (Q.GE.UMX)Y=0.
      RETURN
      END
```

DEMONSTRATION LISTING: SPMSTEST

```
      FUNCTION SIMILE (X,T)
C THIS SUBPROGRAM DOES ALL SIMILARITY CURVE WORK
CURVES ARE PARALLEL LINES IF (LNORPR.EQ.0)
CURVES ARE COAXIAL PARABOLAS IF (LNORPR.NE.0)
      COMMON/SMILES/XMUL,TMUL,LNORPR
C SIMILE IS THE XI CORRESPONDING TO (X,T)
      IF (LNORPR.NE.0) GOTO 20
      SIMS=XMUL*X+TMUL*T
      GOTO 25
   20 SIMS=(X-XMUL)/SQRT(T-TMUL)
   25 SIMILE=SIMS
      RETURN
C SIMSEX IS THE SPMS INTEGRATING FACTOR
      ENTRY SIMSEX (XI)
      SIMS = 1.
      SIMSEX=SIMS
      RETURN
C SIMSEG IS G(XI)*SIMSEX
      ENTRY SIMSEG (XI)
      IF (LNORPR.NE.0) GOTO 40
      SIMS=-TMUL/(XMUL*XMUL)
      GOTO 45
   40 SIMS=.5*XI
   45 SIMSEG=SIMS
      RETURN
C SIMSID IS D(SIMSEG)/D(XI)
      ENTRY SIMSID (XI)
      IF (LNORPR.NE.0) GOTO 50
      SIMS=0.
      GOTO 55
   50 SIMS=.5
   55 SIMSID=SIMS
      RETURN
C SIMPLE PRINTS THE CURVE TYPE
      ENTRY SIMPLE (XI)
      WRITE(6,10)XMUL,TMUL
      IF (LNORPR.NE.0) GOTO 60
      WRITE(6,11)
      GOTO 65
   60 WRITE(6,12)
   65 SIMPLE=XI
      RETURN
   10 FORMAT ('0GEOMETRIC FACTORS',/,' XMUL    ',F10.5,/,
     1                               ' TMUL    ',F10.5)
   11 FORMAT (' PARALLEL LINES')
   12 FORMAT (' COAXIAL PARABOLAS')
      END
```

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DEMONSTRATION LISTING: SPMSTEST
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```

```
      SUBROUTINE SPMSFD
C
C  SUBROUTINE SPMSFD APPROXIMATES THE SLOPES OF THE
C  U AND V CURVES AT HI (INITIAL X VALUE) TO BE USED
C  IN THE SHOOTING METHOD.  THESE VALUES COME ABOUT
C  THROUGH SUCCESSIVE SUBSTITUTION METHODS.
C  SPMSFD FIRST USES A LINEAR APPROXIMATION OF U'S
C  AND V'S.  IT CALLS COEFFS TO GET COEFFICIENTS OF
C  THE TRI-DIAGONAL MATRIX AND CALLS TRIPLE TO
C  EVALUATE THE EQUATIONS AND GET NEW U'S AND V'S
C  AT SOME SUCCESSIVE SUBSTITUTION STEP.
C  SPMSFD PRINTS U AND V VALUES CALCULATED AT THE
C  SECOND AND NEXT-TO-LAST NODES FOR EACH
C  SUCCESSIVE SUBSTITUTION STEP.
C
C                    LIST OF VARIABLES
C  HL:  SIZE OF THE INTERVAL
C  H:   THE X-VALUE AT WHICH U AND V ARE CALCULATED
C     FOR THE FIRST ESTIMATE, I.E. LINEAR ESTIMATE.
C  U:   THE VALUE OF U AT SOME POINT X (NODES) FOR
C     ANY SUCCESSIVE SUBSTITUTION STEP
C  V:   VALUE OF V AT SOME POINT X (NODES) FOR ANY
C     SUCCESSIVE SUBSTITUTION STEP
C  I:   COUNTER FOR U AND V VECTORS AT ANY
C     SUCCESSIVE SUBSTITUTION STEP
C  NMT: NO. OF INTERIOR POINTS. ALSO NO. OF POINTS
C     TO BE CALCULATED.
C  J:   ALSO COUNTER FOR U AND V VECTORS. USED IN
C  FINDING NEXT U AND V VALUES. ONE MORE THAN I.
C N:   NUMBER OF X POINTS AT WHICH WE APPROXIMATE
C      U AND V VECTORS.
      COMMON/NOTICE/ITM,NI,NMT,KUT,NOW
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      COMMON/VECTOR/U(129),V(129),UU(129),VV(129)
      COMMON/WORKER/AU(127),BU(127),CU(127),
     1              AV(127),BV(127),CV(127)
   16 FORMAT (1H0,22X,23HSUCCESSIVE SUBSTITUTION,/,
     1 22H   U(HI+DH)   HV(HI+DH),
     2 42H   U(HM-DH)   V(HM-DH)   IT   MAX.ABS INCR.)
   17 FORMAT (1H ,4F10.5,I5,E15.6)
      HL=HM-HI
      H=HI
      U(1)=UA
      V(1)=VA
CHOOSE LINEAR INTERPOLATION FOR INITIAL ESTIMATES.
      DO 50 I=1,NMT
      H=H+DH
      J=I+1
      U(J)=((H-HI)*UB+(HM-H)*UA)/HL
      V(J)=((H-HI)*VB+(HM-H)*VA)/HL
   50 CONTINUE
```

```
      N=NI+1
      U(N)=UB
      V(N)=VB
      IT=0
      WRITE(6,16)
      WRITE(6,17)U(2),V(2),U(NI),V(NI),IT
COMPUTE COEFFICIENTS FOR TRI-DIAGONAL SYSTEMS.
   60 CALL COEFFS (N)
   65 CALL TRIPLE (AU,BU,CU,UU,NMT)
      CALL TRIPLE (AV,BV,CV,VV,NMT)
      ACT=0.
      DO 250 I=1,NMT
      J=I+1
      DU=UU(I)-U(J)
      ACT=AMAX1(ACT,ABS(DU))
      DV=VV(I)-V(J)
      ACT=AMAX1(ACT,ABS(DV))
      U(J)=UU(I)
  250 V(J)=VV(I)
      IT=IT+1
      WRITE(6,17)U(2),V(2),U(NI),V(NI),IT,ACT
      IF (IT.LT.ITM) GOTO 60
      RETURN
C
C IT: COUNTER FOR NUMBER OF SUCCESSIVE SUBSTITUTIONS
C NI: NUMBER OF INTERVALS
C HI: INITIAL ENDPOINT VALUE (X VALUE)
C HM: FINAL ENDPOINT VALUE (X VALUE)
C UA: INITIAL U VALUE AT INITIAL X VALUE, HI
C VA: INITIAL V VALUE AT INITIAL X VALUE, HI
C DH: DELTA H = DELTA X = INTERVAL SIZE
C UB: INITIAL U VALUE AT FINAL X VALUE, HM
C VB: INITIAL V VALUE AT FINAL X VALUE, HM
C AU: SUB-DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C BU: DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C CU: SUPER-DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C UU: VECTOR OF SOLUTIONS TO TRI-DIAGONAL EQUATIONS.
C AV: SUB-DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C BV: DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C CV: SUPER-DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C VV: VECTOR OF SOLUTIONS TO TRI-DIAGONAL EQUATIONS.
C ACT: THE MAXIMAL U OR V CHANGE FROM ONE
C SUCCESSIVE SUBSTITUTION TO THE NEXT.
C DU-THE CHANGE IN U AT THE JTH NODE FROM ONE
C SUCCESSIVE SUBSTITUTION TO THE NEXT.
C DV-THE CHANGE IN V AT THE JTH NODE FROM ONE
C SUCCESSIVE SUBSTITUTION TO THE NEXT.
C
      END
```

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DEMONSTRATION LISTING: SPMSTEST
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      SUBROUTINE SPMSIV (U,UP,UX,VX,YZM)
C
C SPMSIV FINDS THE U AND V VALUES FOR SOLUTION CURVES.
C IT CALLS OUTPUT TO GIVE A PRINTOUT OF INITIAL
C VALUES AND INFORMATION ABOUT THE PROBLEM.
C IT CALLS PROPS TO GET VALUES FOR
C                 U(3), U(4) (W=U(3),X=U(4)),
C UP(1), UP(2), UP(3), UP(4), AND Y.
C IT CALLS RUNGE FOR EACH H TO GET VALUES FOR U AND V
C               LIST OF VARIABLES
C U:  VALUES FOR U AND V AT ONE CALCULATION STEP
C UP:  IS THE VALUE OF THE DERIVATIVE OF U'S
C UX:  VALUES FOR U AT EACH RUNGE-KUTTA STEP
C VX:  VALUES FOR V AT EACH RUNGE-KUTTA STEP
C YZM:  .01:  USED HERE TO KNOW WHEN TO STOP RUNGE
C   QUIT IF      Y*(1 - Y) .LE. YZM
C I:  COUNTER FOR STORING U AND V VALUES
C H:  X VALUE AT WHICH RUNGE CALCULATES U AND V
C     MUST FIND W,X,W',X' (UB=UP(1)=U';VB=UP(2)=V')
C     (UB=UP(1)=INITIAL VALUE OF U AT HM;
C      VB=UP(2)=INITIAL VALUE OF V AT HM)
C Y:  SATURATION OBTAINED FROM PROPS FOR USE IN RUNGE.
C K:  COUNTER FOR CHECKING OUR DESIRE TO PRINT;
C                    PRINT IF K=KUT
C DH:  CHANGE IN X VALUE, H INCREMENT FOR
C       USE IN RUNGE STEP SIZE
C KUT:  THE NUMBER OF ITERATIONS DESIRED,
C           PRINTOUT TO PRINTOUT
C HS = HM - .25*DH . USED TO STOP U & V CALCULATIONS.
C
      COMMON/NOTICE/ITM,NI,NMT,KUT,NOW
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      DIMENSION UX(2), VX(2)
      DIMENSION U(4),UP(4)
   16 FORMAT (1H1,5X,3HETA,8X,1HY,9X,1HU,9X,
     1                 1HV,9X,1HW,9X,1HX)
   17 FORMAT (1H ,6F10.5)
      I=0
      CALL PROPS (U,UP,HI,Y,0)
   39 H=HI
      WRITE(6,16)
      WRITE(6,17)H,Y,U
C     MODIFY FORD PROGRAM
      I=I+1
      UX(I)=U(1)
      VX(I)=U(2)
      K=0
   40 CALL RUNGE (U,UP,H,Y,DH)
      IF (Y*(1.-Y).GT.YZM) GOTO 42
      WRITE(6,17)H,Y,U
      GOTO 38
```

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      DEMONSTRATION LISTING: SPMSTEST
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```

```
42 I=I+1
   UX(I)=U(1)
   VX(I)=U(2)
   K=K+1
   IF (K.LT.KUT) GOTO 50
   WRITE(6,17)H,Y,U
   K=0
50 IF (H.LT.HS) GOTO 40
   GOTO 30
37 FORMAT (33HOVANISHING PHASE; GO TO NEXT CASE)
38 WRITE(6,37)
30 RETURN
   END
```

```
================================
DEMONSTRATION LISTING: SPMSTEST
================================

      SUBROUTINE TRIPLE (A,B,C,D,N)
      DIMENSION A(2),B(2),C(2),D(2)
      D(1)=D(1)/B(1)
      IF (N.LT.2) GOTO 30
      DO 10 J=2,N
      I=J-1
      C(I)=C(I)/B(I)
      B(J)=B(J)-A(J)*C(I)
      D(J)=(D(J)-A(J)*D(I))/B(J)
   10 CONTINUE
      DO 20 K=2,N
      I=N-K+1
      J=I+1
      D(I)=D(I)-C(I)*D(J)
   20 CONTINUE
   30 RETURN
      END
```

SAMPLE INPUT : PROGRAM PMSTEST

```
   10 FORMAT (6F5.1)
   11 FORMAT (5I5)
C                      PROGRAM NAME : PMSTEST
      READ(5,10) ALFA,VISA,QUE,BETA,VISB,ELL
      READ(5,10) RAMBDA
      READ(5,10) XO,X1
      READ(5,11) LNORPR
      READ(5,11) NDX,NDT,IRQT
      READ(5,10) UA,VA,UB,VB,XMUL,TMUL
C
COLUMNS  1          2          3
C23456789012345678901234567890
C
//GO.SYSIN DD *
  2.9   1.   1.   1.   1.   1.
   .1
   0.   1.
    0
   10   50    2
   1.   4.   3.   1.   1.-  1.
  1.
//
```

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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
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NO. X INTERVALS =  10    LEFT U =  1.0000    RIGHT U =  3.0000
NO. T INTERVALS =  50    LEFT V =  4.0000    RIGHT V =  1.0000
LAMBDA =0.1000
GEOMETRIC FACTORS
XMUL       1.00000
TMUL      -1.00000
PARALLEL LINES
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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
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FUNCTIONS DEFINING PHYSICAL QUANTITIES
--------- -------- -------- ----------
      P(U) = ALFA*INVSINH(QUE*U)
      Q(V) = BETA*V
      DPHI(U) = ALFA/VISA
      DPSI(V) = BETA/VISB
      SIG(Y) = Y*Y
      TAU(Z) = Z*Z
      DPC/DS EXPONENT IS TWO

XI EXTREMES    -0.050000     1.000000
DELTA XI        0.016406
                          SUCCESSIVE SUBSTITJTION
  U(HI+DH)   HV(HI+DH)    U(HM-DH)   V(HM-DH)    IT    MAX.ABS INCR.
   1.03125    3.95313    2.96875    1.04688     0
   1.13497    3.90851    2.99373    0.99495     1    0.912348E 00
   1.16706    3.90648    2.98578    1.00033     2    0.352858E 00
   1.16534    3.90611    2.98548    0.99961     3    0.160713E-01
   1.16510    3.90614    2.98555    0.99957     4    0.333118E-02
   1.16513    3.90614    2.98555    0.99958     5    0.256538E-03
   1.16514    3.90614    2.98555    0.99958     6    0.391006E-04
   1.16514    3.90614    2.98555    0.99958     7    0.391006E-04
   1.16514    3.90614    2.98555    0.99958     8    0.181198E-04
   1.16514    3.90614    2.98555    0.99958     9    0.219345E-04
   1.16513    3.90614    2.98555    0.99958    10    0.314713E-04
                          NEWTON'S METHOD
    WO        XO        UR        VR        WR        XR       UPA       VPA

   3.6802   0.1420   2.7569   0.7854   3.6802   0.1420  10.0653  -5.7212
   3.7302   0.1420   2.7810   0.7877   3.7302   0.1420  10.2159  -5.7212
   3.6802   0.1920   2.7777   1.0129   3.6802   0.1920  10.0653  -5.6070

   4.1483   0.1845   2.9994   1.0071   4.1483   0.1845  11.4752  -5.6240
   4.1983   0.1845   3.0232   1.0100   4.1983   0.1845  11.6258  -5.6240
   4.1483   0.2345   3.0222   1.2282   4.1483   0.2345  11.4752  -5.5098

   4.1511   0.1829   3.0000   0.9997   4.1511   0.1829  11.4836  -5.6278
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==========
FULL DEMONSTRATION OUTPUT: PMSTEST
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ETA	Y	U	V	W	X
-0.05000	0.33836	1.00000	4.00000	4.15113	0.18290
-0.01719	0.41639	1.29637	3.80917	4.15113	0.18290
0.01563	0.48107	1.49461	3.60776	4.15113	0.18290
0.04844	0.53565	1.64141	3.39843	4.15113	0.18290
0.08125	0.58169	1.75737	3.18424	4.15113	0.18290
0.11406	0.62033	1.85330	2.96871	4.15113	0.18290
0.14687	0.65259	1.93548	2.75566	4.15113	0.18290
0.17969	0.67944	2.00779	2.54885	4.15113	0.18290
0.21250	0.70175	2.07278	2.35172	4.15113	0.18290
0.24531	0.72026	2.13217	2.16716	4.15113	0.18290
0.27812	0.73564	2.18720	1.99728	4.15113	0.18290
0.31094	0.74842	2.23872	1.84341	4.15113	0.18290
0.34375	0.75906	2.28739	1.70613	4.15113	0.18290
0.37656	0.76795	2.33370	1.58533	4.15113	0.18290
0.40937	0.77538	2.37802	1.48040	4.15113	0.18290
0.44219	0.78163	2.42063	1.39032	4.15113	0.18290
0.47500	0.78689	2.46176	1.31384	4.15113	0.18290
0.50781	0.79135	2.50159	1.24955	4.15113	0.18290
0.54062	0.79515	2.54027	1.19606	4.15113	0.18290
0.57344	0.79840	2.57790	1.15196	4.15113	0.18290
0.60625	0.80120	2.61459	1.11597	4.15113	0.18290
0.63906	0.80363	2.65042	1.08689	4.15113	0.18290
0.67187	0.80575	2.68543	1.06367	4.15113	0.18290
0.70468	0.80761	2.71970	1.04535	4.15113	0.18290
0.73750	0.80927	2.75326	1.03112	4.15113	0.18290
0.77031	0.81075	2.78616	1.02029	4.15113	0.18290
0.80312	0.81208	2.81842	1.01226	4.15113	0.18290
0.83593	0.81329	2.85007	1.00652	4.15113	0.18290
0.86875	0.81439	2.88114	1.00266	4.15113	0.18290
0.90156	0.81541	2.91165	1.00031	4.15113	0.18290
0.93437	0.81635	2.94162	0.99920	4.15113	0.18290
0.96718	0.81723	2.97107	0.99907	4.15113	0.18290
1.00000	0.81806	3.00001	0.99973	4.15113	0.18290

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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
==================================

FUNCTIONS DEFINING PHYSICAL QUANTITIES
--------- -------- -------- ----------
      P(U) = ALFA*INVSINH(QUE*U)
      Q(V) = BETA*V
      DPHI(U) = ALFA/VISA
      DPSI(V) = BETA/VISB
      SIG(Y) = Y*Y
      TAU(Z) = Z*Z
      DPC/DS EXPONENT IS TWO

VISCOSITY AND DENSITY PARAMETERS
--------- --- -------- ----------
      ALFA =    2.9      VISA =    1.0      QUE =    1.0
      BETA =    1.0      VISB =    1.0      ELL =    1.0

NUMERICAL INPUT VALUES
--------- ----- ------
      DT = 0.0010        DX = 0.1000      LAMBDA = 0.1000
      XO =    0.0        X1 =    1.0
      NT =     51        NX =     11
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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
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          COMPARE SPMS EXTENSION TO PMS SOLUTION
              WITH   2 TIME STEPS PER PRINT
 X       U PMS   U SPMS   DELT U    V PMS   V SPMS   DELT V
   T= 0.0
0.0              1.40855                  3.70483
0.100  1.81417  1.81417  0.0      3.06100  3.06100  0.0
0.200  2.04876  2.04876  0.0      2.42548  2.42548  0.0
0.300  2.22189  2.22189  0.0      1.89287  1.89287  0.0
0.400  2.36554  2.36554  0.0      1.50882  1.50882  0.0
0.500  2.49222  2.49222  0.0      1.26383  1.26383  0.0
0.600  2.60767  2.60767  0.0      1.12226  1.12226  0.0
0.700  2.71485  2.71485  0.0      1.04769  1.04769  0.0
0.800  2.81537  2.81537  0.0      1.01292  1.01292  0.0
0.900  2.91021  2.91021  0.0      1.00039  1.00039  0.0
1.000            3.00001                  0.99973
   T= 0.002
0.0              1.39651                  3.71710
0.100  1.80793  1.80838-0.00044   3.07478  3.07415  0.00063
0.200  2.04495  2.04483  0.00011  2.43817  2.43744  0.00073
0.300  2.21890  2.21878  0.00013  1.90256  1.90211  0.00045
0.400  2.36293  2.36286  0.00007  1.51528  1.51504  0.00024
0.500  2.48984  2.48981  0.00003  1.26771  1.26758  0.00013
0.600  2.60547  2.60545  0.00002  1.12440  1.12432  0.00007
0.700  2.71278  2.71277  0.00001  1.04876  1.04872  0.00004
0.800  2.81342  2.81342  0.00000  1.01337  1.01335  0.00002
0.900  2.90837  2.90836  0.00001  1.00051  1.00050  0.00001
1.000            2.99832                  0.99963
   T= 0.004
0.0              1.38421                  3.72934
0.100  1.80181  1.80252-0.00071   3.08862  3.08729  0.00133
0.200  2.04101  2.04089  0.00012  2.45086  2.44944  0.00141
0.300  2.21588  2.21565  0.00022  1.91232  1.91142  0.00090
0.400  2.36032  2.36017  0.00014  1.52181  1.52132  0.00050
0.500  2.48747  2.48739  0.00008  1.27164  1.27138  0.00027
0.600  2.60327  2.60323  0.00004  1.12656  1.12642  0.00014
0.700  2.71072  2.71070  0.00002  1.04984  1.04977  0.00008
0.800  2.81148  2.81146  0.00001  1.01383  1.01379  0.00004
0.900  2.90655  2.90652  0.00003  1.00064  1.00062  0.00002
1.000            2.99661                  0.99955
   T= 0.006
0.0              1.37165                  3.74155
0.100  1.79569  1.79661-0.00092   3.10247  3.10044  0.00202
0.200  2.03701  2.03692  0.00010  2.46357  2.46149  0.00209
0.300  2.21281  2.21252  0.00029  1.92213  1.92078  0.00135
0.400  2.35769  2.35748  0.00022  1.52841  1.52765  0.00076
0.500  2.48510  2.48497  0.00012  1.27562  1.27522  0.00040
0.600  2.60106  2.60100  0.00006  1.12876  1.12854  0.00022
0.700  2.70865  2.70862  0.00004  1.05094  1.05083  0.00012
0.800  2.80954  2.80950  0.00003  1.01430  1.01424  0.00006
0.900  2.90472  2.90466  0.00006  1.00076  1.00073  0.00003
1.000            2.99487                  0.99949
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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
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   T= 0.008
0.0             1.35882                     3.75372
0.100 1.78954 1.79063-0.00110 3.11629 3.11359 0.00269
0.200 2.03299 2.03292 0.00007 2.47633 2.47357 0.00276
0.300 2.20970 2.20937 0.00034 1.93201 1.93021 0.00181
0.400 2.35506 2.35478 0.00028 1.53507 1.53404 0.00102
0.500 2.48272 2.48255 0.00017 1.27964 1.27910 0.00055
0.600 2.59886 2.59877 0.00009 1.13098 1.13068 0.00030
0.700 2.70659 2.70654 0.00006 1.05206 1.05190 0.00016
0.800 2.80759 2.80755 0.00005 1.01478 1.01470 0.00008
0.900 2.90288 2.90281 0.00007 1.00090 1.00086 0.00004
1.000           2.99310                     0.99944
   T= 0.010
0.0             1.34572                     3.76585
0.100 1.78333 1.78459-0.00126 3.13007 3.12674 0.00333
0.200 2.02894 2.02891 0.00003 2.48913 2.48570 0.00344
0.300 2.20658 2.20621 0.00037 1.94196 1.93969 0.00227
0.400 2.35240 2.35207 0.00033 1.54178 1.54049 0.00129
0.500 2.48034 2.48012 0.00022 1.28372 1.28302 0.00070
0.600 2.59667 2.59654 0.00013 1.13324 1.13286 0.00038
0.700 2.70453 2.70445 0.00008 1.05320 1.05299 0.00020
0.800 2.80565 2.80558 0.00007 1.01527 1.01517 0.00010
0.900 2.90104 2.90096 0.00008 1.00103 1.00098 0.00004
1.000           2.99133                     0.99940
   T= 0.012
0.0             1.33235                     3.77795
0.100 1.77706 1.77849-0.00142 3.14381 3.13989 0.00392
0.200 2.02485 2.02486-0.00001 2.50197 2.49787 0.00411
0.300 2.20343 2.20304 0.00040 1.95198 1.94924 0.00274
0.400 2.34974 2.34936 0.00038 1.54856 1.54700 0.00156
0.500 2.47795 2.47769 0.00027 1.28783 1.28699 0.00085
0.600 2.59446 2.59430 0.00016 1.13553 1.13506 0.00046
0.700 2.70247 2.70236 0.00011 1.05435 1.05410 0.00025
0.800 2.80370 2.80362 0.00008 1.01577 1.01565 0.00012
0.900 2.89918 2.89911 0.00008 1.00117 1.00112 0.00005
1.000           2.98954                     0.99937
   T= 0.014
0.0             1.31870                     3.79001
0.100 1.77073 1.77231-0.00158 3.15751 3.15303 0.00448
0.200 2.02075 2.02079-0.00005 2.51485 2.51007 0.00477
0.300 2.20027 2.19985 0.00042 1.96207 1.95885 0.00322
0.400 2.34706 2.34664 0.00043 1.55541 1.55357 0.00184
0.500 2.47556 2.47525 0.00031 1.29200 1.29100 0.00100
0.600 2.59226 2.59206 0.00020 1.13784 1.13729 0.00055
0.700 2.70040 2.70027 0.00013 1.05552 1.05523 0.00029
0.800 2.80175 2.80166 0.00009 1.01628 1.01613 0.00015
0.900 2.89732 2.89725 0.00007 1.00131 1.00125 0.00006
1.000           2.98775                     0.99934
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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
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```
   T= 0.016
0.0             1.30477                     3.80204
0.100 1.76433 1.76607-0.00174 3.17118 3.16617 0.00500
0.200 2.01661 2.01670-0.00009 2.52775 2.52232 0.00544
0.300 2.19709 2.19666 0.00043 1.97223 1.96852 0.00370
0.400 2.34437 2.34391 0.00046 1.56232 1.56019 0.00213
0.500 2.47316 2.47281 0.00035 1.29621 1.29505 0.00116
0.600 2.59005 2.58981 0.00024 1.14018 1.13955 0.00063
0.700 2.69833 2.69818 0.00015 1.05671 1.05637 0.00034
0.800 2.79979 2.79969 0.00010 1.01680 1.01663 0.00017
0.900 2.89545 2.89539 0.00007 1.00146 1.00140 0.00006
1.000           2.98596                     0.99932
   T= 0.018
0.0             1.29056                     3.81404
0.100 1.75786 1.75975-0.00189 3.18482 3.17931 0.00550
0.200 2.01244 2.01258-0.00014 2.54069 2.53460 0.00610
0.300 2.19389 2.19345 0.00045 1.98245 1.97825 0.00420
0.400 2.34167 2.34117 0.00050 1.56929 1.56687 0.00242
0.500 2.47075 2.47036 0.00039 1.30047 1.29915 0.00133
0.600 2.58784 2.58757 0.00027 1.14256 1.14184 0.00072
0.700 2.69626 2.69608 0.00018 1.05792 1.05753 0.00039
0.800 2.79783 2.79772 0.00011 1.01733 1.01714 0.00019
0.900 2.89358 2.89353 0.00006 1.00162 1.00154 0.00007
1.000           2.98418                     0.99929
   T= 0.020
0.0             1.27604                     3.82600
0.100 1.75133 1.75337-0.00204 3.19842 3.19245 0.00598
0.200 2.00824 2.00844-0.00019 2.55366 2.54692 0.00675
0.300 2.19068 2.19023 0.00045 1.99274 1.98804 0.00470
0.400 2.33896 2.33843 0.00053 1.57633 1.57361 0.00272
0.500 2.46834 2.46791 0.00043 1.30478 1.30329 0.00149
0.600 2.58562 2.58531 0.00030 1.14497 1.14416 0.00081
0.700 2.69418 2.69399 0.00020 1.05915 1.05870 0.00044
0.800 2.79587 2.79574 0.00012 1.01787 1.01765 0.00022
0.900 2.89171 2.89166 0.00005 1.00178 1.00170 0.00008
1.000           2.98240                     0.99926
   T= 0.022
0.0             1.26117                     3.83792
0.100 1.74472 1.74691-0.00218 3.21200 3.20557 0.00643
0.200 2.00402 2.00426-0.00024 2.56666 2.55927 0.00739
0.300 2.18746 2.18699 0.00046 2.00310 1.99789 0.00521
0.400 2.33624 2.33568 0.00056 1.58343 1.58041 0.00302
0.500 2.46591 2.46545 0.00046 1.30914 1.30747 0.00167
0.600 2.58339 2.58306 0.00033 1.14741 1.14650 0.00091
0.700 2.69210 2.69188 0.00022 1.06039 1.05990 0.00049
0.800 2.79390 2.79377 0.00013 1.01842 1.01818 0.00024
0.900 2.88985 2.88979 0.00005 1.00195 1.00186 0.00009
1.000           2.98063                     0.99923
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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
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   T= 0.024
0.0             1.24589                     3.84980
0.100 1.73804 1.74037-0.00233 3.22555 3.21869 0.00686
0.200 1.99977 2.00006-0.00029 2.57968 2.57166 0.00802
0.300 2.18421 2.18375 0.00047 2.01352 2.00780 0.00572
0.400 2.33351 2.33292 0.00059 1.59061 1.58727 0.00334
0.500 2.46349 2.46299 0.00050 1.31354 1.31170 0.00184
0.600 2.58116 2.58080 0.00036 1.14988 1.14888 0.00100
0.700 2.69002 2.68978 0.00024 1.06166 1.06111 0.00055
0.800 2.79193 2.79179 0.00014 1.01898 1.01871 0.00027
0.900 2.88798 2.88793 0.00005 1.00212 1.00202 0.00011
1.000           2.97886                     0.99919
   T= 0.026
0.0             1.23016                     3.86163
0.100 1.73127 1.73376-0.00249 3.23906 3.23181 0.00725
0.200 1.99549 1.99583-0.00034 2.59273 2.58409 0.00865
0.300 2.18095 2.18049 0.00047 2.02401 2.01777 0.00624
0.400 2.33077 2.33016 0.00061 1.59785 1.59418 0.00366
0.500 2.46105 2.46052 0.00053 1.31800 1.31598 0.00202
0.600 2.57893 2.57854 0.00039 1.15238 1.15128 0.00110
0.700 2.68793 2.68767 0.00026 1.06294 1.06234 0.00060
0.800 2.78996 2.78981 0.00015 1.01955 1.01926 0.00029
0.900 2.88611 2.88606 0.00006 1.00231 1.00219 0.00012
1.000           2.97709                     0.99916
   T= 0.028
0.0             1.21393                     3.87341
0.100 1.72440 1.72706-0.00266 3.25254 3.24492 0.00762
0.200 1.99118 1.99158-0.00040 2.60581 2.59655 0.00926
0.300 2.17768 2.17721 0.00047 2.03457 2.02780 0.00677
0.400 2.32802 2.32739 0.00063 1.60515 1.60116 0.00399
0.500 2.45861 2.45805 0.00055 1.32251 1.32030 0.00221
0.600 2.57669 2.57627 0.00041 1.15492 1.15371 0.00120
0.700 2.68584 2.68556 0.00028 1.06424 1.06359 0.00065
0.800 2.78799 2.78783 0.00016 1.02013 1.01981 0.00032
0.900 2.88425 2.88418 0.00007 1.00249 1.00236 0.00013
1.000           2.97532                     0.99913
   T= 0.030
0.0             1.19715                     3.88514
0.100 1.71742 1.72029-0.00287 3.26597 3.25801 0.00796
0.200 1.98683 1.98729-0.00047 2.61890 2.60904 0.00986
0.300 2.17439 2.17393 0.00046 2.04519 2.03789 0.00730
0.400 2.32526 2.32461 0.00065 1.61252 1.60820 0.00433
0.500 2.45616 2.45558 0.00058 1.32706 1.32467 0.00239
0.600 2.57444 2.57401 0.00044 1.15749 1.15618 0.00131
0.700 2.68375 2.68345 0.00030 1.06556 1.06485 0.00071
0.800 2.78602 2.78584 0.00018 1.02073 1.02038 0.00035
0.900 2.88239 2.88231 0.00008 1.00268 1.00254 0.00014
1.000           2.97356                     0.99910
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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
==================================

   T= 0.032
0.0             1.17977                    3.89681
0.100 1.71032 1.71342-0.00310 3.27936 3.27110 0.00826
0.200 1.98243 1.98298-0.00055 2.63200 2.62156 0.01044
0.300 2.17108 2.17063 0.00045 2.05587 2.04803 0.00784
0.400 2.32250 2.32182 0.00067 1.61996 1.61529 0.00467
0.500 2.45370 2.45310 0.00061 1.33167 1.32908 0.00259
0.600 2.57220 2.57173 0.00046 1.16009 1.15868 0.00141
0.700 2.68166 2.68134 0.00032 1.06690 1.06613 0.00076
0.800 2.78405 2.78386 0.00019 1.02133 1.02096 0.00037
0.900 2.88052 2.88043 0.00008 1.00288 1.00273 0.00016
1.000           2.97179                    0.99908
   T= 0.034
0.0             1.16173                    3.90842
0.100 1.70308 1.70648-0.00339 3.29270 3.28418 0.00852
0.200 1.97799 1.97863-0.00064 2.64513 2.63412 0.01100
0.300 2.16775 2.16731 0.00044 2.06661 2.05824 0.00838
0.400 2.31972 2.31903 0.00069 1.62747 1.62245 0.00502
0.500 2.45124 2.45061 0.00063 1.33633 1.33354 0.00279
0.600 2.56995 2.56946 0.00049 1.16272 1.16120 0.00152
0.700 2.67956 2.67922 0.00034 1.06826 1.06744 0.00082
0.800 2.78208 2.78187 0.00021 1.02194 1.02154 0.00040
0.900 2.87865 2.87856 0.00010 1.00309 1.00292 0.00017
1.000           2.97001                    0.99906
   T= 0.036
0.0             1.14304                    3.91997
0.100 1.69571 1.69944-0.00374 3.30599 3.29724 0.00875
0.200 1.97350 1.97426-0.00076 2.65826 2.64671 0.01155
0.300 2.16440 2.16398 0.00042 2.07741 2.06850 0.00891
0.400 2.31693 2.31623 0.00070 1.63505 1.62966 0.00539
0.500 2.44878 2.44812 0.00066 1.34104 1.33805 0.00299
0.600 2.56769 2.56718 0.00051 1.16539 1.16376 0.00163
0.700 2.67746 2.67710 0.00036 1.06964 1.06876 0.00088
0.800 2.78010 2.77988 0.00022 1.02257 1.02214 0.00043
0.900 2.87678 2.87668 0.00011 1.00330 1.00311 0.00018
1.000           2.96823                    0.99904
   T= 0.038
0.0             1.12375                    3.93147
0.100 1.68818 1.69232-0.00413 3.31923 3.31029 0.00894
0.200 1.96896 1.96985-0.00090 2.67140 2.65933 0.01207
0.300 2.16103 2.16064 0.00039 2.08827 2.07882 0.00945
0.400 2.31413 2.31342 0.00071 1.64268 1.63694 0.00575
0.500 2.44630 2.44563 0.00068 1.34580 1.34260 0.00320
0.600 2.56543 2.56490 0.00054 1.16809 1.16635 0.00174
0.700 2.67535 2.67497 0.00038 1.07104 1.07010 0.00094
0.800 2.77812 2.77788 0.00024 1.02321 1.02275 0.00047
0.900 2.87491 2.87479 0.00012 1.00351 1.00332 0.00020
1.000           2.96645                    0.99903
```

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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
==================================

   T= 0.040
0.0             1.10395                     3.94293
0.100 1.68052 1.68510-0.00458 3.33244 3.32333 0.00911
0.200 1.96436 1.96542-0.00106 2.68454 2.67198 0.01256
0.300 2.15763 2.15728 0.00034 2.09918 2.08920 0.00998
0.400 2.31131 2.31060 0.00071 1.65039 1.64427 0.00612
0.500 2.44382 2.44313 0.00070 1.35061 1.34720 0.00341
0.600 2.56316 2.56261 0.00056 1.17082 1.16897 0.00186
0.700 2.67325 2.67285 0.00040 1.07246 1.07146 0.00100
0.800 2.77614 2.77589 0.00025 1.02386 1.02337 0.00050
0.900 2.87303 2.87291 0.00012 1.00374 1.00352 0.00021
1.000           2.96467                     0.99902
   T= 0.042
0.0             1.08371                     3.95437
0.100 1.67273 1.67778-0.00505 3.34563 3.33636 0.00927
0.200 1.95970 1.96095-0.00125 2.69769 2.68465 0.01303
0.300 2.15420 2.15391 0.00029 2.11015 2.09964 0.01051
0.400 2.30848 2.30777 0.00071 1.65816 1.65167 0.00650
0.500 2.44133 2.44062 0.00071 1.35548 1.35185 0.00363
0.600 2.56090 2.56032 0.00058 1.17359 1.17162 0.00197
0.700 2.67114 2.67072 0.00041 1.07390 1.07284 0.00107
0.800 2.77415 2.77389 0.00027 1.02453 1.02400 0.00053
0.900 2.87115 2.87102 0.00013 1.00397 1.00374 0.00023
1.000           2.96288                     0.99901
   T= 0.044
0.0             1.06312                     3.96578
0.100 1.66484 1.67037-0.00553 3.35880 3.34937 0.00943
0.200 1.95499 1.95645-0.00146 2.71084 2.69736 0.01349
0.300 2.15074 2.15053 0.00021 2.12116 2.11013 0.01103
0.400 2.30564 2.30494 0.00070 1.66600 1.65912 0.00687
0.500 2.43883 2.43811 0.00072 1.36040 1.35655 0.00385
0.600 2.55862 2.55803 0.00060 1.17640 1.17430 0.00209
0.700 2.66902 2.66859 0.00043 1.07537 1.07423 0.00113
0.800 2.77216 2.77189 0.00028 1.02520 1.02464 0.00057
0.900 2.86927 2.86913 0.00014 1.00420 1.00396 0.00025
1.000           2.96109                     0.99901
   T= 0.046
0.0             1.04225                     3.97718
0.100 1.65687 1.66286-0.00599 3.37199 3.36237 0.00962
0.200 1.95023 1.95191-0.00168 2.72402 2.71009 0.01393
0.300 2.14725 2.14713 0.00013 2.13223 2.12068 0.01155
0.400 2.30277 2.30209 0.00068 1.67390 1.66664 0.00726
0.500 2.43633 2.43559 0.00073 1.36537 1.36129 0.00408
0.600 2.55634 2.55573 0.00061 1.17924 1.17702 0.00222
0.700 2.66690 2.66645 0.00045 1.07685 1.07565 0.00120
0.800 2.77017 2.76988 0.00029 1.02589 1.02529 0.00060
0.900 2.86738 2.86724 0.00014 1.00444 1.00419 0.00026
1.000           2.95930                     0.99901
```

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==================================
FULL DEMONSTRATION OUTPUT: PMSTEST
==================================

   T= 0.048
0.0             1.02118                 3.98858
0.100 1.64884 1.65524-0.00640 3.38519 3.37535 0.00984
0.200 1.94544 1.94734-0.00191 2.73721 2.72285 0.01436
0.300 2.14375 2.14371 0.00004 2.14335 2.13129 0.01206
0.400 2.29990 2.29924 0.00065 1.68186 1.67422 0.00764
0.500 2.43381 2.43307 0.00074 1.37039 1.36609 0.00431
0.600 2.55406 2.55343 0.00063 1.18212 1.17977 0.00234
0.700 2.66478 2.66432 0.00047 1.07835 1.07709 0.00126
0.800 2.76818 2.76788 0.00030 1.02659 1.02595 0.00064
0.900 2.86550 2.86535 0.00015 1.00469 1.00442 0.00027
1.000           2.95750                 0.99902
   T= 0.050
0.0             1.00000                 4.00000
0.100 1.64077 1.64752-0.00675 3.39842 3.38831 0.01010
0.200 1.94062 1.94274-0.00212 2.75043 2.73563 0.01480
0.300 2.14021 2.14028-0.00006 2.15453 2.14195 0.01258
0.400 2.29700 2.29638 0.00062 1.68989 1.68186 0.00803
0.500 2.43128 2.43055 0.00073 1.37547 1.37093 0.00454
0.600 2.55176 2.55112 0.00064 1.18503 1.18256 0.00247
0.700 2.66266 2.66217 0.00048 1.07988 1.07855 0.00133
0.800 2.76618 2.76587 0.00032 1.02730 1.02663 0.00067
0.900 2.86361 2.86345 0.00015 1.00495 1.00466 0.00029
1.000           2.95570                 0.99903
```

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          ====================================
          FULL DEMONSTRATION LISTING: PMSTEST
          ====================================

C
C                    PROGRAM NAME : PMSTEST
C              DEMONSTRATION OF FULL TESTING OF MODEL
C                 MODIFIED : W. T. FORD : 10/17/83
C
      COMMON/ARAYS/ XPTS(129),UOLD(129),VOLD(129),
     1                          UNEW(129),VNEW(129)
      COMMON/INPUT/ DT,DX,XO,X1,NX,NDX,NT,IRQT,RAMBDA
      COMMON/NOTICE/ITM,NDH,NMT,KUT,NOW
      COMMON/PARAMS/ ALFA,VISA,QUE,BETA,VISB,ELL
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      COMMON/SMILES/XMUL,TMUL,LNORPR
      COMMON/VECTOR/U(129),V(129),UU(129),VV(129)
      COMMON/WORKER/AU(127),BU(127),CU(127),
     1                AV(127),BV(127),CV(127)
      CALL ERRSET (208,256,-1)
   10 FORMAT(6F5.1)
   11 FORMAT(5I5)
C
C PARAMETERS FOR CALCULATIONS OF PHYSICAL QUANTITIES
C
      READ(5,10) ALFA,VISA,QUE,BETA,VISB,ELL
C
C STABILIZING CONTROL PARAMETER
C
      READ(5,10)RAMBDA
C
C (X,T) DOMAIN VARIATION
C
      READ(5,10) XO,X1
C
C SIMILARITY - LINES OR PARABOLAS
C          PARALLEL LINES IF (LNORPR.EQ.0)
C          COAXIAL PARABOLAS IF (LNORPR.NE.0)
C
      READ(5,11)LNORPR
      READ(5,11) NDX,NDT,IRQT
      JRQT=0
C
C SIMILARITY PARAMETERS
C
      READ(5,10) UA,VA,UB,VB,XMUL,TMUL
C
C OPERATIONAL CONTROL
C
      NX = NDX + 1
      NT = NDT + 1
      DX = (X1 - XO)/NDX
      DT = RAMBDA * (DX**2)
      CALL INITPM
C
```

```
      ZERO = 0.0
      X = DX
C
C  USES OF BOUNDU AND BOUNDV ARE COMMENTS RECALLING
C  INITIAL AND BOUNDARY CONDITIONS IN MODEL TESTED
C
C  IMPOSE SIMILARITY INITIAL VALUES
C
      DO 20 I = 2,NDX
      CALL SPSVAL (X,ZERO,UOLD(I),VOLD(I))
C        UOLD(I) = BOUNDU(X,ZERO)
C        VOLD(I) = BOUNDV(X,ZERO)
         X = X + DX
   20 CONTINUE
C
C  IMPOSE SIMILARITY CORNER VALUES
C
      CALL SPSVAL (X0,ZERO,UOLD(1),VOLD(1))
C     UOLD(1) = BOUNDU(X0,ZERO)
C     VOLD(1) = BOUNDV(X0,ZERO)
      CALL SPSVAL (X1,ZERO,UOLD(NX),VOLD(NX))
C     UOLD(NX) = BOUNDU(X1,ZERO)
C     VOLD(NX) = BOUNDV(X1,ZERO)
C
C  PRINT TITLE PAGE, FUNCTIONS, INPUT VALUES
C
      CALL TITLE
C
C SOLVE PROBLEM
C
      T = DT
      DO 21 I = 2,NT
         CALL SOLVE(DT,DX,NT,NDX,UOLD,VOLD,UNEW,VNEW)
         DO 22 J = 2,NDX
            UOLD(J) = UNEW(J)
            VOLD(J) = VNEW(J)
   22    CONTINUE
C
C  IMPOSE SIMILARITY BOUNDARY VALUES
C
      CALL SPSVAL (X0,T,UOLD(1),VOLD(1))
C        UOLD(1) = BOUNDU(X0,T)
C        VOLD(1) = BOUNDV(X0,T)
      CALL SPSVAL (X1,T,UOLD(NX),VOLD(NX))
C        UOLD(NX) = BOUNDU(X1,T)
C        VOLD(NX) = BOUNDV(X1,T)
C
      JRQT=JRQT+1
      IF(JRQT.NE.IRQT)GOTO 111
         CALL OUTSPS(T,NX,UOLD,VOLD,XPTS)
      JRQT=0
```

```
C
  111     T = T + DT
   21 CONTINUE
C
      END
```

```
      ====================================
      FULL DEMONSTRATION LISTING: PMSTEST
      ====================================

      SUBROUTINE BURTON (YU,ZV,U,V,Y,Z)
C THE PRODUCTS YU AND ZV, BEING GIVEN, BURTON FINDS THE
C FACTORS, Y, U, Z AND V, CONSISTENT WITH THE PRODUCTS.
      DATA E,TOL,KMAX/.001,.00001,20/
      K=0
   20 Z = 1.  -Y
      F = PC(Y) - PRESA(YU/Y) + PRESB(ZV/Z)
      IF (ABS(F).LE.TOL) GO TO 30
      K = K + 1
      IF (K.LT.KMAX) GO TO 25
   22   CONTINUE
        P1=PC(Y)
        P2=PRESA(YU/Y)
        P3=PRESB(ZV/Z)
        U1=YU/Y
        V1=ZV/Z
        WRITE(6,24)
        WRITE(6,23) F,G,Y,P1,P2,P3
        WRITE(6,36)
        WRITE(6,35) U1,V1,K,YU,ZV
   35   FORMAT(' ',2(5X,F12.9),2X,I2,2(5X,F12.9))
   36   FORMAT(' ',8X,'U',16X,'V',10X,'K',
     1                    10X,'YU',14X,'ZV')
   23   FORMAT(' ',6(5X,F12.9))
   24   FORMAT(8X,'F',16X,'G',16X,'Y',14X,
     1               'PC(Y)',10X,'PRESA(YU/Y)',
     X       10X,'PRESB(ZV/Z)',/)
        STOP
   25 CONTINUE
      D = SIGN(E,F)
      S = Y - D
      T = 1. - S
      G = PC(S) - PRESA(YU/S) + PRESB(ZV/T)
      IF (ABS(G-F) .LT. TOL) GOTO 22
      DY = D*F/(G-F)
      Y = Y + DY
      IF (Y*(1.-Y) .LT. 0.) GOTO 70
      GO TO 20
   30 U = YU/Y
      V=ZV/Z
      RETURN
   70 WRITE(6,71)
   71 FORMAT(' ',5X,'OUT OF INTERVAL')
      RETURN
      ENTRY BURINV (YU,ZV,U,V,Y,Z)
C THE QUANTITIES U AND V, BEING GIVEN, BURINV FINDS
C Y, Z, AND THE PRODUCTS, YU AND ZV.
      Y=PCINV(PRESA(U)-PRESB(V))
      Z=1.-Y
      YU=Y*U
      ZV=Z*V
```

```
RETURN
END
```

```
      ====================================
      FULL DEMONSTRATION LISTING: PHYSICS
      ====================================

      FUNCTION HEADER (X)
      COMMON/PARAMS/ ALFA,VISA,QUE,BETA,VISB,ELL
   10 FORMAT('1','FUNCTIONS DEFINING PHYSICAL',
     1            ' QUANTITIES',/,
     2           ' ---------- -------- --------',
     3            ' ----------',/,
     4      6X,31HP(U) = ALFA*INVSINH(QUE*U)      ,/,
     5      6X,31HQ(V) = BETA*V                   ,/,
     6      6X,31HDPHI(U) = ALFA/VISA             ,/,
     7      6X,31HDPSI(V) = BETA/VISB             ,/,
     8      6X,31HSIG(Y) = Y*Y                    ,/,
     9      6X,31HTAU(Z) = Z*Z                    ,/,
     A      6X,31HDPC/DS EXPONENT IS TWO          ,/)
C
CALCULATIONS OF PHYSICAL QUANTITIES FROM FUNCTIONS,
CORRELATIONS, ETC. ARE TO BE THRU THIS SUBPROGRAM ONLY.
CHANGE IN FORMULATION BELOW REQUIRES
CORRESPONDING CHANGE IN FORMAT ABOVE.
CONSISTENCY IS REQUIRED IN PC AND PCINV ENTRIES.
C
      WRITE(6,10)
      HEADER=X
      RETURN
C
      ENTRY PRESA (X)
      Z=QUE*X
      PRESA=ALFA*ALOG(Z+SQRT(Z*Z+1))
      RETURN
C
      ENTRY PRESB (X)
      PRESB=BETA*X
      RETURN
C
      ENTRY DPHI  (X)
      DPHI=ALFA/VISA
      RETURN
C
      ENTRY DPSI  (X)
      DPSI=BETA/VISB
      RETURN
C
      ENTRY SIG   (X)
      SIG=X*X
      RETURN
C
      ENTRY TAU   (X)
      TAU=X*X
      RETURN
C
      ENTRY PCINV (X)
      Z=2.+(-X+SQRT(4.+X*X))
```

```
      PCINV=2./Z
      RETURN
C
      ENTRY PC (X)
      PC=1./(1.-X)-1./X
      RETURN
      END
```

```
      ====================================
      FULL DEMONSTRATION LISTING: PMSTEST
      ====================================

      SUBROUTINE SOLVE (DT,DX,NT,NDX,UOLD,VOLD,UNEW,VNEW)
      DIMENSION UOLD(129),UNEW(129),VOLD(129),VNEW(129)
      RAMBDA=DT/DX**2
      J=0
      GOTO 15
   11 DQ=EQ
      DR=ER
   13 J=J+1
      UC=UR
      VC=VR
      YC=YR
      ZC=ZR
   15 UR=UOLD(J+1)
      VR=VOLD(J+1)
      CALL BURINV (YU,ZV,UR,VR,YR,ZR)
      IF (J.LT.1) GOTO 13
      UBR=(UC+UR)/2.
      VBR=(VC+VR)/2.
      YBR=(YC+YR)/2.
      ZBR=1.-YBR
      EQ=SIG(YBR)*DPHI(UBR)*(UR-UC)
      ER=TAU(ZBR)*DPSI(VBR)*(VR-VC)
      IF (J.EQ.1) GOTO 11
      YU=YC*UC+RAMBDA*(EQ-DQ)
      ZV=ZC*VC+RAMBDA*(ER-DR)
      Y = YR
      CALL BURTON (YU,ZV,UNEW(J),VNEW(J),Y,Z)
      IF (J.LT.NDX) GOTO 11
      RETURN
      END
```

```
      ====================================
      FULL DEMONSTRATION LISTING: PMSTEST
      ====================================

      SUBROUTINE COEFFS (N)
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      COMMON/VECTOR/U(129),V(129),UU(129),VV(129)
      COMMON/WORKER/AU(127),BU(127),CU(127),
     1              AV(127),BV(127),CV(127)
C     CREATE COEFFICIENTS FOR PAIR OF TRI-DIAGONAL SYSTEMS.
      NMT=N-2
      HC=HI
      I=-2
      GOTO 210
  200 PUL=PUR
      PVL=PVR
      QUL=QUR
      QVL=QVR
      HC=HC+DH
  205 UC=UR
      VC=VR
      YC=YR
      ZC=ZR
  210 I=I+1
      UR=U(I+2)
      VR=V(I+2)
      CALL DCOUIN (YU,ZV,UR,VR,YR,ZR)
      IF (I.LT.0) GOTO 205
      UBR=(UC+UR)/2.
      VBR=(VC+VR)/2.
      YBR=(YC+YR)/2.
      ZBR=(ZC+ZR)/2.
      HR=HC+DH/2.
      Q=SIMSEX(HR)
      PUR=Q*SIG(YBR)*DPHI(UBR)
      PVR=Q*TAU(ZBR)*DPSI(VBR)
      Q=SIMSEG(HR)
      QUR=Q*DH*YBR/2.
      QVR=Q*DH*ZBR/2.
      IF (I.EQ.0) GOTO 200
      Q=SIMSID(HC)
      BU(I)=QUR-QUL-PUL-PUR-Q*DH*DH*YC
      BV(I)=QVR-QVL-PVL-PVR-Q*DH*DH*ZC
      UU(I)=0.
      VV(I)=0.
      IF (I.NE.1) GOTO 220
      AU(1)=0.
      AV(1)=0.
      UU(1)=U(1)*(QUL-PUL)
      VV(1)=V(1)*(QVL-PVL)
  230 CU(I)=PUR+QUR
      CV(I)=PVR+QVR
      GOTO 200
  220 AU(I)=PUL-QUL
      AV(I)=PVL-QVL
```

```
IF (I.LT.NMT) GOTO 230
CU(I)=0.
CV(I)=0.
UU(I)=-U(N)*(PUR+QUR)
VV(I)=-V(N)*(PVR+QVR)
RETURN
END
```

```
      ==================================
      FULL DEMONSTRATION LISTING: PMSTEST
      ==================================

      SUBROUTINE DCOUPL (YU,ZV,U,V,Y,Z)
C THE PRODUCTS YU AND ZV, BEING GIVEN, DCOUPL FINDS THE
C FACTORS, Y, U, Z AND V, CONSISTENT WITH THE PRODUCTS.
      DATA E,TOL,KMAX/.001,.00001,20/
C INITIAL ESTIMATE OF Y PRESUMED GIVEN IN CALL STATEMENT
      IF ((YU.LE.0.).OR.(ZV.LE.0.)) GOTO 90
      K=0
   20 Z = 1. - Y
      F = PC(Y) - PRESA(YU/Y) + PRESB(ZV/Z)
      IF (ABS(F).LE.TOL) GO TO 30
      K = K + 1
      IF (K.GE.KMAX) GO TO 30
      D = SIGN(E,F)
      S = Y - D
      T = 1. - S
      G = PC(S) - PRESA(YU/S) + PRESB(ZV/T)
      DY = D*F/(G-F)
      Y = Y + DY
      GO TO 20
   30 U = YU/Y
      V=ZV/Z
      RETURN
C
   89 FORMAT ('0PROBABLE INSTABILITY - FATAL ERROR')
   90 WRITE(6,89)
      STOP
C
      ENTRY DCOUIN (YU,ZV,U,V,Y,Z)
C THE QUANTITIES U AND V, BEING GIVEN, DCOUIN FINDS
C Y, Z, AND THE PRODUCTS, YU AND ZV.
      Y=PCINV(PRESA(U)-PRESB(V))
      Z=1.-Y
      YU=Y*U
      ZV=Z*V
      RETURN
      END
```

==================================
FULL DEMONSTRATION LISTING: PMSTEST
==================================

```
      SUBROUTINE SPSVAL (XZ,TZ,UZ,VZ)
      COMMON/NOTICE/ITM,NDH,NMT,KUT,NOW
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      COMMON/VECTOR/U(129),V(129),UU(129),VV(129)
      COMMON/WORKER/AU(127),BU(127),CU(127),
     1              AV(127),BV(127),CV(127)
      XIZ=SIMILE(XZ,TZ)
      CALL SPLINE (U,V,UU,VV,HI,DH,NDH,XIZ,UZ,VZ)
      RETURN
      END
```

```
      ====================================
      FULL DEMONSTRATION LISTING: PMSTEST
      ====================================

      SUBROUTINE INITPM
      COMMON/INPUT/ DT,DX,X0,X1,NX,NDX,NT,IRQT,RAMBDA
      COMMON/SMILES/XMUL,TMUL,LNORPR
      COMMON/NOTICE/ITM,NDH,NMT,KUT,NOW
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      COMMON/VECTOR/U(129),V(129),UU(129),VV(129)
      COMMON/WORKER/AU(127),BU(127),CU(127),
     1              AV(127),BV(127),CV(127)
    1 FORMAT ('0XI EXTREMES',2F12.6,/,' DELTA XI    ',F12.6)
    3 FORMAT('1NO. X INTERVALS =',I4,6X,'U AT LEFT =',
     1   F8.4,6X,'U AT RIGHT =',F8.4)
    4 FORMAT(' NO. T INTERVALS =',I4,6X,'V AT LEFT =',
     1   F8.4,6X,'V AT RIGHT =',F8.4)
    5 FORMAT('0LAMBDA =',F6.4)
      NDT = NT - 1
      WRITE(6,3) NDX,UA,UB
      WRITE(6,4) NDT,VA,VB
      WRITE(6,5) RAMBDA
      KUT=2
      ITM=10
      W=SIMPLE(W)
      W=HEADER(W)
      NDH = 64
      NMT=NDH-1
      YZM=.01
      TF = (NT - 1) * DT
      HI=SIMILE(0.,TF)
      HM=SIMILE(1.,0.)
      DH=(HM-HI)/NDH
      WRITE(6,1)HI,HM,DH
      HS=HM-DH/4.0
      CALL SPMSFD
      BU(1) = (U(2)-U(1))/DH
      BU(2) = (V(2)-V(1))/DH
      CALL NEWTON(AU,BU,YZM,L)
      IF (L.LT.0) GOTO 100
      CALL SPMSIV (AU,BU,U,V,YZM)
      CALL DCOUIN (YU,ZV,AU(1),AU(2),Y,Z)
      IF (Y*(1.-Y).LE.YZM) GOTO 100
      CALL SPLINT (AV,BV,CV,VV,V,HI,HM,NDH)
      CALL SPLINT (AU,BU,CU,UU,U,HI,HM,NDH)
  100 RETURN
      END
```

```
      ====================================
      FULL DEMONSTRATION LISTING: PMSTEST
      ====================================

      SUBROUTINE NEWTON (U,UP,YZM,L)
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      DIMENSION U(4),UP(4),UT(3),VT(3)
      DATA CV,DWX/.00002,.05/
   15 FORMAT (1HO,23X,15HNEWTON'S METHOD,/,4X,
     1        2HWO,6X,2HXO,6X,2HUR,6X,2HVR,6X,
     2        2HWR,6X,2HXR,5X,3HUPA,5X,3HVPA)
   17 FORMAT (1H ,8F8.4)
      U(1)=UA
      U(2)=VA
      CALL PROPS (U,UP,HI,Y,-1)
      WRITE(6,15)
      L=0
      WA=U(3)
      XA=U(4)
  220 K=0
      WRITE(6,17)
  230 H=HI
      U(3)=WA
      U(4)=XA
      CALL PROPS (U,UP,H,Y,1)
      UPA=UP(1)
      VPA=UP(2)
  240 CALL RUNGE (U,UP,H,Y,DH)
      IF (Y*(1.-Y).LE.YZM) GOTO 38
      IF (H.LT.HS) GOTO 240
      K=K+1
      WRITE(6,17)WA,XA,U,UPA,VPA
      UT(K)=U(1)
      VT(K)=U(2)
      U(1)=UA
      U(2)=VA
      IF (K-2) 260,270,280
  260 L=L+1
      IF (L.EQ.11) GOTO 290
      Q=(UB-UT(1))**2+(VB-VT(1))**2
      IF (Q.LE.CV) GOTO 290
      WA=WA+DWX
      GOTO 230
  270 WA=WA-DWX
      XA=XA+DWX
      GOTO 230
  280 XA=XA-DWX
      A1=(UT(2)-UT(1))/DWX
      A2=(UT(3)-UT(1))/DWX
      A3=(VT(2)-VT(1))/DWX
      A4=(VT(3)-VT(1))/DWX
      D=A1*A4-A2*A3
      WA=WA+(A4/D)*(UB-UT(1))-(A2/D)*(VB-VT(1))
      XA=XA-(A3/D)*(UB-UT(1))+(A1/D)*(VB-VT(1))
      GOTO 220
```

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==================================
FULL DEMONSTRATION LISTING: PMSTEST
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 38 L=-1
290 U(3)=WA
    U(4)=XA
    RETURN
    END
```

```
      ====================================
      FULL DEMONSTRATION LISTING: PMSTEST
      ====================================

      SUBROUTINE OUTSPS (T,NX,UPMS,VPMS,XPTS)
C
      DIMENSION UPMS(129),VPMS(129),XPTS(129)
C
      X = XPTS(1)
      WRITE(6,6) T,X,UPMS(1),VPMS(1)
      NDX = NX - 1
      DO 22 J = 2,NDX
         X = XPTS(J)
         CALL SPSVAL (X,T,U,V)
         UIB = UPMS(J) - U
         VIB = VPMS(J) - V
         WRITE(6,7) X,UPMS(J),U,UIB,VPMS(J),V,VIB
   22 CONTINUE
      CALL SPSVAL (XPTS(NX),T,U,V)
      WRITE(6,8) XPTS(NX),U,V
C
    6 FORMAT('0   T=',F6.3,/,1X,F5.3,8X,F8.5,16X,F8.5)
    7 FORMAT(' ',F5.3,6F8.5)
    8 FORMAT(' ',F5.3,8X,F8.5,16X,F8.5)
C
      RETURN
      END
```

```
      ====================================
      FULL DEMONSTRATION LISTING: PMSTEST
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      SUBROUTINE PROPS (U,UP,H,Y,IT)
C
C  SUBROUTINE PROPS CALCULATES VALUES CORRESPONDING
C  TO U, V, X, W, Y, AND/OR Z DEPENDING ON THE VALUE
C  OF THE PARAMETER, IT. GENERALLY, WE ARE FINDING
C  STARTING OR INITIAL VALUES FOR RUNGE.  WE FIND
C  Y AND Z AND CERTAIN OTHER VALUES AT SOME NODE.
C
C  PROPS CALLS THE BUILT-IN FUNCTIONS
C  SIG(X), TAU(X), DPHI(X), DPSI(X) FROM HEADER.
C                  LIST OF VARIABLES
C  IT: VARIABLE TELLING US WHICH WAY TO GO.
C     IT<=-1 => WE KNOW U(1),U(2),UP(1),UP(2);
C               SO WE NEED  U(3),U(4),UP(3),UP(4).
C       WHERE {U(3)=W, U(4)=X, UP(3)=W', UP(4)=X'}
C     IT=0,1 => WE KNOW U(1),U(2),U(3),U(4)
C               SO WE NEED UP(1),UP(2),UP(3),UP(4).
C  Y:  SATURATION OF OUR POROUS MEDIA. Y:(U,V)=>R
C  Z:  PART NOT SATURATED (OF THE POROUS MEDIA).
C  U:  VECTOR OF VALUES FOR U,V,W,AND X AT SOME H
C  UP: VECTOR OF VALUES FOR U',V',W',AND X' AT SOME H
C  H:  X-VALUE WHERE WE HAVE U AND V VALUES (IT<=1).
C
      DIMENSION U(4),UP(4)
      CALL DCOUIN (YU,ZV,U(1),U(2),Y,Z)
      Q=SIMSEX(H)
      E=Q*SIG(Y)*DPHI(U(1))
      F=Q*TAU(Z)*DPSI(U(2))
      Q=SIMSID(H)
      UP(3)=Q*YU
      UP(4)=Q*ZV
      HR=SIMSEG(H)
      IF (IT.GE.0) GOTO 10
      U(3)=E*UP(1)+HR*YU
      U(4)=F*UP(2)+HR*ZV
      GOTO 20
   10 UP(1)=(U(3)-HR*YU)/E
      UP(2)=(U(4)-HR*ZV)/F
   20 RETURN
      END
```

```
==================================
FULL DEMONSTRATION LISTING: PMSTEST
==================================

      SUBROUTINE RUNGE (U,UP,H,Y,DH)
      DIMENSION U(4),UP(4),UO(4),DU(4)
      DATA UMX/3600./
      DO 50 I=1,4
      UO(I)=U(I)
      DU(I)=UP(I)
   50 U(I)=UO(I)+DH*UP(I)/2.
      H=H+DH/2.
      CALL PROPS (U,UP,H,Y,1)
      DO 60 I=1,4
      DU(I)=DU(I)+2.*UP(I)
   60 U(I)=UO(I)+DH*UP(I)/2.
      CALL PROPS (U,UP,H,Y,1)
      DO 70 I=1,4
      DU(I)=DU(I)+2.*UP(I)
   70 U(I)=UO(I)+DH*UP(I)
      H=H+DH/2.
      CALL PROPS (U,UP,H,Y,1)
      DO 80 I=1,4
      DU(I)=DU(I)+UP(I)
   80 U(I)=UO(I)+DH*DU(I)/6.
      CALL PROPS (U,UP,H,Y,1)
      Q=U(1)*U(1)+U(2)*U(2)
      IF (Q.GE.UMX)Y=0.
      RETURN
      END
```

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==================================
FULL DEMONSTRATION LISTING: PMSTEST
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      FUNCTION SIMILE (X,T)
C THIS SUBPROGRAM DOES ALL SIMILARITY CURVE WORK
CURVES ARE PARALLEL LINES IF (LNORPR.EQ.0)
CURVES ARE COAXIAL PARABOLAS IF (LNORPR.NE.0)
      COMMON/SMILES/XMUL,TMUL,LNORPR
C SIMILE IS THE XI CORRESPONDING TO (X,T)
      IF (LNORPR.NE.0) GOTO 20
      SIMS=XMUL*X+TMUL*T
      GOTO 25
   20 SIMS=(X-XMUL)/SQRT(T-TMUL)
   25 SIMILE=SIMS
      RETURN
C SIMSEX IS THE SPMS INTEGRATING FACTOR
      ENTRY SIMSEX (XI)
      SIMS = 1.
      SIMSEX=SIMS
      RETURN
C SIMSEG IS G(XI)*SIMSEX
      ENTRY SIMSEG (XI)
      IF (LNORPR.NE.0) GOTO 40
      SIMS=-TMUL/(XMUL*XMUL)
      GOTO 45
   40 SIMS=.5*XI
   45 SIMSEG=SIMS
      RETURN
C SIMSID IS D(SIMSEG)/D(XI)
      ENTRY SIMSID (XI)
      IF (LNORPR.NE.0) GOTO 50
      SIMS=0.
      GOTO 55
   50 SIMS=.5
   55 SIMSID=SIMS
      RETURN
C SIMPLE PRINTS THE CURVE TYPE
      ENTRY SIMPLE (XI)
      WRITE(6,10)XMUL,TMUL
      IF (LNORPR.NE.0) GOTO 60
      WRITE(6,11)
      GOTO 65
   60 WRITE(6,12)
   65 SIMPLE=XI
      RETURN
   10 FORMAT ('0GEOMETRIC FACTORS',/,' XMUL    ',F10.5,/,
     1                               ' TMUL    ',F10.5)
   11 FORMAT (' PARALLEL LINES')
   12 FORMAT (' COAXIAL PARABOLAS')
      END
```

```
==================================
FULL DEMONSTRATION LISTING: PMSTEST
==================================

SUBROUTINE SPLINE (U,V,S,T,HI,DH,NDH,X,Y,Z)
DIMENSION U(2), V(2), S(2), T(2)
I=(X-HI)/DH
I=1+MAXO(I,0)
I=MINO(NDH,I)
P=(X-HI-(I-1)*DH)/DH
Q=1.-P
Y=U(I)*Q+U(I+1)*P-P*Q*(S(I)*(Q+1.)+S(I+1)*(P+1.))
Z=V(I)*Q+V(I+1)*P-P*Q*(T(I)*(Q+1.)+T(I+1)*(P+1.))
RETURN
END
```

```
      ====================================
      FULL DEMONSTRATION LISTING: PMSTEST
      ====================================

      SUBROUTINE SPLINT (A,B,C,D,U,HI,HM,NDH)
      DIMENSION A(2), B(2), C(2), D(2), U(2)
      NMT=NDH-1
      DH=(HM-HI)/NDH
      Q=DH/(6.*(HM-HI))
      K=0
      TO=0.
      CC=U(NDH+1)-U(NDH)-U(2)+U(1)
   25 CA=CC+4.*TO
   30 K=K+1
      DO 35 I=1,NMT
      A(I)=1.
      B(I)=4.
      C(I)=1.
   35 D(I+1)=U(I)-2.*U(I+1)+U(I+2)
      A(1)=0.
      C(NMT)=0.
      D(2)=D(2)-TO
      D(NDH)=D(NDH)-TO
      CALL TRIPLE (A,B,C,D(2),NMT)
      IF(K .GT. 2)GO TO 100
      G=(CA+D(NDH)+D(2))*Q
      IF(K .GT. 1)GO TO 40
      AL=G
      TO=1.
      GO TO 25
   40 TO=AL/(1.-G+AL)
      GO TO 30
  100 D(1)=TO
      D(NDH+1)=TO
      RETURN
      END
```

FULL DEMONSTRATION LISTING: PMSTEST

```
      SUBROUTINE SPMSFD
C
C  SUBROUTINE SPMSFD APPROXIMATES THE SLOPES OF THE
C  U AND V CURVES AT HI (INITIAL X VALUE) TO BE USED
C  IN THE SHOOTING METHOD.  THESE VALUES COME ABOUT
C  THROUGH SUCCESSIVE SUBSTITUTION METHODS.
C  SPMSFD FIRST USES A LINEAR APPROXIMATION OF U'S
C  AND V'S.  IT CALLS COEFFS TO GET COEFFICIENTS OF
C  THE TRI-DIAGONAL MATRIX AND CALLS TRIPLE TO
C  EVALUATE THE EQUATIONS AND GET NEW U'S AND V'S
C  AT SOME SUCCESSIVE SUBSTITUTION STEP.
C  SPMSFD PRINTS U AND V VALUES CALCULATED AT THE
C  SECOND AND NEXT-TO-LAST NODES FOR EACH
C  SUCCESSIVE SUBSTITUTION STEP.
C
C                    LIST OF VARIABLES
C  HL:  SIZE OF THE INTERVAL
C  H:   THE X-VALUE AT WHICH U AND V ARE CALCULATED
C     FOR THE FIRST ESTIMATE, I.E. LINEAR ESTIMATE.
C  U:   THE VALUE OF U AT SOME POINT X (NODES) FOR
C     ANY SUCCESSIVE SUBSTITUTION STEP
C  V:   VALUE OF V AT SOME POINT X (NODES) FOR ANY
C     SUCCESSIVE SUBSTITUTION STEP
C  I:   COUNTER FOR U AND V VECTORS AT ANY
C     SUCCESSIVE SUBSTITUTION STEP
C  NMT: NO. OF INTERIOR POINTS. ALSO NO. OF POINTS
C       TO BE CALCULATED.
C  J:   ALSO COUNTER FOR U AND V VECTORS. USED IN
C  FINDING NEXT U AND V VALUES. ONE MORE THAN I.
C N:   NUMBER OF X POINTS AT WHICH WE APPROXIMATE
C      U AND V VECTORS.
      COMMON/NOTICE/ITM,NDH,NMT,KUT,NOW
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      COMMON/VECTOR/U(129),V(129),UU(129),VV(129)
      COMMON/WORKER/AU(127),BU(127),CU(127),
     1              AV(127),BV(127),CV(127)
   16 FORMAT (1HO,22X,23HSUCCESSIVE SUBSTITUTION,/,
     1 22H   U(HI+DH)  HV(HI+DH),
     2 42H   U(HM-DH)  V(HM-DH)   IT   MAX.ABS INCR.)
   17 FORMAT (1H ,4F10.5,I5,E15.6)
      HL=HM-HI
      H=HI
      U(1)=UA
      V(1)=VA
CHOOSE LINEAR INTERPOLATION FOR INITIAL ESTIMATES.
      DO 50 I=1,NMT
      H=H+DH
      J=I+1
      U(J)=((H-HI)*UB+(HM-H)*UA)/HL
      V(J)=((H-HI)*VB+(HM-H)*VA)/HL
   50 CONTINUE
```

```
      N=NDH+1
      U(N)=UB
      V(N)=VB
      IT=0
      WRITE(6,16)
      WRITE(6,17)U(2),V(2),U(NDH),V(NDH),IT
COMPUTE COEFFICIENTS FOR TRI-DIAGONAL SYSTEMS.
   60 CALL COEFFS (N)
   65 CALL TRIPLE (AU,BU,CU,UU,NMT)
      CALL TRIPLE (AV,BV,CV,VV,NMT)
      ACT=0.
      DO 250 I=1,NMT
      J=I+1
      DU=UU(I)-U(J)
      ACT=AMAX1(ACT,ABS(DU))
      DV=VV(I)-V(J)
      ACT=AMAX1(ACT,ABS(DV))
      U(J)=UU(I)
  250 V(J)=VV(I)
      IT=IT+1
      WRITE(6,17)U(2),V(2),U(NDH),V(NDH),IT,ACT
      IF (IT.LT.ITM) GOTO 60
      RETURN
C
C IT: COUNTER FOR NUMBER OF SUCCESSIVE SUBSTITUTIONS
C NI: NUMBER OF INTERVALS
C HI: INITIAL ENDPOINT VALUE (X VALUE)
C HM: FINAL ENDPOINT VALUE (X VALUE)
C UA: INITIAL U VALUE AT INITIAL X VALUE, HI
C VA: INITIAL V VALUE AT INITIAL X VALUE, HI
C DH: DELTA H = DELTA X = INTERVAL SIZE
C UB: INITIAL U VALUE AT FINAL X VALUE, HM
C VB: INITIAL V VALUE AT FINAL X VALUE, HM
C AU: SUB-DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C BU: DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C CU: SUPER-DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C UU: VECTOR OF SOLUTIONS TO TRI-DIAGONAL EQUATIONS.
C AV: SUB-DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C BV: DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C CV: SUPER-DIAGONAL ELEMENTS OF TRI-DIAGONAL MATRIX.
C VV: VECTOR OF SOLUTIONS TO TRI-DIAGONAL EQUATIONS.
C ACT: THE MAXIMAL U OR V CHANGE FROM ONE
C SUCCESSIVE SUBSTITUTION TO THE NEXT.
C DU-THE CHANGE IN U AT THE JTH NODE FROM ONE
C SUCCESSIVE SUBSTITUTION TO THE NEXT.
C DV-THE CHANGE IN V AT THE JTH NODE FROM ONE
C SUCCESSIVE SUBSTITUTION TO THE NEXT.
C
      END
```

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FULL DEMONSTRATION LISTING: PMSTEST
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      SUBROUTINE SPMSIV (U,UP,UX,VX,YZM)
C
C SPMSIV FINDS THE U AND V VALUES FOR SOLUTION CURVES.
C IT CALLS OUTSPS TO GIVE A PRINTOUT OF INITIAL
C VALUES AND INFORMATION ABOUT THE PROBLEM.
C IT CALLS PROPS TO GET VALUES FOR
C                U(3), U(4) (W=U(3),X=U(4)),
C UP(1), UP(2), UP(3), UP(4), AND Y.
C IT CALLS RUNGE FOR EACH H TO GET VALUES FOR U AND V
C               LIST OF VARIABLES
C U:  VALUES FOR U AND V AT ONE CALCULATION STEP
C UP:  IS THE VALUE OF THE DERIVATIVE OF U'S
C UX:  VALUES FOR U AT EACH RUNGE-KUTTA STEP
C VX:  VALUES FOR V AT EACH RUNGE-KUTTA STEP
C YZM:  .01:  USED HERE TO KNOW WHEN TO STOP RUNGE
C   QUIT IF      Y*(1 - Y) .LE. YZM
C I:  COUNTER FOR STORING U AND V VALUES
C H:  X VALUE AT WHICH RUNGE CALCULATES U AND V
C     MUST FIND W,X,W',X' (UB=UP(1)=U';VB=UP(2)=V')
C     (UB=UP(1)=INITIAL VALUE OF U AT HM;
C      VB=UP(2)=INITIAL VALUE OF V AT HM)
C Y:  SATURATION OBTAINED FROM PROPS FOR USE IN RUNGE.
C K:  COUNTER FOR CHECKING OUR DESIRE TO PRINT;
C                    PRINT IF K=KUT
C DH:  CHANGE IN X VALUE, H INCREMENT FOR
C       USE IN RUNGE STEP SIZE
C KUT:  THE NUMBER OF ITERATIONS DESIRED,
C           PRINTOUT TO PRINTOUT
C HS = HM - .25*DH . USED TO STOP U & V CALCULATIONS.
C
      COMMON/NOTICE/ITM,NDH,NMT,KUT,NOW
      COMMON/SCALAR/HI,HM,DH,HS,UA,VA,UB,VB
      DIMENSION UX(2), VX(2)
      DIMENSION U(4),UP(4)
   16 FORMAT (1H1,5X,3HETA,8X,1HY,9X,1HU,
     1         9X,1HV,9X,1HW,9X,1HX)
   17 FORMAT (1H ,6F10.5)
      I=0
      CALL PROPS (U,UP,HI,Y,0)
   39 H=HI
      WRITE(6,16)
      WRITE(6,17)H,Y,U
C     MODIFY FORD PROGRAM
      I=I+1
      UX(I)=U(1)
      VX(I)=U(2)
      K=0
   40 CALL RUNGE (U,UP,H,Y,DH)
      IF (Y*(1.-Y).GT.YZM) GOTO 42
      WRITE(6,17)H,Y,U
      GOTO 38
```

```
42 I=I+1
   UX(I)=U(1)
   VX(I)=U(2)
   K=K+1
   IF (K.LT.KUT) GOTO 50
   WRITE(6,17)H,Y,U
   K=0
50 IF (H.LT.HS) GOTO 40
   GOTO 30
37 FORMAT (33HOVANISHING PHASE; GO TO NEXT CASE)
38 WRITE(6,37)
30 RETURN
   END
```

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      FULL DEMONSTRATION LISTING: PMSTEST
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      SUBROUTINE TRIPLE (A,B,C,D,N)
      DIMENSION A(2),B(2),C(2),D(2)
      D(1)=D(1)/B(1)
      IF (N.LT.2) GOTO 30
      DO 10 J=2,N
      I=J-1
      C(I)=C(I)/B(I)
      B(J)=B(J)-A(J)*C(I)
      D(J)=(D(J)-A(J)*D(I))/B(J)
   10 CONTINUE
      DO 20 K=2,N
      I=N-K+1
      J=I+1
      D(I)=D(I)-C(I)*D(J)
   20 CONTINUE
   30 RETURN
      END
```