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THE DECONFINEMENT TRANSITION ON A BCH LATTICE*

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ABSTRACT

The results of a Monte Carlo simulation of pure SU(2) gauge theory at finite temperature on a body-centered hypercubic(BCH) lattice are presented. The behavior of the transition temperature as a function of the critical lattice coupling g^* is compared with the prediction of asymptotic scaling as given by the perturbative β function. A comparison of the results for the BCH lattice and the simple hypercubic(SH) lattice provides important tests of universality.

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I INTRODUCTION

It is generally believed that at sufficiently high temperature, in the absence of dynamical quarks, confined QCD undergoes a deconfining transition to a plasma-like phase containing liberated quarks and gluons. The effects of such a transition should be manifest in heavy ion collisions and the early universe. It is convenient to study the QCD predictions for the behavior of this transition by performing Monte Carlo simulations of lattice QCD on an asymmetric lattice of size $N_T \times N_S^3$, where N_T and N_S denote the number of sites in the temporal and the spatial directions respectively. If periodic boundary conditions are imposed in the temporal direction, the finiteness of N_T introduces the effects of a non-zero temperature defined by $T=1/N_T a$, where a is the lattice spacing. The value of N_S is finite only for practical convenience and satisfies the condition $N_S \gg N_T$. To date, most studies¹⁻³⁾ have been carried out on simple hypercubic(SH) lattices with the Wilson action. However, other discretizations of the theory are possible. In particular, it is interesting to study finite temperature QCD formulated on a body-centered hypercubic(BCH) lattice⁴⁾

The BCH lattice⁵⁾ is constructed by filling four-space with hypercubes and choosing the lattice sites to be at the corners and at the center of each hypercube, as shown (for convenience in three dimensions) in Fig. 1. The resulting lattice is seen to consist of two staggered SH lattices, one formed from the corners of each hypercube, the other formed from the body-centers. The links joining the lattice sites form a web of equilateral triangles, the elementary plaquettes of the theory. It is a special property of four dimensions that all the links have length one. The generating vectors for the BCH lattice are the unit vectors $(1,0,0,0)$, $(0,1,0,0)$, $(0,0,1,0)$ and $(1/2,1/2,1/2,1/2)$. Each site has 24 nearest neighbors, compared with 8 in the SH case. Similarly, each BCH link is shared by 8 plaquettes as opposed to 6 in the SH case. It is possible to construct both triangular and rectangular Wilson loops on this lattice.

The choice of such a complicated structure is motivated by considerations of rotational invariance and universality. Whereas the SH lattice has only 4 symmetry axes and is invariant under a 384 element subgroup of the rotation group, the BCH lattice has 12 symmetry axes and a

point symmetry group consisting of 1152 elements. In fact, BCH lattice gauge theory possesses the maximal rotational invariance of any four dimensional lattice theory and consequently may provide the closest approximation to the continuum limit. Monte Carlo measurements of quantities such as masses (or temperatures) are expected to exhibit scaling at sufficiently weak coupling. That is, the behavior of a mass, m_i , as a function of the coupling constant g is given by

$$m_i a = c_i f(g), \quad (1)$$

where c_i is a constant and the function f , which is non-universal in that it depends on the choice of lattice action, is the same for all the m_i . Asymptotic scaling occurs when $f(g)$ becomes universal and depends only on the first two terms of the perturbative beta function. The onset of both scaling and asymptotic scaling are important phenomena to investigate for the BCH theory. Furthermore, scaling implies that the ratio of two masses, m_i/m_j , is just the ratio of the constants, c_i/c_j . Since this ratio is a physical quantity, universality, which states that physical quantities are independent of the precise nature of the lattice action, can be tested by comparing the results obtained for the BCH lattice with those obtained for the SH lattice.

II DEFINITIONS AND CALCULATIONS

The BCH symmetric action used in this study is given by

$$S_{\text{BCH}} = \beta/8 \sum_{\Delta} \text{Tr}(U_{\Delta}) . \quad (2)$$

where U_{Δ} is the triangular plaquette formed from the product of links joining nearest neighboring sites and β is the usual $SU(2)$ inverse coupling, $4/g^2$. A standard technique for measuring the critical temperature T_c , is to observe the behavior of an order parameter, the thermal Wilson loop $L(\vec{x})$, which consists of the product of links along the t -direction at site \vec{x} :

$$L(\vec{x}) = 1/2 \text{Tr} \prod_{i=1}^{N_t} U_i(\vec{x}). \quad (3)$$

The free energy F_q of a static quark at location \vec{x} is given by the expectation value of $L(\vec{x})$. Namely,

$$\langle L(\vec{x}) \rangle = \exp(-F_q/T), \quad (4)$$

so that in the confined phase where F_q is infinite, $\langle L \rangle$ is zero, and in the deconfined phase where F_q is finite, $\langle L \rangle$ is non-zero. Universality arguments indicate that the transition is second order, so that for a fixed lattice size, $\langle L \rangle$ changes continuously as β is increased. The critical coupling β^* is the point at which $\langle L \rangle$ becomes non-zero.

Another operator of interest is the modified Wilson line \tilde{L} , which, roughly speaking, is the product of the average environment of the links along the t-direction and is defined as

$$L = \frac{1}{2} \text{Tr} \prod_{i=1}^{N_\tau} \left[\left(I_2(k_i \beta/2) W_i \right) / \left(I_1(k_i \beta/2) k_i \right) \right], \quad (5)$$

where I_n denotes the modified Bessel function of order n , W_i denotes the sum of wedges contiguous to link U_i and $k_i = \sqrt{\det W_i}$. A wedge contiguous to link U_i is the product of any two links which form a triangular plaquette when multiplied with the link U_i . There are 8 such wedges associated with each link. \tilde{L} has the useful property that its expectation value is equal to that of L , but its variance is reduced. Unfortunately, the importance of this effect is partially offset by the large sweep-to-sweep correlation lengths present in the data. Careful analysis is required.

Our prediction of asymptotic scaling for the behavior of T_c as a function of the critical coupling $\beta^* = 4/g^*{}^2$ is given by

$$T_c a = N_\tau^{-1} = T_c / \Lambda_{\text{BCH}} (6\pi^2 \beta^* / 11)^{51/121} \exp(-3\pi^2 \beta^* / 11). \quad (6)$$

The simulations are being done on a CRAY-1 and a CRAY-XMP. The update time on the former is about 12 usec/link. The XMP runs a factor of 2 or so faster. The study has used approximately 14 hours of CRAY-1 time to date. The lattices considered have $2 \leq N_\tau \leq 6$, with $N_s \geq 2N_\tau$. Toroidal boundary conditions have been implemented in the spatial directions. One

technical point worthy of note is the fact that BCH Monte Carlo algorithms are relatively easy to vectorize because parallel links do not have plaquettes in common. Hence, it is possible to update all links in a given direction at once.

III RESULTS

Although the Monte Carlo measurements and data analysis are incomplete, it is possible to extract preliminary estimates for T_c in order to test asymptotic scaling and universality. Fig. 2 shows, for $N_\tau = 3$, the behavior of the absolute value of the modified Wilson line $\langle |L| \rangle$ as a function of β . The sharp rise in the expectation value signals the presence of the phase transition at $\beta^* \approx 2.77$. Fig. 3 shows the estimates of β^* for all the values of N_τ considered. The solid line is the prediction of asymptotic scaling given by Eq. (6), with the value of T_c/Λ_{BCH} adjusted so that the prediction passes through the point corresponding to $N_\tau = 6$. However, it is clear from Fig. 3 that asymptotic scaling is not satisfied, so that any fit to T_c/Λ_{BCH} is not very meaningful. Universality can be tested by comparing estimates of the ratio $T_c/\sqrt{\sigma}$ (σ is the string tension) for the BCH and SH lattices. Measurements of σ on the BCH lattice⁷⁾ show scaling violations that are very similar to those depicted in Fig. 3. Consequently, $T_c/\sqrt{\sigma}$ is roughly constant over the range of β considered and in agreement⁷⁾ with the values of $T_c/\sqrt{\sigma}$ for the SH lattice given in Table I. These estimates were calculated using recent measurements of T_c/Λ_{SH} ⁸⁾ and $\Lambda_{\text{SH}} = 0.018 \pm .001 \sqrt{\sigma}$.⁹⁾

It appears that, although asymptotic scaling is violated, ratios of physical quantities are constant, indicating the presence of a scaling region which is also consistent with universality.

Table I
Estimates for $T_c/\sqrt{\sigma}$ for the SH Lattice

N_τ	T_c/h_{SH}	$T_c/\sqrt{\sigma}$
2	$29.0 \pm .1$	$.52 \pm .03$
3	$43.2 \pm .3$	$.78 \pm .05$
4	$41.6 \pm .4$	$.75 \pm .05$
5	$39.9 \pm .6$	$.72 \pm .05$

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FIGURE CAPTIONS

Fig. 1. The BCH lattice in three dimensions.

Fig. 2. The absolute value of the modified Wilson line versus β for $N_\tau = 3$.

Fig. 3. The critical coupling for $2 < N_\tau < 6$. The solid line is the asymptotic scaling prediction with $T_c/\Lambda_{\text{BCH}} = 146$.

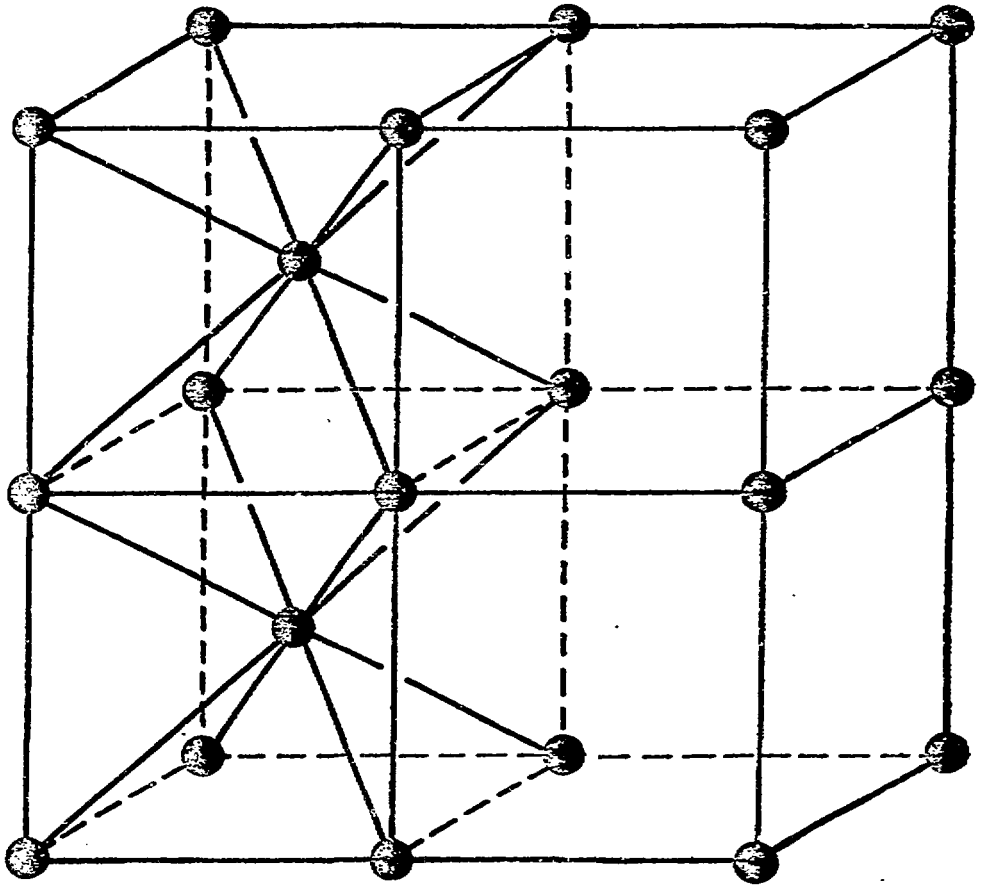


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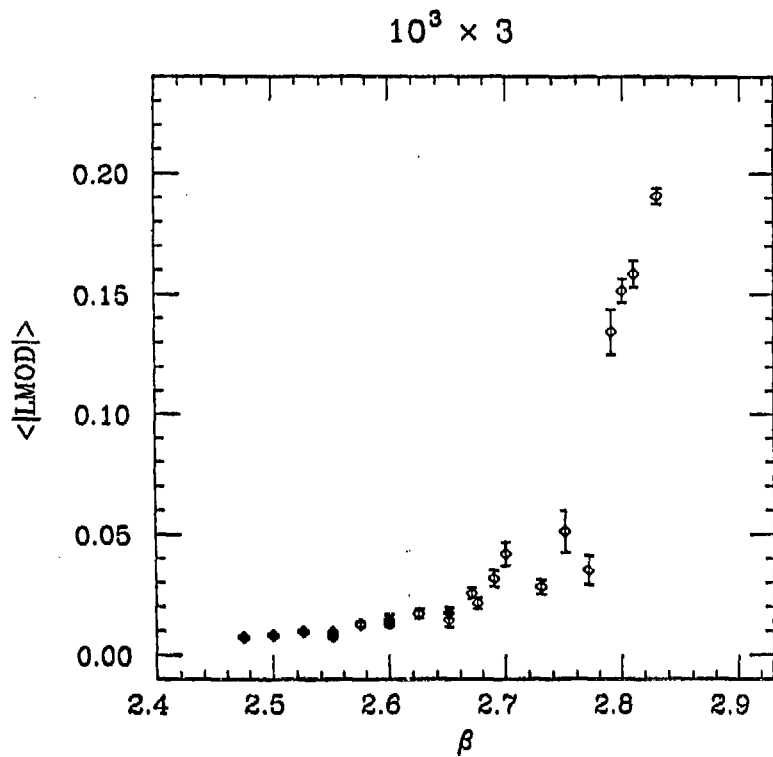


Fig. 2. The absolute value of the modified Wilson line versus β for $N_t = 3$.

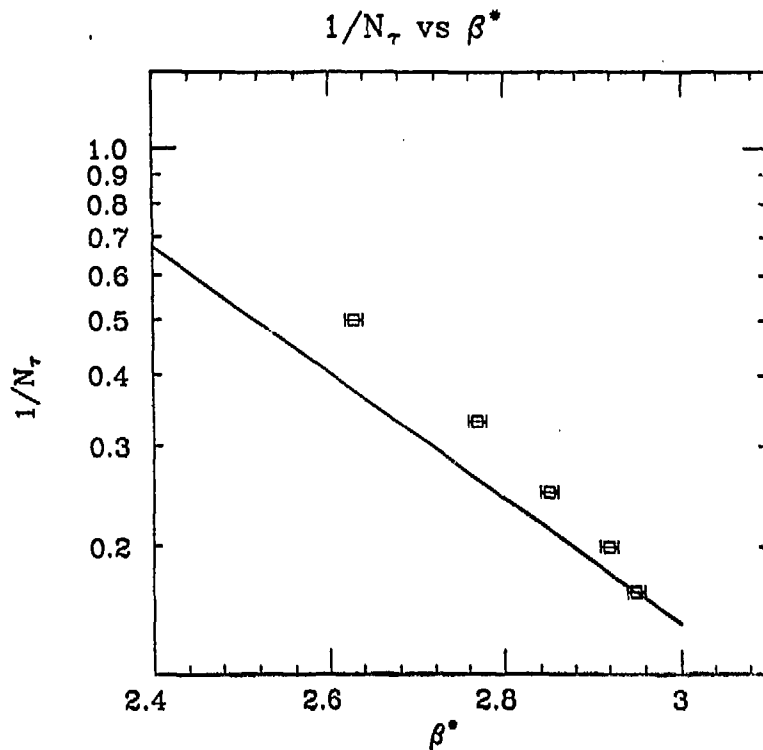


Fig. 3. The critical coupling for $2 < N_\tau < 6$. The solid line is the asymptotic scaling prediction with $T_c/\Lambda_{\text{BCH}} = 146$.