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**Latin Square Three-Dimensional
Ball Plate (A Practical Approach to
Evaluation of Coordinate Measuring
Machine Capability)**

By L. L. Jones

Published October 1981

Topical Report

Prepared for the United States Department of Energy
Under Contract Number DE-AC04-76-DP00613.



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LATIN SQUARE THREE-DIMENSIONAL BALL PLATE (A Practical Approach to Evaluation of Coordinate Measuring Machine Capability)

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Prepared by L. L. Jones

Electronic coordinate measuring machines (CMM) represent the state-of-the-art in automatic three-dimensional measuring equipment. Current techniques for determining measurement capability of CMM are complex. A Latin square three-dimensional ball plate has been developed to evaluate the measurement capability of a CMM in three dimensions. The unique allocation of tooling balls in an XYZ space permits data to be obtained that is balanced statistically and can be analyzed quickly by using the analysis of variance techniques for a Latin square experimental design. The analysis of this data is used to determine quantitative measures of the precision and systematic error of CMM in three dimensions and to identify sources of error in machine geometry. This ball plate currently is used to determine CMM capability and to identify source(s) of error in machine geometry. This device requires 70 percent less time than some conventional techniques to estimate CMM capability in three dimensions.

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SUMMARY

Electronic coordinate measuring machines (CMMs) represent the state-of-the-art in automatic three-dimensional measuring equipment. Current techniques to determine the measurement capability (precision plus systematic error) of a CMM are complex. Knowledge of this capability is important because it can affect machine productivity. A Latin square three-dimensional ball plate (LS3DBP) has been developed to obtain data that can be analyzed to determine the measurement capability of a CMM. The determination is achieved by a unique allocation of the heights of tooling balls in three dimensions. This allocation is achieved by using the concept of a Latin square experimental design. Data obtained from this configuration can be analyzed quickly by using the standard analysis of variance technique for a Latin square experimental design. The LS3DBP can reduce the time required to determine CMM measurement capability by 70 percent, compared to some conventional methods.

The information determined from the analysis is precision, systematic error, and identification of error sources in CMM geometry. This information provides a quantitative basis to determine whether the CMM can adequately measure a product and to assure that the decision regarding product acceptability has a high probability of being correct.

DISCUSSION

SCOPE AND PURPOSE

The Latin square three-dimensional ball plate (LS3DBP) is an innovation in the current technology of determining directly the measurement capability of coordinate measuring machines (CMMs). The advantage of this innovation to earlier art lies in the unique allocation of the heights of tooling balls in three dimensions (Figure 1). This allocation is achieved by using the concept of a Latin square experimental design. The heights of the tooling balls (cones) are arranged in a three-dimensional pattern. Data obtained from using this configuration are balanced statistically and can be analyzed quickly by using the standard analysis of variance technique for a Latin square experimental design. The analysis of data obtained by use of this device is used to determine quantitative measures of the precision and systematic error of CMMs in three dimensions and to identify error sources in machine geometry.

ACTIVITY

CMM

Electronic coordinate measuring machines represent the state-of-the-art in three-dimensional measuring equipment. Although thousands of CMMs are in use throughout the world, current techniques for determining precision and systematic error are complex and not readily available or used by most users and manufacturers.

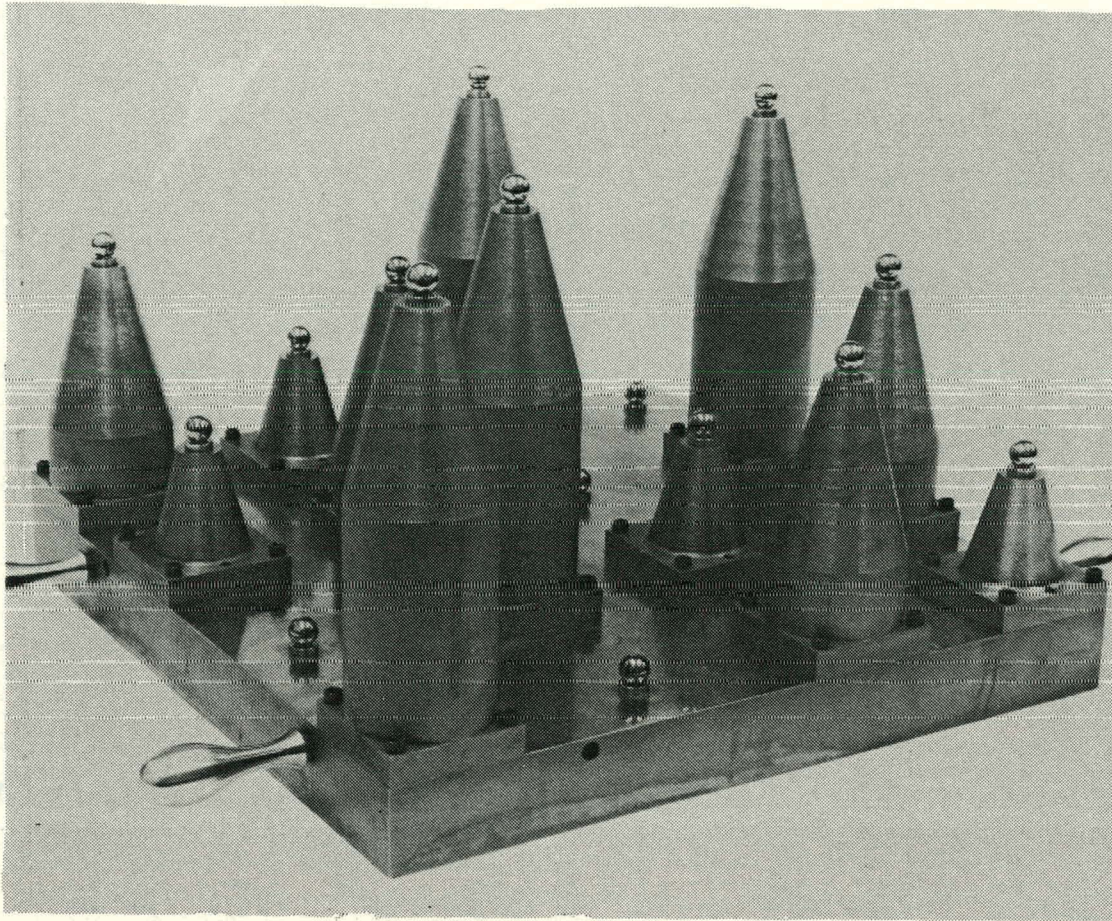


Figure 1. Latin Square Three-Dimensional Ball Plate

One report¹ describes the significance and versatility of CMMs. Appendix A lists sources that can supply information about examples of CMMs. Classifications of CMMs are in Appendix B. Knowledge of CMM measurement capability is important because it can affect machine productivity, which will be demonstrated in a subsequent section.

Uses of LS3DBP

The LS3DBP is used to obtain data that is analyzed by using the analysis of variance (AOV) technique to determine the measurement capability of a CMM and to identify source(s) of error in machine geometry. The analysis technique is simple, straightforward, and can be performed in about one hour by an individual with no background in statistics who uses a hand calculator. (The AOV computational procedure can also be programmed into the micro-processor interfaced with a CMM.) The LS3DBP can be used with any CMM with a working space that will accommodate its configuration. The information that can be determined through analysis of data from the LS3DBP follows.

Measurement Capability

Measurement capability (accuracy) of a coordinate measuring machine is characterized by precision and systematic error (bias), determined relative to a given calibrated fixture. Precision is a random (inherent) error in the measurement process of a CMM. This error is determined by the inability of the measurement process of a CMM to obtain the same values (X and/or Y and/or Z) when the same point is measured more than once. The magnitude of this error may vary from one location to another within the working space of a CMM.

Systematic error (bias) is the difference between the true value of a characteristic and the average of repeated measurements of

that characteristic. This study deals with systematic error that is caused by error in CMM geometry. Thus, systematic error is determined by bias introduced into the measurement process by movement along the X-, Y-, and Z-axes. Other factors that may contribute to systematic error are environments, operators, and calibration error. These factors are not considered in this study and subsequent analyses. CMM measurement capability is variation in the measurement process that consists of precision plus systematic error. The characterization of this capability in this study is denoted by σ^2_{CMM} and is a univariate representation of the observed variability in a three-dimensional space. This characterization provides the CMM user with a quantitative measure of the machine's measurement capability and a basis for determining whether a CMM can measure product adequately and for assuring that a decision regarding product acceptability has a high probability of being correct.

Identification of Errors in CMM Geometry

There are 21 sources of error in machine geometry that can introduce inaccuracy into the measurement process. Such inaccuracy results in misclassification of parts. (Misclassification means to accept a defective part or to reject a good part.) Each source of error produces a unique pattern of systematic errors (bias) in the deviations from nominal. These patterns may be characterized as to which axis movement affects the deviations, and the form of the effect on the deviations of that axis movement. An advantage

of the LS3DBP is that the analysis of variance, from which quantitative estimates of the factors affecting machine capability are obtained, also can be used to identify sources of error in CMM geometry. Balanced data from an experimental design (Latin square) supply quantitative estimation of systematic bias, both in magnitude and form. A study involving simulated data from a cantilever design CMM indicates that the type of error can be identified by proper interpretation of the Latin square analysis of variance.² (This will be discussed further in the section on identification of machine geometry error.) Rapid identification of the source of error reduces machine downtime for adjustments and thus increases productivity. Proper adjustment of the machine will reduce inaccuracy, which will in turn reduce the probability of misclassifying a part.

Description of the LS3DBP

Design Concept

Statistically balanced data are the most efficient means of obtaining information about a system. Other current methods of evaluating CMMs in three dimensions cannot adequately determine deviations from nominal values simultaneously in XYZ space that are balanced and that can be analyzed by using simple statistical techniques. The inadequacy of other methods exists because the associated devices must be moved to obtain balanced data. This movement introduces an additional source of variability into the measurement process that cannot be isolated. The data obtained

by methods that require moving such devices fail to provide the best determination of the measurement capability of the CMM.

The LS3DBP overcomes this inadequacy caused by movement by using the concept of a Latin square experimental design in a single construction. A distinct advantage of the single construction is the reduction of CMM time required to obtain data to determine machine capability in XYZ space. Current methods can require the use of a two-dimensional ball plate evaluated three times at one location in the XY plane; then tilted on its longest side and evaluated three times in the YZ plane; then rotated 90° and evaluated one time in the XZ plane. Tilting in ball plate on its side can be a potential source of error because the plate may deform in this position. The total time required is three and one-half hours (30 minutes for each evaluation). The LS3DBP requires no movement and provides simultaneous data in XYZ space. Two evaluations require only one hour (30 minutes for each evaluation), a 70 percent reduction in the time required to obtain data in XYZ space. This time savings increases CMM productivity.

A Latin square of side n , or an n by n Latin square, is an arrangement of n letters, each repeated n times, in a square array of side n in such a manner that each letter appears only once in each row and column. An example of a 4 by 4 Latin square ($n = 4$) is shown in Figure 2. A two-dimensional representation of a LS3DBP ($n = 4$) is presented in Figure 3. The actual device is shown in Figure 1.

ROW 4	B	C	A	D
ROW 3	C	B	D	A
ROW 2	D	A	C	B
ROW 1	A	D	B	C
	COL 1	COL 2	COL 3	COL 4

A, B, C, AND D DENOTE 4 DIFFERENT FACTORS OR 4 DIFFERENT LEVELS OF A SINGLE FACTOR OR TREATMENT.

Figure 2. A Four by Four Latin Square

Y_4	Z_2	Z_3	Z_1	Z_4
Y_1	Z_3	Z_2	Z_4	Z_1
Y_2	Z_4	Z_1	Z_3	Z_2
Y_1	Z_1	Z_4	Z_2	Z_3
	X_1	X_2	X_3	X_4

X_i - DENOTES DISTANCE ALONG THE X-AXIS ($i=1, 2, 3, 4$)

Y_j - DENOTES DISTANCE ALONG THE Y-AXIS ($j=1, 2, 3, 4$)

Z_k - DENOTES DISTANCE ALONG THE Z-AXIS ($k=1, 2, 3, 4$)

Figure 3. Two-Dimensional Representation of a Latin Square Three-Dimensional Ball Plate

Data obtained from a Latin square experimental design are balanced statistically and therefore can be analyzed by using the analysis of variance technique. (This technique will be illustrated in a subsequent section.) The balance concept implies that each level of each factor occurs equally often at each level of all remaining factors (Figure 2). The levels are axis distance and the factors are axes. In Figure 3, the distance X_1 occurs exactly once with each distance of $Y(Y_1, Y_2, Y_3, Y_4)$, and with each distance of $Z(Z_1, Z_2, Z_3, Z_4)$. The same is true for X_2 , X_3 , and X_4 , and about Y and Z . The advantage of the "balancing" concept is that determination of precision and accuracy for the analysis of data is applicable over the total range the three axes were moved.

Physical Characteristics

The dimensions of the LS3DBP is 18 by 18 by 11 in. (X by Y by Z). These dimensions were selected to be compatible with CMMs at Bendix Kansas City. The cone heights are approximately 9, 6, and 3 in., respectively. The approximate height of the cored bench plate is 2 in. The bench plate and cones are made from the alloy meehanite to provide strength and stability. The cones are 3 in. in diameter at the base, and the walls are 1/2-in. thick. The cones have been cored and tapered to a 1-in. diameter at the top to reduce weight and to provide a solid base to maintain stability. The cones, grouped together (9-and 0-in. heights, and 3-and 6-in. heights), provide even weight distribution to each quadrant of

the plate. (In Figure 3, $Z_1 = 9$ in.; $Z_2 = 6$ in., $Z_3 = 3$ in. and $Z_4 = 0$ in. heights.) Each cone is fastened to the bench plate with four cap screws and two dowell pins.

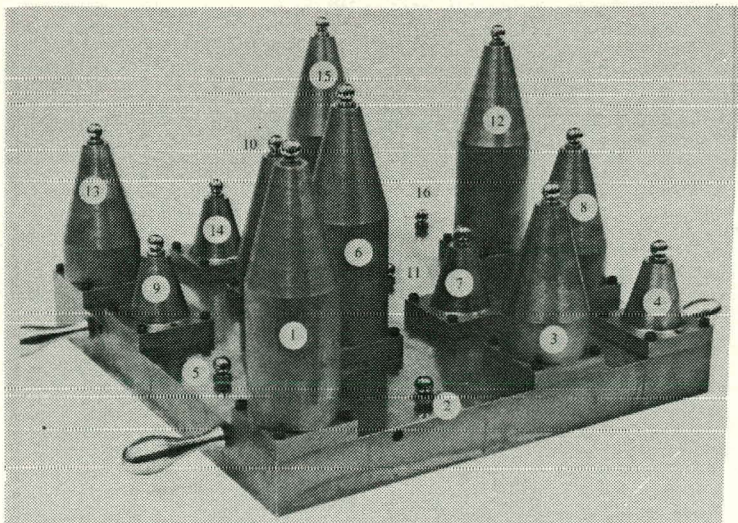
The tooling balls are 5/8-in.-diameter tungsten carbide balls, grade five, and the uniformity between balls for diameter and sphericity measures within 0.000010 in. These balls are press-fit into the cones and bench plate. Holes have been drilled into each side of the plate to provide a location for plate temperature to be monitored. Other holes have been drilled at three locations in the bottom for feet to establish three points of surface contact to define a plane. If stress relief occurs, the deformation will always be consistent if the plate is resting on three points.

Determining the point coordinates of the tooling balls (Table 1) should be accomplished as accurately as possible. The point coordinates in Table 1 were obtained by using a Sheldon Coordinate Measuring Machine (Figure 4) and have been assigned an accuracy (uncertainty) of ± 0.000180 in. It is expected that this uncertainty may be reduced by additional measurements of these coordinates. In practice, the estimate of coordinate accuracy would be combined with the estimates of precision and systematic error of the CMM to completely describe the accuracy of the CMM. The point coordinates become the nominal values.

Table 1. Point Coordinates for Gage Master for the Three-Dimensional Measuring Machine*

Point Number	X Coordinates (In.)	Y Coordinates (In.)	Z Coordinates (In.)
1	0.000000	0.000000	0.000000
2	5.000328	0.001090	-8.999770
3	10.001227	-0.000417	-2.999062
4	15.001312	0.000000	-5.999860
5	0.000884	5.001870	-8.999847
6	5.000450	5.000704	0.000838
7	10.003708	4.997918	-5.998178
8	15.001899	5.000274	-2.998789
9	0.000616	10.002434	-5.999860
10	5.001917	10.001809	-2.998642
11	10.001670	9.999530	-8.997912
12	15.002967	10.000883	0.000786
13	0.001012	15.002576	-3.000078
14	5.000853	15.002546	-6.001035
15	10.000411	15.005720	0.000000
16	15.001188	14.999662	-9.000279

*Accuracy of a given coordinate is ± 0.000180 in.



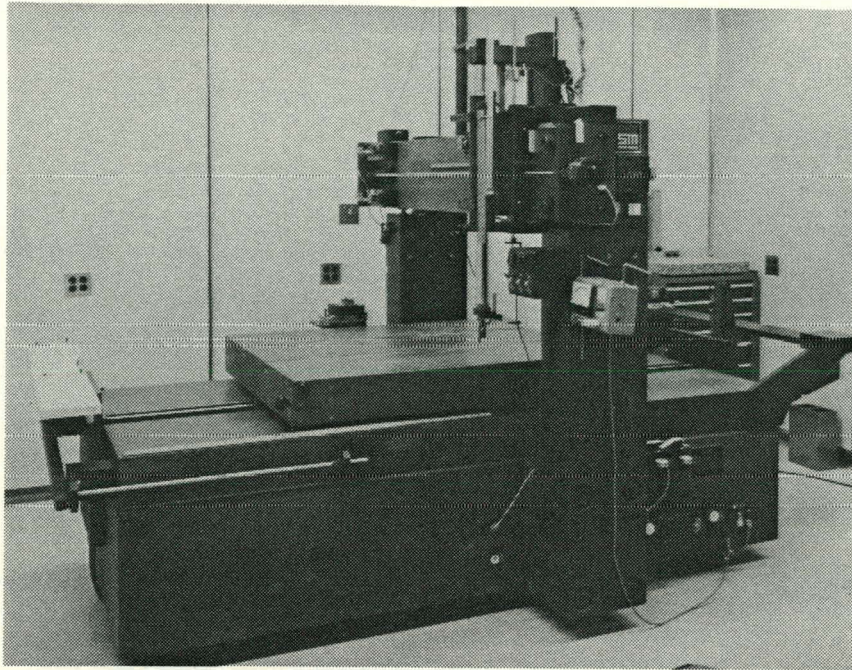


Figure 4. Sheldon Coordinate Measuring Machine

Data Collection

Data is collected by using the LS3DBP in the following manner. The device is placed on the table of a CMM (Figures 5 through 7). In this example, cone one is selected as the reference ball. The machine is started at this location. The probe is then moved from ball to ball in sequence (1, 2, 3, 4, 5, 6,..., 16). At each ball location, X, Y, and Z observed values are obtained automatically. (In most machines the nominal values are read into the CMM computer so that deviations from nominal can be obtained automatically.)

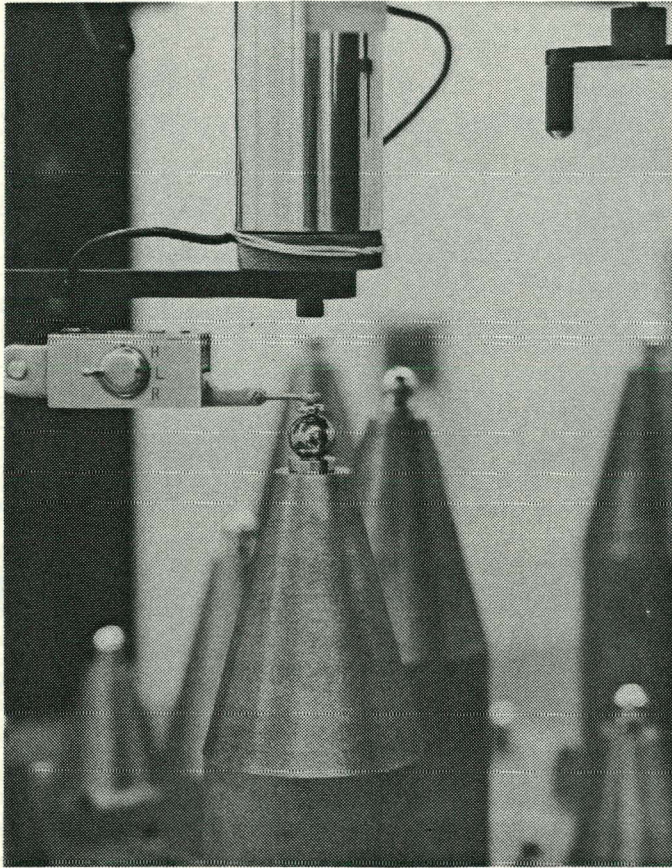


Figure 5. Close-Up of Probe at Point One

Notation

The following notation is used in this analysis.

σ_{XDEV}^2 = A quantitative measure of variation of deviations from
X nominal.

σ_{YDEV}^2 = A quantitative measure of variation of deviations from
Y nominal.

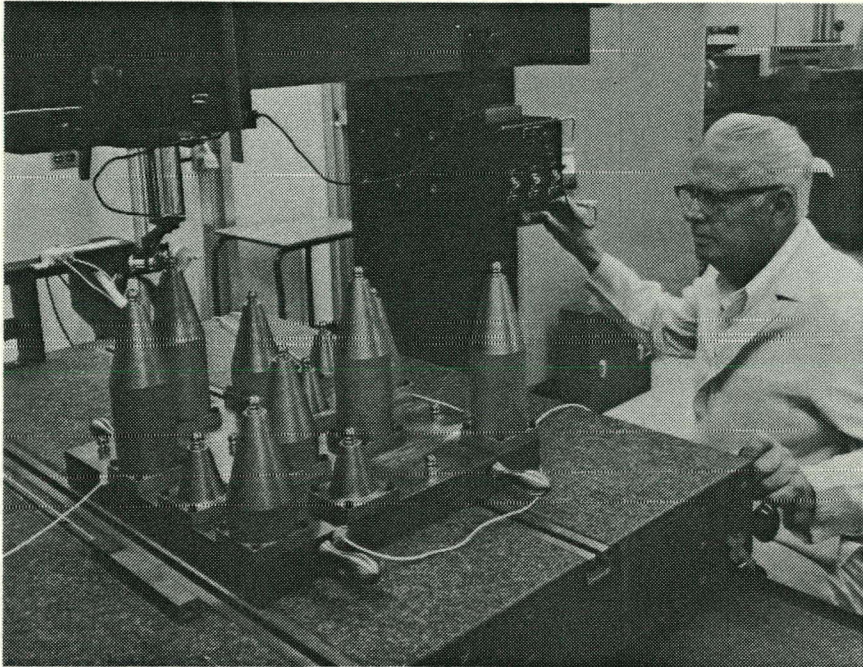


Figure 6. LS3DBP on Table of CMM With Probe at Point 12

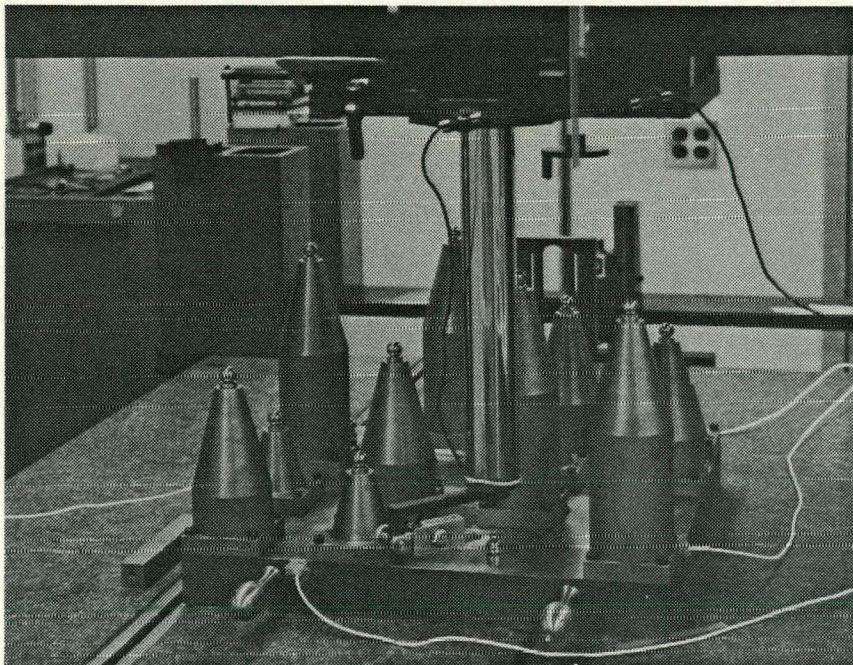


Figure 7. LS3DBP on Table of CMM With Probe at Point Five

- σ_{ZDEV}^2 = A quantitative measure of variation of deviations from Z nominal.
- σ_R^2 = A quantitative measure of inherent random variation in the measuring process, determined by both repeated observations and/or residual interaction.
- σ_X^2 = A quantitative measure of systematic error in deviations from nominal because of movement of the probe along the X-axis.
- σ_Y^2 = A quantitative measure of systematic error in deviations from nominal because of movement of the probe along the Y-axis.
- σ_Z^2 = A quantitative measure of systematic error in deviations from nominal because of movement of the probe along the Z-axis.
- σ_{CMM}^2 = A quantitative measure of capability of a coordinate measuring machine.

Determining the Capability of CMMs

An example of the analysis technique is presented. In Table 2 are a set of nominal values and sample data readings from a coordinate measuring machine obtained by using the LS3DBP. For

Table 2. Sample Data to Demonstrate Analysis
Technique for Data From LS3DBP

Pt	Axis	Nominal Values	Observed Values	OBS-NOM (μ in.)
1	X	0.000000	0.000000	0
	Y	0.000000	0.000000	0
	Z	0.000000	0.000000	0
2	X	4.999490	4.999500	10
	Y	0.001590	0.001590	0
	Z	-9.000310	-9.000310	0
3	X	10.000910	10.000920	10
	Y	-0.000450	-0.000490	-40
	Z	-2.999960	-2.999960	0
4	X	15.000750	15.000770	20
	Y	0.000020	-0.000020	-40
	Z	-6.001260	-6.001300	-40
5	X	0.000200	0.000220	20
	Y	5.002580	5.002580	0
	Z	-8.999640	-8.999640	0
6	X	5.000510	5.000450	-60
	Y	5.000690	5.000650	-40
	Z	0.000710	0.000750	40
7	X	10.003230	10.003210	-20
	Y	4.998210	4.998170	-40
	Z	-5.998780	-5.998820	-40
8	X	15.001730	15.001750	20
	Y	5.000260	5.000180	-80
	Z	-2.999800	-2.999800	0
9	X	0.000470	0.000490	20
	Y	10.002890	10.002890	0
	Z	-5.999370	-5.999450	-80
10	X	5.001930	5.001990	60
	Y	10.001950	10.001910	-40
	Z	-2.998460	-2.998580	-120
11	X	10.001180	10.001200	20
	Y	9.999860	9.999940	80
	Z	-8.998270	-8.998350	80
12	X	15.003230	15.003330	100
	Y	10.000570	10.000570	0
	Z	0.000040	0.000000	-40

Table 2 Continued. Sample Data to Demonstrate
Analysis Technique for Data
From LS3DBP

Pt	Axis	Nominal Values	Observed Values	OBS-NOM (μ in.)
13	X	0.001140	0.001160	20
	Y	15.003010	15.002970	-40
	Z	-2.999210	-2.999290	-80
14	X	5.000590	5.000650	60
	Y	15.002970	15.003010	40
	Z	-6.000630	-6.000710	-80
15	X	10.000670	10.000690	20
	Y	15.005610	15.005650	40
	Z	0.000040	0.000000	-40
16	X	15.000750	15.000770	20
	Y	15.000020	14.999940	-80
	Z	-9.000830	-9.000940	-110

simplification in this study, the subsequent analyses assume that the nominal values in Table 2 are the true values. In practice, the true values are not known and the uncertainty of the point coordinates would be combined with the measures of precision and systematic error of the CMM to completely describe the accuracy of the CMM. A statistical layout of the data is presented in Figure 8 for the X, Y, and Z deviations from nominal. The objective of the analysis will be to estimate the precision and systematic error of the CMM being evaluated. For this study $\hat{\sigma}_{\text{CMM}}^2 = \hat{\sigma}_{\text{XDEV}}^2 + \hat{\sigma}_{\text{YDEV}}^2 + \hat{\sigma}_{\text{ZDEV}}^2$, where:

$$\hat{\sigma}_{XDEV}^2 = \hat{\sigma}_X^2 + \hat{\sigma}_Y^2 + \hat{\sigma}_Z^2 + \hat{\sigma}_{RX}^2;$$

$$\hat{\sigma}_{YDEV}^2 = \hat{\sigma}_X^2 + \hat{\sigma}_Y^2 + \hat{\sigma}_Z^2 + \hat{\sigma}_{RY}^2; \text{ and}$$

$$\hat{\sigma}_{ZDEV}^2 = \hat{\sigma}_X^2 + \hat{\sigma}_Y^2 + \hat{\sigma}_Z^2 + \hat{\sigma}_{RZ}^2.$$

The symbol $\hat{}$ denotes a value estimated from using observed data.

These components (σ^2) are determined directly from the analysis of variance table. Table 3 shows a generalized analysis of variance table for a 4 by 4 Latin square. The computation procedure for data from the LS3DBP is also in Figure 9 for deviations from X nominal. Table 4 shows the AOV table for deviations from X nominal. The components used to determine machine capability are obtained by simple algebra, indicated by using the mean-square column and the expected mean-square column. For example, the estimate of precision is $\hat{\sigma}_{RX}^2 = 1250 \mu\text{in.}^2$. To estimate the component caused by movement along the Z-axis,

$$\hat{\sigma}_Z^2 = \frac{Z_{MS} - E_{MS}}{4}.$$

If this value is negative, set $\hat{\sigma}_Z^2 = 0$. Similarly,

DEVIATIONS FROM X NOMINAL (μ IN.)

		DISTANCE ALONG THE X-AXIS (INCHES)			
		$X_1 = 0$	$X_2 = 5$	$X_3 = 10$	$X_4 = 15$
DISTANCE ALONG THE Y-AXIS (INCHES)	$Y_1 = 0$	Z_1 0	Z_4 10	Z_2 10	Z_3 20
	$Y_2 = 5$	Z_4 20	Z_1 -60	Z_3 -20	Z_2 20
	$Y_3 = 10$	Z_3 20	Z_2 60	Z_4 20	Z_1 100
	$Y_4 = 15$	Z_2 20	Z_3 60	Z_1 20	Z_4 20

DEVIATIONS FROM Y NOMINAL (μ IN.)

		DISTANCE ALONG THE X-AXIS (INCHES)			
		$X_1 = 0$	$X_2 = 5$	$X_3 = 10$	$X_4 = 15$
DISTANCE ALONG THE Y-AXIS (INCHES)	$Y_1 = 0$	Z_1 0	Z_4 0	Z_2 -40	Z_3 -40
	$Y_2 = 5$	Z_4 0	Z_1 -40	Z_3 -40	Z_2 -80
	$Y_3 = 10$	Z_3 0	Z_2 -40	Z_4 80	Z_1 0
	$Y_4 = 15$	Z_2 -40	Z_3 40	Z_1 40	Z_4 -80

Figure 8. Data From Table 2 in Latin Square Matrix (Reference Weight Z_1 [9-in. cone] = 0, $Z_2 = -3$, $Z_3 = -6$, $Z_4 = -9$.)

$$\hat{\sigma}_Y^2 = \frac{Y_{MS} - E_{MS}}{4}$$

DEVIATIONS FROM Z NOMINAL (μ IN.)

		DISTANCE ALONG THE X-AXIS (INCHES)			
		$X_1 = 0$	$X_2 = 5$	$X_3 = 10$	$X_4 = 15$
DISTANCE ALONG THE Y-AXIS (INCHES)	$Y_1 = 0$	Z_1 0	Z_4 0	Z_2 0	Z_3 -40
	$Y_2 = 5$	Z_4 0	Z_1 40	Z_3 -40	Z_2 0
	$Y_3 = 10$	Z_3 -80	Z_2 -120	Z_4 80	Z_1 -40
	$Y_4 = 15$	Z_2 -80	Z_3 -80	Z_1 -40	Z_4 -110

Figure 8 Continued. Data From Table 2 in Latin Square Matrix (Reference Height Z_1 [9-in. cone] = 0, $Z_2 = -3$, $Z_3 = -6$, $Z_4 = -9$.)

and

$$\hat{\sigma}_X^2 = \frac{X_{MS} - E_{MS}}{4}.$$

The analyses of deviations from Y and Z nominal are similar to those for deviations from X nominal. Replace the deviations from X nominal in Figure 9 by the appropriate set of deviations and compute the AOVs. The AOVs are presented in Tables 5 and 6 for

$$C = (\text{Sum of } X_{\text{dev}})^2 / 16 = [(0) + (10) + (10) + \dots + (20)]^2 / 16 = \frac{(320)^2}{16} = 6400$$

$$T_{ss} = \text{Sum of } (X_{\text{dev}})^2 - C = [(0)^2 + (10)^2 + (10)^2 + \dots + (20)^2] - 6400 = 24600 - 6400 = 18200$$

$$X_{ss} = \frac{X_1^2 + X_2^2 + X_3^2 + X_4^2}{4} - C = \frac{(60)^2 + (70)^2 + (30)^2 + (160)^2}{4} - C = 8750 - 6400 = 2350$$

$$Y_{ss} = \frac{Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2}{4} - C = \frac{(40)^2 + (-40)^2 + (200)^2 + (120)^2}{4} - C = 14400 - 6900 = 8000$$

$$Z_{ss} = \frac{Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2}{4} - C = \frac{(60)^2 + (110)^2 + (80)^2 + (70)^2}{4} - C = 6750 - 6400 = 350$$

$$E_{ss} = T_{ss} - X_{ss} - Y_{ss} - Z_{ss} = 18200 - 2350 - 8000 - 350 = 7500$$

Where:

X_1 = Sum of all deviation from X Nominal at X = 0

$$X_1 = 0 + 20 + 20 + 20 = 60$$

X_2 = Sum of all deviation from X Nominal at X = 5

$$X_2 = 10 + (-60) + 60 + 60 = 70$$

$$X_3 = 10 + (-20) + 20 + 20 = 30$$

$$X_4 = 20 + 20 + 100 + 20 = 160$$

Y_1 = Sum of all deviation from X Nominal at Y = 0

$$Y_1 = 0 + 10 + 10 + 20 = 40$$

$$Y_2 = 20 + (-60) + (-20) + 20 = -40$$

$$Y_3 = 20 + 60 + 20 + 100 = 200$$

$$Y_4 = 20 + 60 + 20 + 20 = 120$$

Z_1 = Sum of all deviation from X Nominal at Z = Base Height (9-inch cone)

$$Z_1 = 0 + (-60) + 20 + 100 = 60$$

$$Z_2 = 10 + 20 + 60 + 20 = 110$$

$$Z_3 = 20 + (-20) + 20 + 60 = 80$$

$$Z_4 = 10 + 20 + 20 + 20 = 70$$

Figure 9. Computational Technique to Compute Sum of Squares for a Four by Four Latin Square

Table 3. Generalized Analysis of Variance for Four by Four Latin Square With No Repeated Runs

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square
Total	15	Tss		
X	3	Xss	$X_{ms}=X_{ss}/3$	$\sigma_R^2 + 4\sigma_X^2$
Y	3	Yss	$Y_{ms}=Y_{ss}/3$	$\sigma_R^2 + 4\sigma_Y^2$
Z	3	Zss	$Z_{ms}=Z_{ss}/3$	$\sigma_R^2 + 4\sigma_Z^2$
Error*	6	Ess	$E_{ms}=E_{ss}/6$	σ_R^2

* residual

deviations from Y and Z nominals, respectively. The results from the AOVs are used to determine CMM measurement capability. (The aforementioned analyses can be programmed into the microcomputer interfaced with the CMM; a procedure that requires 300 to 400 programming steps. Calculations could begin as soon as the first data point is taken, and the complete analysis would be finished almost as soon as the last data point was taken. Otherwise, about one hour is required, using a non-programmable scientific hand calculator.)

Precision is defined as $\pm 2\hat{\sigma}_{CMMR}$, where $\hat{\sigma}_{CMMR} = \sqrt{\sigma_{XR}^2 + \sigma_{YR}^2 + \sigma_{ZR}^2}$. For the data in this example, $\hat{\sigma}_{CMMR} = \sqrt{1250 + 1467 + 2490} = \sqrt{5207} = 72 \mu\text{in.} = 0.000072 \text{ in.}$ Therefore, precision = $\pm 0.000144 \text{ in.}$

Table 4. Deviation From X Nominal ($\mu\text{in.}^2$)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square
Total	15	18200		
X	3	2350	783	$\sigma_{RX}^2 + 4\sigma_X^2$
Y	3	8000	2667	$\sigma_{RX}^2 + 4\sigma_Y^2$
Z	3	350	117	$\sigma_{RX}^2 + 4\sigma_Z^2$
Error*	6	7500	1250	σ_{RX}^2

*residual

Component Estimation

$$\hat{\sigma}_{RX}^2 = 1250$$

$$\hat{\sigma}_Z^2 = 0$$

$$\hat{\sigma}_Y^2 = 354$$

$$\hat{\sigma}_X^2 = 0$$

$$\hat{\sigma}_{XDEV}^2 = 1604$$

Machine capability is defined as $\pm 2\hat{\sigma}_{CMM}$, where $\hat{\sigma}_{CMM} = \sqrt{\hat{\sigma}_{XDEV}^2 + \hat{\sigma}_{YDEV}^2 + \hat{\sigma}_{ZDEV}^2}$. For the data in this example, $\hat{\sigma}_{CMM} = \sqrt{1604 + 1999 + 3190} = \sqrt{6793} = 82 \mu\text{in.} = 0.000082 \text{ in.}$ Therefore, machine capability = $\pm 0.000164 \text{ in.}$

This information is then used to determine whether the machine has the capability of correctly classifying parts submitted to be inspected. Parts whose "specifications" or tolerance requirements

Table 5. Deviations From Y Nominal ($\mu\text{in.}^2$)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square
Total	15	28400		
X	3	7600	2533	$\sigma_{RY}^2 + 4\sigma_X^2$
Y	3	5200	1733	$\sigma_{RY}^2 + 4\sigma_Y^2$
Z	3	6800	2267	$\sigma_{RY}^2 + 4\sigma_Z^2$
Error*	6	8800	1467	σ_{RY}^2

*residual

Component Estimation

$$\hat{\sigma}_{RY}^2 = 1467$$

$$\hat{\sigma}_Z^2 = 200$$

$$\hat{\sigma}_Y^2 = 66$$

$$\hat{\sigma}_Z^2 = 266$$

$$\hat{\sigma}_{YDEV}^2 = 1999$$

are less than or close to the machine tolerance cannot be adequately inspected by using the machine; if they are inspected, misclassification may result. To illustrate, suppose, for example, that the diameter of a critical part has a tolerance requirement of ± 0.000164 in. For the measurement machine, in the aforementioned study, the capability is ± 0.000164 in. Based on these tolerances, if the variation in the process is such that the process is making five percent defective parts, then 14 percent

Table 6. Deviations From Z Nominal ($\mu\text{in.}^2$)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square
Total	15	43844		
X	3	14569	4856	$\sigma_{RZ}^2 + 4\sigma_X^2$
Y	3	5569	1856	$\sigma_{RZ}^2 + 4\sigma_Y^2$
Z	3	8769	2923	$\sigma_{RZ}^2 + 4\sigma_Z^2$
Error*	6	14937	2490	σ_{RZ}^2

*residual

Component Estimation

$$\hat{\sigma}_{RZ}^2 = 2490$$

$$\hat{\sigma}_Z^2 = 108$$

$$\hat{\sigma}_Y^2 = 0$$

$$\hat{\sigma}_X^2 = 592$$

$$\hat{\sigma}_{ZDEV}^2 = 3190$$

of the parts will be misclassified because of the magnitude of machine accuracy. Then, suppose that, based on this information, this magnitude of accuracy was determined to be unacceptable for this part. By using the techniques described in the subsequent section, it was determined that there was yaw in the X-axis. The CMM was then adjusted and its capability was determined to be ± 0.000041 in. Now, the misclassification error rate would be only two percent. Therefore, knowledge of the measurement capability of a CMM can be used to determine whether the CMM can correctly measure inspection items.

Identification of Errors in Machine Geometry

The Latin square AOV has another important use in evaluating CMMs. If a CMM is out of calibration or exhibits excessive variation or both, it is desirable to make the necessary adjustments to bring the CMM back into calibration (reduce its inaccuracy). Currently, the approach to determine the necessary adjustments is based on experience and trial and error. An advantage of the LS3DBP is that the analysis of variance, from which quantitative estimates of the factors affecting machine measurement capabilities are obtained, can also be used to identify sources of error in CMM geometry. A study involving simulated data from a cantilever design coordinate measuring machine indicates that the type of error--yaw, pitch, roll in X/Y/X, point-to-point positioning in X/Y/Z, and out-of-squareness (XY, XZ/and YX) can be identified by proper interpretation of the Latin square analysis of variance.² The results of this study are summarized in Table 7. Each source of error produces a unique pattern regarding which deviations from nominal are affected, which axis affects the deviation, and the form (linear, quadratic and/or cubic) that the effect of axis movement has. From Table 7, yaw in the X-axis is identified by linear and cubic ($L_{1,c}$) effects on X deviations from nominal when movement along the X-axis is present. In addition, movement along the X-axis causes a small quadratic effect on deviations from Y nominal.

Table 7. Summary of Analysis Results on Simulated Data From Cantilever Design CMM

Classification of Error	Source of Error	XDEV Axis Movement			YDEV Axis Movement			ZDEV Axis Movement		
		X-Axis	Y-Axis	Z-Axis	X-Axis	Y-Axis	Z-Axis	X-Axis	Y-Axis	Z-Axis
Point to Point	In X	L_1								
	In Y				L_1					
	In Z									L_1
YAW	In X	$L_{1,c}$			S_q					
	In Y		L_q			S_q				
	In Z			L_q						S_1
Pitch	In X	$L_{1,c}$						S_q		
	In Y				$L_{1,c}$				S_q	
	In Z						L_q			S_1
Roll	In X				L_q	S_1	S_1	$L_q(xy)$	$L_1(xy)$	$S_{1,c}$
	In Y		$L_q(yz)$	$L_1(yz)$					$L_q(yz)$	$L_1(yz)$
	In Z									
Out of Squareness	XY		L_1							
	XZ			L_1						
	YZ						L_1			

NOTATION: L - Denotes Large Effect
 S - Denotes Small Effect
 l - Denotes Linear Effect
 q - Denotes Quadratic Effect
 c - Denotes Cubic Effect

The determination of the linear, quadratic, and cubic effects is as follows. Consider the AOV for deviations from X nominal in Table 4. The three degrees of freedom for movement along each axis can be separated into three independent effects--linear, quadratic, and cubic. This separation is achieved by the use of orthogonal comparisons for equally spaced data.³ Figure 10 illustrates the coefficients and divisors with the computational procedure to obtain the three effects. A complete separation of the axes (X, Y, and Z) sums of squares is in Table 8.

To judge which sources are significant, consider first only these axes whose mean square is of some predetermined magnitude greater than the error mean square. The predetermined magnitude can be a tabulated F to determine statistical significance.⁴ (Regardless of the statistical significance, this magnitude should be judged for practical significance relative to specified CMM precision.) A conservative test of significance is to consider those sources whose mean square is more than twice the magnitude of error mean square. By applying the conservative test to the mean square in Table 8, the Y-axis is significant and Y_1 and Y_c would be candidates. In Table 9, no axes are significant. In Table 10, only the Y-axis approaches twice the error mean square, and Y_1 is a potential candidate.

Coefficient and divisors for separation of sums of squares of axis movement for data from LS3DBP.

<u>Contrast</u>	<u>Degrees of Freedom</u>	<u>Coefficient</u>	<u>Divisor</u>
Linear (l)	1	-3,-1,+1,+3	20
Quadratic (q)	1	+1,-1,-1,+1	4
Cubic (c)	1	-1,+3,-3,+1	20

Computational Procedure

1. Take the 4 totals for the X-axis (Figure 9) (Deviation from X nominal)

$$\frac{X_1}{60} \quad \frac{X_2}{70} \quad \frac{X_3}{30} \quad \frac{X_4}{160}$$

$$X_1 = [-3(60) - 1(70) + 1(30) + 3(160)]^2 / (20)(4) = 845$$

$$X_q = [+1(60) - 1(70) - 1(30) + 1(160)]^2 / (4)(4) = 900$$

$$X_c = [-1(60) + 3(70) - 3(30) + 1(160)]^2 / (20)(4) = 605$$

2. The previous three sums of square equal the sums of squares for X in Table A-5 (845+900+605=2350)

3. For the effect of Y-axis on deviation from X nominal

$$\frac{Y_1}{40} \quad \frac{Y_2}{-40} \quad \frac{Y_3}{200} \quad \frac{Y_4}{120}$$

$$Y_1 = [-3(40) - 1(-40) + 1(200) + 3(120)]^2 / (20)(4) = 2880$$

$$Y_q = [+1(40) - 1(-40) - 1(200) + 1(120)]^2 / (4)(4) = 0$$

$$Y_c = [-1(40) + 3(-40) - 3(200) + 1(120)]^2 / (20)(4) = 5120$$

4. For separation of contrasts for Z-axis on deviation from X nominal

$$\frac{Z_1}{60} \quad \frac{Z_2}{110} \quad \frac{Z_3}{80} \quad \frac{Z_4}{70}$$

$$Z_1 = [-3(60) - 1(110) + 1(80) + 3(70)]^2 / (20)(4) = 0$$

$$Z_q = [+1(60) - 1(110) - 1(80) + 1(70)]^2 / (4)(4) = 225$$

$$Z_c = [-1(60) + 3(110) - 3(80) + 1(70)]^2 / (20)(4) = 125$$

The complete analysis breakdown is presented in Table 8.

Figure 10. Coefficient and Divisors for Separation of Sums of Squares of Axis Movement for Data From LS3DBP and Computational Procedure

If the previous data had been taken on a cantilever design CMM, the candidates (Y_1 and Y_c on deviations from X nominal and Y_1 on deviations from Z nominal) could be compared to the results in Table 7 to determine the source of errors in machine geometry. However, the data that were used in this discussion to demonstrate the analysis procedure is from a bridge design CMM. A comparison is not meaningful because the patterns of deviations are expected to be different for each of the five major design configurations (Appendix B). The philosophy used to evaluate error identification for a cantilever design can be applied to other CMM design configurations to develop results for each configuration, as shown in Table 7. To date, only the results of the cantilever design have been determined. An investigation to define error patterns in other design configurations is underway.

CONCLUSIONS

Coordinate measuring machines are an important factor in meeting the challenge to improve productivity in the field of quality assurance. These machines can fulfill this function only if they are measuring to their designed capabilities. The Latin square three-dimensional ball plate (LS3DBP) can be used to determine the measurement capabilities of a CMM in three dimensions. Data

Table 8. Expanded Analysis of Variance for Deviations From X Nominal for Separation of Axis Sum of Squares Into Three Orthogonal Contrasts for Use in Identification of Sources of Error in Machine Geometry

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Total	15	18200	
X	3	2350	783
X_1	1	845	845
X_q	1	900	900
X_c	1	605	605
Y	3	8000	2667
Y_1	1	2880	2880
Y_q	1	0	0
Y_c	1	5120	5120
Z	3	350	117
Z_1	1	0	0
Z_q	1	225	225
Z_c	1	125	125
Error*	6	7500	1250
*residual			

obtained by use of the LS3DBP are statistically balanced and therefore can be analyzed quickly by using the analysis of variance technique. The information that can be determined from the analyses follows:

Table 9. Expanded Analysis of Variance for Deviations From Y Nominal for Separation of Axis Sum of Squares Into Three Orthogonal Contrasts for Use in Identification of Sources of Error in Machine Geometry

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Total	15	28400	
X	3	7600	2533
X_1	1	2000	2000
X_q	1	3600	3600
X_c	1	2000	2000
Y	3	5200	1733
Y_1	1	1280	1280
Y_q	1	0	0
Y_c	1	3920	3920
Z	3	6800	2267
Z_1	1	320	320
Z_q	1	3600	3600
Z_c	1	2880	2880
Error*	6	8800	1467
*residual			

- Precision;
- Systematic error (bias);
- Capability; and
- Identification of sources of error in CMM geometry.

Table 10. Expanded Analysis of Variance for Deviations From Z Nominal for Separation of Axis Sum of Squares Into Three Orthogonal Contrasts for Use in Identification of Sources of Error in Machine Geometry

Source Variation	Degrees of Freedom	Sum of Squares	Mean Square
Total	15	43844	
X	3	5569	1856
X_1	1	61	61
X_q	1	2257	2257
X_c	1	3251	3251
Y	3	14569	4856
Y_1	1	11762	11762
Y_q	1	2256	2256
Y_c	1	551	551
Z	3	8769	2923
Z_1	1	1	1
Z_q	1	8557	8557
Z_c	1	211	211
Error*	6	14937	2490
*residual			

This information provides a quantitative basis to determine whether the CMM can adequately measure a product and to assure that the decision to accept product has a high probability of being correct.

REFERENCES

¹"CMM Stands for Productivity Improvement," Quality, Volume 18, Number 3, March 1979, pp 38-54.

²L. L. Jones, J. V. Grice, and K. E. Moser, Use of Latin Square Analysis of Variance to Identify Sources of Error in Cantilever Design Coordinate Measuring Machines (Final Report). Bendix Kansas City: BDX-613-2679, October 1981 (Available from NTIS).

³R. G. Steel and J. H. Torrie, Principles and Procedures of Statistics. New York: McGraw Hill, 1960, pp 222.

⁴Steel and Torrie, pp 103-104.

Appendix A

INFORMATION ABOUT EXAMPLES OF COORDINATE MEASURING MACHINES

Information about various types of coordinate measuring machines can be obtained from these companies:

Digital Electronic Automation, Incorporated
36310 Ecorse Road
Romulus, MI 48174

Mitutoyo Instrument Center
14847-B Proctor Avenue
City of Industry, CA 91746

Boice Division
Mechanical Technology, Incorporated
968 Albany-Shaker Road
Latham, NY 12210

Carl Zeiss, Incorporated
IMT Division
1 Skyline Drive
Hawthorne, NY 10532

Bendix Automation & Measurement Division
Springfield and Thomas Streets
Dayton, OH 45401

Appendix B
CLASSIFICATIONS OF CMMs

(The material in this appendix has been taken from an initial draft of ASME B89.1.12 Committee--CMM Industrial Standard.

This material is included for general information regarding various types of CMMs. Permission to incorporate this material was granted by Erwin Loewen, Chairman of ASME B89 Committee.)

CLASSIFICATION - The following descriptions relate to Coordinate Measuring Devices employing moving mechanical components, the relative motions of which are measured by a suitable encoding means. The devices incorporate probing components utilized to interface with the object being measured, which is restrained on a support table. Variations in the methods of moving the mechanical components, the encoding means and the probing components may be varied within each classification to further expand the types of Coordinate Measuring Devices.

CANTILEVER - A measuring device employing three mechanical components moving along mutually perpendicular axes. The probing device is attached to the component with motion in the vertical direction, which in turn is attached to the first of the two horizontal components. The first is fixed to the second at one end only, in cantilever fashion, over the support table. (see Fig. A)

BRIDGE - A measuring device employing three mechanical components moving along mutually perpendicular axes. The probing device is attached to the component with motion in the vertical direction, which in turn is attached to the first of the two horizontal components. The first is attached to the second at two locations forming a bridge over the support table. (see Fig. B)

COLUMN - A measuring device employing one mechanical component moving in the vertical direction to which the probing device is attached. The support table is movable in two mutually perpendicular horizontal directions. (see Fig. C)

HORIZONTAL ARM - A measuring device employing three mechanical components moving along mutually perpendicular axes. The probing device is attached to the first component movable in the horizontal direction. This component is attached to the vertical component in cantilever fashion over the support table. The vertical component is attached to the second horizontal component. (see Fig. D)

GANTRY - A measuring device employing three mechanical components moving along mutually perpendicular axes. The probing device is attached to the component moving in the vertical direction, which in turn is attached to the first horizontal component. The first component is attached to the second horizontal in two locations bridging the support table (support jacks) in a location elevated above the support table. (see Fig. E)

OTHER - Any machine utilizing axes configuration different than stated in the above categories.

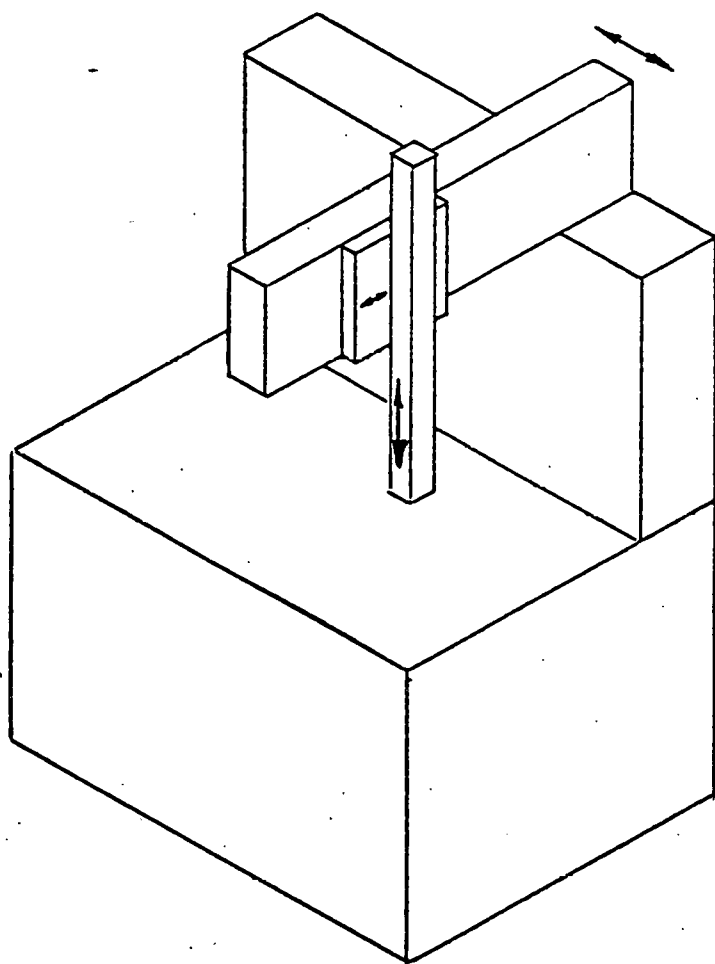


FIG. "A"

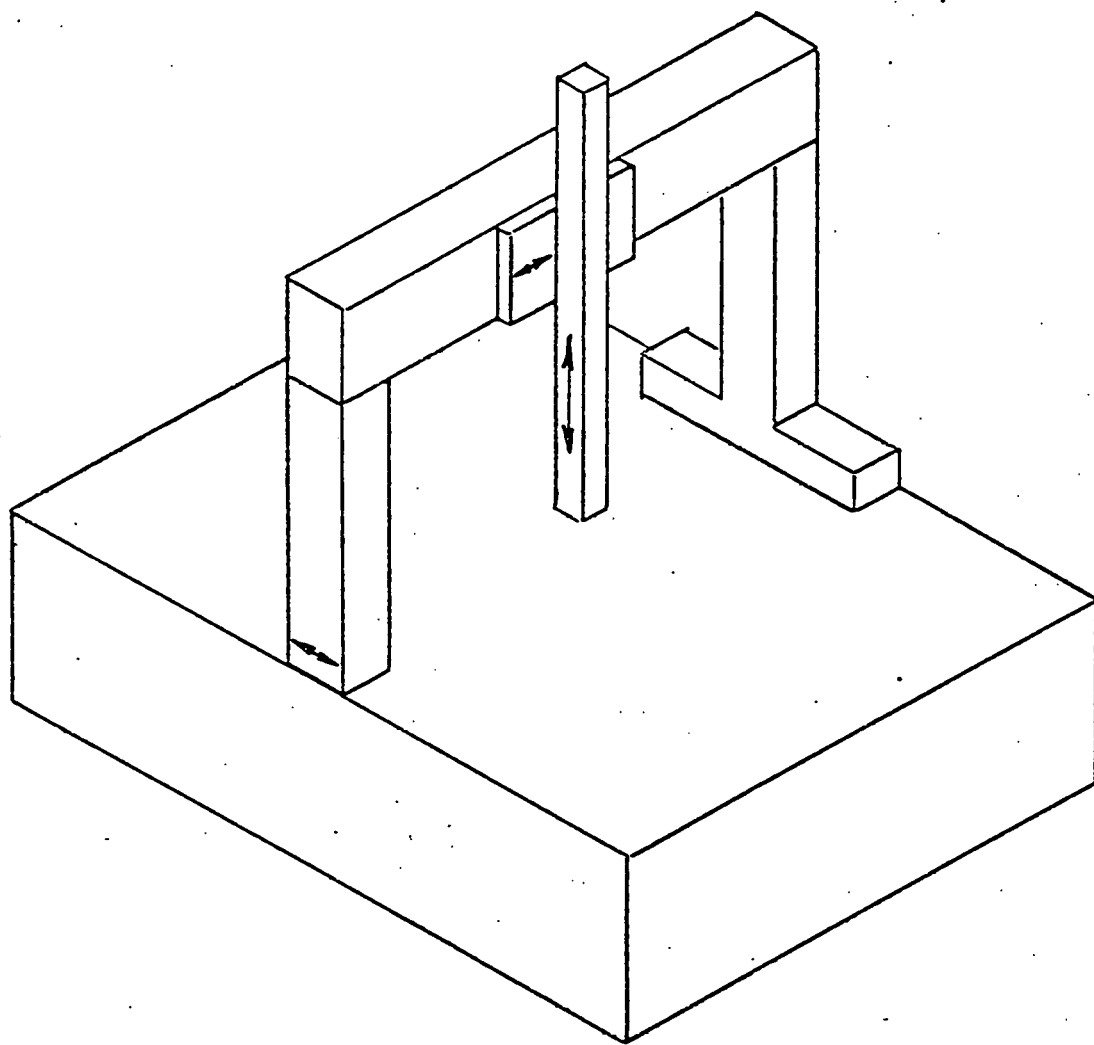


FIG. "B"

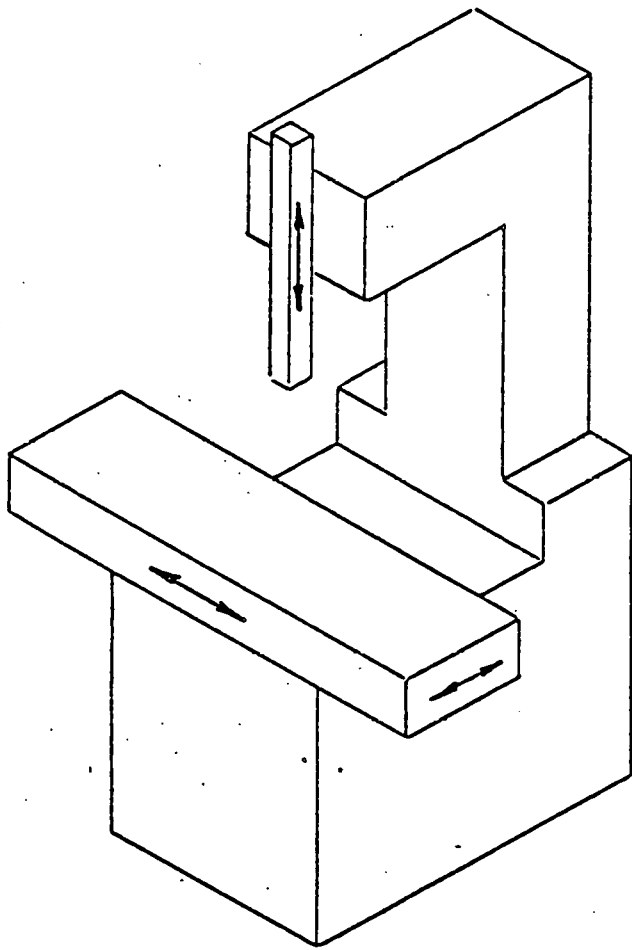


FIG. "C"

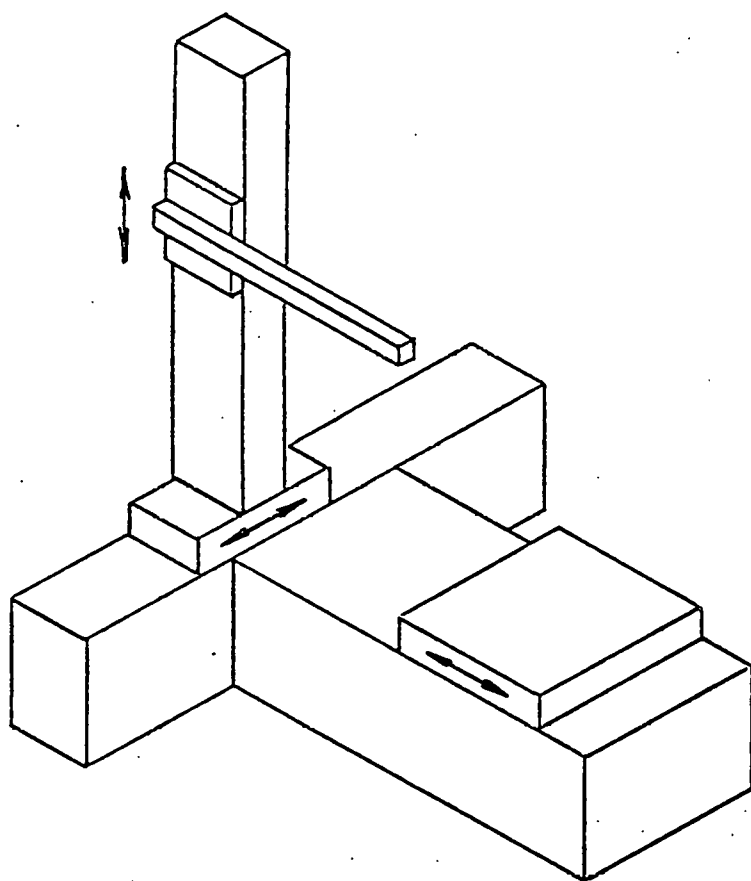


FIG. "D"

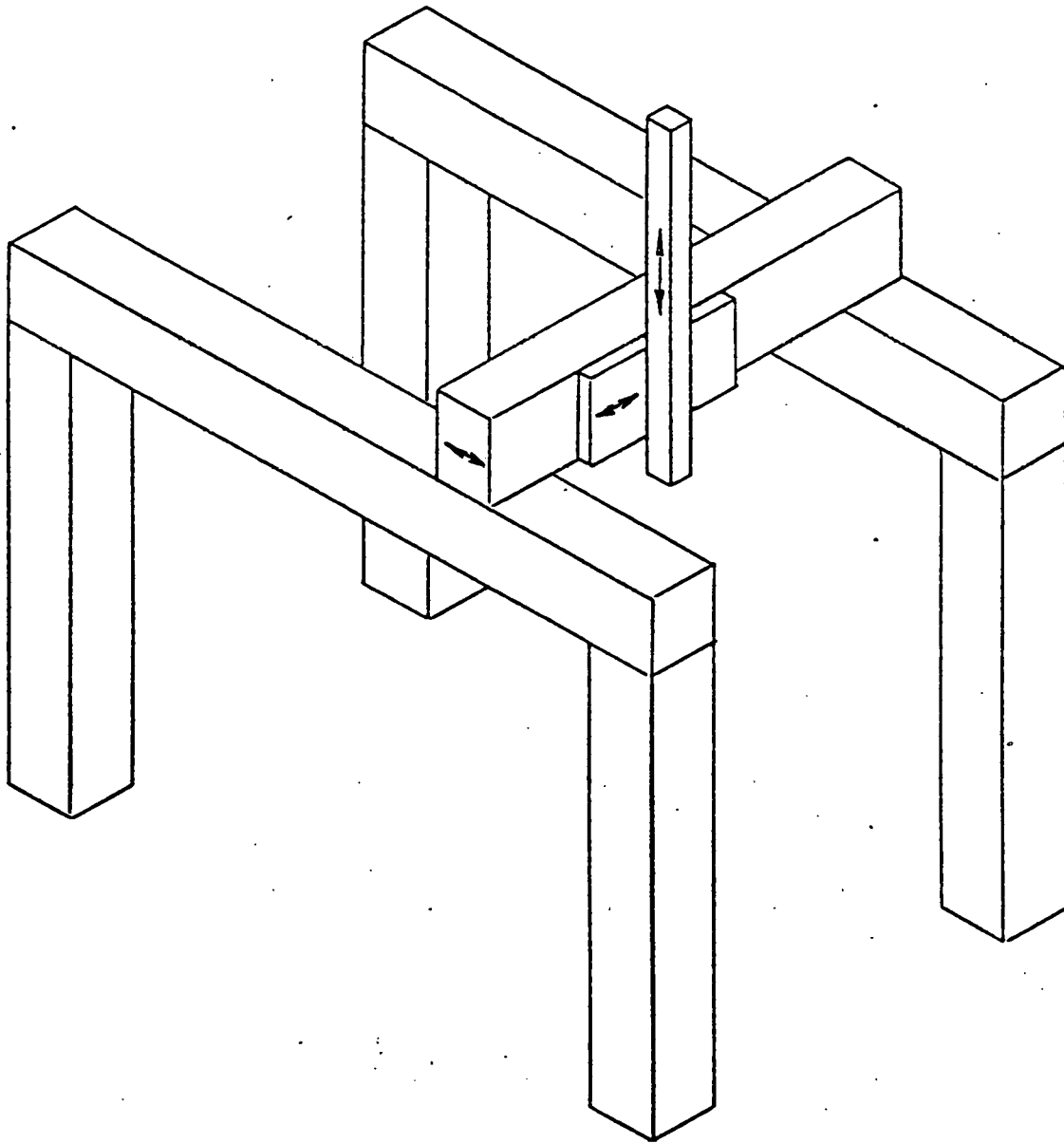


FIG. "E"

BDX-613-2682

LATIN SQUARE THREE-DIMENSIONAL BALL PLATE,
L. Jones, Topical, October 1981

Electronic coordinate measuring machines (CMM) represent the state-of-the-art in automatic three-dimensional measuring equipment. Current techniques for determining measurement capability of CMM are complex. A Latin square three-dimensional ball plate has been developed to evaluate the measurement capability of a CMM in three dimensions. The unique allocation of tooling balls in an XYZ space permits data to be obtained that is balanced statistically and can be analyzed quickly by using the analysis of

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variance techniques for a Latin square experimental design. The analysis of this data is used to determine quantitative measures of the precision and systematic error of CMM in three dimensions and to identify sources of error in machine geometry. This ball plate currently is used to determine CMM capability and to identify source(s) of error in machine geometry. This device requires 70 percent less time than some conventional techniques to estimate CMM capability in three dimensions.

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